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## Optimal disassembly of complex products

A. J. D. LAMBERT†

This paper presents a method for determining the optimum disassembly sequence for selective disassembly of discarded complex products. This methodology has been developed within the framework of the increasing need for formulation of a theory on selective disassembly and the various purposes of it. The objective of the optimization is maximizing the economic performance of the disassembly process within given technical and environmental constraints. It is demonstrated that the method, that is based on graphs, can be used to generate optimum sequences according to various objectives in a straightforward way. Also a ranking of the most favourable alternative sequence is possible. This is important to the inclusion of other criteria in the model and in adaptation of the disassembly system to fluctuating parameters like material yields and constraints. Contrary to earlier work on generation of disassembly sequences, in selective disassembly all possible incomplete disassembly sequences are included. This considerably increases the number of disassembly sequences to be studied with respect to the number of possible assembly sequences. According to the method described here, the number of viable sequences is determined in a straightforward way. On the basis of a specific example the general methodology is discussed.

### 1. Introduction

Past research on disassembly sequences has been carried out from several points of view. First of all, disassembly sequences were considered as reverse assembly sequences and were studied within the framework of research on robotized assembly processes. Sekiguchi *et al.* (1983) investigated the process of automatically verifying assembly drawings based on geometrical properties and connective relations consistent with CAD/CAM. Subsequently, the liaison diagram was introduced as a tool to get around less viable liaisons and, consequently, to omit sequences that include the realization of such connections (De Fazio and Whitney 1987) and to reduce the size of the problem. Disassembly analysis was primarily used as a tool to generate assembly sequences. It resulted in an interactive computer aid that allowed designers to exclude those assembly sequences that were not viable according to their specific criteria (Baldwin *et al.* 1991). Homem de Mello and Sanderson presented an algorithm to generate all assembly sequences based on the geometry of the device to be assembled (Homem de Mello and Sanderson 1990, 1991). They introduced the AND/OR graph that became the basis of many further investigations including the present work. They also considered disassembly sequences as the reverse of assembly sequences and used them to reduce the problem of finding assembly sequences. The lumping principle (considering a group of parts as one subassembly) and heuristics considerably decreased the amount of calculation time.

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The problem of parallelism has been also studied (Lee and Shin 1990). Parallelism implies the assembly of two or more subassemblies and subsequently joining them, rather than building the assembly by adding separate parts one-by-one. In the case of disassembly, parallelism appeared essential in most of the cases.

Secondly, disassembly sequences were specially studied for service purposes, e.g. the maintenance of aeroplanes. An important feature in this field is remote manipulation for disassembly operations at hazardous sites, as in nuclear fission reactors, tokamak installations and, more recently, the dismantling of nuclear weapons. Principal objectives of the optimum disassembly sequences included optimal robot motion, security, stability of (sub-) assemblies and speed (Subramani and Dewhurst 1991; Lee and Kumara 1992). These studies also served the product design process by obtaining an improved serviceability and thus optimizing life-cycle costs. Linear programming techniques were applied to optimize disassembly sequences (Kanehara *et al.* 1993). Although incomplete sequences were not considered by these authors because their work focused on assembly planning, some interesting calculations, like that of determining the disassembly sequence with minimum energy use, were successfully carried out.

At present, there is a strong incentive towards recycling of discarded complex products. This is boosted by intentions of the authorities to make the producers responsible for their products when they are discarded by the consumer. Intended legislation includes the obligation to recollect and upgrade such products in an environmentally conscious way. Because of the growing quantity, diversity and complexity of complex products including cars, electric and electronic consumer goods, and capital goods, this obligation is accompanied by restrictions on disposal of the complete products and shredder residue, and the duty to remove harmful fluids and parts from the product. This leads to a new approach of optimum disassembly sequences, viz. the sequences that offer maximum economic feasibility within given environmental constraints. Within this class of problems the emphasis lies on precedence relationships that have been analysed in previous studies as only one aspect of the problem. Some proposals with respect to optimal disassembly sequence generation have been made without going into much detail (Wang and Johnson 1995). Important within this framework is the curve indicating a break-even point for costs and revenues at a disassembly grade somewhere in between 0% and 100%. This type of curve, however, cannot be properly applied when parallelism is required, or when many disassembly sequences should be taken into account.

## 2. The recycling process

Apart from logistics like recollection, transport and storage, the typical order of production processes for upgrading discarded products is:

- stripping, to recover parts that may serve as spare parts,
- selective disassembly, which serves multiple purposes,
- dismantling,
- sorting,
- unlocking, usually by shredding,
- separation,
- upgrading,
- waste treatment, e.g. incineration,

Stripping is not included in the model described here because it is largely specific for car disassembly. The difference between *disassembly* and *dismounting* is the quasi-reversible character of the former process.

Selective disassembly serves the following purposes:

- recovery of valuable parts and assemblies, to be applied in the production process,
- recovery of valuable materials,
- removal of harmful fluids and/or parts,
- increasing the purity of the remainder of the product,
- decreasing the amount of shredder residue, and
- providing accessibility to other parts that should be removed.

The distinction between disassembly and dismantling is not very strict. In the model under discussion all actions are considered as disassembly processes in which exactly one part is removed. In practice, such a process may be more or less destructive (drilling, breaking, cutting, sawing, etc.). Although much attention has been paid to robotizing the disassembly process, this seems not to be very promising in the short term. The occurrence of many different product types, the possible damage and/or contamination of the discarded products, and their complexity are obviously forbidding restrictions to large-scale application of this technology. Handcraft, in combination with well designed tools and accessories, eventually supported by some electronic knowledge base, will be most commonly applied until products, specially designed for automatized dismantling, will appear.

Because disassembly takes place for several reasons and not only for the recovery of parts as such, not all disassembled parts will remain unseparated. Different parts with related composition may be put together in the same container in order to recover the materials. To which extent this should occur depends on quantity, costs, prices and the arrangement of subsequent operations. The desired amount of shredder residue, the desired purity, and the availability of certain separation processes may be of importance. Processing the materials requires on the one hand a large and stable supply, on the other hand homogeneity and purity. These two requirements are contradictory and this usually leads to an optimum somewhere in between. This is an extension of the *clustering problem* (Wang and Johnson 1995).

In the case when the acquired material flows are inhomogeneous, unlocking and separation operations may be carried out. Unlocking is usually performed by shredding, and here the size of the fragments is of importance. A fine size requires more energy but enables a more selective separation process.

The separation process usually consists of magnetic separation, eddy current separation and, eventually, float-and-sink processes and the like. In this way, ferro materials, non-ferro materials and some of the plastics are recovered. Further sophisticated separation techniques may also be applied. In a shredder, a blower and cyclones are present to separate the light particles. This flow, together with the remaining material that has not been separated, is called the shredder residue. It usually consists of some undefined mixture of low-valued components that is difficult to upgrade or apply and that may be contaminated. Therefore restrictions are set to its disposal, and the alternative option of incineration under controlled circumstances is expensive. To have an impression of the amount of shredder residue it should be mentioned that the shredding of a car generates about 25 weight-% of shredder residue. In the case of electronic devices, this figure is even higher.

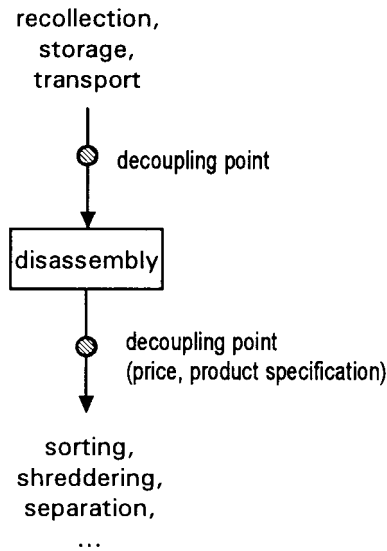


Figure 1. Location of the information decoupling points.

Separation is followed by a range of various materials recovery processes. Some of these processes, being very sophisticated, are carried out by specialized enterprises. An example is the recovery of precious metals from granulate of printed circuit boards.

### 3. The optimum disassembly sequence

#### 3.1. Positioning of selective disassembly

As has been stated in the preceding section, the disassembly process is the precursor of a chain of other recovery production steps. These determine how the disassembly process should be equipped and how its internal parameters should be set. The disassembly system should be feasible with respect to criteria on costs, energy use, environmental burden and so on.

- Economic feasibility requires that, within the constraints imposed by legislature, agreements, licences and the like, the revenues of the total recovery process should be as high as possible.
- Feasibility with respect to energy use implies that the energy conservation potential of the recovery process, obtained by replacement of virgin materials with secondary ones and by avoidance of waste should be larger than the energy requirements of the recovery process.
- Environmental feasibility means that the total amount of waste, weighed according to its harmfulness, should decrease when applying selective disassembly.

This work deals with the optimum *disassembly* sequence. This implies that operations that precede and follow the disassembly operation, are not considered. The input and output flows of the disassembly process are both characterized by information decoupling points (see Fig. 1). This means that all the information concerning preceding production steps like recollection, and subsequent production steps

like shredding, separation and waste treatment, are condensed in the costs and revenues of the respective materials, and of the components that are obtained by the disassembly process. Although the sorting process takes place in combination with the disassembly process, it is not considered in the problem statement, for the sake of simplicity. The optimization of the combined disassembly and sorting process will be subject to further research.

Disassembly systems are usually characterized by many internal degrees of freedom, that should be adjusted in such a way that the process is optimal subject to some definite objective. This may be the achievement of maximum revenue within the constraints the system is subjected to. The problem is different from the classical studies on assembly and disassembly sequences, because the requirements of the disassembly process are not as strict as in the case of assembly and maintenance. Even if parts recovery is practised, this will concern only a restricted number of parts of the product. When, however, materials recovery is intended, most of the actions allow some destruction of the respective parts and should not be performed with as much care as the corresponding actions in assembly or maintenance processes. Although many attempts have been made to robotize the disassembly process, in practice this will often be inevitable because robotizing presupposes the supply of large quantities of well-defined products. Absence or impossibility of extensively implemented automation and a minor necessity for exactness and care makes a detailed emphasis on geometry, direction of operation, stability of subassemblies, the character of liaisons and the like of secondary importance. In the model that is described in this paper, the characteristics of disassembly operations will be condensed in the cost of the operation. The emphasis will therefore be on the precedence relationships in the disassembly operation. A second principal difference between assembly and selective disassembly is the possibility of skipping a number of disassembly steps. This considerably increases the number of possible disassembly sequences as are visualized in appendix 1.

### 3.2. Liaison diagram and disassembly graph

In determining the full set of (dis-)assembly sequences, *liaison analysis* is of importance. Herewith the order is studied of the liaisons that are about to be (dis)established. Some examples have been worked out on the basis of a liaison diagram (De Fazio and Whitney 1987). The liaisons are numbered and for each liaison it is determined what precedent disconnection operations are required as a condition to make disconnection of the specific liaison possible. The possible liaison sequences are graphically represented and assembly sequences can be selected in which the more difficult steps (simultaneous establishing of multiple liaisons) are avoided. This considerably simplifies the problem but may lead to sub-optimization.

To demonstrate the method, a 10-parts ballpoint pen has been taken as an example (see Fig. 2). Its parts are numbered from 1–10 and their respective material composition is indicated in the figure. In the liaison diagram of this example (see Fig. 3) the liaisons are indicated by capitals. The disassembly operation starts with disconnection of liaison C, or with simultaneous disconnection of liaison D, E, and L. However, contrary to assembly, in selective disassembly the problem of simultaneous disconnection (De Fazio and Whitney 1987) is of minor importance.

With an emphasis on precedence relationships, the concept of the *disassembly graph* (see Fig. 4) is useful (Lambert 1994). It is based on the work on AND/OR graphs (Homem de Mello and Sanderson 1990, 1991). The only features from geom-

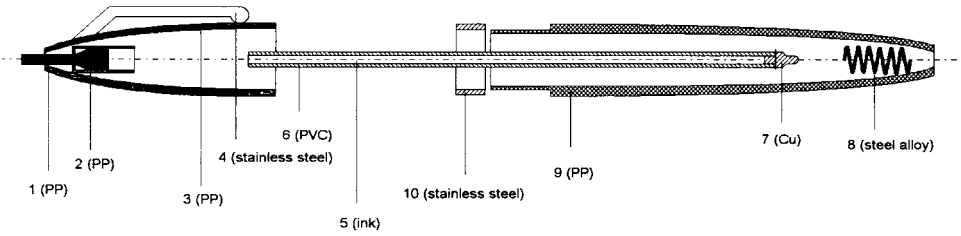


Figure 2. Configuration of the example assembly.

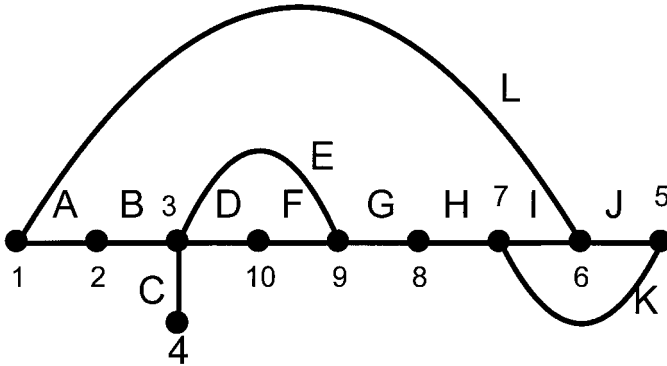


Figure 3. Liaison diagram of the example assembly.

etry and liaisons that remain present in this graph are the precedence relationships. The nodes of this graph are subassemblies and not the connections between them. This is in accordance with the aim of the selective disassembly process, namely getting definite parts, or getting parts of a definite composition. More recently, this concept has been borrowed and modified (Penev and De Ron 1996) to a somewhat more heuristic version.

The disassembly graph is set up by inventory of the subassemblies that may appear in disassembling the product in a proper way. The plane is divided in columns that are ordered according to the number of parts that are present in the respective subassemblies. These are indicated as circles. The numbers within the circles are a short notation of the parts that are present in the subassembly. Here 5/9 means 5 to 9 and 5,9 means 5 and 9. Hence the complete assembly (1/10) is at the left hand side, while at the extreme right the separate parts are indicated. Filled dots at the right of the circle are used when a choice can be made between different actions. At each action two lines run to the right, each arriving at a new subassembly with fewer parts than the original one. Actions, i.e. separations of two subassemblies, are indicated by lower case *a* to *t*. A disassembly process consists of a sequence of actions, e.g. *a-k-t*. As is clearly seen from Fig. 4, parallelism is allowed. Action *a*, for instance, separates assembly 1/10 in subassemblies 1/4 and 5/10, each of which will be further treated separately. A restriction is the supposition that at each step the (sub-)assembly under consideration falls apart in exactly *two* new subassemblies. The optimal sequence that will be calculated and discussed in this paper, is indicated by a bold line. It can be indicated by the action sequence: *b-c-d-h-n*. Although at some actions two subassemblies are created, in this case it is only one of them that will be further

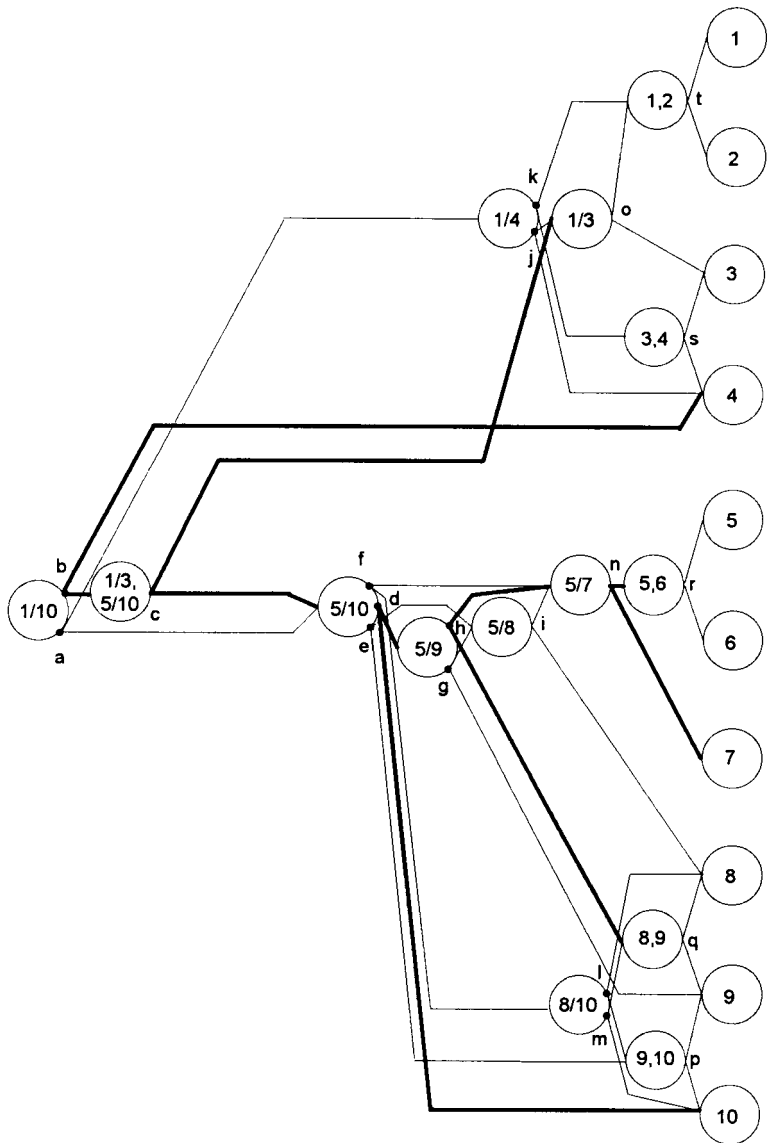


Figure 4. Disassembly graph of the example assembly, with optimal disassembly sequence indicated.

disassembled because of economic reasons. In the general case, parallelism in the sequences is not excluded at all. The optimal sequence  $b-c-d-h-n$  is an incomplete disassembly sequence because actions  $o, t, q$  and  $r$  are not performed here. In appendix A the theoretical maximum number of subassemblies and complete sequences, and the sum of complete and incomplete sequences, is tabulated as a function of the number of parts of the original assembly. The appropriate formulae to calculate them are also represented in this appendix. Although the figures presented in appendix A are impressive, they are much lower in practice, because of restrictions set by the real configuration of the product to be disassembled. In the case of the 10-part



ballpoint pen the number of possible complete disassembly sequences is 15, that of all complete and incomplete sequences is 387. A straightforward way to derive these figures from the graphical representation is explained in the next subsection.

Real products to be dismantled commonly have many more than 10 parts. The number of viable disassembly sequences increases sharply with the number of parts. Analysis of such systems make sense only when considerable simplifications have been carried through:

- *Clustering*: Related groups of disassembly activities are taken together. In Spengler (1994) a model is developed for obtaining optimal combinations of dismantling, upgrading and application of materials resulting from demolition of built structures.
- *Lumping*: The product is considered as hierarchical and modular system. The product is first separated in its principal modules, and these are subsequently dismantled in their submodules and so on. A personal computer, for example, is always disassembled in system, keyboard, cables, and monitor, Subsequently the monitor is disassembled in housing, tube, and electronic circuit boards, and so on. Modules are defined as strongly interconnected subassemblies.
- *Parallel clustering*. This means that all similar actions are performed at the same instance. In disassembling a keyboard, for example, all keys are considered as one single part.

In the simple 10-parts example, lumping is possible to some extent as follows from the disassembly graph of Fig. 4. Subassemblies 1/4, 5/7 and 8/10 can be considered as modules. 1,2 is a submodule. In the liaison diagram, modules should be characterized by strong internal and relatively weak external relationships. It follows from the optimal disassembly sequence in Fig. 4 that one should be careful with lumping. If subassembly 1/4 is strictly esteemed as a strongly connected module, the actions *b* and *c* are not considered and a suboptimal disassembly sequence is the result.

### 3.3. The process graph

The criterion for the optimum disassembly here is the maximum revenue of the activities. An activity is a single separation step, with index *k*, that runs here from *a* through *t*. The revenue  $r_k$  of activity *k* is given by:

$$r_k = \sum (p_{ok} \cdot m_{ok}) - (p_{ik} \cdot m_{ik}) - c_k$$

Here  $p_{ok}$  and  $p_{ik}$  represent the specific prices (in \$/kg) of output and input respectively;  $m_{ok}$  and  $m_{ik}$  are their respective masses, and  $c_k$  is the cost of the disassembly operation (labour, energy, equipment and the like). The summation sign indicates that there are two separate output flows, each characterized by mass and specific price. Their respective revenues should be added.

The formula states that at each disassembly step the value of the original subassembly is destroyed, and the value of the two new subassemblies is created. This process takes place at the cost  $c_k$ . It will be clear that some actions, even when their revenues are negative, may, nevertheless, be economically viable when their execution is indispensable for carrying out further activities. One should *a priori* only refrain from unprofitable 'final' actions, i.e. actions at the extreme right of the tree structure of Fig. 4, for these are never conditional for subsequent profitable actions.

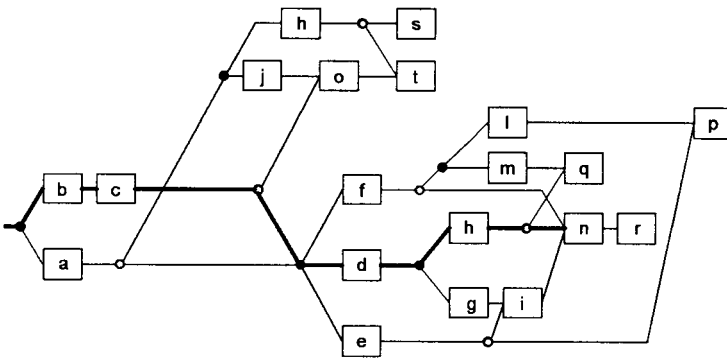


Figure 5. Process graph of the example assembly with optimal disassembly sequence indicated.

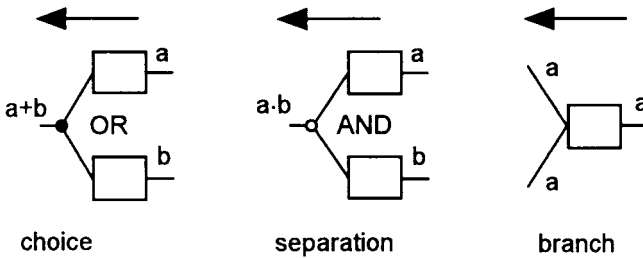


Figure 6. Calculation rules at the different types of nodes.

This consideration opens the way to an action-oriented graphical approach, called the *process graph*. Starting from the disassembly graph, where the nodes represent subassemblies and the actions are indicated by minuscules at the right-hand-side of the nodes, here the mere actions are represented by branches. In the process graph (see Fig. 5) they are indicated by rectangles. There are three types of nodes: choices, separations and branches. A *choice* is represented by a filled dot ●, and a *separation* by a circle ○. A separation means that two subassemblies are created, each containing more than one part. They should be disassembled in parallel. A third feature is a *branch*. Two or more sequences arrive here at the left side of the action. This indicates that multiple different sequences may precede this action.

Starting from the process graph it is possible to obtain the number of complete disassembly sequences. This calculation is carried out from the right towards the left, using multiplication rules at separation points and summation rules at choices, see Fig. 6. In this figure it is indicated that a choice has an OR-character, a separation has an AND-character, and a branch is indifferent. If there is a choice between one leaving branch comprising  $a$  possible subsequences and the other leaving branch comprising  $b$  possible subsequences, then upstream from the choice node there are  $a + b$  subsequences possible. At a separation point one should multiply in a similar way the number of possible subsequences. At a branch the information is passed unchanged from the leaving branch to the entering branches. At the extreme right, the number of possible sequences obviously equals 1.

### 3.4. The extended process graph

The process graph, however, is not an appropriate tool by itself because it does not deal with incomplete sequences. To include these, the *extended process graph* is used. This graph is also action-oriented. It is constructed moving from the right (single parts) towards the left (full assembly) in the disassembly graph. For each possible disassembly a sub-graph is composed, adding new actions and a shunt to the sub-graphs that are already determined (see Fig. 7). The subsequent steps include choice points and sometimes separation points. Straightforward application of the rules of calculation leaves us with all possible complete and incomplete disassembly sequences. The figures at the branch indicate the number of possible sequences. For instance,  $t$  is a final action. It can either be executed or not, creating two possible sequences in this way. Action  $t$  acts on subassembly 1,2 as is indicated in the right column of Fig. 7. Subassemblies are indicated by bold rectangles. Thus, subassembly 1,2 can be processed in two ways. If one proceeds with action  $o$  it is clear that combination  $o-t$  should be considered. There are 3 possible sub-sequences, one is no action at all, the other is performing  $o$  and subsequently stopping, and the third is performing the sequence  $o-t$ . In this way it is possible to make a complete listing of the possible sequences. In this example case there are 387 of them. In appendix B this figure is calculated in a more formal way, without application of graphics at all.

To obtain the optimum disassembly sequence with respect to maximum revenue, the revenues should be inserted within the actions and subassemblies in a systematic way, again proceeding from the right towards the left. In Table 1 a complete set of data is presented on masses and prices of all subassemblies and parts, and on the costs of the possible disassembly steps. The data contained in Table 1 have been

Mass (kg/item)	Price of part (\$/kg)	Price of subassembly (\$/kg)	Separation cost (\$/item)	Separation revenue (\$/item)
$m_1 = 0.38$	$p_1 = 0.5$	$p_{12} = 0.3$	$c_a = 0.1$	$r_a = 2.061$
$m_2 = 0.11$	$p_2 = 0.5$	$p_{34} = 0.08$	$c_b = 0.15$	$r_b = 2.326$
$m_3 = 1.49$	$p_3 = 0.1$	$p_{56} = -0.1$	$c_c = 0.2$	$r_c = 1.175$
$m_4 = 1.06$	$p_4 = 1.5$	$p_{89} = 0.4$	$c_d = 0.25$	$r_d = 0.294$
$m_5 = 0.19$	$p_5 = 0.0$	$p_{9,10} = 0.4$	$c_e = 0.3$	$r_e = 2.322$
$m_6 = 0.19$	$p_6 = 0.5$	$p_{123} = 0.05$	$c_f = 0.35$	$r_f = 1.363$
$m_7 = 0.38$	$p_7 = 2.5$	$p_{567} = -0.2$	$c_g = 0.4$	$r_g = 2.216$
$m_8 = 0.29$	$p_8 = 1.0$	$p_{8...10} = 0.4$	$c_h = 0.45$	$r_h = 2.006$
$m_9 = 2.59$	$p_9 = 0.5$	$p_{1...4} = 0.0$	$c_i = 0.5$	$r_i = -0.047$
$m_{10} = 0.09$	$p_{10} = 1.5$	$p_{5...8} = -0.3$	$c_j = 0.55$	$r_j = 1.139$
		$p_{5...9} = -0.4$	$c_k = 0.6$	$r_k = -0.249$
		$p_{5...10} = -0.5$	$c_l = 0.65$	$r_l = 0.596$
		$p_{1...3,5,10} = -0.55$	$c_m = 0.7$	$r_m = -0.601$
		$p_{1...10} = -0.6$	$c_n = 0.75$	$r_n = 0.314$
			$c_o = 0.8$	$r_o = -0.603$
			$c_p = 0.4$	$r_p = -0.042$
			$c_q = 0.45$	$r_q = -0.017$
			$c_r = 0.5$	$r_r = -0.367$
			$c_s = 0.55$	$r_s = 0.985$
			$c_t = 0.6$	$r_t = -0.502$

Table 1. Data belonging to the example of Figure 2.

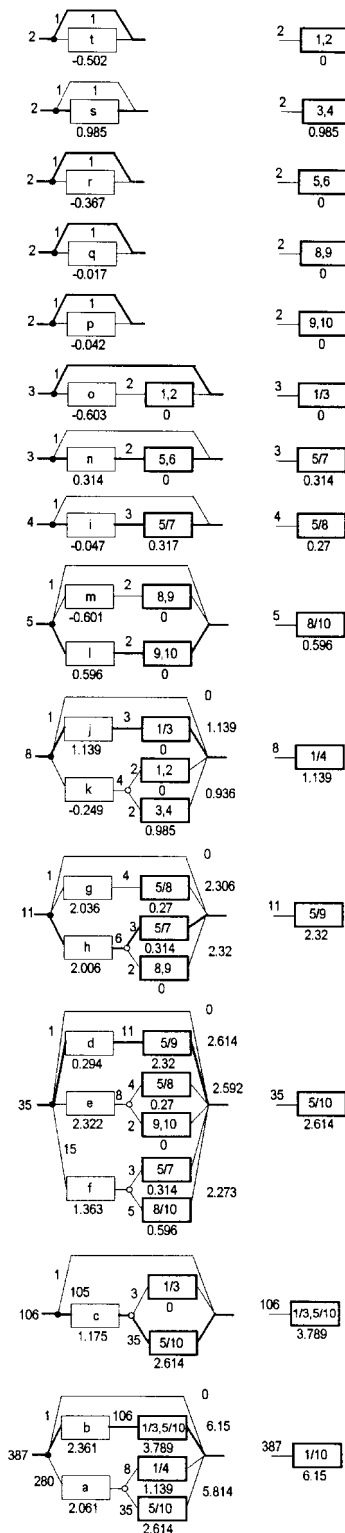


Figure 7. Graphical determination of the optimal disassembly sequence.

chosen only to illustrate the method of calculation. In practice they depend on the material composition and purity of the subassemblies, and the complexity of the disassembly operation. Starting from these data, for each action the revenue is calculated according to the relation presented in § 3.3, and the results are tabulated in the rightmost column of the same table. In the graphics of Fig. 7 the revenues are represented as figures below the actions.

From this data one is able to work out the extended separation process graph starting with the final actions  $p, q, r, s$ , and  $t$ . Each action, represented by a rectangle, is shunted. This means that sequences in which the action is not performed are allowed. This introduces an extra choice and increases the number of possibilities according to the OR-rule. The choice with the highest revenue will be made, and the whole subgraph is summarized by a bold rectangle including the respective subassembly code and the optimal cost of processing this subassembly. Subsequently the preceding actions like  $l, m, n$ , and  $o$  are studied. They are placed in series with the action whose cost is already determined, and the entirety is shunted again. Of this set of actions again the optimum is determined. If there are separations, both branches of the separation can be carried out, but when a branch shows revenue zero, it means that no actions along this branch are performed because they only bring losses. An example of this feature is in the subgraph of disassembly (5/9) in Fig. 7. In the subgraph belonging to subassembly (5/10), action  $f$  is followed by a separation point, resulting in subassemblies 5/7 and 8/10 with optimal revenues +0.314 and 0.596 respectively. The revenue of action  $f$ , i.e. 1.363, is added to this figure, resulting in a total revenue of 2.273 if choice  $f$  has been made.

Proceeding in this way, the optimum disassembly sequence is readily determined, viz  $b-c-d-h-n$ , with a total revenue of 6.15. This figure is composed of the creation of a positive value of 3.888 (parts and subassemblies), the destruction of a negative value of 4.062 (the original assembly) and the process costs of -1.8. The optimum disassembly sequence is indicated by a bold line. In a similar way it is possible to list alternative favourable sequences. The two best sequences are  $b-c-d-h-n-q$  (net revenue = 6.133) and  $b-c-e-i-n$  (net revenue = 6.128).

An example of an environmentally constrained problem is given by inclusion of the obligation to remove one or more hazardous parts or fluids. In the example of the ballpoint pen this may be the ink, indicated by part 5. In this case it follows from the disassembly graph that action  $r$  is indispensable, although it is a final action with a negative revenue of -0.361. The optimum sequence then is  $b-c-d-h-n-r$  with results in a net revenue of 5.783. Within this constraint the number of viable sequences is decreased down to 100. Although the resulting optimal sequence here is trivial, in practice the parameters can be such that a totally different optimal sequence results.

#### 4. Conclusions and recommendations

Although the obligation for companies to recollect and upgrade the discarded complex products they once produced is dictated by environmental arguments, quantitative knowledge of the environmental benefit of this measure is weak or lacking. Moreover, extensive recollection and recycling is accompanied by an appreciable economic impact. Therefore it is useful to make an optimal trade-off between economy and environment. A means to achieve this objective is adjustment of the internal degrees of freedom of the recycling processes. Because there are many degrees of freedom and the problem is complex, there exists a need for an appro-

appropriate simplification and outline of product recycling systems in order to obtain quantitative insight and to support decision making.

In the present study recycling systems have been analysed and subsystems have been defined. With respect to one of these subsystems, the disassembly system, the problem of finding an optimum disassembly sequence has been elaborated. A systematic method has been described that returns optimum disassembly sequences. This method is an adaptation of the existing theory concerning optimum (dis)assembly sequences to a new class of problems. Sensitivity analysis and the introduction of extra constraints is possible here. It is also possible to change objectives and, consequently, to apply multiple-criteria analysis to the different disassembly sequences. The approach of this paper is a first step to cope with problems connected with disassembly of complex consumer devices like automotive vehicles, consumer electronics and so on. Further research, however, is required, especially on classes of products rather than defined product types, and on optimization of a combination of the disassembly system to other determinative processes (especially, sorting, shredding and separation) of the recycling system. Sensitivity analyses (Wolters and Mareschal 1995) should be carried out to study the influence of minor changes in product structure, changes in costs and prices, introduction of new technology and other internal and external criteria on the optimal disassembly sequences.

## Appendix A

In this appendix expressions for the theoretical maximum of the number of subassemblies and disassembly sequences are presented as a function of the number of parts of the original assembly:

Number of parts	Number of subassemblies	Complete disassembly sequences	Total number of disassembly sequences
1	1	1	1
2	3	1	2
3	7	3	7
4	15	15	41
5	31	105	346
6	63	945	3 797
7	127	10 395	51 157
8	255	135 135	816 357
9	511	2 027 025	15 050 590
10	1023	34 459 425	314 726 297

*Calculation of the contents of the Table:*

If the device has  $n$  parts, the number of possible subassemblies  $N_n$  is:

$$N_n = (2 \cdot N_{n-1}) + 1$$

The number of complete disassembly paths  $C_n$  is:

$$c_n = (2n - 3) \cdot C_{n-1}.$$

The total number of disassembly paths  $P_n$  is:

$$P_1 = 1$$

$$P_2 = \frac{1}{2} \cdot \binom{1}{2} \cdot P_1 + 1$$

$$P_3 = \binom{1}{3} \cdot P_2 + 1$$

$$\vdots$$

$$P_{10} = \binom{1}{10} \cdot P_9 + \binom{2}{10} \cdot P_2 \cdot P_8 + \binom{3}{10} \cdot P_3 \cdot P_7 + \binom{4}{10} \cdot P_4 \cdot P_6 \\ + \frac{1}{2} \cdot \binom{5}{10} \cdot P_5 \cdot P_5 + 1$$

## Appendix B

In this appendix the number of possible disassembly sequences is calculated straight from the disassembly graph, as an alternative to the graphics of Fig. 7, in a formal way:

$$(1/10) = (1/3, 5/10) \text{ OR } \{(1/4) \text{ AND } (5/10)\} \text{ OR } 1$$

$$(1/3, 5/10) = \{(1/3) \text{ AND } (5/10)\} \text{ OR } 1$$

$$(5/10) = (5/9) \text{ OR } \{(5/8) \text{ AND } (9,10)\} \text{ OR } \{(5,7) \text{ AND } (8/10)\} \text{ OR } 1$$

$$(5/9) = (5/8) \text{ OR } \{(5/7) \text{ AND } (8,9)\} \text{ OR } 1$$

$$(1/4) = (1/3) \text{ OR } \{(1,2) \text{ AND } (3,4)\} \text{ OR } 1$$

$$(5/8) = (5/7) \text{ OR } 1$$

$$(1/3) = (1,2) \text{ OR } 1$$

$$(5/7) = (5,6) \text{ OR } 1$$

$$(8/10) = (8,9) \text{ OR } (9,10) \text{ OR } 1$$

$$(1/2) = \{(1) \text{ AND } (2)\} \text{ OR } 1$$

$$(3,4) = \{(3) \text{ AND } (4)\} \text{ OR } 1$$

$$(5,6) = \{(5) \text{ AND } (6)\} \text{ OR } 1$$

$$(8,9) = \{(8) \text{ AND } (9)\} \text{ OR } 1$$

$$(9,10) = \{(9) \text{ AND } (10)\} \text{ OR } 1$$

Next one proceeds in the reverse direction. Single parts have value 1. AND is replaced by a multiplication, OR is replaced by an addition according to figure

$$(9,10) \rightarrow 1 \cdot 1 + 1 = 2$$

$$(8,9) \rightarrow 1 \cdot 1 + 1 = 2$$

$$(5,6) \rightarrow 1 \cdot 1 + 1 = 2$$

$$(3,4) \rightarrow 1 \cdot 1 + 1 = 2$$

$$(1,2) \rightarrow 1 \cdot 1 + 1 = 2$$

$$(8,10) \rightarrow 2 + 2 + 1 = 5$$

$$(5,7) \rightarrow 2 + 1 = 3$$

$$(1,3) \rightarrow 2 + 1 = 3$$

$$(5,8) \rightarrow 3 + 1 = 4$$

$$(1/4) \rightarrow 3 + (2 \cdot 2) + 1 = 8$$

$$(5/9) \rightarrow 4 + (3 \cdot 2) + 1 = 11$$

$$(5/10) \rightarrow 11 + (4 \cdot 2) + (3 \cdot 5) + 1 = 35$$

$$(1/3, 5/10) \rightarrow (3 \cdot 35) + 1 = 106$$

$$(1/10) \rightarrow 106 + (8 \cdot 35) + 1 = 387$$

Proceeding along these lines the number of possible complete and incomplete disassembly sequences is determined according to a systematic method.

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