OPTIMAL DEVELOPMENT POLICIES WITH FINANCIAL FRICTIONS

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ABSTRACT

We study optimal dynamic Ramsey policies in a standard growth model with financial frictions. For developing countries with low financial wealth, the optimal policy intervention increases labor supply and lowers wages, resulting in higher entrepreneurial profits and faster wealth accumulation. This in turn relaxes borrowing constraints in the future, leading to higher labor productivity and wages. The use of additional policy instruments, such as subsidized credit, may be optimal as well. In the long run, the optimal policy reverses sign. Taking advantage of the tractability of our framework, we extend the model to study its implications for optimal exchange rate and sectoral industrial policies.

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1   Introduction

Is there a role for governments in underdeveloped countries to accelerate economic development by intervening in product and factor markets? Should they use taxes and subsidies? If so, which ones? To answer these questions, we study optimal policy intervention in a standard growth model with financial frictions. In our framework, forward-looking heterogeneous producers face borrowing (collateral) constraints which result in a misallocation of capital and depressed labor productivity. It is therefore similar to the one studied in a number of recent papers relating financial frictions to aggregate productivity (see e.g. Banerjee and Duflo, 2005; Jeong and Townsend, 2007; Buera and Shin, 2013, as well as other papers discussed below). Our paper is the first to study the implications of Ramsey-optimal policies for a country's development dynamics in such an economy.[1]

We tackle the design of optimal policy using a simple and tractable model, which allows us to obtain sharp analytical characterizations. Our economy is populated by two types of agents: a continuum of heterogeneous entrepreneurs and a continuum of homogeneous workers. Entrepreneurs differ in their wealth and their productivity, and borrowing constraints limit the extent to which capital can reallocate from wealthy to productive individuals. In the presence of financial frictions, productive entrepreneurs make positive profits; they then optimally choose how much of these to consume and how much to retain for wealth accumulation. Workers decide how much labor to supply to the market and how much to save. Section 2 lays out the structure of the economy and characterizes the decentralized laissez-faire equilibrium. We specialize our benchmark analysis to the case of a small open economy, and consider the closed-economy extension in Section 6.2.

As a result of financial frictions, marginal products of capital are not equalized in equilibrium, and if a redistribution of capital from unproductive towards productive entrepreneurs were possible, it could be used to construct a Pareto improvement for all entrepreneurs and workers. But also simpler deviations from the decentralized equilibrium may result in a Pareto improvement. In Section 3 we provide a first example, namely a wealth transfer between workers and all entrepreneurs, *independently* of their productivity, taking advantage of the gap between the average return to capital of entrepreneurs and the interest rate available to workers. While we think that transfers may not be a realistic policy option for a number of reasons discussed later in the paper, this perturbation illustrates sharply the nature of inefficiency in the model, and provides a natural benchmark for thinking about

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[1]Two other papers by Caballero and Lorenzoni (2007) and Angeletos, Collard, Dellas, and Diba (2013) study Ramsey policies in economies with financial frictions, but with a different focus, as we discuss below.

the other policy interventions we analyze.

In Section 4, we explore policy interventions more systematically: we introduce various tax instruments into this economy and study the optimal Ramsey policies given the available set of instruments. We consider the problem of a benevolent planner that seeks to maximize the welfare of workers.[2] Importantly, we view the financial friction as a *technological* feature of the economic environment so that the planner faces the same constraints present in the decentralized economy. We first consider the case with only three tax instruments, which effectively allow the planner to manipulate worker savings and labor supply decisions, and then show how the results generalize to cases with a much greater number of instruments, including a capital (credit) subsidy and a transfer from workers to entrepreneurs.

Our main result is that the optimal policy intervention involves distorting labor supply of workers, but that it looks rather different for developing countries (far below their steady state in terms of financial wealth) and developed countries (in the vicinity of the steady state).[3] In particular, it is optimal to *increase* labor supply in the initial phase of transition, when entrepreneurs are undercapitalized, and *reduce* labor supply once the economy comes close enough to the steady state, where entrepreneurs are well capitalized. Greater labor supply reduces equilibrium wages paid by entrepreneurs, increasing their profits and accelerating wealth accumulation. This, in turn, makes future financial constraints less binding, resulting in greater labor productivity and higher wages. The only case in which there is no benefit from increasing labor supply in the initial transition phase, is when an unbounded transfer from workers to entrepreneurs is available, that is when the planner can transfer so much wealth from workers to entrepreneurs that the economy reaches its steady state immediately.

While our benchmark analysis focuses on a labor supply subsidy for concreteness, there are of course many equivalent ways of implementing the optimal allocation, including non-tax market regulation (as we discuss in Section 4.5). The common feature of such policies is that they increase labor supply in the short run, thereby hurting workers, and simultaneously increase profits, thereby benefitting entrepreneurs. We show that such *pro-business* development policies are optimal even when the planner puts zero weight on the welfare of

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[2]As will become clear, this constitutes a conservative benchmark for our analysis, and we generalize the results to the case with an arbitrary positive Pareto weight on the welfare of entrepreneurs in Section 6.1.

[3]In the simplest version of the model, countries only differ in their financial wealth (the only state variable), and other factors affecting the steady state are held constant. In this special case the level of financial wealth and overall economic development (income per capita) are one-to-one. In a richer environment with cross-country steady state (or balanced growth path, BGP) differences, the precise determinant of the optimal policy is the distance of financial wealth from its steady state (or BGP) level, which nonetheless is likely positively correlated with income per capita.

entrepreneurs. Indeed, the planner finds it optimal to hurt workers in the short-run so as to reward them with high wages in the long-run. An alternative way of thinking about this result is that the labor supply decision of workers involves a dynamic *pecuniary externality* (see Greenwald and Stiglitz, 1986): workers do not internalize the fact that working more leads to faster wealth accumulation by entrepreneurs and higher wages in the future. The planner corrects this using a Pigouvian subsidy.[4]

The goal of our paper is to develop a tractable model for the analysis of optimal development policies under financial frictions. This objective motivates a number of modeling choices. In Section 5 we discuss these in detail, in particular which ones are made purely for tractability and which ones are necessary for our results. The tractability of our model makes it easily amenable to a number of extensions and generalizations, which we discuss in Section 6. In particular, we introduce a nontradable sector to study the real exchange rate implications of optimal development policies, and extend the model to multiple tradable sectors with a comparative advantage to illustrate the implications for optimal industrial policies.

Related Literature   As mentioned in the first paragraph of this introduction, our paper is related to the large theoretical literature studying the role of financial market imperfections in economic development, and in particular the more recent literature relating financial frictions to aggregate productivity.[5] We contribute to this literature by studying optimal

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[4]In fact, a reduced form of our setup is mathematically equivalent to a setup in which production is subject to a learning-by-doing externality, whereby working more today increases future productivity, as in Krugman (1987), Young (1991), Matsuyama (1992), Lucas (1993) and more recently in Korinek and Serven (2010) and Benigno and Fornaro (2012). While mathematically equivalent, the economics are quite different: the dynamic externality in our framework is a pecuniary one stemming from the presence of financial frictions and operating via the (mis)allocation of resources, rather than a technological externality.

[5]Apart from the papers already cited there, see the early contributions by Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997) and Piketty (1997), the more recent papers by Erosa and Hidalgo-Cabrillana (2008), Caselli and Gennaioli (2013), Quintin (2008), Amaral and Quintin (2010), Buera, Kaboski, and Shin (2011), Song, Storesletten, and Zilibotti (2011), Midrigan and Xu (2014) and the recent surveys by Matsuyama (2008) and Townsend (2010). These papers are part of a growing literature exploring the macroeconomic effects of micro distortions, in particular their effects on aggregate total factor productivity (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Bartelsman, Haltiwanger, and Scarpetta, 2013), which are, in turn, a part of a large literature arguing that cross-country income differences are primarily accounted for by low TFP in developing countries (Hall and Jones, 1999; Klenow and Rodríguez-Clare, 1997; Caselli, 2005). The modeling of financial frictions in the paper also follows the tradition in the recently burgeoning macro-finance literature (e.g. Kiyotaki and Moore 1997; see Brunnermeier, Eisenbach, and Sannikov 2013 for a comprehensive survey). The environment we study is also similar to that in the literature on entrepreneurship and wealth distribution, e.g. Cagetti and De Nardi (2006), Bassetto, Cagetti, and Nardi (2013) and in many of the papers surveyed by Quadrini (2011).

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Ramsey policies and the resulting implications for a country's transition dynamics.[6]

Our paper is closely related to the work of Caballero and Lorenzoni (2007) who analyze the Ramsey-optimal response to a cyclical preference shock in a two-sector small open economy with financial frictions in the tradable sector. Similarly to our framework, the financial frictions in their work give rise to a pecuniary externality, which justifies a policy intervention that distorts the allocation of resources across sectors.[7] Our paper differs from theirs in three respects. First, we study long-run development policies, whereas they consider cyclical policies. Second, we focus on interventions that distort labor supply, whereas they focus on interventions that distort consumption across sectors. Third, we study a different framework, building on Moll (2014), which follows the tradition of the macro-development literature and departs minimally from the neoclassical growth model by introducing financial frictions, and is therefore particularly well-suited for studying transition dynamics.[8] Another related paper by Angeletos, Collard, Dellas, and Diba (2013) studies Ramsey policies in a setup with heterogeneous producers and within-period liquidity constraints, but their focus on optimal public debt management as supply of collateral is very different from ours.

In terms of methodology, we follow the dynamic public finance literature and study a Ramsey problem (see e.g. Lucas and Stokey, 1983). In particular, we analyze a Ramsey problem in an environment with idiosyncratic risk and incomplete markets as in Aiyagari (1995) and Shin (2006) among others. In contrast to most papers in this literature, however, we are neither concerned with capital taxation (e.g. Chamley, 1986; Judd, 1985; Aiyagari, 1995), nor with optimal financing of government expenditure and debt management (e.g. Barro, 1979; Lucas and Stokey, 1983).

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[6]A few papers in this literature evaluate the effects of various (not necessarily optimal) policies: for example, Banerjee and Newman (2003) analyze trade policy, Buera, Moll, and Shin (2013) subsidies targeted to individual entrepreneurs, and Buera and Nicolini (2013) monetary policy. Buera, Kaboski, and Shin (2012) study the macroeconomic effects of a large-scale micro finance program and argue that it may lead to higher wages, lower profits and lower wealth accumulation (the reverse logic of the optimal policy intervention in our paper). But none study Ramsey-optimal policies like we do.

[7]A related strand of work emphasizes a different type of pecuniary externality that operates through prices in borrowing constraints, for example Lorenzoni (2008), Jeanne and Korinek (2010) and Bianchi (2011).

[8]Caballero and Lorenzoni study an economy in which entrepreneurs with linear preferences operate technologies that are Leontief in capital and labor, face sunk costs of investing and maximally tight financial frictions (i.e., cannot borrow at all). Given these assumptions, optimal policy intervention during deterministic transitions (in the absence of aggregate shocks) is always completely frontloaded, i.e. all taxation is done in the first period. In our framework, in contrast, entrepreneurs have strictly concave preferences, operate Cobb-Douglas technologies, and can borrow up to a multiple of their wealth. As a result, the time path of optimal taxes and transition dynamics are more gradual. Finally, we also analyze in detail the role of transfers between workers and entrepreneurs, and the robustness of our main result to the introduction of other tax instruments.

Empirical relevance There exists a body of anecdotal evidence that the sort of policies prescribed by our normative analysis have historically been used in countries with successful development experiences. For example, Lin (2012, p. 191) provides a detailed discussion of the distortionary macro-development policies that were implemented in China, and in particular the "policy of low input prices—which included nominal wage rates for workers and prices of raw materials, energy and transportation" to "enable firms to generate profits large enough to repay the loans or accumulate enough funds for reinvestment".[9] Related, Kim and Leipziger (1997) state that low labor costs in early stages of development have been instrumental to the rapid development of South Korea, and that this was an official goal of government policy.[10]

From a more historical perspective, Ventura and Voth (2013) provide evidence that the rapid economic growth in the 18th century Britain was in part due to reduced labor and land prices, caused by expanding government borrowing which crowded out unproductive agricultural investment and reduced factor demand by this declining sector. Lower factor prices, in turn, increased profits in the new industrial sectors, allowing the capitalists in these sectors to build up wealth, which in the absence of an efficient financial system was the major source of reinvestment.

One of the most commonly advocated development policies is (real) exchange rate devaluation. Rodrik (2008) provides a recent systematic study of the effects of this policy across many developing countries (see also Woodford, 2008, for a cautious interpretation of the evidence). The argument in favor of such intervention rests on the assumption that the scale of the tradable sector is inefficiently small, and an exchange rate devaluation is aimed at reducing labor costs and increasing international competitiveness of this sector. Our paper spells out one particular friction which can speak to the potential benefits of a real exchange

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[9]Similarly, Lin (2013) notes that in China "big companies and rich people are receiving subsidies from the depositors who have no access to banks' credit services and are relatively poor" and that these distortions "led to a high rate of investment and quick building up of production capacity." In line with our result that such policies should be "stage-dependent," Lin also raises the possibility that such distortions may have outlived their usefulness and should be removed. He also discusses the effects of such policies on income inequality, an important topic that we do not address in this paper.

[10]While the evidence of explicit wage suppression policies is harder to come by, the *absence* of any regulation or policies protecting workers arguably contributed to reduced labor costs in the early stages of Korea's transition. This absence of worker protection is also a pervasive feature in many other developing countries. Besides Korea, typical examples include Japan in the 1950s and 60s and many of the nowadays South-East Asian countries. See also Cole and Ohanian (2004) and Alder, Lagakos, and Ohanian (2013) who argue that the pervasiveness of unionization had a detrimental effect on development in the United States after the Great Depression. Note, however, that we are not advocating the abandonment of worker protection in developing countries, as our framework is silent on the exact implementation of the optimal employment allocation, and there are other equivalent implementations like the wage subsidy in our benchmark model.

rate devaluation (see Section 6.3).

Another commonly advocated and implemented policy involves support to comparative advantage sectors (see e.g. Harrison and Rodriguez-Clare, 2010; Lin, 2012). Similarly Growth Commission (2008) argues that *export promotion policies* may be beneficial, at least as long as they are only temporary.[11] We evaluate such policies in a multisector extension in Section 6.4, where we show that it is optimal to temporarily subsidize the comparative advantage sector to speed up the reallocation of resources across sectors.[12]

2 An Economy with Financial Frictions

In this section we describe our baseline economy with financial frictions. We consider a one-sector small open economy populated by two types of agents: workers and entrepreneurs. The economy is set in continuous time with an infinite horizon and no aggregate shocks so as to focus on the properties of transition paths. We first describe the problem of workers, followed by that of entrepreneurs. We then characterize some aggregate relationships and the decentralized equilibrium in this economy.

2.1 Workers

The economy is populated by a representative worker with preferences

$$\int_0^\infty e^{-\rho t} u\big(c(t), \ell(t)\big) \mathrm{d}t, \tag{1}$$

where ρ is the discount rate, c is consumption, and ℓ is market labor supply. We assume that $u(\cdot)$ is increasing and concave in its first argument and decreasing and convex in its second argument, with a positive and finite Frisch elasticity of labor supply (see Appendix A.1). Where it leads to no confusion, we drop the time index t.

The household takes the market wage $w(t)$ as given, as well as the price of the consumption good which we choose as the numeraire. It can borrow and save using non-state-contingent bonds which pay risk free interest rate $r(t) \equiv r^*$. As a result, the flow budget

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[11]The Growth Commission report studies the growth experiences and policies of the thirteen economies that have grown at an average rate of seven percent a year or more for 25 years or longer since 1950 (Botswana, Brazil, China, Hong Kong, Indonesia, Japan, South Korea, Malaysia, Malta, Oman, Singapore, Taiwan and Thailand). It argues that all of these benefited substantially from exports and many promoted exports in some way or another.

[12]See Bardhan (1971), Redding (1999) and Melitz (2005) for related analyses of optimal dynamic trade policy in environments with physical learning-by-doing externalities, and the references in footnote 4.

constraint of the household is:

$$c + \dot{b} \leq w\ell + r^*b, \tag{2}$$

where $b(t)$ is the household asset position. The solution to the household problem is characterized by an Euler equation

$$\frac{\dot{u}_c}{u_c} = \rho - r^*, \tag{3}$$

and a static optimality (labor supply) condition:

$$-\frac{u_\ell}{u_c} = w, \tag{4}$$

where subscripts denote respective partial derivatives.

2.2 Entrepreneurs

There is a unit mass of entrepreneurs that produce the homogeneous tradable good. Entrepreneurs are heterogeneous in their wealth a and productivity z, and we denote their joint distribution at time t by $\mathcal{G}_t(a, z)$. In each time period of length Δt, entrepreneurs draw a new productivity from a Pareto distribution $G_z(z) = 1 - z^{-\eta}$ with shape parameter $\eta > 1$, with lower values of η corresponding to a greater heterogeneity of the productivity draws. We consider the limit economy in which $\Delta t \to 0$, so we have a continuous time setting in which productivity shocks are *iid* over time.[13] Finally, we assume a law of large numbers so the share of entrepreneurs experiencing any particular sequence of shocks is deterministic.

Entrepreneurs have preferences

$$\mathbb{E}_0 \int_0^\infty e^{-\delta t} \log c_e(t)\mathrm{d}t \tag{5}$$

where δ is their discount rate. Each entrepreneur owns a private firm which can use k units of capital and n units of labor to produce

$$A(t)(zk)^\alpha n^{1-\alpha} \tag{6}$$

units of output where $\alpha \in (0, 1)$ and $A(t)$ is aggregate productivity following an exogenous

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[13]Moll (2014) shows how to extend the environment to the case where shocks are persistent at the expense of some extra notation and mathematical complication. Persistent shocks generate some additional endogenous dynamics for aggregate total factor productivity, but the qualitative properties of the decentralized equilibrium are otherwise unchanged. As explained in Moll (2014), an iid process in continuous time can also be viewed as the limit of a mean-reverting process as the speed of mean reversion goes to infinity.

path. Entrepreneurs hire labor in the competitive labor market at wage $w(t)$ and purchase capital in a capital rental market at rental rate $r(t) \equiv r^*$.[14]

Entrepreneurs face collateral constraints:

$$k \leq \lambda a, \tag{7}$$

where $\lambda \geq 1$ is an exogenous parameter. By placing a restriction on an entrepreneur's leverage ratio k/a, the constraint captures the common prediction from models of limited commitment that the amount of capital available to an entrepreneur is limited by his personal assets.[15] At the same time, the particular formulation of the constraint is analytically convenient and allows us to derive most of our results in closed form. As shown in Moll (2014), the constraint could also be generalized in a number of ways at the expense of some extra notation.[16] We can also allow for evolution of λ over time, and show below that this is isomorphic to changes in the exogenous aggregate productivity, A. Finally, note that by varying $\lambda \in [1, \infty)$, we can trace out all degrees of efficiency of capital markets, with λ therefore capturing the degree of financial development.

An entrepreneur's wealth evolves according to

$$\dot{a} = \pi(a, z) + r^* a - c_e, \tag{8}$$

where $\pi(a, z)$ are his profits

$$\pi(a, z) \equiv \max_{\substack{n \geq 0, \\ 0 \leq k \leq \lambda a}} \left\{ A(zk)^\alpha n^{1-\alpha} - wn - r^* k \right\}.$$

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[14]The setup with a rental market is chosen solely for simplicity. As shown by Moll (2014), it is equivalent to a setup in which entrepreneurs own and accumulate capital k and can trade in a risk-free bond, provided the entrepreneurs know their productivity one period in advance (see also Buera and Moll, 2012).

[15]The constraint can be derived from the following limited commitment problem. Consider an entrepreneur with wealth a who rents k units of capital. The entrepreneur can steal a fraction $1/\lambda$ of rented capital. As a punishment, he would lose his wealth. In equilibrium, the financial intermediary will rent capital up to the point where individuals would have an incentive to steal the rented capital, implying a collateral constraint $k/\lambda \leq a$, or $k \leq \lambda a$. See Banerjee and Newman (2003) and Buera and Shin (2013) for a similar motivation of the same form of constraint. Note, however, that the constraint is essentially static because it rules out optimal long-term contracts (as in Kehoe and Levine, 2001, for example). On the other hand, as Banerjee and Newman put it, "there is no reason to believe that more complex contracts will eliminate the imperfection altogether, nor diminish the importance of current wealth in limiting investment."

[16]For example, we could allow the maximum leverage ratio λ to be an arbitrary function of productivity so that (7) becomes $k \leq \lambda(z)a$. The maximum leverage ratio may also depend on the interest rate and wages, calendar time and other aggregate variables. What *is* crucial is that the collateral constraint is linear in wealth.

Maximizing out the choice of labor, n, profits are linear in capital, k. It follows that the optimal capital choice is at a corner: it is zero for entrepreneurs with low productivity, and the maximal amount allowed by the collateral constraints, λa, for those with high enough productivity. Throughout the paper we assume that along all transition paths considered, there always exist entrepreneurs with low enough productivity that choose to be inactive. In this case, we have the following characterization:[17]

Lemma 1 *Factor demands and profits are linear in wealth and can be written as:*

$$k(a, z) = \lambda a \cdot \mathbf{1}_{\{z \geq \underline{z}\}}, \tag{9}$$

$$n(a, z) = \left((1 - \alpha)A/w\right)^{1/\alpha} z k(a, z), \tag{10}$$

$$\pi(a, z) = \left[\frac{z}{\underline{z}} - 1\right] r^* k(a, z), \tag{11}$$

where the productivity cutoff $\underline{z}$ is defined by:

$$\alpha \left(\frac{1 - \alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} A^{1/\alpha} \underline{z} = r^*. \tag{12}$$

The marginal entrepreneurs with productivity $\underline{z}$ break even and make zero profits, while entrepreneurs with productivity $z > \underline{z}$ receive Ricardian rents given by (11), which depend on both their productivity edge and the scale of operation determined by their wealth through the collateral constraint. The labor demand depends on both entrepreneur's productivity and capital choice, with the marginal product of labor equalized across active entrepreneurs. At the same time, the choice of capital among the active entrepreneurs is shaped by the collateral constraint, which depends only on the assets of the entrepreneurs, and not on their productivity. Therefore, the marginal product of capital increases with entrepreneurs' productivity z, reflecting the misallocation of resources in the economy.

Finally, entrepreneurs choose consumption and savings to maximize (5) subject to (8) and (11). Under our assumption of log utility combined with the linearity of profits in wealth, there exists an analytic solution to their consumption policy function, $c_e = \delta a$, and therefore

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[17]Proof: Equation (10) is the first order condition of profit maximization with respect to n, which substituted into the profit equation results in $\pi(a, z) = \max_{0 \leq k \leq \lambda a} \left\{ \left(\alpha\left((1-\alpha)/w\right)^{(1-\alpha)/\alpha} A^{1/\alpha} z - r^*\right) k \right\}$. Equations (9) and (12) characterize the solution to this problem of maximizing a linear function of k. Finally, we substitute (12) into the expression for profits to obtain (11). The assumption that the least productive entrepreneur is inactive can be ensured, for any initial conditions, by choosing sufficient amount of productivity heterogeneity (η small enough), however, in the limit without heterogeneity ($\eta \to \infty$), this assumption is necessarily violated. The analysis of the case when all entrepreneurs produce ($\underline{z} = 1$) yields similar qualitative results.

the evolution of wealth satisfies (see Appendix A.2 which spells out the value function of entrepreneurs and proves the result):

$$\dot{a} = \pi(a, z) + (r^* - \delta)a, \tag{13}$$

which completes our description.

2.3 Aggregation and equilibrium

We first provide a number of useful equilibrium relationships. Aggregating (9) and (10) over all entrepreneurs, we obtain the aggregate capital and labor demand:[18]

$$\kappa = \lambda x \underline{z}^{-\eta}, \tag{14}$$

$$\ell = \frac{\eta}{\eta - 1} \big((1 - \alpha)A/w\big)^{1/\alpha} \lambda x \underline{z}^{1-\eta}, \tag{15}$$

where $x(t) \equiv \int a \mathrm{d}\mathcal{G}_t(a, z)$ is aggregate (or average) entrepreneurial wealth. Note that we have made use of the assumption that productivity shocks are iid over time which implies that, at each point in time, wealth a and productivity z are independent in the cross-section of entrepreneurs.

Aggregate output in the economy can be characterized by a production function:[19]

$$y = Z\kappa^\alpha \ell^{1-\alpha} \qquad \text{with} \quad Z \equiv A\left(\frac{\eta}{\eta - 1}\, \underline{z}\right)^\alpha, \tag{16}$$

where Z is aggregate total factor productivity (TFP) which is the product of aggregate technology A and the average productivity of active entrepreneurs, $\mathbb{E}\{z|z \geq \underline{z}\} = \eta\underline{z}/(\eta-1)$. Using (14)–(16) together with the productivity cutoff condition (12), we characterize the equilibrium relationship between average wealth x, labor supply ℓ and aggregate output y, and express other equilibrium objects as functions of these three variables:[20]

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[18]Specifically, $\kappa(t) = \int k_t(a, z)\mathrm{d}\mathcal{G}_t(a, z)$ and $\ell(t) = \int n_t(a, z)\mathrm{d}\mathcal{G}_t(a, z)$.

[19]Aggregate output equals $y(t) = \int A(t)\big(zk_t(a, z)\big)^\alpha n_t(a, z)^{1-\alpha}\mathrm{d}\mathcal{G}_t(a, z)$, i.e., integral of individual outputs using production function (6). Aggregate production function (16) combines this definition with aggregate capital and labor demand in (14) and (15).

[20]Proof: Combine cutoff condition (12) and labor demand (15) to solve for (18). Substitute the resulting expression (18) and capital demand (14) into aggregate production function (16) to obtain (17). The remaining equation are a result of direct manipulation of (14)–(16) and (18), after noting that aggregate profits are an integral of individual profits in (11) and equal to $\Pi(t) = \kappa(t)/(\eta - 1)$.

Lemma 2 (a) *Equilibrium aggregate output satisfies:*

$$y = y(x, \ell) \equiv \Theta \, x^\gamma \ell^{1-\gamma}, \tag{17}$$

where

$$\Theta \equiv \frac{r^*}{\alpha} \left[\frac{\eta \lambda}{\eta - 1} \left(\frac{\alpha A}{r^*} \right)^{\frac{\eta}{\alpha}} \right]^\gamma \qquad and \qquad \gamma \equiv \frac{\alpha/\eta}{\alpha/\eta + (1 - \alpha)}.$$

(b) *The productivity cutoff $\underline{z}$ and the division of income in the economy can be expressed as follows:*

$$\underline{z}^\eta = \frac{\eta \lambda}{\eta - 1} \frac{r^*}{\alpha} \frac{x}{y}, \tag{18}$$

$$w\ell = (1 - \alpha)y, \tag{19}$$

$$r^* \kappa = \alpha \frac{\eta - 1}{\eta} y, \tag{20}$$

$$\Pi = \frac{\alpha}{\eta} y, \tag{21}$$

where $\Pi(t) \equiv \int \pi_t(a, z) \mathrm{d}\mathcal{G}_t(a, z)$ are aggregate profits of the entrepreneurs.

Lemma 2 expresses equilibrium aggregates as functions of the state variable x and labor supply ℓ. Note that given (17), both the equilibrium wage rate, $w = (1 - \alpha)y/\ell$, and the productivity cutoff, $\underline{z}$, are increasing functions of x/ℓ. High entrepreneurial wealth, x, increases capital demand and allows a given labor supply to be absorbed by a smaller subset of more productive entrepreneurs, raising both the average productivity of active entrepreneurs and aggregate labor productivity (hence wages). If labor supply, ℓ, increases, less productive entrepreneurs become active to absorb it which in turn reduces average productivity and wages.[21] Nonetheless, both higher x and higher ℓ lead to an increase in aggregate output and aggregate incomes of all groups in the economy—workers, entrepreneurs and rentiers.

The presence of financial frictions results in active entrepreneurs making positive profits, and therefore a fraction of national income is received by entrepreneurs.[22] Note from Lemma 2 that the parameter γ is increasing in capital intensity (α) and productivity heterogeneity (i.e., decreasing in η), and equals the share of profits in the total income of

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[21]Note that the effect of increased labor supply on the marginal product of labor through declining productivity, $\underline{z}$, is *partly* offset by the expansion in demand for capital, κ, as can be seen from (14).

[22]This happens at the expense of rentiers, whose share of income falls below α due to the decreased demand for capital in a frictional environment and despite the maintained return on capital of r^*. The share of labor still equals $(1 - \alpha)$, as in the frictionless model (with an arbitrary *bounded* productivity distribution, $z \leq z_{\max}$), where $\gamma = 0$, $y = \Theta \ell$, $\Theta = \left[A z_{\max}^\alpha (\alpha/r)^\alpha \right]^{1/(1-\alpha)}$, $w\ell = (1 - \alpha)y$, $r^* \kappa = \alpha y$ and $\Pi = 0$.

imperfectly-mobile factors (i.e., labor and entrepreneurial wealth): $\gamma = \Pi/(w\ell + \Pi)$. This parameter measures the severity of financial frictions and therefore plays a crucial role in the analysis of optimal policies in Section 4.

Finally, integrating (13) across all entrepreneurs, aggregate entrepreneurial wealth evolves according to:

$$\dot{x} = \frac{\alpha}{\eta}y + (r^* - \delta)x, \tag{22}$$

where from Lemma 2 the second term on the right-hand side equals aggregate entrepreneurial profits $\Pi = \alpha y/\eta$. Therefore, greater labor supply increases output, which raises entrepreneurial profits and speeds up wealth accumulation.

A *competitive equilibrium* in this small open economy is defined in the usual way: Taking prices as given, (i) workers maximize their utility (1) subject to their budget constraint (2); (ii) entrepreneurs maximize their respective utility (5) subject to their respective budget constraint (8), which involves the optimal production decisions characterized in Lemma 1; and (iii) the path of the wage rate, $w(t)$ clears the labor market at each point in time, while capital is in perfectly-elastic supply at interest rate r^*. Given our analysis in the preceding sections, a competitive equilibrium can be summarized as a time path for $\{c, \ell, b, y, x, w, \underline{z}\}_{t \geq 0}$ satisfying (2)–(4) and (17)–(22), given an initial asset position of the household b_0, initial entrepreneurial wealth x_0, and a path of exogenous productivity $\{A\}_{t \geq 0}$.

3 Inefficiency of Laissez-faire Equilibrium

In our economy, financial frictions limit the ability of resources to relocate towards the most productive entrepreneurs resulting in dispersion of the marginal product of capital and inefficiency of the laissez-faire equilibrium allocation. However, the equilibrium fails to satisfy much weaker notions of efficiency, which do not require the transfers of wealth from unproductive towards productive entrepreneurs.

In particular, consider a transfer between workers and all entrepreneurs independently of their productivity. Availability of such transfers necessarily leads to a Pareto improvement for all agents in the economy because workers and entrepreneurs face different rates of return, which fail to equalize due to the financial friction. Indeed, the workers face a rate of return r^*, while an entrepreneur with productivity z faces a rate of return $R(z) \equiv r^*\left(1 + \lambda\left[z/\underline{z} - 1\right]^+\right)$, with $R(z) \geq r^*$ for all z and $R(z) > r^*$ for $z > \underline{z}$. Because of the collateral constraint, an entrepreneur with productivity $z > \underline{z}$ cannot expand his capital to drive down his return

to r^*. The expected rate of return across entrepreneurs is given by:

$$\mathbb{E}_z R(z) = r^* \left(1 + \frac{\lambda \underline{z}^{-\eta}}{\eta - 1} \right) = r^* + \frac{\alpha}{\eta} \frac{y}{x} > r^*, \tag{23}$$

where the first equality integrates $R(z)$ using the Pareto distribution $G(z)$ and the second equality uses the equilibrium cutoff expression (18). Due to this lack of equalization of returns, a transfer of resources from workers to entrepreneurs at $t = 0$ and a reverse transfer at $t' > 0$ with interest accumulated at a rate $r_\omega = r^* + \omega \int_0^{t'} \frac{\alpha}{\eta} \frac{y(t)}{x(t)} \mathrm{d}t > r^*$ for some $\omega > 0$ necessarily leads to a Pareto improvement for all workers and entrepreneurs (see Appendix A.3).

We chose the above perturbation to illustrates sharply the nature of inefficiency in the model, and provides a natural benchmark for thinking about various other policy interventions. Yet, transfers may not be a realistic policy option for a number of reasons, as we discuss in Section 4.5 below. Furthermore, the type of transfer policy discussed here effectively allows the government to get around the specific financial constraint we have adopted, and hence it is not particularly surprising that it results in a Pareto improvement. Such a transfer policy may also not prove robust to alternative formulations of the financial friction. It is for these two reasons that the main focus of the paper is on optimal Ramsey taxation with a given set of simple policy tools. These have a similar capability to Pareto-improve the equilibrium allocation, but constitute a more realistic and, we think, more robust policy alternative to transfers.

4 Optimal Ramsey Taxation

In this section we study optimal interventions with a given set of policy tools. We start our analysis with the following tax instruments: a labor tax $\tau_\ell(t)$, a savings tax on workers $\tau_b(t)$, a savings (asset) subsidy to entrepreneurs $\varsigma_x(t)$, and a lump-sum transfer to workers $T(t)$. We show that the latter two instruments closely reproduce a transfer between workers and entrepreneurs. In section 4.4, we then extend our analysis to include additional tax instruments directly affecting the decisions of entrepreneurs, including a subsidy to the cost of capital. We rule out any direct redistribution of wealth *among* entrepreneurs, which would clearly be desirable given the inefficient allocation of capital. Instead, we ask how a planner can improve upon the competitive equilibrium with a limited set of aggregate tax instruments, which do not condition on entrepreneurs' productivity.

4.1 Economy with taxes

In the presence of labor and savings taxes (τ_ℓ, τ_b) and subsidies/transfers (ς_x, T), the budget constraints of the agents change from (2) and (8) to:

$$c + \dot{b} \leq (1 - \tau_\ell)w\ell + (r^* - \tau_b)b + T, \tag{24}$$

$$\dot{a} = \pi(a, z) + (r^* + \varsigma_x)a - c_e, \tag{25}$$

Without loss of generality due to the Ricardian equivalence, we assume that the government budget is balanced period-by-period, and therefore:

$$\tau_\ell w\ell + \tau_b b = \varsigma_x x + T. \tag{26}$$

In the presence of taxes, the optimality conditions of households become:

$$\frac{\dot{u}_c}{u_c} = \rho - r^* + \tau_b, \tag{27}$$

$$-\frac{u_\ell}{u_c} = (1 - \tau_\ell)w, \tag{28}$$

while the consumption policy function for entrepreneurs remains $c_e = \delta a$ and wages still satisfy the labor demand relationship (19).

The following result simplifies considerably the analysis of the optimal policies:[23]

Lemma 3 *Any aggregate allocation* $\{c, \ell, b, x\}_{t \geq 0}$ *satisfying*

$$c + \dot{b} = (1 - \alpha)y(x, \ell) + r^* b - \varsigma_x x, \tag{29}$$

$$\dot{x} = \frac{\alpha}{\eta}y(x, \ell) + (r^* + \varsigma_x - \delta)x, \tag{30}$$

and transversality conditions, where $y(x, \ell)$ *is defined in* (17)*, can be supported as a competitive equilibrium under appropriately chosen policies* $\{\tau_\ell, \tau_b, \varsigma_x, T\}_{t \geq 0}$*, and the equilibrium characterization in Lemma 2 still applies.*

Intuitively, equations (29) and (30) are the respective aggregate budget constraints of

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[23]Proof: The introduced policy instruments do not directly affect the static choices (profit maximization) of entrepreneurs given their wealth a, productivity z, and wage rate w, and therefore the aggregation results in Lemma 2 still apply, which we use to express out the aggregate wage bill and entrepreneurial profits as functions of aggregate output. With a proportional subsidy to assets, $c_e = \delta a$ is still optimal, and therefore aggregate entrepreneurial wealth must satisfy (30). Combining (24) and (26) results in (29), and any allocation $\{c, \ell\}_{t \geq 0}$ satisfying (29) and a transversality condition can be supported by an appropriate choice of $\{\tau_b, \tau_\ell\}_{t \geq 0}$.

workers and entrepreneurs, where we have substituted the government budget constraint (26) and the expressions for the aggregate wage bill and profits as a function of aggregate income (output) from Lemma 2, which still applies in this environment. The additional two constraints on the equilibrium allocation are the optimality conditions of workers, (27) and (28), but they can always be ensured to hold by an appropriate choice of labor and savings subsidies for workers. Finally, given a dynamic path of ℓ and x, we can recover all remaining aggregate variables supporting the allocation from Lemma 2.

Similarly to the *primal approach* in the Ramsey taxation literature (e.g. Lucas and Stokey, 1983), Lemma 3 allows us to replace the problem of choosing a time path of the policy instruments subject to a corresponding dynamic equilibrium outcome by a simpler problem of choosing a dynamic aggregate allocation satisfying the *implementability constraints* (29) and (30). These two constraints differ somewhat from those one would obtain following the standard procedure of the primal approach because we exploit the special structure of our model to derive more tractable conditions.[24]

Finally, note from (29)–(30) that the asset (savings) subsidy to entrepreneurs, $\varsigma_x x$, acts as a tool for redistributing wealth from workers to entrepreneurs (or vice versa when $\varsigma_x < 0$). In fact, the asset subsidy is essentially equivalent to a lump-sum transfer to entrepreneurs, as it does not distort the policy functions of either workers or entrepreneurs. The only difference with a lump-sum transfer is that a proportional tax to assets does not affect the consumption policy rule of the entrepreneurs, in contrast to a lump-sum transfer which makes the savings decision of entrepreneurs analytically intractable.[25] In what follows we refer to ς_x as *transfers to entrepreneurs* to emphasize that it is a very direct tool for wealth redistribution towards entrepreneurs. Note from (26) that *a priori* we do not restrict whether it is workers or entrepreneurs who receive revenues from the use of the distortionary taxes τ_ℓ and τ_b (or who pay lump-sum taxes in the case of subsidies).

4.2   Optimal policies without transfers to entrepreneurs

We start by analyzing the planner's problem in the absence of transfers (asset subsidies) to entrepreneurs, that is under the restriction that $\varsigma_x \equiv 0$. This allows us to isolate most clearly

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[24]In particular, the implementability condition for households differs. The standard approach substitutes the household first order conditions (27)–(28) back into their budget constraint and integrates it over time, while we only use the aggregate labor demand (19) instead and work with the flow version of the constraint. Using the standard, less tractable, version of the implementability constraint yields identical results.

[25]The savings rule of entrepreneurs stays unchanged when lump-sum transfers are unanticipated. In this case the savings subsidy and lump-sum transfers are exactly equivalent, however, the assumption of unanticipated lump-sum transfers is unattractive for several reasons.

the forces that shape optimal labor and savings subsidies to workers, which in this case are financed by the lump-sum tax on workers. In Section 4.3 we relax this assumption to show the qualitative robustness of our findings to the presence or absence of transfers.

We assume that the planner maximizes the welfare of households and puts zero weight on the welfare of entrepreneurs. As will become clear, this is the most conservative benchmark for our results. We relax this assumption in Section 6.1, where we also show that the optimal policy not only achieves a redistribution of welfare from entrepreneurs to workers, but also can ensure a Pareto improvement.

The Ramsey problem in this case is to choose policies $\{\tau_\ell, \tau_b, T\}_{t \geq 0}$ to maximize household utility (1) subject to the resulting allocation being a competitive equilibrium. From Lemma 3, this problem is equivalent to maximizing (1) with respect to the aggregate allocation $\{c, \ell, b, x\}_{t \geq 0}$ subject to (29)–(30) after imposing $\varsigma_x \equiv 0$, which we reproduce as:

$$\max_{\{c,\ell,b,x\}_{t \geq 0}} \int_0^\infty e^{-\rho t} u(c, \ell) \mathrm{d}t \tag{P1}$$

$$\text{subject to} \quad c + \dot{b} = (1 - \alpha) y(x, \ell) + r^* b, \tag{31}$$

$$\dot{x} = \frac{\alpha}{\eta} y(x, \ell) + (r^* - \delta) x, \tag{32}$$

and given initial b_0 and x_0. (P1) is a standard optimal control problem with controls (c, ℓ) and states (b, x), and we denote the corresponding co-state vector by $(\mu, \mu\nu)$. To ease the characterization of the solution and ensure the existence of a finite steady state, in what follows we assume $\delta < \rho = r^*$. However, neither the first inequality nor the second equality are essential for the pattern of optimal policies along the transition path, which is our focus.

Before characterizing the solution to (P1), we provide a brief discussion of the nature of this planner's problem. Apart from the Ramsey interpretation that we adopt as the main one, this planner's problem admits two additional interpretations. First, it corresponds to the planner's problem adopted in Caballero and Lorenzoni (2007), which rules out any transfers or direct interventions into the decisions of agents, and only allows for aggregate market interventions which affect agent behavior by moving equilibrium prices (wages in our case). Second, the solution to this planner's problem is a constrained efficient allocation under the definition developed in Dávila, Hong, Krusell, and Ríos-Rull (2012) for economies with exogenously incomplete markets and borrowing limits, as ours, where standard notions of constrained efficiency are hard to apply. Under this definition, the planner can choose policy functions for all agents respecting, however, their budget sets and exogenous borrowing

constraints. Indeed, in our case the planner does not want to change the policy functions of entrepreneurs, but chooses to manipulate the policy functions of households exactly in the way prescribed by the solution to (P1). The implication is that the planner in this case does not need to identify who are the entrepreneurs in the economy, relaxing the informational requirement of the Ramsey policy. As we show in later sections, the baseline structure of the planner's problem (P1) is maintained in a number of extensions we consider.

By examining (P1), we observe that the planner has no reason to distort the worker's choice of c, but there are two reasons to distort their choice of ℓ. First, the workers take wages as given and do not internalize that $w = (1-\alpha)y/\ell$ (see Lemma 2), that is, by restricting labor supply the workers can increase their wages. As the planner only cares about the welfare of workers, this *monopoly effect* induces the planner to reduce labor supply. Second, the workers do not internalize the positive effect of their labor supply on entrepreneurial profits and wealth accumulation, which affects future output and wages. This *dynamic productivity effect* through wealth accumulation forces the planner to increase labor supply. The interaction between these two forces shapes the optimal policy of the planner, which we now characterize more formally.

The optimality conditions for (P1) can be simplified to yield:[26]

$$\frac{\dot{u}_c}{u_c} = \rho - r^* = 0, \tag{33}$$

$$-\frac{u_\ell}{u_c} = \big(1 - \gamma + \gamma\nu\big) \cdot (1-\alpha)\frac{y}{\ell}, \tag{34}$$

$$\dot{\nu} = \delta\nu - \big(1 - \gamma + \gamma\nu\big)\frac{\alpha}{\eta}\frac{y}{x} \tag{35}$$

An immediate implication of (33) is that the planner does not distort the intertemporal margin and the consumption path still satisfies the Euler equation (3). There is no need to distort the workers' savings decision since, holding labor supply constant, consumption does not have a direct effect on productivity and wages. In terms of implementation, this requires no use of the savings tax on workers, $\tau_b \equiv 0$.

In contrast, the laissez-faire allocation of labor according to the labor supply condition (4) is in general suboptimal. Indeed, combining planner's optimality (34) with (19) and (28),

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[26]The present-value Hamiltonian for (P1) is given by:

$$\mathcal{H} = u(c, \ell) + \mu\big[(1-\alpha)y(x, \ell) + r^* b - c\big] + \mu\nu\big[\alpha y(x, \ell)/\eta + (r^* - \delta)x\big].$$

The optimality for c and b, $\mathcal{H}_c = 0$ and $\dot{\mu} - \rho\mu = -\mathcal{H}_b$ result in (33). (34) and (35) correspond to the optimality with respect to ℓ and x, $\mathcal{H}_\ell = 0$ and $(\dot{\mu\nu}) - \rho\mu\nu = -\mathcal{H}_x$, which we simplify using the properties of $y(\cdot)$ given in (17) and the definition of γ.
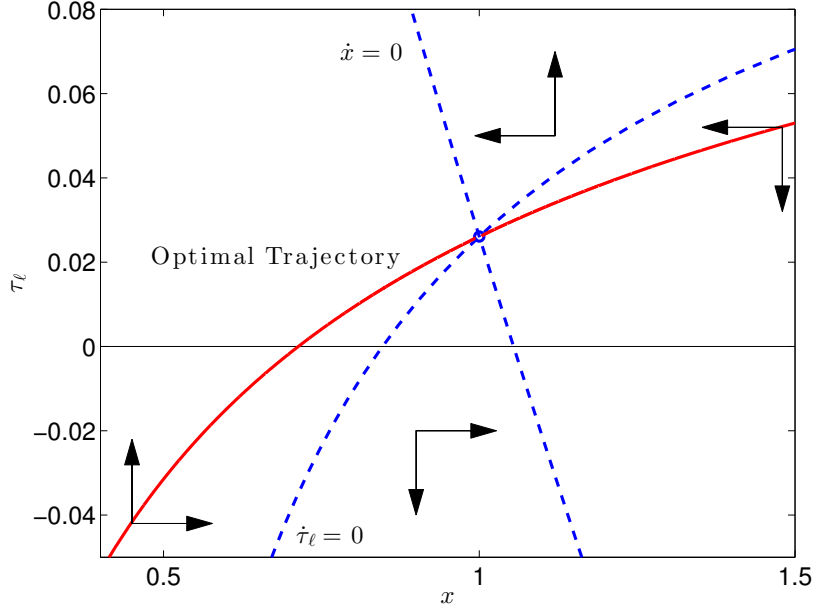
Figure 1: Planner's allocation: phase diagram for transition dynamics

the *labor wedge* (tax) can be expressed in terms of the co-state ν as:

$$\tau_\ell \equiv \gamma(1 - \nu). \tag{36}$$

Whether labor supply is subsidized or taxed depends on whether ν is greater than one, which emphasizes the interaction between the two forces outlined above. The statically optimal monopolistic labor tax equals $\tau_\ell = \gamma$. The offsetting force is the dynamic productivity gain from increased labor supply, which is reflected by the term $\gamma\nu > 0$ in (36). When entrepreneurial wealth is scarce, its shadow value for the planner is high ($\nu > 1$), and the planner subsidizes labor.[27] Recall that γ is a measure of the distortion arising from the financial frictions, and in the frictionless limit with $\gamma = 0$, the planner does not need to distort any margin.

We rewrite the optimality conditions (34) and (35), replacing the co-state ν with the labor tax $\tau_\ell = \gamma(1 - \nu)$:

$$-\frac{u_\ell}{u_c} = (1 - \tau_\ell) \cdot (1 - \alpha)\frac{y(x, \ell)}{\ell}, \tag{37}$$

$$\dot{\tau}_\ell = \delta(\tau_\ell - \gamma) + \gamma(1 - \tau_\ell)\frac{\alpha}{\eta}\frac{y(x, \ell)}{x} \tag{38}$$

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[27]Note that (35) can be solved forward to express ν as a net present value of future marginal products of wealth, $y_x = \gamma y/x$, which are monotonically decreasing in x with $\lim_{x\to 0} y_x = \infty$ (see Appendix A.5).

The planner's allocation $\{c, \ell, b, x\}_{t \geq 0}$ solving (P1), satisfies (31)–(33) and (37)–(38). With $r^* = \rho$, the marginal utility of consumption is constant over time, $u_c(t) \equiv \bar{\mu}$, and the system separates in a convenient way. Given a level of $\bar{\mu}$, which can be pinned down from the intertemporal budget constraint, the optimal labor wedge can be characterized by means of two ODEs in (τ_ℓ, x), (32) and (38), together with the static optimality condition (37).[28] These can be analyzed by means of a phase diagram (Figure 1) and other standard tools (see Appendix A.5) to yield:

Proposition 1 *The solution to the planner's problem* (P1) *corresponds to the globally stable saddle path of the ODE system* (32) *and* (38)*, as summarized in Figure 1. In particular, starting from $x_0 < \bar{x}$, both $x(t)$ and $\tau_\ell(t)$ increases over time towards the unique positive steady state $(\bar{\tau}_\ell, \bar{x})$, with labor supply taxed in steady state:*[29]

$$\bar{\tau}_\ell = \frac{\gamma}{\gamma + (1 - \gamma)(\delta/\rho)} > 0. \tag{39}$$

Labor supply is subsidized, $\tau_\ell(t) < 0$, when entrepreneurial wealth, $x(t)$, is low enough. The planner does not distort the workers' intertemporal margin, $\tau_b(t) \equiv 0$.

The optimal steady state labor wedge is strictly positive, meaning that in the long-run the planner suppresses labor supply rather than subsidizing it. This tax is however smaller than the optimal *monopoly tax* equal to γ (i.e., $0 < \bar{\tau}_\ell < \gamma$), because with $\delta > r^*$ the entrepreneurial wealth accumulation is bounded and the financial friction is never resolved (i.e., even in steady state the shadow value of entrepreneurial wealth is positive, $\bar{\nu} > 0$). Nonetheless, in steady state the redistribution force necessarily dominates the dynamic productivity considerations. This, however, is not the case along the entire transition path to steady state, as we prove in Proposition 1 and as can be seen from the phase diagram in Figure 1. Consider a country that starts out with entrepreneurial wealth considerably below its steady-state level, i.e. in which entrepreneurs are initially severely undercapitalized. Such a country finds it optimal to subsidize labor supply during the initial transition phase, until entrepreneurial wealth reaches a high enough level.

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[28]Interestingly, the system of ODEs (32) and (38) depends on the form of the utility function (and, in particular, on preference parameters such as the Frisch elasticity of labor supply) only indirectly through labor supply $\ell(x, \tau_\ell)$ defined by (37), as we discuss further in Appendix A.5. Furthermore, these parameters do not affect the value of the steady state tax.

[29](39) follows from (32) and (38) evaluated in steady state (for $\dot{x} = \dot{\tau}_\ell = 0$). Using the definition of $y(\cdot)$, steady-state versions of (32), (37) and $u_c \equiv \bar{\mu}$ determine $(\bar{x}, \bar{\ell}, \bar{c})$ as a function of $\bar{\mu}$, which is then recovered as a fixed point from the intertemporal budget constraint of the households.
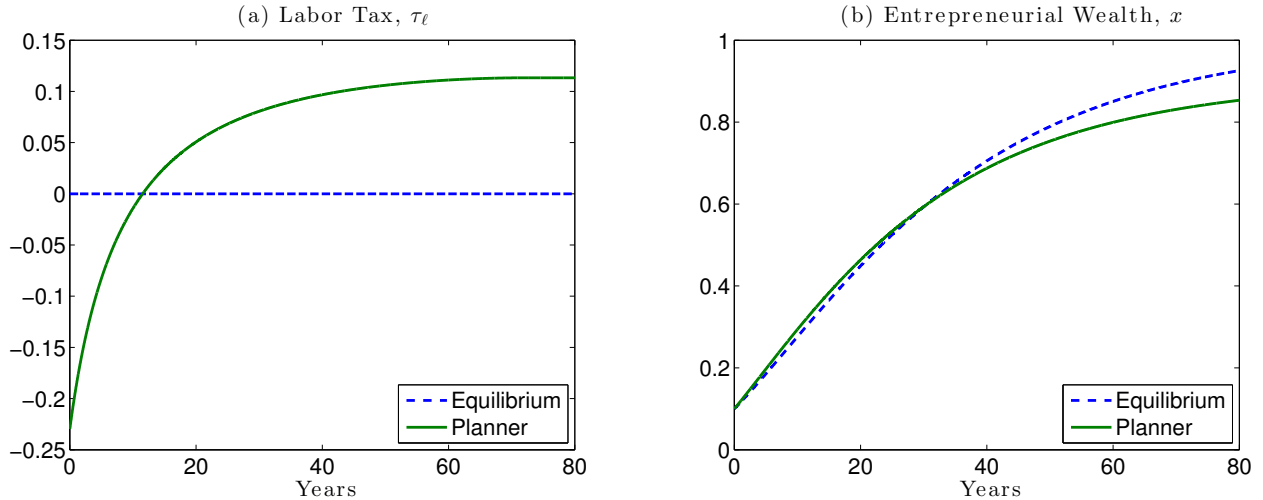
Figure 2: Planner's allocation: labor tax $\tau_\ell(t)$ and entrepreneurial wealth $x(t)$

Note: steady state entrepreneurial wealth in the laissez-faire equilibrium is normalized to 1 in panel (b).

Figures 2 and 3 illustrate the transition dynamics for key variables, comparing the allocation chosen by the planner to the one that would obtain in a laissez-faire equilibrium.[30] The left panel of Figure 2 plots the optimal labor tax, which is negative in the early phase of transition (i.e., a labor supply subsidy), and then switches to being positive in the long run. This is reflected in the initially increased and eventually depressed labor supply in the planner's allocation in Figure 3a. The purpose of the labor supply subsidy is to speed up entrepreneurial wealth accumulation (Figures 2b and 3b), which in turn translates into higher productivity and wages in the medium run, at the cost of their reduction in the short run (Figures 3c and 3d). The labor tax and suppressed labor supply in the long run are used to redistribute the welfare gains from entrepreneurs towards workers through the resulting increase in wages.[31]

Figure 3e shows that during the initial phase of the transition, the optimal policy increases GDP, as well as the incomes of all groups of agents—workers, active entrepreneurs and rentiers (inactive entrepreneurs)—according to Lemma 2. Output y is higher both due to a higher labor supply ℓ and increased capital demand κ (the capital-output ratio, κ/y, remains

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[30]Our numerical examples use the following benchmark parameter values: $\alpha = 1/3$, $\delta = 0.1$, $\rho = 0.03$, $\eta = 1.06$ and $\lambda = 2$, as well as balanced growth preferences with a constant Frisch elasticity $1/\phi$: $u(c,\ell) = \log c - \psi \ell^{1+\phi}/(1+\phi)$, with $\psi = 1$ and $\phi = 1$. The initial condition x_0 is 10% of the steady state level in the laissez-faire equilibrium, and the initial wealth of workers is $b_0 = 0$.

[31]This long-run policy reversal is not surprising given that the planner puts no weight on the welfare of entrepreneurs, an assumption that we relax in Section 6.1. Yet, even if this reversal was ruled out (by imposing a restriction $\tau_\ell \leq 0$), the planner still wants to subsidize labor during the early transition, emphasizing that the purpose of this policy is not merely a reverse redistribution at a later date.

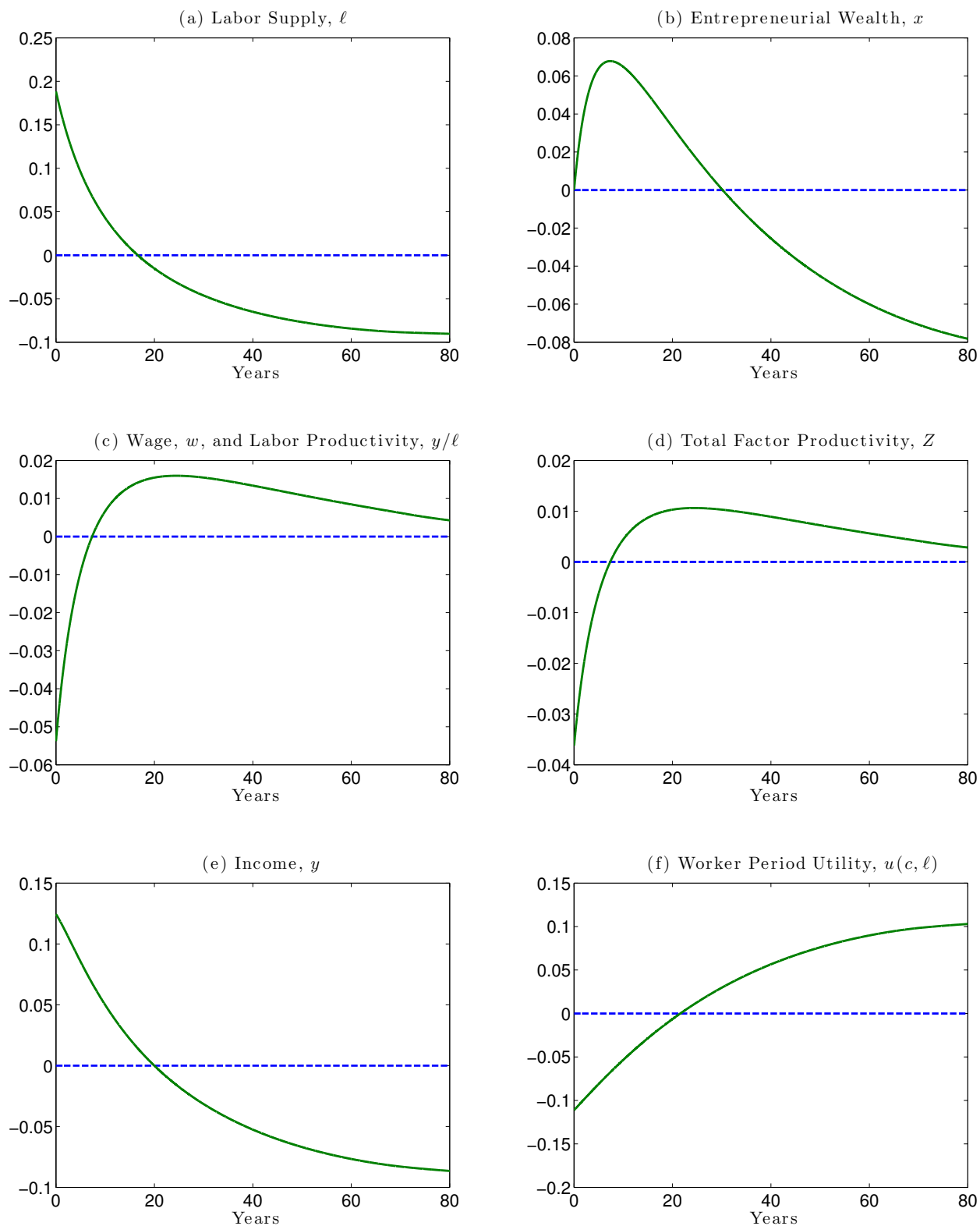Figure 3: Planner's allocation: proportional deviations from the laissez-faire equilibrium

Note: in (d), the deviations in TFP are the same as deviation in $\underline{z}^{\alpha}$, as follows from (16); in (e), income deviations characterize simultaneously the deviations in output (y), wage bill ($w\ell$), profits (Π), and capital income ($r^{*}\kappa$), and hence capital (κ), as follows from Lemma 2.

constant according to (20)). This increase in demand is met by an inflow of capital, which is in perfectly elastic supply in a small open economy. The effect of the increase in inputs ℓ and κ is partly offset by a reduction in the TFP due to a lower productivity cutoff $\underline{z}$.

Although our numerical example is primarily illustrative, it can be seen that the transition dynamics in this economy may take a very long time, and the quantitative effects of the Ramsey policies may be quite pronounced.[32] In particular, in our example the Ramsey policy increases labor supply by up to 18% and GDP by up to 12% during the initial phase of the transition, which last around 20 years. This is supported by an initial labor supply subsidy of over 20%, which switches to a 12% labor tax in the long run. Despite the increased labor income, workers initially suffer in flow utility terms (Figure 3f) due to increased labor supply. Workers are compensated with a higher utility in the future, reaping the benefits of both higher wages and lower labor supply, and gain on net intertemporally.[33]

Implementation The Ramsey-optimal allocation can be implemented in a number of different ways. For concreteness, we focus here on the early phase of transition, when the planner wants to increase labor supply. The way we set up the problem, the optimal allocation during this initial phase is implemented with a labor supply subsidy, $\varsigma_\ell(t) \equiv -\tau_\ell(t) > 0$, financed by a lump-sum tax on workers. In this case, workers' gross labor income including subsidy is $(1+\varsigma_\ell)(1-\alpha)y$, while their net income subtracting the lump-sum tax is still given by $(1-\alpha)y$, hence resulting in no direct change in their budget set. Note that increasing labor supply unambiguously increases net labor income $(w\ell)$, but decreases the net wage rate (w) paid by the firms. This is why we sometimes refer to this policy as *wage suppression*.

An equivalent implementation is to give a wage bill subsidy to firms financed by a lump-sum tax on workers. In this case, the equilibrium wage rate increases, but the firms pay only a fraction of the wage bill, and the resulting allocation is the same. There are of course alternative implementations that rely on directly controlling the *quantity* of labor supplied, rather than its *price*; for example, *forced labor*—a forced increase in the hours worked relative to the competitive equilibrium. Such a non-market implementation pushes

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[32]Slow transition dynamics are a generic feature of models in which heterogeneous producers face collateral constraints. Such models therefore have the potential to explain observed growth episodes such as the post-war miracle economies. This is in contrast to transitions in the neoclassical growth model which are characterized by very fast convergence. See Buera and Shin (2013) who make this argument by means of a quantitative theory of endogenous TFP dynamics in the presence of financial frictions.

[33]While we found the patterns in Figures 3a–e to be robust to alternative setups, the specific pattern of the flow-utility changes in Figure 3f is more sensitive to the assumptions, and in particular changes substantially when workers are assumed to be hand-to-mouth (as well as in the closed economy), in which case the utility gains come in significantly sooner due to the increasing path of consumption tied to that of output.

workers off their labor supply schedule and the wage is determined by moving along the labor demand schedule of the business sector. Our theory is silent on the relative desirability of one form of intervention over another. See Weitzman (1974) for a discussion. The general feature of all these implementation strategies is their *pro-business* tilt in the sense that they reduce the effective labor costs to firms, allowing them to expand production and generate higher profits, in order to facilitate the accumulation of entrepreneurial wealth in the absence of direct transfers to entrepreneurs. In Section 4.5 we discuss in more detail the implementation strategies for this and other related allocations.

Learning-by-doing analogy   One alternative way of looking at the planner's problem (P1) is to note that from (17), GDP depends on current labor supply $\ell(t)$ and entrepreneurial wealth $x(t)$. But from (22), entrepreneurial wealth accumulates as a function of past profits, which are a constant fraction of past aggregate incomes, or outputs. Therefore, current output depends on the entire history of past labor supplies, $\{\ell\}_{t \geq 0}$, and the initial level of wealth, x_0. This setup, hence, is isomorphic at the aggregate to a neoclassical growth model in which productivity is a function of past labor supplies, and hence is a special case of a general formulation with learning-by-doing externality in production (see, for example, Krugman, 1987). As a result, some of our policy implications have a lot in common with those that emerge in economies with learning-by-doing externalities, as we discuss in Sections 6.3–6.4. The detailed micro-structure of our environment not only provides discipline for the aggregate planning problem, but also differs in qualitative ways from an environment with learning by doing. In particular, we now switch to the characterization of optimal policies in the presence of transfers, which are a powerful tool in our environment, but have no bite in an economy with learning by doing.

4.3   Optimal policy with transfers to entrepreneurs

We now show that the conclusions obtained in the previous section, in particular that optimal Ramsey policy involves a labor subsidy when entrepreneurial wealth is low, are robust to allowing for transfers to entrepreneurs as long as these are constrained to be finite. Formally, we extend the planner's problem (P1) to allow for an asset subsidy to entrepreneurs, ς_x. The planner now chooses a sequence of three taxes, $\{\tau_b, \tau_\ell, \varsigma_x\}_{t \geq 0}$ to maximize household utility (1) subject to the resulting allocation being a competitive equilibrium. We again make use of Lemma 3, which allows us to recast this problem as the one of choosing a dynamic allocation $\{c, \ell, b, x\}_{t \geq 0}$ and a sequence of transfers $\{\varsigma_x\}_{t \geq 0}$ which satisfy household

budget constraint (29) and aggregate wealth accumulation equation (30).

We impose an additional constraint on the aggregate transfer:[34]

$$s \leq \varsigma_x(t)\,x(t) \leq S, \tag{40}$$

where $s \leq 0$ and $S \geq 0$. The previous section analyzed the special case of $s = S = 0$. The case with unrestricted transfers corresponds to $S = -s = +\infty$, which we consider as a special case now, but in general we allow s and S to be bounded.

We reproduce the planning problem in this case as:

$$\max_{\substack{\{c,\ell,b,x\}_{t\geq 0}, \\ \{\varsigma_x:\ s\leq \varsigma_x x\leq S\}_{t\geq 0}}} \int_0^\infty e^{-\rho t} u(c,\ell)\mathrm{d}t \tag{P2}$$

$$\text{subject to} \quad c + \dot{b} = (1-\alpha)y(x,\ell) + r^* b - \varsigma_x x,$$

$$\dot{x} = \frac{\alpha}{\eta}y(x,\ell) + (r^* + \varsigma_x - \delta)x,$$

given the initial conditions b_0 and x_0. We still denote the two co-states by μ and $\mu\nu$. Appendix A.4 sets up the Hamiltonian for (P2) and provides the full set of equilibrium conditions, following the same steps outlined in footnote 26. In particular, the optimality conditions (33)–(35) still apply, but now with two additional complementary slackness conditions:

$$\nu \geq 1, \quad \varsigma_x x \leq S \qquad \text{and} \qquad \nu \leq 1, \quad \varsigma_x x \geq s. \tag{41}$$

This has two immediate implications. First, as before, the planner never distorts the intertemporal margin of workers, that is $\tau_b \equiv 0$. Second, whenever the bounds on transfers are slack, $s < \varsigma_x x < S$, the co-state for the wealth accumulation constraint is unity, $\nu = 1$. In particular, this is always the case when transfers are unbounded, $S = -s = +\infty$. Note that $\nu = 1$ means that the planner's shadow value of wealth, x, equals $\bar{\mu}$—the shadow value of extra funds in the household budget constraint. This equalization of marginal values is intuitive given that the planner has access to a transfers between the two groups of agents. From (34) and (36), $\nu = 1$ immediately implies that the labor supply condition is undistorted, that is $\tau_\ell = 0$.[35] This discussion allows us to characterize the planner's

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[34]Why transfers may be constrained in reality is discussed in detail in Section 4.5. Given the reasons discussed there, for example political economy considerations limiting aggregate transfers from workers to entrepreneurs, we find a constraint on the aggregate transfer ($\varsigma_x x$) more realistic than one on the subsidy rate (ς_x). However, the analysis of the alternative case is almost identical and we leave it out for brevity. In fact, it is straightforward to generalize (40) to allow s and S to be functions of aggregate wealth, $x(t)$.

[35]Note that when transfers are unbounded, (P2) can be replaced with a simpler optimal control problem
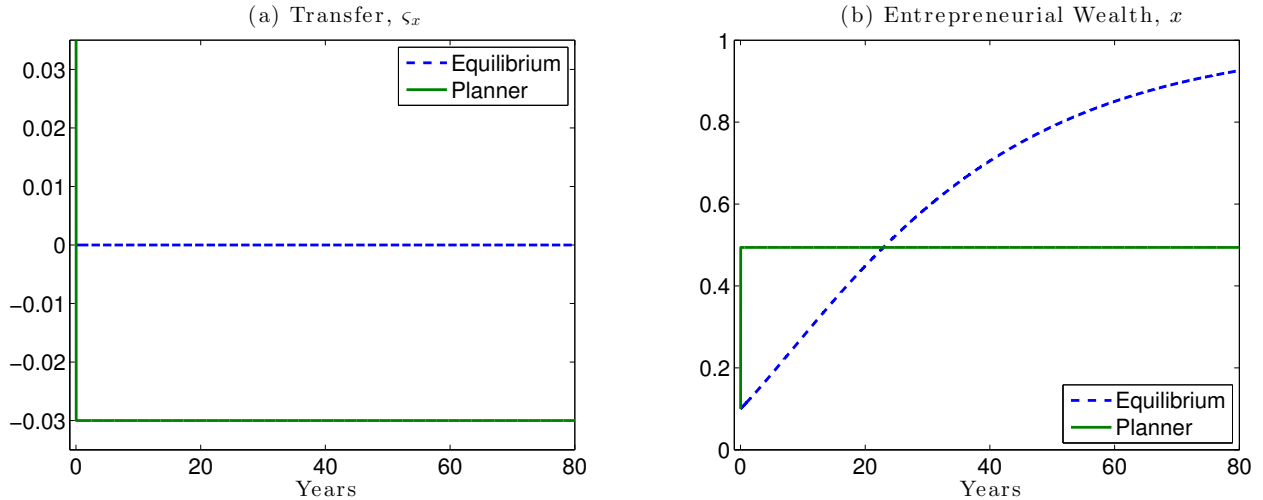
Figure 4: Planner's allocation with unlimited transfers

Note: in (a), *transfer* refers to the asset (savings) subsidy to entrepreneurs, which equals $\varsigma_x(0) = \infty$ and $\varsigma_x(t) = -r^*$, financed by a lump-sum tax on workers, and resulting in the path of entrepreneurial wealth $x(t)$ depicted in (b); other variables instantaneously reach their steady state values, while labor and savings wedges (taxes) for workers are set to zero.

allocation when unbounded transfers are available (see illustration in Figure 4):

Proposition 2 *In the presence of unbounded transfers ($S = -s = +\infty$), the planner distorts neither intertemporal consumption choice, nor intratemporal labor supply along the entire transition path: $\tau_b(t) = \tau_\ell(t) = 0$ for all t. The steady state is achieved in one instant, at $t = 0$, and the steady state asset subsidy equals $\varsigma_x(t) = \bar{\varsigma}_x = -r^*$ for $t > 0$, i.e. a transfer of funds from entrepreneurs to workers. When $x(0) < \bar{x}$, the planner makes an unbounded transfer from workers to entrepreneurs at $t = 0$, i.e. $\varsigma_x(0) = +\infty$, to ensure $x(0+) = \bar{x}$.*[36]

Proposition 2 shows that the asset subsidy to entrepreneurs dominates the other instruments at the planner's disposal, as long as it is unbounded. When the planner can freely reallocate wealth between households and entrepreneurs, he no longer faces the need to distort the labor supply or savings decisions of the workers. Clearly, the infinite transfer in the initial period, $\varsigma_x(0)$, is an artifact of the continuous time environment. In discrete time,

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with a single state variable $m \equiv b + x$ and one aggregate dynamic constraint:

$$\dot{m} = (1 - \alpha + \alpha/\eta)y(x, \ell) + r^*m - \delta x - c.$$

The choice of x in this case becomes static, maximizing the right-hand side of the dynamic constraint at each point in time, and the choice of labor supply can be immediately seen to be undistorted. The results of Proposition 2 can be obtained directly from this simplified formulation (see Appendix A.4).

[36]The steady state entrepreneurial wealth is determined from (30) substituting in $\bar{\varsigma}_x$: $\delta = \alpha/\eta \cdot y(\bar{x}, \bar{\ell})/\bar{x}$, where $\bar{\ell}$ satisfies the labor supply condition (37) with $\tau_\ell = 0$ and $u_c = \bar{\mu}$.

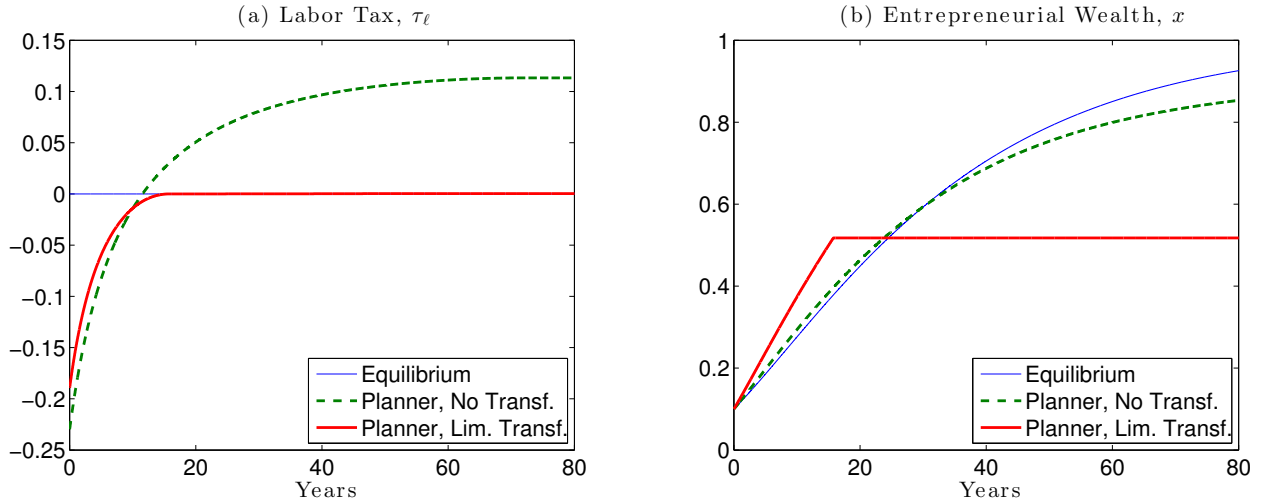(a) Labor Tax, τ_ℓ (b) Entrepreneurial Wealth, x

Figure 5: Planner's allocation with limited transfers

Note: evolution of the labor tax $\tau_\ell(t)$ and entrepreneurial wealth $x(t)$ when transfers to entrepreneurs are bounded by $\varsigma_x(t)x(t) \leq S < \infty$, while s is not binding (i.e., $s \leq -r^*\bar{x}$); until the steady state is reached, the transfer is maxed out ($\varsigma_x(t) = S/x(t)$); in steady state, $\bar{x}$ and $\bar{\varsigma}_x = -r^*$ are the same as in Figure 4.

the required transfer is simply the difference between initial and steady state wealth, which however can be very large if the economy starts far below its steady state in terms of entrepreneurial wealth. There is a variety of reasons why large redistributive transfers may be undesirable or infeasible in reality, as we discuss in detail in Section 4.5, and already alluded to in Section 3. We, therefore, turn now to the analysis of the case with bounded transfers.

For brevity, we consider here the case in which the upper bound is binding, $S < \infty$, but the lower bound is not binding, that is $s \leq -r^*\bar{x}$, while Appendix A.4 presents the general case. The planner's allocation in this case is characterized by $u_c = \bar{\mu}$, (30), (34), (35) and (41), and the transition dynamics has two phases. In the first phase, $x(t) < \bar{x}$ and $\tau_\ell(t) < 0$ (as $\nu(t) > 1$), while the planner simultaneously chooses the maximal possible transfer from workers to entrepreneurs each period, $\varsigma_x(t)x(t) = S$. During this phase, the characterization is the same as in Proposition 1, but with the difference that a transfer S is added to the entrepreneurs' wealth accumulation constraint (32) and subtracted from the workers' budget constraint (31). That is, starting from $x_0 < \bar{x}$, entrepreneurial assets accumulate over time and the planner distorts labor supply upwards at a decreasing rate: $x(t)$ increases and $\tau_\ell(t) < 0$ decreases in absolute value towards zero. The second phase is reached at some finite time $\bar{t} > 0$, and corresponds to a steady state described in Proposition 2: $x(t) = \bar{x}$, $\nu(t) = 1$, $\tau_\ell(t) = 0$ and $\varsigma_x(t) = -r^*$ for all $t \geq \bar{t}$. Throughout the entire transition the intertemporal margin of workers is again not distorted, $\tau_b(t) = 0$ for all t.

We illustrate the planner's dynamic allocation in this case in Figure 5 and summarize its

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properties in the following Proposition:

Proposition 3 *Consider the case with $S < \infty$, $s \leq -r^*\bar{x}$, and $x(0) < \bar{x}$. Then there exists $\bar{t} \in (0, \infty)$ such that: (1) for $t \in [0, \bar{t})$, $\varsigma_x(t)x(t) = S$ and $\tau_\ell(t) < 0$, with the dynamics of $\big(x(t), \tau_\ell(t)\big)$ described by a pair of ODEs (30) and (38) together with a static equation (37), with a globally-stable saddle path as in Proposition 1; (2) for $t \geq \bar{t}$, $x(t) = \bar{x}$, $\tau_\ell(t) = 0$ and $\varsigma_x(t) = -r^*$, corresponding to the steady state in Proposition 2. For all $t \geq 0$, $\tau_b(t) = 0$.*

Therefore, our main result that optimal Ramsey policy involves a labor supply subsidy when entrepreneurial wealth is low is robust to allowing for transfers from workers to entrepreneurs as long as these transfers are bounded. Applying this logic to a discrete-time environment, whenever the transfers cannot be large enough to jump entrepreneurial wealth immediately to its steady state level (therefore, resulting in a transition period with $\nu > 1$), the optimal policy involves a pro-business intervention of increasing labor supply.

4.4   Other tax instruments

In order to evaluate the robustness of our conclusions, we now briefly consider the case with additional tax instruments which directly affect the decisions of entrepreneurs. Specifically, in addition to an asset subsidy ς_x, we introduce a profit subsidy ς_π, a revenue subsidy ς_y, a wage-bill subsidy ς_w, and a capital subsidy ς_k, all financed by a lump-sum tax on households, so that the budget set of the entrepreneurs is now given by:

$$\dot{a} = (1 + \varsigma_\pi)\pi(a, z) + (r^* + \varsigma_x)a - c_e, \tag{42}$$

$$\pi(a, z) = \max_{\substack{n \geq 0, \\ 0 \leq k \leq \lambda a}} \Big\{ (1 + \varsigma_y)A(zk)^\alpha n^{1-\alpha} - (1 - \varsigma_w)w\ell - (1 - \varsigma_k)r^*k \Big\},$$

and the entrepreneurs' consumption-saving decision still satisfies $c_e = \delta a$.

Note that the capital subsidy ς_k is similar to a credit subsidy ς_r, i.e. a subsidy on $(k - a)$ rather than k. In fact, in our framework, where all active entrepreneurs have the same leverage ratio λ, the two instruments have identical effects (operating through reducing the cost of capital) when $\varsigma_k = (\lambda - 1)\varsigma_r/\lambda$. Hence, our exclusive focus on ς_k is without loss of generality. Credit and capital subsidies are, arguably, a natural tax instrument to address the financial friction, and they have been an important element of real world industrial policies (see McKinnon, 1981; Diaz-Alejandro, 1985; Leipziger, 1997).

In the presence of the additional subsidies to entrepreneurs, the equilibrium characterization in Lemma 2 no longer applies and needs to be generalized, as we do in Appendix A.6.

In particular, the aggregate output function is now:

$$y(x, \ell) = \left(\frac{1 + \varsigma_y}{1 - \varsigma_k} \right)^{\gamma(\eta - 1)} \Theta x^\gamma \ell^{1 - \gamma},$$

with γ and Θ defined as before. Furthermore, the planner's problem has a similar structure to (P1) and (P2) with the added optimization over the choice of the additional subsidies.

We prove the following in Appendix A.6:

Proposition 4 *The profit subsidy ς_π, as well as a synthetic profit subsidy $\varsigma_y = -\varsigma_k = -\varsigma_w$, have the same effect as a transfer from workers to entrepreneurs, just like the asset subsidy ς_x.*
(i) When the available instruments can be combined to engineer an unbounded transfer from workers to entrepreneurs, no other instrument is used, and in particular the labor margin is undistorted (as in Proposition 2).
(ii) Otherwise, there is a period of transition during which all available policy instruments are used to speed up the accumulation of entrepreneurial wealth, and in particular the labor supply margin is distorted (as in Propositions 1 and 3).[37]

The profit subsidy, just like the asset subsidy, does not affect the policy rules of entrepreneurs, and therefore approximates the effect of a pure transfer between workers and entrepreneurs. When either of these instruments is available and unbounded, Proposition 2 applies and other taxes are not used. A revenue subsidy can be combined with taxes on capital and labor to replicate the effect of a profit subsidy.

Taken separately, the effects of the revenue subsidy ς_y are similar to those of the wage bill subsidy ς_w (which, in turn, is equivalent to the labor supply subsidy, $\varsigma_\ell \equiv -\tau_\ell$), however, not identical, as ς_y leads to a larger increase in entrepreneurial revenues and profits for a given increase in labor supply. The effects of the capital subsidy ς_k are quite different from those of the labor subsidy ς_w, as ς_k increases entrepreneurial profits by means of distorting the extensive margin of active entrepreneurs (rather than the labor supply decision of workers). All of these instruments are inferior to transfers as they introduce their specific distortions to the allocation, and hence in general they cannot be ranked. Yet, either of these instruments (and their combinations) are beneficial during the transition by enhancing entrepreneurial profits and wealth accumulation to reduce the severity of the financial constraints.

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[37]Using the planner's problem (P4) set up in Appendix A.6, we show that when only ς_y or only ς_k is available, the planner sets them proportional to $(\nu - 1)$, in parallel with the optimal labor supply subsidy analyzed in Section 4.2. When only ς_k and ς_w are available jointly, the planner sets $\varsigma_k = \varsigma_w \propto (\nu - 1)$. Similarly, when only ς_y and ς_w are available jointly, the planner sets $\varsigma_y = -\varsigma_w \propto (\nu - 1)$. In all cases, ν gradually declines during the transition, similar to the patterns described in Figure 1 and Proposition 1.

The general conclusion from this analysis is that, whenever an unbounded transfer between workers and entrepreneurs cannot be engineered, there is a period of transition during which all available policy instruments are used in a *pro-business* manner to speed up the accumulation of entrepreneurial wealth in the least distortive way.

4.5 Constraints on implementation

The analysis above suggests the superiority of transfers to alternative policy tools. Here we discuss a number of arguments why transfers may not constitute a feasible or desirable policy option, as well as other constraints on implementation, which justify our focus on the optimal policy under a restricted set of instruments.

First, large transfers may be infeasible simply due to the budget constraint of the government (or the household sector), when the economy starts far away from its long-run level of wealth. Furthermore, unmodeled distributional concerns in a richer environment with heterogeneous workers may make large transfers—which are large lump-sum taxes from the point of view of workers—undesirable or infeasible (see Werning, 2007). Note that, in contrast, the policy of subsidizing labor supply, while in the short run also shifting gains towards the entrepreneurial sector, has the additional advantage of increasing GDP and incomes of all groups of agents in the economy. If not just entrepreneurs but also the household sector were financially constrained, or if there were an occupational choice such that workers had the option to become entrepreneurs, large lump-sum taxes on households would be even more problematic and the argument in favor of a labor supply subsidy would be even stronger.

Second, large transfers from workers to entrepreneurs may be infeasible for political economy reasons. This limitation is particularly relevant under socialist or populist governments of many developing countries, but even for more technocratic governments a policy of direct financial injections into the business sector, often labelled as a bailout, may be hard to justify. In contrast, it is probably easier to ensure broad public support of more indirect policies, such as labor supply subsidies or competitive exchange rate devaluations. Another political economy concern is that transfers to businesses may become entrenched once given out, e.g. due to political connections. As a result originally "well-intended" transfers may persist far beyond what is optimal from the point of view of a benevolent planner (see Buera, Moll, and Shin, 2013).

Third, the information requirement associated with transfers is likely to be unrealistically strict. Indeed, the government needs to be able to separate entrepreneurs from workers, as every agent in the economy will have an incentive to declare himself an entrepreneur when the

government announces the policy of direct subsidies to business. As a result, the government is likely to be forced to condition its support on some easily verifiable observables. One potential observable is the amount of labor hired by entrepreneurs, and the labor supply subsidy implicitly does just that.[38]

Furthermore, and as already mention in Section 3, transfers constitute such a powerful tool in our environment because they allow the government to effectively side-step the collateral constraint in the economy, by first inflating entrepreneurial wealth and later imposing a lump-sum tax on entrepreneurs to transfer the resources back to the households. Such a policy may be infeasible if entrepreneurs can hide their wealth from the government. In contrast, labor supply taxes are less direct, affecting entrepreneurs only via the equilibrium wage rate, and hence less likely to trigger such deviations.

Finally, the general lesson from our analysis is the optimality of a pro-business stance of government policy during the initial phase of the transition, which may be achieved to some extent with whatever instrument the government has at its disposable. It is possible that the government has very limited flexibility in the use of any tax instruments, and hence has to rely on alternative non-tax market regulation. For example, the government can choose how much market and bargaining power to leave to each group of agents in the economy, or affect the market outcomes by means of changing the value of the outside options of different agents.[39] Such interventions may allow the government to implement some of the Ramsey-optimal allocations without the use of explicit taxes and transfers.

5 Discussion of Assumptions

The goal of this paper is to develop a model of transition dynamics with financial frictions in which we can analyze optimal government interventions. This motivates a number of the assumptions we adopt, which allow for tractable aggregation of the economy and result in a simple characterization of equilibrium under various government policies (see Lemma 3).

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[38]For tractability, the way we set up the Ramsey problem without transfers, the subsidy to labor supply is financed by a lump-sum tax on workers. An alternative formulation is to levy the lump-sum tax on all agents in the economy without discrimination. The two formulations yield identical results in the limiting case when the number (mass) of entrepreneurs is diminishingly small relative to the number (mass) of workers, which we take to be a realistic benchmark.

[39]During the New Deal policies of Franklin D. Roosevelt, the government increased the monopoly power of unions in the labor market and businesses in the product markets. Many Asian countries, for example Korea, have taken an alternative pro-business stance in the labor markets, by halting unions and giving businesses an effective monopsony power. The governments of relatively rich European countries, on the other hand, tilt the bargaining power in favor of labor by providing generous unemployment insurance and a strict regulation of hiring and firing practices.

We now discuss these assumptions systematically, emphasizing which ones are made purely for tractability and which ones are necessary for our results.

Functional forms Our results are robust to many different functional forms for the utility function of households, as long as these feature a positive and finite Frisch elasticity of labor supply. For entrepreneurs, we assume logarithmic utility as it delivers a simple closed-form consumption policy function and makes it easy to characterize wealth dynamics, but the analysis can be generalized to CRRA utility (see Moll, 2014).[40] The Pareto productivity distribution is useful for tractable aggregation and in order to maintain log-linearity of the equilibrium conditions, but is not essential for any of the results, and in certain applications may be conveniently replaced with other distributions. The time-invariant paths of exogenous productivity ($A_t \equiv A$) and maximum leverage ratio ($\lambda_t \equiv \lambda$) can be immediately generalized to arbitrary deterministic or stochastic time series processes without major consequences for the results, as they affect the planner's problem only through the reduced-form productivity Θ_t defined in (17).[41]

The three functional form assumptions that are essential for tractability are the constant returns to scale (CRS) in production, CRRA utility of entrepreneurs which implies linear savings rules, and the linearity of the collateral constraint in the wealth of entrepreneurs. Together they result in optimal production and accumulation decisions that are linear in the wealth of the entrepreneurs, allowing for tractable aggregation and substantial reduction in the size of the state space. Indeed, in general, the state space of the model should include the time-varying joint distribution of endogenous wealth and exogenous productivity, $\mathcal{G}_t(a, z)$. Yet, with our assumptions, the state space reduces to a single variable—the aggregate (or average) wealth of all entrepreneurs x_t—and we only need to keep track of its dynamics characterized by (30). In the earlier literature, the enormous state space typical in the models with financial frictions and heterogeneity has been the main impediment to the optimal policy analysis outside of stationary equilibria.[42] While there is a sharp discontinuity in the

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[40]We additionally adopt a technical assumption that entrepreneurs are more impatient than households ($\delta > \rho = r^*$) in order to insure the existence of a steady state. This assumption can be dropped if one is willing to stick to the analysis of the transition path in a model without a steady state. Alternatively, this assumption is not needed if workers are hand-to-mouth (in equilibrium) or subject to idiosyncratic income risk, in which case $\delta = \rho > r^*$ is a natural assumption in a small open economy and would arise endogenously in a closed economy (Aiyagari, 1994).

[41]For example, the analysis could be extended at little cost to the case where the economy is on a balanced growth path (BGP), with growth driven by sustained improvements in productivity (A_t) and/or sustained financial development (growth in λ_t). In this case it is not the absolute level of entrepreneurial wealth, but rather its deviation from the BGP level, that matters for whether the planner subsidizes or taxes labor.

[42]Some existing analyses of optimal policy do take into account transition dynamics but restrict tax

tractability of the framework once one departs from CRS in production, there is no reason to expect such discontinuity in optimal policies, as the equilibrium allocation itself is continuous in returns to scale (see Moll, 2014). For example, with CRS all active firms are financially constrained, while with returns to scale slightly below one, *almost* all firms are constrained. Therefore, the CRS economy is simply the tractable limiting case of a decreasing returns economy. And standard calibrated values for returns to scale are relatively close to one (see e.g. Atkeson and Kehoe, 2007; Buera and Shin, 2013). More generally, the conceptual requirement for our policy prescriptions to hold is that a non-trivial share of output is produced by financially constrained firms during the transition period.

Heterogeneity We develop a particularly tractable model of heterogeneity and aggregation. Many of our results can be illustrated in economies without heterogeneity, i.e. with a single productivity type of entrepreneurs. There are three main reasons why we opt in favor of a model with heterogeneity. First, this makes our framework closer to the canonical model of financial frictions used in the macro-development literature (see references in the Introduction) to which we want to relate our optimal policy analysis. Second, it allows us to capture misallocation and endogenous TFP dynamics, as well as their response to optimal policies, along the transition path. Financial frictions introduce two distortions in the allocation of resources: one, the economy hires less capital in the aggregate, which reduces labor productivity and depresses wages; and, two, capital is misallocated across firms, which reduces aggregate TFP and also depresses wages. The two effects reinforce each other in our framework, while a model without heterogeneity captures only the first effect of the depressed demand for capital at the aggregate. Third, and somewhat surprisingly, the model with a continuum of heterogeneous entrepreneurs is more tractable than its analogue without heterogeneity. This is because continuous heterogeneity is *regularizing*, adding smoothness to the equilibrium conditions without complicating them. This in turn allows us to capture the declining force of the financial frictions as the economy accumulates wealth and approaches the steady state, but at the same time avoids the need to keep track of different binding patterns of the financial constraint and the corresponding switches in the equilibrium

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instruments to be constant over time, making the optimal policy choice effectively a static problem (see e.g. Conesa, Kitao, and Krueger, 2009). Our result that the sign of the optimal policy differs according to whether an economy is close to or far away from steady state underlines the importance of examining time-varying optimal policy. The tools developed in Lucas and Moll (2013) and Nuño (2013) in simpler environments, should eventually make it possible to extend our analysis of time-varying Ramsey-optimal policies to cases in which the joint distribution of productivity and wealth is a state variable.

regime.[43]

The *iid* assumption for the productivity process delivers particular tractability to the model by reducing the state space to a single variable—the aggregate wealth of the entrepreneurs. Our results are robust to the alternative extreme of constant productivity types (and, in particular, to the special case with no heterogeneity). Further, as opposed to the CRS assumption, the *iid* assumption can be relaxed without a dramatic increase in the size of the state space. In particular, if productivity follows a Markov process, the number of state variables equals the number of possible productivity realizations, as we only need to characterize the aggregate wealth dynamics for each productivity group (see Moll, 2014).

Finally, we rule out endogenous occupation choice, i.e. there are no transitions between the group of workers and the group of entrepreneurs. This is important for analytical tractability of the model, in particular the savings policy function of the entrepreneurs. In the data, the probability of entering entrepreneurship over a time period of a year is typically small (see e.g. Hurst and Lusardi, 2004; Cagetti and De Nardi, 2009). Furthermore and as already discussed, the wage subsidy prescribed in our analysis increases labor income and wealth accumulation of workers and therefore it may have additional beneficial effects in a richer environment with occupational choice. Therefore, we expect our results to also hold in a model with occupational choice, though such a model would be substantially more complicated and would have to be solved numerically.

Financial frictions Following the tradition in the literature (see references in the Introduction), we model financial frictions as the interaction between incomplete markets and collateral constraints, both exogenously imposed. The particular form of the collateral constraint can be generalized in a number of ways, in particular letting the financial friction parameter λ be a function of time or other individual or aggregate variables (see Moll, 2014). Note that in our model even when λ is constant, the severity of the financial frictions reduces over time as wealth x accumulates, endogenously making more developed countries less financially constrained.

The key conceptual assumption, however, is that the use of capital and production require a certain minimal *skin in the game*, and thus the effects generalize to a model with a richer set of available assets, including equity.[44] More generally, the effects we emphasize are likely to

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[43]In the case without heterogeneity, such regime switches would happen, for example, if the financial frictions stopped to bind altogether around or at the steady state. In the case with a discrete number of productivity types such regime switches occur more often throughout the transition. These regime switches make the complete characterization of dynamics substantially less tractable.

[44]Note that issuing equity does not replicate transfers between agents, which have the ability to sidestep

be present as long as the scale of production of a non-trivial share of businesses is constrained by their net worth or the wealth of their owners.[45] A large development literature (see e.g. Banerjee and Duflo, 2005, and the references cited therein) has documented the importance of such constraints for developing countries.

6   Extensions

In this section we discuss ways in which our analysis can be generalized and extended. In particular, we offer four extensions to our benchmark analysis. First, we consider the case when the planner puts an arbitrary positive Pareto weight on the welfare of entrepreneurs. Second, we characterize optimal policies in a closed economy. Third, we consider a reinterpretation of the baseline model which introduces a non-tradable sector and allows for a discussion of the policy implications for the real exchange rate. And, finally, we extend our analysis to multiple tradable sectors to allow for comparative advantage and the discussion of optimal industrial policies.

6.1   Pareto weight on entrepreneurs

Our analysis generalizes in a natural way to the case where the planner puts an arbitrary non-zero Pareto weight on the welfare of the entrepreneurs. In Appendix A.2, we show that the expected present value of an entrepreneur with assets a_0 at time $t = 0$ is given by:

$$V_0(a_0) = v_0 + \frac{1}{\delta} \log a_0 + \frac{1}{\delta} \int_0^\infty e^{-\delta t} \frac{\alpha}{\eta} \frac{y\big(x(t), \ell(t)\big)}{x(t)} \mathrm{d}t, \tag{43}$$

where v_0 is a constant, and the last term represent the discounted present value of expected entrepreneurial returns (see (23)). In Appendix A.7 we extend the baseline planner's problem (P1) to allow for an arbitrary Pareto weight, $\theta \geq 0$, on the utilitarian welfare function for all entrepreneurs, $\mathbb{V}_0 = \int V_0(a)\mathrm{d}G_{a,0}(a)$. We show that the resulting optimal policy parallels that characterized in our main Proposition 1, with the optimal labor tax now given by:

$$\tau_\ell^\theta(t) = \gamma\big(1 - \nu(t)\big) - e^{(\rho-\delta)t} \frac{\theta}{\delta \bar{\mu}} \frac{\gamma}{x(t)}. \tag{44}$$

the financial constraints. This is because equity does not increase the net worth (assets) of the entrepreneurs, which we assume to be the relevant variable for the collateral constraint. In other words, it is the net worth of entrepreneurs that limits borrowing, not the absence of markets in risky assets.

[45]Indeed, the model identifies the derivative of aggregate output with respect to aggregate wealth of the business sector, $\partial y/\partial x = \gamma \cdot y/x$, as the key statistic determining the benefits of a pro-business policy intervention (see footnote 27)—an insight we expect to persist beyond the specific environment of our model.

Therefore, the optimal tax schedule simply shifts down (for a given value of ν) in response to a greater weight on the entrepreneurs in the social objective. That is, the transition is associated with a larger subsidy to labor supply initially and a smaller tax on labor later on.[46] In this sense, we view our results in Section 4 as a conservative benchmark, since even when the planner does not care about entrepreneurs, he still chooses a pro-business policy tilt during the early transition.

6.2 Closed economy

We can also extend our analysis to the case of a closed economy in which the total supply of capital equals the sum of assets held by workers and entrepreneurs, $\kappa(t) = x(t) + b(t)$, and the interest rate, $r(t)$, is determined endogenously to equalize the demand and supply of capital.[47] In Appendix A.8 we set up the closed economy. In particular, we generalize Lemmas 2 and 3 to show that the constraints on allocations (29)–(30) in the closed economy become:

$$\dot{b} = \left[(1 - \alpha) + \alpha \frac{\eta - 1}{\eta} \frac{b}{\kappa} \right] y(\kappa, x, \ell) - c - \varsigma_x x, \tag{45}$$

$$\dot{x} = \left[1 + (\eta - 1) \frac{x}{\kappa} \right] \frac{\alpha}{\eta} y(\kappa, x, \ell) + (\varsigma_x - \delta) x, \tag{46}$$

and where the output function is now:

$$y(\kappa, x, \ell) = \Theta^c \left(\kappa^{\eta - 1} x \right)^\alpha \ell^{1 - \alpha} \qquad \text{with} \qquad \Theta^c \equiv A \left(\frac{\eta}{\eta - 1} \lambda^{1/\eta} \right)^\alpha, \tag{47}$$

instead of (17). The only other difference between (45)–(46) and (29)–(30) is that we have substituted in the expression for the equilibrium interest rate from (20), $r = \alpha(\eta - 1)/\eta \cdot y/\kappa$, which continues to hold in the closed economy. The closed economy dynamics depend on an additional state variable—the capital stock, κ.

Appendix A.8 further solves the planner's problem and characterizes the optimal policies in the closed economy. The main new result is that the planner no longer keeps the intertemporal margin undistorted, and chooses to encourage worker's savings in the early phase of transition, provided x/κ is low enough. This allows the economy to accumulate

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[46]Interestingly, the long-run optimal tax rate is the same for all $\theta \geq 0$, as a consequence of our assumption that entrepreneurs are more impatient than workers, $\delta > \rho$. When $\delta = \rho$, the long-run tax depends on θ and can be negative for θ large enough.

[47]Another interesting case, which we do not consider here, is that of a large open economy, in which the optimal unilateral policy additionally factors in the incentives to manipulate the country's intra- and intertemporal terms of trade (see, for example, Costinot, Lorenzoni, and Werning, 2013).

capital, κ, faster, which in turn raises output and profits, and speeds up entrepreneurial wealth accumulation. The long-run intertemporal wedge may be positive, negative or zero, depending on how large x/κ is in the steady state. The qualitative prediction for the labor wedge remain the same as in the small open economy: an initial labor supply subsidy is replaced eventually by a labor supply tax after entrepreneurs have accumulated enough wealth.[48]

6.3   Nontradables and the real exchange rate

In this section we reinterpret our framework to feature two sectors—a tradable sector with financially constrained entrepreneurs and a frictionless non-tradable sector. Although very stylized, the advantage of this formulation is that it maps directly into our setup of Section 2 without any adjustment to the modeling structure. Furthermore, it is a realistic first approximation if one thinks of the non-tradable sector as less capital intensive and with firms operating on a smaller scale, hence less subject to financing constraints. The assumption that the non-tradable sector is frictionless is also adopted by Caballero and Lorenzoni (2007). We here present this reinterpretation of our framework for illustrative purposes and leave a full treatment of a multisector economy with both tradable and non-tradable sectors that are all subject to financial constraints for future research.

Specifically, we think of an environment with workers having utility over tradable and non-tradable goods, $U\big(c(t), c_N(t)\big)$, who inelastically supply one unit of labor. For simplicity, entrepreneurs consume only tradables, but this assumption can be easily relaxed. Labor supply is split between the tradable sector, $\ell(t)$, and the non-tradable sector, $\ell_N(t) = 1 - \ell(t)$. Production in the non-tradable sector uses only labor with a constant returns technology, $y_N(t) = A_N(t)\ell_N(t)$, and the market for non-tradables is competitive. Assuming constant non-tradable productivity and normalizing $A_N(t) \equiv 1$, this economy is mathematically isomorphic to the one described in Section 2, with leisure replaced by non-tradable consumption, $c_N = y_N = 1 - \ell$. The wage rate now equals the price of non-tradables, $p_N(t) = w(t)$, maintaining the tradable good as numeraire. The equilibrium characterization in Lemma 2 still applies with $y(t)$ now denoting tradable output, or aggregate revenues of the tradable sector.

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[48]Formally, we show that, in the absence of transfers, the optimal tax on labor supply and worker savings satisfy:

$$\tau_\ell^c(t) = \left(1 + (\eta - 1)\frac{x(t)}{\kappa(t)}\right)\frac{\alpha}{\eta}\big(1 - \nu(t)\big) \qquad \text{and} \qquad \tau_b^c(t) = r(t)\left(1 - \frac{1}{\gamma}\frac{x(t)}{\kappa(t)}\right)\frac{\alpha}{\eta}\big(1 - \nu(t)\big),$$

where $\nu(t)$ is again the co-state for $x(t)$.

Furthermore, the planner's problems studied in Section 4 also stays unchanged and Lemma 3 still applies, with an interpretational change that instead of subsidizing labor supply the planner is using a tax on non-tradable goods, $\tau_N(t) \equiv -\tau_\ell(t)$, to manipulate the demand for non-tradables (which is the counterpart to the labor supply condition (28)):

$$\frac{U_N}{U_c} = (1 + \tau_N)p_N,$$

with the tax revenues rebated lump-sum back to the households. An alternative equivalent implementation uses a labor tax in the non-tradable sector, creating a wedge between the wage rate and the cost of labor for non-tradable firms.

Since this environment is mathematically isomorphic to the one studied in Sections 2–4, the characterization of the optimal policy in Propositions 1–3 still applies, and the planner now optimally taxes the non-tradable sector (either consumption or labor supply) in the early phases of transition, which is equivalent to a labor supply subsidy to the tradable sector. This has additional implications for the real exchange rate, which in this model is pinned down by the net (after-tax) price of non-tradables, $(1+\tau_N)p$. Since the planner makes non-tradables more expensive during the initial phase of transition, the economy faces an *appreciated* real exchange rate.

This conclusion contrasts with the prescriptions to *devalue* the exchange rate obtained by Rodrik (2008), Korinek and Serven (2010) and Benigno and Fornaro (2012) in economies featuring a version of learning-by-doing externality. As we argued in the end of Section 4.2, the reduced-form version of our economy looks similar to an economy with learning-by-doing externality, so what is the source of this difference? Indeed, it arises only because we do not rule out a static instrument (i.e., a tax on non-tradables), which as we show in Proposition 1 dominates the intertemporal instrument (e.g., a savings subsidy, or capital controls under an alternative implementation, as in Jeanne, 2012). In Appendix A.9, we show that if the static tax is not available, the planner indeed encourages savings in the early transition, which in turn leads to a *depreciated* real exchange rate.[49] Therefore, the real exchange rate implications of the optimal policy crucially depend on which instruments are available, even when the nature of inefficiency remains the same. This emphasizes that the real exchange rate might not be a particularly useful guide for policymakers, as there is

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[49]The static tax instrument may be infeasible because of binding trade agreements or if it is hard to distinguish tradable from non-tradable labor supply for taxation purposes. The intertemporal wedge tilts the path of consumption and aims to both reduce the demand for non-tradables and increase the labor supply to the tradable sectors via an income effect. This leads to higher exports and net foreign asset accumulation, which is however accompanied by an inflow of productive capital to satisfy increased capital demand.

no robust theoretical link between this variable and growth-promoting policy interventions.

6.4 Comparative advantage and industrial policies

To illustrate the main new insight, we consider here the simplest multisector extension of the model which allows for intratemporal trade with comparative advantage, while Appendix A.10 describes a more general environment. In particular, we consider a small open economy with perfect capital mobility producing two goods traded at the world prices, p_1^* and p_2^*. Aggregate labor is supplied inelastically and split between the two sectors. Each sector is exactly as described in Section 2, and in particular Lemma 2 applies now at the sectoral level. Specifically, the sectoral output, by analogy with (17), is a function of sectoral labor supply ℓ_i and sectoral entrepreneurial wealth x_i: $y_i = \Theta_i x_i^\gamma \ell_i^{1-\gamma}$ for $i = 1, 2$.

We assume that sectors are symmetric in everything except their *latent* comparative advantage which is given by $p_i^* \Theta_i$.[50] As reflected in its definition following (17), Θ_i may differ across sectors due to either physical productivity A_i or financial constraints λ_i, which for example depend on the pledgeability of sectoral assets. The *actual* comparative advantage is also shaped by the allocation of sectoral entrepreneurial wealth, x_i, which accumulates according to analogues of (22). In the short run, the country may specialize against its latent comparative advantage, if entrepreneurs in that sector are poorly capitalized (as was pointed out in Wynne, 2005). In the long-run, the latent comparative advantage forces dominate, and entrepreneurial wealth relocates towards the sector with the highest $p_i^* \Theta_i$.

Appendix A.10 characterizes the decentralized allocation in this economy, and sets up the planner's problem that maximizes the welfare of workers by choosing the dynamic allocation of labor supply across sectors. We show that the planner chooses to distort the decentralized allocation, and instead of equalizing marginal revenue products of labor across the two sectors, tilts the labor supply towards the latent comparative advantage sector. This is because the planner's allocation is not only shaped by the current labor productivity, which is increasing in current x_i, but also takes into account the shadow value of the sectoral entrepreneurial wealth (denoted ν_i and defined by the analogs of (35)), which depends on the latent comparative advantage $p_i^* \Theta_i$. Figure 6 provides an illustration by plotting the

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[50]The difference between autarky and open economy relative prices determines comparative advantage. Assuming the final good is homothetic in the two sectoral outputs, and if both sectors are active in the long-run (as, for example, with a Cobb-Douglas final good), the long-run autarky relative price is given by $p_1^A/p_2^A = \Theta_2/\Theta_1$. A small open economy instead takes international relative price p_1^*/p_2^* as given. Hence the ratio $(p_1^* \Theta_1)/(p_2^* \Theta_2)$ determines the country's long-run comparative advantage. See Section 5 of Lucas (1988) for a general analysis of relative price dynamics in an environment with a learning-by-doing externality.
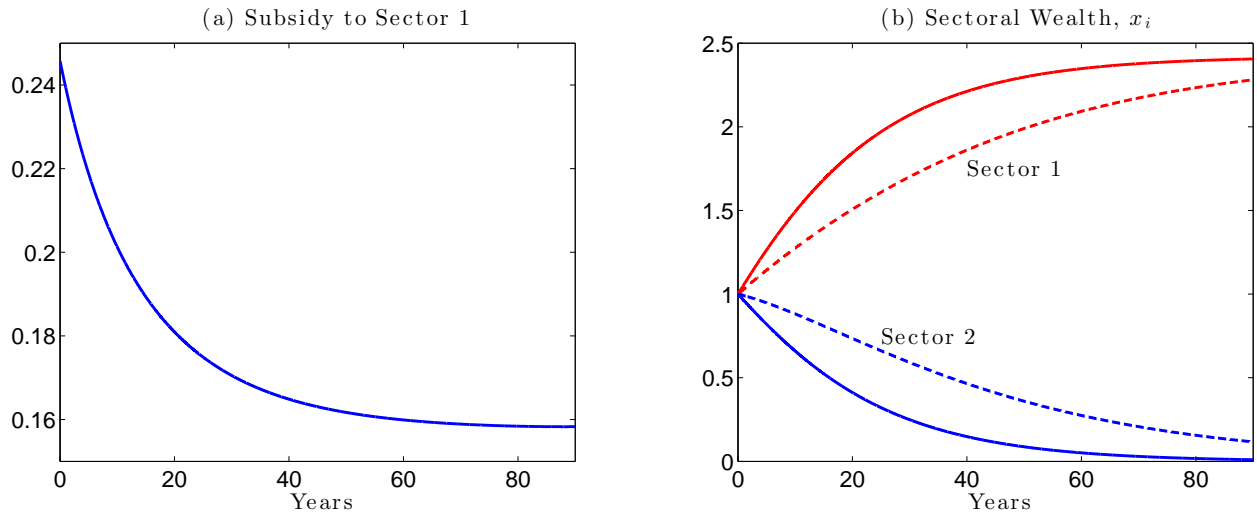
Figure 6: Planner's allocation in a two-sector economy

Note: The sectors are symmetric in everything (including the initial entrepreneurial wealth), but their latent comparative advantage: $p_1^* \Theta_1 > p_2^* \Theta_2$, i.e. Sector 1 has the latent comparative advantage. Panel (a) plots the labor supply subsidy to the comparative advantage Sector 1; the long-run level of the subsidy is not consequential, as the comparative disadvantage Sector 2 shrinks to zero. Panel (b) plots the evolution of the sectoral entrepreneurial wealth under the decentralized allocation (dashed lines) and under the planner's allocation (solid lines).

dynamics of the optimal labor subsidy to the comparative advantage sector and the resulting paths of the sectoral entrepreneurial wealth. The planner favors the comparative advantage sector and speeds up the relocation of factors towards it, consistent with some popular policy prescriptions (see Lin, 2012, and other references in the Introduction).

7 Conclusion

The presence of financial frictions opens the door for welfare-improving government interventions in product and factor markets. We develop a framework to study the Ramsey optimal interventions which accelerate economic development in financially underdeveloped economies. In this framework, financial frictions justify a policy intervention that reduces wages and increases labor supply in the early stages of transition so as to speed up entrepreneurial wealth accumulation and to generate higher labor productivity and wages in the long-run.

To gain a better understanding of the optimal development policies and their implications for a country's growth dynamics, we set up our Ramsey problem in as tractable an environment as possible. By making a number of strong assumptions, we obtain a sharp

analytical characterization of the optimal policies and a precise qualitative understanding of the mechanisms at play, which are likely to persist in more detailed and complex quantitative models with financial frictions.

Our framework is also tractable enough to be extended in a number of different directions. For example, we can study the Ramsey-optimal policies in a heterogeneous multisector economy where each sector differs in the extent of financial frictions, comparative advantage and tradability of the output, so as to relate to the popular discussion of exchange rate and industrial policies. Another natural application of our framework is an analysis of the optimal policy response to transitory shocks and the resulting cyclical fluctuations.

A Appendix

A.1 Frisch labor supply elasticity

For any utility function $u(c, \ell)$ defined over consumption c and labor ℓ, consider the system of equations

$$u_c(c, \ell) = \mu, \tag{A1}$$

$$u_\ell(c, \ell) = -\mu w. \tag{A2}$$

These two equations define ℓ and c as a function of the marginal utility μ and the wage rate w. The solution for ℓ is called the *Frisch labor supply* function and we denote it by $\ell = \ell^F(\mu, w)$. We assumed that the utility function features a positive and finite Frisch labor supply elasticity for all (μ, w):

$$\varepsilon(\mu, w) \equiv \frac{\partial \log \ell^F(\mu, w)}{\partial \log w} = \frac{1}{\frac{u_{\ell\ell}\ell}{u_\ell} - \frac{(u_{c\ell})^2\ell}{u_{cc}u_\ell}} \in (0, \infty), \tag{A3}$$

where the second equality comes from a full differential of (A1)–(A2) under constant μ, which we simplify using $w = -u_\ell/u_c$ implied by the ratio of (A1) and (A2). Therefore, the condition we impose on the utility function is:

$$\frac{u_{\ell\ell}\ell}{u_\ell} > \frac{(u_{c\ell})^2\ell}{u_{cc}u_\ell} \qquad \Leftrightarrow \qquad u_{\ell\ell}u_{cc} > (u_{c\ell})^2 \tag{A4}$$

for all possible pairs (c, ℓ). Due to convexity of $u(\cdot)$, this in particular implies $u_{\ell\ell} < 0$.

A.2 Value and policy functions of entrepreneurs

Lemma A1 *Consider an entrepreneur with logarithmic utility, discount factor δ and budget constraint $\dot{a} = R_t(z)a - c_e$ for some $R_t(z)$, where z is iid over time. Then his consumption policy function is $c_e = \delta a$ and his expected value starting from initial assets a_0 is*

$$V_0(a_0) = -\frac{1}{\delta}(1 - \log \delta) + \frac{1}{\delta}\log a_0 + \frac{1}{\delta}\int_0^\infty e^{-\delta t}\mathbb{E}_z R_t(z)\mathrm{d}t. \tag{A5}$$

Proof: This derivation follows the proof of Lemma 2 in Moll (2014). Denote by $v_t(a, z)$ the value to an entrepreneur with assets a and productivity z at time t, which can be expressed recursively as (see Chapter 2 in Stokey, 2009):

$$\delta v_t(a, z) = \max_{c_e}\left\{\log c_e + \frac{1}{\mathrm{d}t}\mathbb{E}\{\mathrm{d}v_t(a, z)\}, \quad \text{s.t. } \mathrm{d}a = [R_t(z)a - c_e]\mathrm{d}t\right\}.$$

The value function depends on calendar time t because prices and taxes vary over time. In the absence of aggregate shocks, from the point of view of entrepreneurs, calendar time is a "sufficient statistic" for the evolution of the distribution $\mathcal{G}_t(a, z)$.

The proof proceeds with a guess and verify strategy. Guess that the value function takes the form $v_t(a, z) = B\tilde{v}_t(z) + B\log a$. Using this guess we have that $\mathbb{E}\{\mathrm{d}v_t(a, z)\} = B\mathrm{d}a/a + B\mathbb{E}\{\mathrm{d}\tilde{v}_t(z)\}$.

Rewrite the value function:

$$\delta B \tilde{v}_t(z) + \delta B \log a = \max_{c_e} \left\{ \log c_e + \frac{B}{a} \left[R_t(z)a - c_e \right] + B \frac{1}{dt} \mathbb{E}\{\mathrm{d}\tilde{v}_t(z)\} \right\}.$$

Take first order condition to obtain $c_e = a/B$. Substituting back in,

$$\delta B \tilde{v}_t(z) + \delta B \log a = \log a - \log B + B R_t(z) - 1 + B \frac{1}{dt} \mathbb{E}\{\mathrm{d}\tilde{v}_t(z)\}.$$

Collecting the terms involving $\log a$, we see that $B = 1/\delta$ so that $c_e = \delta a$ and $\dot{a} = [R_t(z) - \delta]a$, as claimed in (13) in the text.

Finally, the value function is

$$v_t(a, z) = \frac{1}{\delta} \big(\tilde{v}_t(z) + \log a \big), \tag{A6}$$

confirming the initial conjecture, where $\tilde{v}_t(z)$ satisfies

$$\delta \tilde{v}_t(z) = \delta(\log \delta - 1) + R_t(z) + \frac{1}{dt} \mathbb{E}\{\mathrm{d}\tilde{v}_t(z)\}. \tag{A7}$$

Next we calculate expected value:

$$V_0(a_0) = \int v_0(a_0, z) g_z(z) \mathrm{d}z = \frac{1}{\delta} \big(\tilde{V}_0 + \log a_0 \big),$$

where $g_z(\cdot)$ is the pdf of z and $\tilde{V}_0 \equiv \int \tilde{v}_0(z) g_z(z) \mathrm{d}z$. Integrating (A7):

$$\delta \tilde{V}_t = \delta(\log \delta - 1) + \int R_t(z) g_z(z) \mathrm{d}z + \dot{\tilde{V}}_t, \tag{A8}$$

where we have used that (under regularity conditions so that we can exchange the order of integration)

$$\int \frac{1}{dt} \mathbb{E}\{\mathrm{d}\tilde{v}_t(z)\} g_z(z) \mathrm{d}z = \frac{1}{dt} \mathbb{E}\left\{ \mathrm{d} \int \tilde{v}_t(z) g_z(z) \mathrm{d}z \right\} = \frac{1}{dt} \mathbb{E}\{\mathrm{d}\tilde{V}_t\} = \dot{\tilde{V}}_t.$$

Integrating (A8) forward in time:

$$\tilde{V}_0 = \log \delta - 1 + \int_0^\infty e^{-\delta t} \left[\int R_t(z) g_z(z) \mathrm{d}z \right] \mathrm{d}t,$$

and hence

$$V_0(a_0) = -\frac{1}{\delta}(1 - \log \delta) + \frac{1}{\delta} \log a_0 + \frac{1}{\delta} \int_0^\infty e^{-\delta t} \mathbb{E}_z\{R_t(z)\} \mathrm{d}t. \qquad \blacksquare$$

We now calculate the average return in our model:

$$\mathbb{E}_z\{R_t(z)\} = \int R_t(z) \mathrm{d}G(z) = \int r^* \left(1 + \lambda \left[\frac{z}{z(t)} - 1 \right]^+ \right) \eta z^{-\eta-1} \mathrm{d}z = r^* \left(1 + \frac{\lambda}{\eta - 1} z^{-\eta} \right),$$

where we used (9) and (11) to express $R_t(z)$ and integrated using the Pareto productivity distribution. Finally, using (18), we can rewrite:

$$\mathbb{E}_z\{R_t(z)\} = r^* + \frac{\alpha}{\eta} \frac{y\big(x(t), \ell(t)\big)}{x(t)},$$

which corresponds to equation (23) in the text. Substituting it into (A5) results in expression (43) in the text. A similar derivation can be immediately applied to the case with an asset subsidy, $\varsigma_x(t)$, as long as it is finite.

A.3 Inefficiency of decentralized equilibrium

Proposition A1 *Consider a (small) transfer of wealth $\hat{x}_0 = -\hat{b}_0 > 0$ at $t = 0$ from a representative household uniformly to all entrepreneurs and a reverse transfer at time $t' > 0$ equal to*

$$\hat{x}_0 \exp\left\{ r^* t' + \gamma \int_0^{t'} \frac{\alpha}{\eta} \frac{y\big(x(t), \ell(t)\big)}{x(t)} \mathrm{d}t \right\} > \hat{x}_0 e^{r^* t'},$$

holding constant $\ell(t)$ and $c_e(t)$ for all $t \geq 0$. Such perturbation strictly improves the welfare of workers and leaves the welfare of all entrepreneurs unchanged, constituting a Pareto improvement.

Proof: For any time path $\{c, \ell, b, x, c_e\}_{t \geq 0}$ satisfying the household and entrepreneurs budget constraints:

$$\dot{b}(t) = (1 - \alpha)y\big(x(t), \ell(t)\big) + r^* b(t) - c(t), \tag{A9}$$

$$\dot{x}(t) = \frac{\alpha}{\eta} y\big(x(t), \ell(t)\big) + r^* x(t) - c_e(t), \tag{A10}$$

starting from (b_0, x_0), consider a perturbation $\tilde{x}(t) \equiv x(t) + \beta \hat{x}(t)$, where β is a scalar and $\hat{x}$ is a differentiable function from $\mathbb{R}_+$ to $\mathbb{R}$, and similarly for other variables. Finally, consider perturbations such that:

$$\hat{x}(0) = -\hat{b}(0) = \hat{x}_0 > 0,$$
$$\hat{\ell}(t) = \hat{c}_e(t) = 0 \qquad \forall t \geq 0,$$
$$\hat{c}(t) = 0 \qquad \forall t \in [0, t'],$$

and $\{\tilde{c}, \tilde{\ell}, \tilde{b}, \tilde{x}, \tilde{c}_e\}_{t \in (0, t')}$ satisfy (A9)–(A10).

For such perturbations, we Taylor-expand (A9)–(A10) around $\beta = 0$ for $t \in (0, t')$:

$$\dot{\hat{b}}(t) = (1 - \alpha)\frac{\partial y\big(x(t), \ell(t)\big)}{\partial x}\hat{x}(t) + r^* \hat{b}(t),$$

$$\dot{\hat{x}}(t) = \frac{\alpha}{\eta}\frac{\partial y\big(x(t), \ell(t)\big)}{\partial x}\hat{x}(t) + r^* \hat{b}(t),$$

with $\hat{x}(0) = -\hat{b}(0) = \hat{x}_0$. Note that these equations are linear in $\hat{x}(t)$ and $\hat{b}(t)$, and we can integrate them on $(0, t)$ for $t \leq t'$ to obtain:

$$\hat{b}(t) = -\hat{x}_0 e^{r^* t} + \int_0^t e^{r^*(t-\tilde{t})} (1-\alpha) \frac{\partial y(x(\tilde{t}), \ell(\tilde{t}))}{\partial x} \hat{x}(\tilde{t}) \mathrm{d}\tilde{t},$$

$$\hat{x}(t) = \hat{x}_0 \exp\left\{ \int_0^t \left(\frac{\alpha}{\eta} \frac{\partial y(x(\tilde{t}), \ell(\tilde{t}))}{\partial x} + r^* \right) \mathrm{d}\tilde{t} \right\},$$

Therefore, by $t = t'$, we have a cumulative deviation in the state variables equal to:

$$\hat{x}(t'_-) + \hat{b}(t'_-) = \hat{x}_0 e^{r^* t'} \left[\left(\exp\left\{ \gamma \int_0^{t'} \frac{\alpha}{\eta} \frac{y(x(t), \ell(t))}{x(t)} \mathrm{d}t \right\} - 1 \right) + (1-\gamma) \int_0^{t'} e^{-r^* t} \frac{\alpha}{\eta} \frac{y(x(t), \ell(t))}{x(t)} \frac{\hat{x}(t)}{\hat{x}_0} \mathrm{d}t \right]$$

where t'_- denotes an instant before t', and we have used the functional form for $y(\cdot)$ and definition of γ in (17), which imply $\partial y / \partial x = \gamma y / x$ and $(1-\alpha)\gamma = (1-\gamma)\alpha/\eta$. Both terms inside the square bracket are positive (since $\hat{x}(t)/\hat{x}_0 > 1$ due to the accumulation of the initial transfer). The first term is positive due to the higher return the entrepreneurs make on the initial transfer $\hat{x}_0$ relative to households. The second term represents the increase in worker wages associated with the higher entrepreneurial wealth, which leads to an improved allocation of resources and higher labor productivity.[51]

At $t = t'$, a reverse transfer from entrepreneurs to workers equal to

$$\hat{x}_0 \exp\left\{ r^* t' + \gamma \int_0^{t'} \frac{\alpha}{\eta} \frac{y(x(t), \ell(t))}{x(t)} \mathrm{d}t \right\}$$

result in $\hat{x}(t') = 0$ and $\hat{b}(t') > 0$, which allows to have $\hat{c}(t) = r^* \hat{b}(t') > 0$ for all $t \geq t'$, with $\hat{\ell}(t) = \hat{c}_e(t) = 0$. This constitutes a Pareto improvement since the new allocation has the same labor supply by workers and consumption by entrepreneurs with a strictly higher consumption for workers: $\tilde{\ell}(t) = \ell(t)$, $\tilde{c}_e(t) = c_e(t)$, $\tilde{c}(t) \geq c(t)$ for all $t \geq 0$ and with strict inequality for $t \geq t'$. ∎

A.4 Optimality conditions for the planner's problem

Consider the more general problem (P2). The present-value Hamiltonian for this problem is given by:

$$\mathcal{H} = u(c, 1-\ell) + \mu\left[(1-\alpha)y(x,\ell) + r^* b - c - \varsigma_x x\right] + \mu\nu\left[\frac{\alpha}{\eta}y(x,\ell) + (r^* + \varsigma_x - \delta)x\right] + \mu\bar{\xi}(S - \varsigma_x x) + \mu\underline{\xi}(\varsigma_x x - s),$$

where we have introduced two additional Lagrange multipliers $\mu\bar{\xi}$ and $\mu\underline{\xi}$ for the corresponding bounds on transfers. The full set of optimality conditions is given by:

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[51]Note that for small t', we have the following limiting characterization:

$$\frac{\hat{x}(t') + \hat{b}(t')}{\hat{x}_0 t'} \to \frac{\alpha}{\eta} \frac{y(x(0), \ell(0))}{x(0)} \quad \text{as} \quad t' \to 0,$$

which corresponds to the average return differential between entrepreneurs and workers, $\mathbb{E}_z R_0(z) - r^*$.

$$0 = \frac{\partial \mathcal{H}}{\partial c} = u_c - \mu, \tag{A11}$$

$$0 = \frac{\partial \mathcal{H}}{\partial \ell} = -u_\ell + \mu\big(1 - \gamma + \gamma\nu\big)(1-\alpha)\frac{y}{\ell}, \tag{A12}$$

$$0 = \frac{\partial \mathcal{H}}{\partial \varsigma_x} = \mu x \left(\nu - 1 - \bar{\xi} + \underline{\xi}\right), \tag{A13}$$

$$\dot{\mu} - \rho\mu = -\frac{\partial \mathcal{H}}{\partial b} = -\mu r^*, \tag{A14}$$

$$(\dot{\mu\nu}) - \rho\mu\nu = -\frac{\partial \mathcal{H}}{\partial x} = -\mu\big(1 - \gamma + \gamma\nu\big)\frac{\alpha}{\eta}\frac{y}{x} - \mu\nu(r^* - \delta) - \mu\varsigma_x\left(\nu - 1 - \bar{\xi} + \underline{\xi}\right), \tag{A15}$$

where we have used the fact that $\partial y/\partial \ell = (1 - \gamma)y/\ell$ and $\partial y/\partial x = \gamma y/x$ which follow from the definition of $y(\cdot)$ in (17), as well as the definition of γ. Additionally, we have two complementary slackness conditions for the bounds-on-transfers constraints:

$$\bar{\xi} \geq 0, \quad \varsigma_x x \leq S \qquad \text{and} \qquad \underline{\xi} > 0, \quad \varsigma_x x \geq s. \tag{A16}$$

Under our parameter restriction $\rho = r^*$, (A14) and (A11) imply:

$$\dot{\mu} = 0 \qquad \Rightarrow \qquad u_c(t) = \mu(t) \equiv \bar{\mu} \quad \forall t.$$

With this, (A12) becomes (34) in the text. Given $\mu \equiv \bar{\mu}$ and $r^* = \rho$ and (A13), (A15) becomes (35) in the text. Finally, (A13) can be rewritten as:

$$\nu - 1 = \bar{\xi} - \underline{\xi}.$$

When both bounds are slack, (A16) implies $\bar{\xi} = \underline{\xi} = 0$, and therefore $\nu = 1$. When the upper bound is binding, $\nu - 1 = \bar{\xi} > 0$, and when the lower bound is binding $\nu - 1 = -\underline{\xi} < 0$. Therefore, we obtain the complementary slackness condition (41) in the text.

The case with no transfers $(S = -s = 0)$ results in planner's problem (P1) with Hamiltonian provided in footnote 26. The optimality conditions in this case are (A11), (A12), (A14) and

$$(\dot{\mu\nu}) - \rho\mu\nu = -\frac{\partial \mathcal{H}}{\partial x} = -\mu\big(1 - \gamma + \gamma\nu\big)\frac{\alpha}{\eta}\frac{y}{x} - \mu\nu(r^* - \delta),$$

which also results in (35) after simplification.

The case with unbounded transfers $(S = -s = +\infty)$ allows to simplify the problem considerably, as described in footnote 35. Indeed, in this case we defined a single state variable $m \equiv b + x$ and combine the two constraints in problem (P2), to write the resulting problem as:

$$\max_{\{c,\ell,x,m\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(c, 1 - \ell)\mathrm{d}t \tag{P3}$$

$$\text{subject to} \qquad \dot{m} = \big(1 - \alpha + \alpha/\eta\big)y(x,\ell) + r^* m - \delta x - c,$$

with a corresponding present-value Hamiltonian:

$$\mathcal{H} = u(c, 1 - \ell) + \mu\big[(1 - \alpha + \alpha/\eta)y(x,\ell) + r^*m - \delta x - c\big],$$

with the optimality conditions given by (A11), (A14) and

$$0 = \frac{\partial \mathcal{H}}{\partial \ell} = -u_\ell + \mu(1 - \alpha)\frac{y}{\ell}, \tag{A17}$$

$$0 = \frac{\partial \mathcal{H}}{\partial x} = \mu\left(-\delta + \frac{\alpha}{\eta}\frac{y}{x}\right). \tag{A18}$$

(A17) immediately implies $\tau_\ell(t) \equiv 0$, and (A18) pins down x/ℓ at each instant. The required transfer is then backed out from the aggregate entrepreneurial wealth dynamics (30).

The case with bounded transfers Consider the case with $S < \infty$. There are two possibilities: (a) $s \leq -r^*\bar{x}$, as discussed in the text; and (b) $r^*\bar{x} < s \leq 0$, which we consider first. In this case there are two regions:

1. for $x < \bar{x}$, $\varsigma_x x = S$ binds, $\bar{\xi} = \nu - 1 > 0$ and $\underline{\xi} = 0$. This immediately implies $\tau_\ell = \gamma(1 - \nu) < 0$, and the dynamics of (x, τ_ℓ) is as in Proposition 1, with the difference that $\dot{x} = \alpha y/\eta + (r^* - \delta)x + S$ with $S > 0$ rather than $S = 0$.

2. when $x = \bar{x}$ is reached, the economy switches to the steady state regime with $\bar{\varsigma}_x\bar{x} = s < 0$ binding, and hence $\nu - 1 = -\underline{\xi} < 0$ and $\bar{\xi} = 0$, in which:

$$\frac{\alpha}{\eta}\frac{y(\bar{x},\bar{\ell})}{\bar{x}} = (\delta - r^*) - \frac{s}{\bar{x}} < \delta,$$

$$\bar{\tau}_\ell = \gamma(1 - \bar{\nu}) = \frac{\gamma}{\gamma + (1 - \gamma)\frac{\delta\bar{x}}{r^*\bar{x}+s}} > 0.$$

When this regime (steady state) is reached, there is a jump from labor supply subsidy to a labor supply tax, as well as a switch in the aggregate transfer to entrepreneurs from S to s.

In the alternative case when $s < -r^*\bar{x}$, the first region is the same, and in steady state $\bar{\varsigma}_x\bar{x} = -r^*\bar{x} > s$ and hence the constraint is not binding: $\underline{\xi} = \bar{\xi} = \bar{\nu} - 1 = \bar{\tau}_\ell = 0$. The steady state in this case is characterized by (A17)–(A18), and $\bar{\varsigma}_x = -r^*$ ensures $\dot{x} = 0$ at $\bar{x}$. In this case, τ_ℓ continuously increases to zero when steady state is reached, and the aggregate transfer to entrepreneurs jumps from S to $-r^*\bar{x}$.

A.5   Proof of Proposition 1

Consider (32), (37) and (38). Under our parameter restriction $\rho = r^*$, the households' marginal utility is constant over time $\mu(t) = u_c(t) = \bar{\mu}$ for all t. Using the definition of the Frisch labor supply function (see Appendix A.1), (19) and (17), (37) can be written as

$$\ell = \ell^F\Big(\bar{\mu}, (1 - \tau_\ell)(1 - \alpha)\Theta(x/\ell)^\gamma\Big).$$

For given $(\bar{\mu}, \tau_\ell, x)$, this is a fixed point problem in ℓ, and given positive and finite Frisch elasticity (A3) (i.e., under the condition on the utility function (A4)) one can show that it has a unique solution, which we denote by $\ell = \ell(x, \tau_\ell)$, where we suppress the dependence on $\bar{\mu}$ for notational simplicity. Note that

$$\frac{\partial \log \ell(x, \tau_\ell)}{\partial \log x} = \frac{\varepsilon\gamma}{1 + \varepsilon\gamma} \in (0, 1), \qquad \frac{\partial \log \ell(x, \tau_\ell)}{\partial \log(1 - \tau_\ell)} = \frac{\varepsilon}{1 + \varepsilon\gamma} \in (0, 1/\gamma) \tag{A19}$$

where the bounds follow from (A3). Substituting $\ell(x, \tau_\ell)$ into (38) and (32), we have a system of two autonomous ODEs in (τ_ℓ, x)

$$\dot{\tau}_\ell = \delta(\tau_\ell - \gamma) + \gamma(1 - \tau_\ell)\frac{\alpha}{\eta}\Theta\left(\frac{\ell(x, \tau_\ell)}{x}\right)^{1-\gamma},$$

$$\dot{x} = \frac{\alpha}{\eta}\Theta x^\gamma \ell(x, \tau_\ell)^{1-\gamma} + (r^* - \delta)x.$$

We now show that the dynamics of this system in (τ_ℓ, x) space can be described with the phase diagram in Figure 1.

Steady State   We first show that there exists a unique positive steady state $(\bar{\tau}_\ell, \bar{x})$, i.e. a solution to

$$\gamma(1 - \bar{\tau}_\ell)\frac{\alpha}{\eta}\Theta\left(\frac{\ell(\bar{x}, \bar{\tau}_\ell)}{\bar{x}}\right)^{1-\gamma} = \delta(\gamma - \bar{\tau}_\ell), \tag{A20}$$

$$\frac{\alpha}{\eta}\Theta\left(\frac{\ell(\bar{x}, \bar{\tau}_\ell)}{\bar{x}}\right)^{1-\gamma} = \delta - r^*. \tag{A21}$$

Substituting (A21) into (A20) and rearranging, we obtain the expression for $\bar{\tau}_\ell$ in (39). From (A21), $\bar{x}$ is then the solution to the fixed point problem

$$\bar{x} = \left(\frac{\alpha}{\eta}\frac{\Theta}{\delta - r^*}\right)^{\frac{1}{1-\gamma}}\ell(\bar{x}, \bar{\tau}_\ell) \equiv \Phi(\bar{x}) \tag{A22}$$

Depending on the properties of the Frisch labor supply function, there may be a trivial solution $\bar{x} = 0$. We instead focus on positive steady states. Consider $\varepsilon(\mu, w)$ from (A3) and define

$$\varepsilon_1 \equiv \min_w \varepsilon(\bar{\mu}, w) > 0, \quad \varepsilon_2 \equiv \max_w \varepsilon(\bar{\mu}, w) < \infty, \quad \theta_1 \equiv \frac{\varepsilon_1\gamma}{1 + \varepsilon_1\gamma} > 0, \quad \theta_2 \equiv \frac{\varepsilon_2\gamma}{1 + \varepsilon_2\gamma} < 1.$$

From (A19), there are constants k_1 and k_2 such that $k_1 x^{\theta_1} \leq \ell(x, \bar{\tau}_\ell) \leq k_2 x^{\theta_2}$. Since $\theta_1 > 0, \theta_2 < 1$, there are $x_1 > 0$ sufficiently small and $x_2 < \infty$ sufficiently large such that $\Phi(x_1) > x_1$ and $\Phi(x_2) < x_2$. Finally, taking logs on both sides of (A22), we have

$$\tilde{x} = \tilde{\Theta} + \tilde{\ell}(\tilde{x}), \quad \tilde{\ell}(\tilde{x}) \equiv \log \ell(\exp(\tilde{x}), \bar{\tau}_\ell), \quad \tilde{\Theta} \equiv \log\left(\frac{\alpha}{\eta}\frac{\Theta}{\delta - r^*}\right)^{\frac{1}{1-\gamma}} \tag{A23}$$

satisfying $\tilde{\Theta} + \tilde{\ell}(\tilde{x}_1) > \tilde{x}_1$ and $\tilde{\Theta} + \tilde{\ell}(\tilde{x}_2) < \tilde{x}_2$, where $\tilde{x}_j \equiv \log x_j$, for $j \in \{1, 2\}$. From (A19), we have $0 < \tilde{\ell}'(\tilde{x}) < 1$ for all $\tilde{x}$ and therefore (A23) has a unique fixed point $\tilde{x}_1 < \log \bar{x} < \tilde{x}_2$.

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Transition dynamics (A21) implicitly defines a function $x = \phi(\tau_\ell)$, which is the $\dot{x} = 0$ locus. We have that

$$\frac{\partial \log \phi(\tau_\ell)}{\partial \log(1 - \tau_\ell)} = \frac{\frac{\partial \log \ell}{\partial \log(1-\tau_\ell)}}{1 - \frac{\partial \log \ell}{\partial \log x}} = \varepsilon \in (0, \infty).$$

Therefore the $\dot{x} = 0$ locus is strictly downward-sloping in (x, τ_ℓ) space, as drawn in Figure 1. The $\dot{\tau}_\ell = 0$ locus may be non-monotonic, but we know that the two loci intersect only once (the steady state is unique). The state space can then be divided into four quadrants. It is easy to see that $\dot{\tau}_\ell > 0$ for all points to the north-west of the $\dot{\tau}_\ell = 0$ locus, and $\dot{x} > 0$ for all points to the south-west of the $\dot{x} = 0$ locus, as indicated by the arrows in Figure 1. It then follows that the system is saddle path stable. Assuming Inada conditions on the utility function and given output function $y(\cdot)$ defined in (17), the saddle path is the unique solution to the planner's problem (P1).

Now consider points (x, τ_ℓ) along the saddle path. There is a threshold $\hat{x}$ such that $\tau_\ell < 0$ whenever $x < \hat{x}$ and vice versa, that is labor supply is subsidized when wealth is sufficiently low. There is an alternative argument for this result along the lines of footnote 27. Equation (35) can be solved forward to yield:

$$\nu(0) = \int_0^\infty e^{-\int_0^t (\delta - \alpha y_x(s)/\eta) \mathrm{d}s} (1 - \alpha) y_x(t) \mathrm{d}t,$$

with $x(0) = x_0$ and where $y_x(t) \equiv \partial y(x(t), \ell(t))/\partial x = \gamma y(x(t), \ell(t))/x(t) \propto (\ell(t)/x(t))^{1-\gamma}$. The marginal product of x, y_x, is unbounded as $x \to 0$. Therefore, for low enough x_0, we must have $\nu(0) > 1$ and hence $\tau_\ell(0) < 0$. ∎

A.6 Additional tax instruments (proof of Proposition 4)

We first prove an equilibrium characterization result, analogous to Lemma 2:

Lemma A2 *When subsidies $(\varsigma_x, \varsigma_\pi, \varsigma_y, \varsigma_k, \varsigma_w)$ are used, the output function is given by:*

$$y = \left(\frac{1 + \varsigma_y}{1 - \varsigma_k}\right)^{\gamma(\eta-1)} \Theta x^\gamma \ell^{1-\gamma}, \tag{A24}$$

where Θ and γ are defined as in Lemma 2, and we have:

$$\underline{z}^\eta = \frac{1 - \varsigma_k}{1 + \varsigma_y} \frac{\eta\lambda}{\eta - 1} \frac{r^*}{\alpha} \frac{x}{y},$$

$$(1 - \varsigma_w) w\ell = (1 - \alpha)(1 + \varsigma_y)y,$$

$$(1 - \varsigma_k) r^* \kappa = \frac{\eta - 1}{\eta} \alpha (1 + \varsigma_y)y,$$

$$\Pi = \frac{\alpha}{\eta}(1 + \varsigma_y)y.$$

Proof: Consider the profit maximization problem (42) in this case. The solution to this problem is given by:

$$k = \lambda a \mathbf{1}_{\{z \geq \underline{z}\}},$$

$$\ell = \left((1-\alpha) \frac{(1+\varsigma^y)A}{(1-\varsigma^w)w} \right)^{1/\alpha} zk,$$

$$\pi = \left[\frac{z}{\underline{z}} - 1 \right] (1-\varsigma^k) r^* k,$$

where the cutoff is defined by the zero-profit condition:

$$\alpha \left[(1+\varsigma^y) A \right]^{1/\alpha} \left(\frac{1-\alpha}{(1-\varsigma^w)w} \right)^{\frac{1-\alpha}{\alpha}} \underline{z} = (1-\varsigma^k) r^*. \tag{A25}$$

Finally, labor demand in the sector is given by:

$$\ell = \left((1-\alpha) \frac{(1+\varsigma^y)A}{(1-\varsigma^w)w} \right)^{1/\alpha} \frac{\eta \lambda}{\eta - 1} x \underline{z}^{1-\eta}, \tag{A26}$$

and aggregate output is given by:

$$y = \left((1-\alpha) \frac{(1+\varsigma^y)}{(1-\varsigma^w)w} \right)^{\frac{1-\alpha}{\alpha}} A^{1/\alpha} \frac{\eta \lambda}{\eta - 1} x \underline{z}^{1-\eta}. \tag{A27}$$

Combining these three conditions, we solve for $\underline{z}$, w and y, which result in the first three equations of the lemma. Aggregate capital demand and profits in this case are still given by:

$$\kappa = \lambda x \underline{z}^{-\eta} \qquad \text{and} \qquad \Pi = \kappa/(\eta - 1),$$

and combining these with the solution for $\underline{z}^\eta$ we obtain the last two equations of the lemma. ∎

The immediate implication of this lemma is that asset and profit subsidies do not affect the equilibrium relationships directly, but do so only indirectly through their affect on aggregate entrepreneurial wealth.

With this characterization, and given that the subsidies are financed by a lump-sum tax on households, we can write the planners problem as

$$\max_{\{c,\ell,b,x,\varsigma_x,\varsigma_\pi,\varsigma_k,\varsigma_w,\varsigma_y\}} \int_0^\infty e^{-\rho t} u\big(c(t), \ell(t)\big) \mathrm{d}t \tag{P4}$$

subject to

$$c + \dot{b} \leq \left[(1-\alpha) - \frac{\varsigma_y}{1+\varsigma_y} - \frac{\varsigma_k}{1-\varsigma_k} \frac{\eta-1}{\eta} \alpha - \varsigma_\pi \frac{\alpha}{\eta} \right] (1+\varsigma_y) y(x, \ell, \varsigma_y, \varsigma_k) + r^* b - \varsigma_x x,$$

$$\dot{x} = (1+\varsigma_\pi) \frac{\alpha}{\eta} (1+\varsigma_y) y(x, \ell, \varsigma_y, \varsigma_k) + (r^* + \varsigma_x - \delta) x,$$

where $y(x, \ell, \varsigma_y, \varsigma_k)$ is defined in (A24) and the negative terms in the square brackets correspond to lump-sum taxes levied to finance the respective subsidies. Note that ς_w drops out from the constraints, and it can be recovered from

$$-\frac{u_c}{u_\ell} = (1 + \varsigma_\ell)w = \frac{1 + \varsigma_\ell}{1 - \varsigma_w} \cdot (1 + \varsigma_y)(1 - \alpha)\frac{y}{\ell},$$

assuming $\varsigma_\ell = 0$, otherwise there is implementational indeterminacy since ς_ℓ and ς_w are perfectly substitutable policy instruments as long as $\varsigma_w = \varsigma_\ell/(1 + \varsigma_\ell)$.

When unbounded asset or profit subsidies are available, we can aggregate the two constraints in (P4) in the same way we did in Appendix A.4 in planner's problem (P3) by defining a single state variable $m \equiv b + x$. The corresponding Hamiltonian in this case is:

$$\mathcal{H} = u(c, \ell) + \mu \left[\left(1 - \alpha + \frac{\alpha}{\eta} - \frac{\varsigma_y}{1 + \varsigma_y} - \frac{\varsigma_k}{1 - \varsigma_k}\frac{\eta - 1}{\eta}\alpha\right) \frac{(1 + \varsigma_y)^{1+\gamma(\eta-1)}}{(1 - \varsigma_k)^{\gamma(\eta-1)}}\Theta x^\gamma \ell^{1-\gamma} + r^* m - \delta x - c \right],$$

where we have substituted (A24) for y. The optimality with respect to $(\varsigma_y, \varsigma_k)$ evaluated at $\varsigma_y = \varsigma_k = 0$ are, after simplification:

$$\left.\frac{\partial \mathcal{H}}{\partial \varsigma_y}\right|_{\varsigma_y = \varsigma_k = 0} \propto -\frac{1}{1 - \alpha + \alpha/\eta} + 1 + \gamma(\eta - 1) = 0,$$

$$\left.\frac{\partial \mathcal{H}}{\partial \varsigma_k}\right|_{\varsigma_y = \varsigma_k = 0} \propto -\frac{\frac{\eta-1}{\eta}\alpha}{1 - \alpha + \alpha/\eta} + \gamma(\eta - 1) = 0,$$

and combining $\partial \mathcal{H}/\partial c = 0$ and $\partial \mathcal{H}/\partial \ell = 0$, both evaluated at $\varsigma_y = \varsigma_k = 0$, we have:

$$u_\ell/u_c = (1 - \alpha)y/\ell.$$

Finally, optimality with respect to m implies as before $\dot{\mu} = 0$ and $u_c(t) = \mu(t) \equiv \bar{\mu}$ for all t. This implies that whenever profit and/or asset subsidies are available and unbounded, other instruments are not used:

$$\varsigma_y = \varsigma_k = (\varsigma_\ell + \varsigma_w) = \varsigma_b = 0.$$

Indeed, both ς_π and ς_y, appropriately chosen, act as transfers between workers and entrepreneurs, and do not affect any equilibrium choices directly, in particular do not affect $y(\cdot)$, as can be seen from (A24). This is the reason why these instruments are favored over other distortionary ways to affect the dynamics of entrepreneurial wealth.

Examining (42), we see that the following combination of taxes $\varsigma_y = -\varsigma_k = -\varsigma_w = \varsigma$ is equivalent to a profit subsidy $\varsigma_\pi = \varsigma$, and therefore whenever these three instruments are jointly available, they are used in this way to replicate a profit subsidy.

Next, in planner's problem (P4) we restrict $\varsigma^a = \varsigma^\pi \equiv 0$, and write the resulting Hamiltonian:

$$\mathcal{H} = u(c, \ell) + \mu \left[r^* b - c + \left((1 - \alpha) - \frac{\varsigma^y}{1 + \varsigma^y} - \frac{\varsigma^k}{1 - \varsigma^k}\frac{\eta - 1}{\eta}\alpha \right)(1 + \varsigma^y)y \right] + \mu\nu \left[(r^* - \delta)a + \frac{\alpha}{\eta}(1 + \varsigma^y)y \right],$$

where y is given in (A24). The optimality conditions with respect to b and c are as before, and result in $u_c = \mu \equiv \bar\mu$. The optimality with respect to x results in a dynamic equation for ν, analogous to (35). The optimality with respect to ς^k, ς^y and ℓ are now given by:

$$\frac{\partial \mathcal{H}}{\partial \varsigma^k} \propto -\left[\frac{\varsigma^y}{1+\varsigma^y} + \frac{\varsigma^k}{1-\varsigma^k}\right] + \frac{\alpha}{\eta}(\nu - 1) = 0,$$

$$\frac{\partial \mathcal{H}}{\partial \varsigma^y} \propto -(\eta - 1)\left[\frac{\varsigma^y}{1+\varsigma^y} + \frac{\varsigma^k}{1-\varsigma^k}\right] + (\nu - 1) = 0,$$

$$\frac{\partial \mathcal{H}}{\partial \ell} \propto \frac{u_\ell}{u_c} + \left(1 - \gamma\frac{\eta}{\alpha}\frac{\varsigma^y}{1+\varsigma^y} - \gamma(\eta-1)\frac{\varsigma^k}{1-\varsigma^k} + \gamma(\nu-1)\right)\frac{(1+\varsigma^y)(1-\alpha)y}{\ell} = 0.$$

We consider the case when there is an additional restriction—either $\varsigma^y = 0$ or $\varsigma^k = 0$—so that a profit subsidy cannot be engineered. We immediately see that in the former case $\varsigma^k/(1 - \varsigma^k) \propto \varsigma_\ell \propto (\nu - 1)$, and in the latter $\varsigma^y/(1 + \varsigma^y) \propto -\varsigma_\ell \propto (\nu - 1)$. The results under the restriction $\varsigma_w = \varsigma_w = 0$ are more tedious to derive, and we omit them for brevity. This completes the proof of the statements in Proposition 4. ∎

A.7 Pareto weight on entrepreneurs

Consider an extension to the planning problem (P1) in Section 4.2 (without transfers, $\varsigma_x \equiv 0$) in which the planner puts a positive Pareto weight $\theta > 0$ on the utilitarian welfare criterion of all entrepreneurs $\mathbb{V}_0 \equiv \int V_0(a)\mathrm{d}G_{a,0}(a)$ where $V_0(\cdot)$ is the expected value to an entrepreneur with initial assets a_0. From Appendix A.2, this value can be written as (43) and therefore

$$\mathbb{V}_0 = v_0 + \frac{1}{\delta}\int \log a\, \mathrm{d}G_{a,0}(a) + \frac{1}{\delta}\int_0^\infty e^{-\delta t}\frac{\alpha}{\eta}\frac{y(x,\ell)}{x}\mathrm{d}t.$$

Since given the instruments the planner cannot affect the first two terms in $\mathbb{V}_0$, the planner's problem in this case can be written as:

$$\max_{\{c,\ell,b,x\}_{t\geq 0}} \int_0^\infty e^{-\rho t}u(c,\ell)\mathrm{d}t + \frac{\theta}{\delta}\int_0^\infty e^{-\delta t}\frac{\alpha}{\eta}\frac{y(x,\ell)}{x}\mathrm{d}t \qquad (P7)$$

$$\text{subject to} \qquad c + \dot{b} = (1-\alpha)y(x,\ell) + r^*b,$$

$$\dot{x} = \frac{\alpha}{\eta}y(x,\ell) + (r^* - \delta)x,$$

The Hamiltonian for this problem is:

$$\mathcal{H} = u(c,\ell) + \frac{\theta}{\delta}e^{-(\delta-\rho)t}\frac{\alpha}{\eta}\frac{y}{x} + \mu\big[(1-\alpha)y(x,\ell) + r^*b - c\big] + \mu\nu\left[\frac{\alpha}{\eta}y(x,\ell) + (r^* - \delta)x\right],$$

and the optimality conditions are $u_c(t) = \mu(t) = \bar\mu$ for all t and:

$$\frac{\partial \mathcal{H}}{\partial \ell} = u_\ell + \bar{\mu} \left[\frac{\theta}{\delta \bar{\mu}} e^{-(\delta - \rho)t} \frac{\gamma}{x} + (1 - \gamma) + \gamma \nu \right] (1 - \alpha) \frac{y}{\ell} = 0,$$

$$\dot{\nu} - \rho \nu = -\frac{1}{\bar{\mu}} \frac{\partial \mathcal{H}}{\partial x} = (\delta - r^*) \nu - \left[\frac{\theta}{\delta \bar{\mu}} e^{-(\delta - \rho)t} \frac{\gamma}{x} + (1 - \gamma) + \gamma \nu \right] \frac{\alpha}{\eta} \frac{y}{x}.$$

The dynamic system characterizing (x, ν) is the same as in Section 4.2 with the exception of an additional term $\frac{\theta}{\delta \bar{\mu}} e^{-(\delta - \rho)t} \frac{\gamma}{x} \geq 0$ in the condition above. Similarly, the optimal labor wedge which we denote by τ_ℓ^θ is given by (44).

A.8 Closed economy

Lemma 1, as well as aggregation equations (14)–(16) and income accounting equations (19)–(21) from Lemma 2, still apply in the closed economy. The difference however is that now r is endogenous and we have an additional equilibrium condition $\kappa = x + b$. Substituting in capital demand (14) into the aggregate production function (16), we obtain (47) which defines $y(x, \kappa, \ell)$ in the text. We can then summarized the planner's problem in the closed economy as:

$$\max_{\{c, \ell, \kappa, b, x\}} \int e^{-\rho t} u(c, \ell) \mathrm{d}t, \tag{PC}$$

$$\text{subject to} \quad \dot{b} = \left[(1 - \alpha) + \alpha \frac{\eta - 1}{\eta} \frac{b}{\kappa} \right] y(x, \kappa, \ell) - c,$$

$$\dot{x} = \left[1 + (\eta - 1) \frac{x}{\kappa} \right] \frac{\alpha}{\eta} y(x, \kappa, \ell) - \delta x$$

and given (b_0, x_0), $\kappa = x + b$, and where we have used (20) to substitute out endogenous interest rate r.

To simplify notation, we replace the first constraint with the sum of the two constants to substitute κ for $b + x$. The Hamiltonian for this problem is:

$$\mathcal{H} = u(c, \ell) + \mu \left[y(\kappa, \ell, x) - c - \delta x \right] + \mu \nu \left(\left[1 + (\eta - 1) \frac{x}{\kappa} \right] \frac{\alpha}{\eta} y(\kappa, \ell, x) - \delta x \right)$$

and the optimality conditions are:

$$0 = \frac{\partial \mathcal{H}}{\partial c} = u_c - \mu,$$

$$0 = \frac{\partial \mathcal{H}}{\partial \ell} = u_\ell + \mu \left[1 + \nu \frac{\alpha}{\eta} \left(1 + (\eta - 1) \frac{x}{\kappa} \right) \right] (1 - \alpha) \frac{y}{\ell},$$

$$\dot{\mu} - \rho \mu = -\frac{\partial \mathcal{H}}{\partial \kappa} = -\mu r - \mu \nu \frac{\alpha}{\eta} \left(1 + (\eta - 1) \frac{x}{\kappa} \right) r + \mu \nu \frac{x}{\kappa} r,$$

$$\dot{(\mu \nu)} - \rho \mu \nu = -\frac{\partial \mathcal{H}}{\partial x} = -\mu \left(\frac{\alpha}{\eta} \frac{y}{x} - \delta \right) - \mu \nu \left(\frac{\alpha}{\eta} \left(1 + (\eta - 1) \frac{x}{\kappa} \right) \frac{\alpha}{\eta} \frac{y}{x} + r - \delta \right).$$

From the second condition we have labor wedge:

$$-\frac{u_\ell}{u_c} = \left[1 + \nu\frac{\alpha}{\eta}\left(1 + (\eta-1)\frac{x}{\kappa}\right)\right](1-\alpha)\frac{y}{\ell} \qquad \Rightarrow \qquad \tau_\ell^c = -\nu\frac{\alpha}{\eta}\left(1 + (\eta-1)\frac{x}{\kappa}\right).$$

Next we use the other conditions to characterize the intertemporal wedge:

$$\frac{\dot{u}_c}{u_c} = \rho - r - \nu r\left[\frac{\alpha}{\eta} - \frac{x}{\kappa}\left(1 - \alpha\frac{\eta-1}{\eta}\right)\right] \qquad \Rightarrow \qquad \tau_b^c = -\nu r\left[\frac{\alpha}{\eta} - \frac{x}{\kappa}\left(1 - \alpha\frac{\eta-1}{\eta}\right)\right].$$

Finally, we have:

$$\dot{\nu} = \left(\delta + \nu r\left[\frac{\alpha}{\eta} - \frac{x}{\kappa}\left(1 - \alpha\frac{\eta-1}{\eta}\right)\right] - \frac{\alpha}{\eta}\left(1 + (\eta-1)\frac{x}{\kappa}\right)\frac{\alpha}{\eta}\frac{y}{x}\right)\nu - \left(\frac{\alpha}{\eta}\frac{y}{x} - \delta\right).$$

This dynamic system can be solved using conventional methods. Note that ν in this problem corresponds to $(\nu-1)$ in the text, as we have used the sum of the two constraint (country aggregate resource constraint) instead of using the household budget constraint.

A.9 Optimal intertemporal wedge

Assume the planner cannot manipulate the labor supply margin, and only can distort the intertemporal margin. The planner's problem in this case can be written as:

$$\max_{\{c,\ell,b,x\}_{t\geq 0}} \int_0^\infty e^{-\rho t} u(c,\ell)\mathrm{d}t \qquad\qquad (\text{P6})$$
$$\text{subject to} \qquad c + \dot{b} = (1-\alpha)y(x,\ell) + r^* b,$$
$$\dot{x} = \frac{\alpha}{\eta}y(x,\ell) + (r^* - \delta)x,$$
$$-\frac{u_c}{u_\ell} = (1-\alpha)\frac{y(x,\ell)}{\ell},$$

where the last constraint implies that the planner cannot distort labor supply, and we denote by $\mu\psi$ the Lagrange multiplier on this additional constraint. We can write the Hamiltonian for this problem as:

$$\mathcal{H} = u(c,\ell) + \mu\left[(1-\alpha)y(x,\ell) + r^* b - c\right] + \mu\nu\left[\frac{\alpha}{\eta}y(x,\ell) + (r^*-\delta)x\right] + \mu\psi\left[(1-\alpha)y(x,\ell) - h(c,\ell)\right],$$

where $h(c,\ell) \equiv -\ell u_\ell(c,\ell)/u_c(c,\ell)$. The optimality conditions are:

$$0 = \frac{\partial\mathcal{H}}{\partial c} = u_c - \mu\left(1 + \psi h_c\right),$$
$$0 = \frac{\partial\mathcal{H}}{\partial\ell} = u_\ell + \mu\left(1 - \gamma + \gamma\nu\right)(1-\alpha)\frac{y}{\ell} + \mu\psi\left((1-\gamma)(1-\alpha)\frac{y}{\ell} - h_\ell\right),$$
$$\dot{\mu} - \rho\mu = -\frac{\partial\mathcal{H}}{\partial b} = -\mu r^*,$$
$$(\dot{\mu\nu}) - \rho\mu\nu = -\frac{\partial\mathcal{H}}{\partial x} = -\mu\left(1 - \gamma + \gamma\nu\right)\frac{\alpha}{\eta}\frac{y}{x} - \mu\nu(r^* - \delta) - \mu\psi(1-\gamma)\frac{\alpha}{\eta}\frac{y}{x}.$$

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Under our parameter restriction $\rho = r^*$, the third condition implies $\dot\mu = 0$ and $\mu(t) \equiv \bar\mu$ for all t, however, now $u_c = \bar\mu(1 + \psi h_c)$ and is no longer constant in general, reflecting the use of the savings subsidy to workers. Combining this with the second optimality condition and the third constraint on the planner's problem, we have:

$$(1 - \alpha)\frac{y}{\ell} = -\frac{u_c}{u_\ell} = \frac{\big((1 - \gamma)(1 + \psi) + \gamma\nu\big)(1 - \alpha)y/\ell - \psi h_\ell}{1 + \psi h_c},$$

which we simplify using $h = (1 - \alpha)y$:

$$\psi = \frac{\gamma(\nu - 1)}{h_c + \ell h_\ell/h - (1 - \gamma)}. \tag{A28}$$

Finally, the dynamics of ν satisfies:

$$\dot\nu = \delta\nu - \big((1 - \gamma)(1 + \psi) + \gamma\nu\big)\frac{\alpha}{\eta}\frac{y}{x},$$

and the distortion to the consumption smoothing satisfies:

$$u_c = \bar\mu(1 + \psi h_c) = \bar\mu(1 + \Gamma(\nu - 1)), \qquad \Gamma \equiv \frac{\gamma h_c}{h_c + \ell h_\ell/h - (1 - \gamma)}. \tag{A29}$$

Recall that under $\rho = r^*$, $\dot u_c/u_c = -\varsigma_b$, and therefore $\varsigma_b > 0$ whenever $\psi h_c = \Gamma(\nu - 1)$ is decreasing.

A.10 Multisector model

We consider the simplest setup with multiple sectors: (i) small open economy with perfect capital mobility; (ii) an arbitrary number of sectors which we denote with $i = 1, \ldots, n$; (iii) all goods are internationally tradable and the international prices $\{p_i^*\}$ are taken as given by the country; (iv) all sectors are symmetric and differ in $(\alpha_i, \eta_i, \lambda_i)$ as well as initial conditions $\{x_i(0)\}$; (v) aggregate labor supply is exogenous and it must be split between labor supply to individual sectors, $\sum_{i=1}^n \ell_i(t) = L$.

Given fixed world prices p_i^*, the planner's problem separates in a convenient way: the planner first maximizes labor income and then chooses the optimal consumption allocation. The labor income maximization problem is

$$\max_{\{\ell_i(\cdot), x_i(\cdot)\}_{i=1}^n} \int_0^\infty e^{-\rho t} \sum_{i=1}^n (1 - \alpha_i)y_i(x_i(t), \ell_i(t))dt,$$

$$\text{s.t.} \quad \dot x_i = \frac{\alpha_i}{\eta_i}p_i^* y_i(x_i, \ell_i) + (r^* - \delta)x_i,$$

$$\sum_{i=1}^n \ell_i \le L.$$

given $\{x_i(0)\}_i$ and where

$$y_i = \Theta_i x_i^{\gamma_i} \ell_i^{1 - \gamma_i}, \qquad \text{with} \qquad \Theta_i = \frac{r^*}{\alpha_i}\left[\frac{\eta_i}{\eta_i - 1}\lambda_i\left(\frac{\alpha_i A_i}{r^*}\right)^{\frac{\eta_i}{\alpha_i}}\right]^{\gamma_i}.$$

Decentralized solution   In the decentralized equilibrium, labor is allocated in such a way to ensure wage equalization across all sectors. More formally, the equilibrium conditions for the decentralized allocation are given by:

$$w = (1 - \alpha_i)\frac{y_i}{\ell_i},$$

for all i, and the total wage bill in the economy is $wL = \sum_{i=1}^{n}(1 - \alpha_i)y_i$.

Planner   Denoting ω the Lagrange multiplier on the labor resource constraint and ν_i the Lagrange multiplier on the x_i accumulation constraint, the optimality conditions are given by (for $i = 1, \ldots, n$):

$$\omega = (1 - \gamma + \gamma\nu_i)(1 - \alpha_i)\frac{y_i(x_i, \ell_i)}{\ell_i},$$

$$\dot{\nu}_i = \delta\nu_i - \left(1 - \gamma + \gamma\nu_i\right)\frac{\alpha_i}{\eta_i}\frac{y_i(x_i, \ell_i)}{x_i}$$

$$\dot{x}_i = \frac{\alpha_i}{\eta_i}y_i(x_i, \ell_i) + (r^* - \delta)x_i$$

The consumption allocation is undistorted, both across goods and across time. Only the labor supply to particular sectors is subsidized when $\nu_i(t) > 1$ and taxed when $\nu_i(t) < 1$.

Two sector example   Suppose sectors are symmetric in everything except $p_i^*\Theta_i$. In particular, assume $\alpha_1 = \alpha_2$, $\eta_1 = \eta_2$, but $p_1^*\Theta_1 > p_2^*\Theta_2$. Then, in the decentralized equilibrium, wage equalization across sectors implies $y_1/\ell_1 = y_2/\ell_2$, and hence:

$$\ell_i = \frac{(p_i^*\Theta_i)^{1/\gamma}x_i}{(p_1^*\Theta_1)^{1/\gamma}x_1 + (p_2^*\Theta_2)^{1/\gamma}x_2}L.$$

From this, it is easy to see that when $p_1^*\Theta_1 > p_2^*\Theta_2$, then $\ell_1(t) > \ell_2(t)$ and $\dot{x}_1(t) > \dot{x}_2(t)$ for all t, and hence $\ell_1(t) \to L$ and $\ell_2(t) \to 0$ as $t \to \infty$ (as well as $x_2(t) \to 0$).

The planner's labor allocation instead satisfies (from the optimality conditions above):

$$\ell_i = \frac{((1 - \gamma + \gamma\nu_i)p_i^*\Theta_i)^{1/\gamma}x_i}{((1 - \gamma + \gamma\nu_1)p_1^*\Theta_1)^{1/\gamma}x_1 + ((1 - \gamma + \gamma\nu_2)p_2^*\Theta_2)^{1/\gamma}x_2}L.$$

The labor supply FOCs can also be rewritten as

$$(1 + \tau)(1 - \alpha)\frac{y_1(x_1, \ell_1)}{\ell_1} = (1 - \alpha)\frac{y_2(x_2, \ell_2)}{\ell_2} \qquad \text{where} \qquad 1 + \tau = \frac{1 - \gamma + \gamma\nu_1}{1 - \gamma + \gamma\nu_2}$$

is the subsidy to Sector 1, and we're interested in its evolution over time. It is convenient to make a substitution of variables $m_i \equiv \ell_i/x_i$, which is well-defined for Sector 2 even as ℓ_2 and x_2 converge to zero over time. The optimality conditions for $\dot{\nu}_i$ and $\dot{x}_i$ can be rewritten in terms of $\{\nu_i, x_i, m_i\}$, and m_i are defined according to the labor supply conditions above. This dynamic system can be analyzed using conventional methods, and Sector 1 is subsidized whenever $\nu_1 > \nu_2$.

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