Optimal Dispatch in a Balancing Market with Intermittent Renewable Generation

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Abstract—In this paper, three single-stage stochastic programs are proposed and compared for optimal dispatch by a System Operator (SO) into balancing markets (BM). The motivation for the models is to represent a possible requirement to undertake system balancing with increasing amounts of intermittent renewable generation. The proposed optimization models are reformulated as tractable Mixed Integer Linear Programs (MILPs) and these consider both fuel cost and intermittency cost of the generators, when the SO activates the up- or down-regulation bids. These three models are based on the main approaches seen in practice: dual-imbalance pricing, single imbalance pricing and single imbalance pricing with spot reversion. A scenario-generation algorithm based on predictive conditional dynamic density distributions is also proposed. We perform a comparative analysis of these three proposed models in terms of how they help the SO to optimize their balancing market actions considering intermittent-renewable generators. The single imbalance pricing is found to be the most market efficient.

Index Terms—Balancing market, Imbalance settlement cost (ISC), Stochastic program, GAMLSS

NOMENCLATURE

The main notation is presented below for a quick reference. Additional symbols are introduced throughout the text.

Indices

i generating unit
ω scenarios of power generation

Parameters

$g_{i,\omega}^m$ metered output of generator i in scenario w
g_i^s scheduled output of generator i
g_i^{up} up-regulation quantity bid of generator i
g_i^{dn} down-regulation quantity bid of generator i
λ_i^{up} up-regulation price bid of generator i
λ_i^{dn} down-regulation price bid of generator i
SIL System Imbalance Level
β CVaR parameter (specified probability level)
π_ω Probability weight of scenario ω
$\mathcal{G}$ Total number of generators
λ^{spot} Spot price
μ Mean of metered output of generators

σ Standard deviation of metered output of generators
$wind_t$ Wind generation in MWh at time t
dem_t Demand in MWh at time t
hyd_t Hydropower in MWh at time t
Ξ Big-M parameter
ϵ Constant used in linearizing 'If' condition

Variables

ϕ_i^{up} Binary variable for up-regulation bid of generator i
ϕ_i^{dn} Binary variable for down-regulation bid of generator i
$\overline{\phi}$ Binary variable for system up-regulation status
$\underline{\phi}$ Binary variable for system down-regulation status
λ_{reg}^{dn} Down-regulation price
λ_{reg}^{up} Up-regulation price
$\delta_{i,\omega}^m$ Imbalance quantity of generator i in scenario w
$\overline{\alpha}_\beta$ Surplus generation for a given probability value β
$\underline{\alpha}_\beta$ Unmet level of demand for a given probability value β
Δ_ω Net deviation in scenario ω
Δ_ω^- Net negative deviation in scenario ω
Δ_ω^+ Net positive deviation in scenario ω
b_ω^- Binary variable to linearize Δ_ω^-
b_ω^+ Binary variable to linearize Δ_ω^+

Variables used in DIP model for Big-M linearization

$I_{i,w}^+$ Bilinear variable for positive deviation and down-regulation
$I_{i,w}^-$ Bilinear variable for negative deviation and up-regulation
s_ω Non-linear term for $\mathbb{E}$(ISC) due to negative deviation
c_ω Non-linear term for $\mathbb{E}$(ISC) due to positive deviation
$I_{i,w}^{up}$ Non-linear variable equal to product of s_ω and ϕ_i^{up}
$I_{i,w}^{dn}$ Non-linear variable equal to product of c_ω and ϕ_i^{dn}
z_ω^{dn} Variable to linearize the product of ϕ_i^{dn} and c_ω
z_ω^{up} Variable to linearize the product of ϕ_i^{up} and s_ω
u_ω^- Non-linear variable for linearizing CVaR constraint for unmet demand
u_ω^+ Non-linear variable for linearizing CVaR constraint for surplus generation
b_ω^1 Binary variable to linearize *Max* operator for surplus generation
b_ω^2 Binary variable to linearize *Max* operator for unmet demand

Variables used in SIP-SR model for Big-M linearization

$N_{i,\omega}^{up}$ Bilinear variable equal to product of ϕ_i^{up} and Δ_ω^-
$N_{i,\omega}^{dn}$ Bilinear variable equal to product of ϕ_i^{dn} and Δ_ω^+
c_ω^{up} Bilinear variable related to up-regulation cost
c_ω^{dn} Bilinear variable related to down-regulation cost

$q_{i\omega}$	Binary variable to identify marginal down-regulating generator
$r_{i\omega}$	Binary variable to identify marginal up-regulating generator
ϕ	Binary variable to indicate equal activated volumes for up- and down-regulation
k_i^{up}	Non-linear variable equal to product of ϕ_i^{up} and ϕ
k_i^{dn}	Non-linear variable equal to product of ϕ_i^{dn} and ϕ
h^+	Bilinear variable equal to product of Δ_ω^+ and ϕ
h^-	Bilinear variable equal to product of Δ_ω^- and ϕ
t_ω^{up}	Bilinear variable equal to product of Δ_ω^+ and $\overline{\phi}$
t_ω^{dn}	Bilinear variable equal to product of Δ_ω^- and $\underline{\phi}$

Set

Ω	Set of decision variables

Operators

$\mathbb{I}$(condition) "If" operator
$\mathbb{E}$(expression) "Expectation" operator.

I. INTRODUCTION

IN electricity markets, renewable generation has been increasing substantially following decreasing technology costs and supportive government policies [1]. In the European Union, the share of renewable generation has increased by 64% from 2007 to 2017 [2]. Reference [3] reports 25 TWh production from renewable generation in the third quarter of 2018 in UK (a 10% increase year-on-year). A similarly increasing pattern is reported for the US electricity system [4]. This increasing share of renewable generation brings major environmental benefits but it also introduces new operational challenges. Although renewable generation has low marginal costs and is CO_2 free, it is highly variable and intermittent [5]. This feature implies a need for more real-time balancing services by the system operator (SO). Authors in [6] and [7] discuss the balancing task of the SO in an electricity system with high penetration of intermittent-renewable generation. They suggest that at some point the intermittent-renewable generators will have to actively participate in the balancing market (BM) and be part of the solution as well as the cause. On the legislation side, the European Union directive 2003/54/EC enforces the non-discriminatory and market-based provision of balancing services [8]. This encourages intermittent-renewable generators to provide balancing services in the BM (see for example [9] for Britain and [10] for Nordic countries).

Accordingly, authors in [11] propose different models for the SO to facilitate the active participation of wind generators in the balancing markets. In [12], a Lyapunov optimization based real-time strategy has been proposed in order to minimize the imbalance costs for a virtual power plant. This formulation does not need the specification of the probability distributions of the renewable power generation. In [13], a combined energy and reserve market model has been proposed to increase the wind producers' revenue by allowing them to optimally bid in both markets and help them reduce their imbalance costs. A mathematical model to maximize the overall profit of a portfolio of a wind farm and a combined heat and power by minimizing their imbalances in a two-price balancing market

has been proposed in [14]. A bidding strategy to minimize the imbalance costs for wind power producers is described in [15] where forecast errors in wind power are represented as stochastic processes. But the risk associated with wind power forecast errors is not accounted for in this model. Reference [16] discusses the lack of economic incentives for the wind power producers in Portugal to minimize their imbalances as the wind imbalance costs are socialised. In [17], a method to choose the generation adjustments and the volumes of elastic demand that match the imbalances in the net inelastic demand (inelastic demand minus RES production), with respect to the day-ahead scheduling, has been proposed. In [18], the link between the power system cost due to wind power imbalances and the imbalance settlement payments to wind power producers is investigated. They draw a comparison between the one and two-price settlement systems.

TABLE I
OVERVIEW OF LITERATURE IN RELATED AREAS

Ref.	BMC +ISC	Imbal. pricing		Role of SO in BM	Prob. constr.	Stoch. gen.	Prob. scen. gen.
		Single	Dual				
[6]	✗	✗	✗	✗	✗	✗	✗
[11]	✗	✓	✓	✗	✗	✗	✗
[15]	✗	✓	✗	✗	✗	✓	✗
[19]	✗	✓	✗	✗	✓	✓	✗
[20]	✗	✗	✗	✓	✗	✗	✗
[21]	✗	✗	✗	✓	✗	✗	✗
[22]	✗	✗	✗	✓	✗	✗	✗
[23]	✗	✓	✓	✗	✗	✓	✗
[24]	✗	✓	✗	✗	✗	✗	✗
[25]	✗	✗	✓	✗	✗	✗	✓
[26]	✗	✓	✗	✗	✗	✗	✗
Our paper	✓	✓	✓	✓	✓	✓	✓

Table I summarizes an overview of the published research on the various aspects of specifications. The first column in Table I, 'BMC+ISC' refers to the approach of considering both balancing market cost and imbalance settlement cost to take the balancing market decisions. The second column, 'Imbal. pricing (Single and Dual)', mentions the imbalance pricing schemes. While some of the literature is limited to the producers'/consumers' perspectives of minimizing the imbalance settlement costs, the third column, 'Role of SO in BM', aims to highlight which papers have discussed about the involvement of the SO in the balancing market. 'Prob. const.' (Probabilistic constraint) emphasizes the approach for considering the uncertainty in the system, which in our case is a CVaR (Conditional Value at Risk) constraint. The fifth column, 'Stoch. gen.' (Stochastic generation), highlights those papers that have considered stochastic generation in their work. 'Prob. scen. gen.' (Probabilistic scenario generation) tries to show if the method chosen for generating scenarios is probabilistic or not.

The above research generally, but implicitly, recognizes the role of the imbalance settlement cost (ISC) influencing the active participation of the intermittent-renewable generators in the BM. This point is further explained in [6] and [20]. Accordingly, proper modeling of the ISC in the SO

optimal-dispatch can ensure the efficient participation of the intermittent-renewable generators in the balancing market and maybe incentivize the intermittent-renewable generators to adapt technologies to reduce their ISC exposure (eg. hybrid facilities with battery-storage devices) or to hedge more effectively (eg. with weather derivatives). Furthermore, in principle, the SO as a regulated entity would be expected to perform a welfare maximizing approach to system balancing, and thus include all relevant costs to participants, including ISCs, but in practice this is not yet widespread. Hence this is the motivation for this paper.

II. BACKGROUND

In this section, we present a brief description of a few electricity markets to explain their basic functioning:

- Nordic electricity market: At the day-ahead (DA) stage, all the producers and consumers willing to participate in the DA market submit their bidding curves to the electricity market operator. Then the market operator clears the DA market based on uniform price auction and defines the DA price, all the producers and consumers below that price are cleared in the DA auction. Based on the updated forecasts, these producers and consumers can adjust their position in the market by trading in the intraday market. At the gate closure (60 minutes before actual delivery of electricity), the position becomes binding and cannot be changed further. However, at the time of actual delivery of electricity, these producers and consumers deviate from their scheduled value. It is then the responsibility of the SO to activate bids in the balancing market to match the supply and demand of electricity. According to the existing method, there is a merit-order dispatch (marginal cost pricing (MCP) auction) performed by the SO, in the balancing market [1], to decide which producer or consumer needs to be called upon, based on their marginal cost of production or marginal utility of consumption. In the imbalance settlement stage, the SO calculates the imbalance settlement cost for each producer or consumer who participated in the market based on their deviation from the committed value and the direction of the system imbalance [29].

- Great Britain (GB): In GB electricity market, there is continuous trading up until the gate closure, which happens one hour before the actual delivery of electricity, and that is when all generators and retailers must nominate to the SO what they expect to generate or consume in the delivery period. But the participants are allowed to trade even after the gate closure for a further hour until the beginning of the actual delivery period. The delivery period length (also known as Settlement Period (SP)) is 30 minutes. It is the responsibility of the SO to ensure 50 Hz frequency, by activating bids and offers continuously

in every SP[2]. For each SP, a single imbalance price (SIP) is calculated. Based on whether the offer volumes exceed the bid volumes or vice versa, the NIV (Net Imbalance Volume) can be either positive or negative. The marginal price for 1 MW of the NIV gives the SIP [19], [31].

- PJM [3]: PJM compiles the bids in ascending price order and auction is performed to clear the bids until the point of intersection of the supply-demand curve. This sets the DA price for a particular hour for the delivery of electricity on the next day. Real-time market is another market that serves electricity for immediate delivery. Supply and demand curves are paired and based on the intersection point (MCP auction), the prices are calculated every five minutes. Based on the situation of fluctuations in the system, PJM informs the producers what their output should be. But if they deviate from PJM's instructions, they are penalized. If they follow the instructions of PJM, they are compensated for that. Another important market in PJM is the ancillary services market, which comprises of regulation and reserves market. The regulation market is in place to correct the small fluctuations in the electricity supply/demand that can affect the stability of power system. It allows to compensate those producers/consumers that have the ability to adjust their production/consumption as per the requirement in the system [33].

III. CONTRIBUTIONS

This paper contributes to the relevant research in this context in several ways:

- Our paper proposes a mechanism for the SO to dispatch generators to activate their up- and down-regulation bids on the basis of having lowest expected costs, taking into account both their production costs and potential deviations from nominated outputs. We propose a stochastic framework for clearing the real time balancing market by the SO.

- In our formulation, we introduce a CVaR constraint, which allows surplus generation or unmet demand, with an appropriate small probability. This produces useful and computationally tractable solutions under supply side uncertainty, where hard constraints with no surplus generation or unmet demand might lead to infeasibility.

- The proposed method has been mathematically applied to compare three imbalance pricing models namely Dual Imbalance Pricing (DIP), Single Imbalance Pricing with Spot Reversion (SIP-SR) and Single Imbalance Pricing (SIP) models.

- The three proposed models have been tested on a stylised five-generator system and a more realistic eight-generator

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[1]Here we refer to the m-FRR (manual Frequency Restoration Reserve) of the balancing market. This is the main balancing resource in the Nordic market apart from this the other reserves are FCR-N (Frequency Containment Reserve-Normal), FCR-D (Frequency Containment Reserves-Disturbance) and a-FRR (automatic Frequency Restoration Reserve). For more information check [27], [28].

[2]Short term operating reserve (STOR) provides the grid in GB with additional power when actual demand in the network is greater than the forecast and/or there is unforeseen generation unavailability [30].

[3]PJM: Pennsylvania-Jersey-Maryland, which is the regional transmission organiziation that is responsible for coordinating the wholesale electricity market in some or all parts of Indiana, Delaware, Virginia, West Virginia, Maryland, Illinois, Pennsylvania, Michigan, Kentucky, North Carolina, Tennessee, New Jersey, Ohio, and the District of Columbia [32].
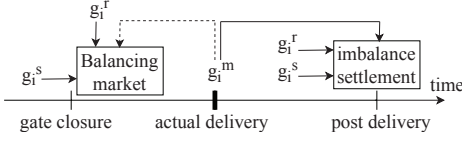
Fig. 1. The proposed balancing market operation (Dotted line represents uncertainty)

Fig. 2. (a) Time series for scheduled generation and depiction of imbalance at a specific time (b) Illustration of δ_i^m

system. In the case of eight-generator system, the uncertain parameters in the stochastic MILP models were estimated and predicted using a state-of-the art algorithm, the Generalized Additive Models for Location, Scale and Shape (GAMLSS). We demonstrate the performance of our proposed MILP models and the GAMLSS-based uncertainty-modeling algorithm with realistic numerical studies.

The rest of the paper is structured as follows. Section IV develops the optimal-dispatch model for the SO. Section V presents the GAMLSS-based forecasting model for generation imbalances. An illustrative example is provided in Section VI. Numerical results based on variations of a Nordic case study is provided in Section VII. Finally, Section VIII concludes the paper.

IV. MATHEMATICAL FORMULATION

We assume three stages in our analysis. At gate closure the scheduled generations (g_i^s) are reported to the SO. Then the SO receives up-regulation offers from generators for increasing output and down-regulation bids for reducing output, (g_i^r). If called, generators receive payments for their generation, or pay for their reduced generation. The SO also forecasts system demand in real time and estimates the imbalance settlement charges. Since the metered output of generators (g_i^m) are available only at the imbalance settlement stage, they are uncertain parameters in balancing-market optimal dispatch model. These stages are illustrated in Fig. 1. We now consider three variations in the formation of imbalance charges.

A. Dual Imbalance Pricing (DIP)

The DIP mechanism uses two imbalance prices, often called system buy price (SBP) and system sell price (SSP) [34]. The SBP is the average price at which the system has to buy electricity during a balancing period and the SSP reflects the average price at which the system has to sell electricity in order to dispose of surplus energy. Participants will then pay SBP or receive SSP depending upon whether their imbalance volumes were short or long. This mechanism has been favoured by regulators who want to deter speculation. It was the mechanism introduced in Great Britain in 2001 (although it has since been changed), also in France, Spain, Italy and elsewhere. At gate closure, let λ_i^{up}, λ_i^{dn}, g_i^{up} and g_i^{dn} represent bid prices and volumes for generator i participating in the balancing market. For notational simplicity, we assume that each generator submits only one bid pair. Let ϕ_i^{up} (respectively, ϕ_i^{dn}) be binary variables which take values 1 or 0 depending whether the bid (respectively, offer) from generator i is accepted or not.
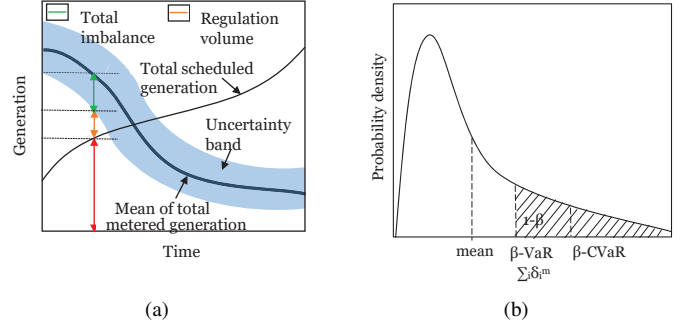
1) Objective function of proposed optimal-dispatch model:
The proposed objective function for the SO optimal-dispatch model is given by (1), where BSC stands for Balance Settlement Cost:

$$\text{BSC} = \sum_i (\lambda_i^{up} g_i^{up} \phi_i^{up} - \lambda_i^{dn} g_i^{dn} \phi_i^{dn}) + \quad (1a)$$

$$\max(\sum_i \delta_i^m, 0) \frac{\sum_i \lambda_i^{dn} g_i^{dn} \phi_i^{dn}}{\sum_i g_i^{dn} \phi_i^{dn}} \mathbb{I}(\sum_i \phi_i^{dn} \geq 1) + \quad (1b)$$

$$\min(\sum_i \delta_i^m, 0) \frac{\sum_i \lambda_i^{up} g_i^{up} \phi_i^{up}}{\sum_i g_i^{up} \phi_i^{up}} \mathbb{I}(\sum_i \phi_i^{up} \geq 1) \quad (1c)$$

In (1), both g_i^{up} and g_i^{dn} are assumed to be positive. g_i^m is the metered output of generator i and is considered to be a random parameter with known means and known continuous joint distribution. g_i^s is the scheduled power dispatch of generator i. The $\delta_i^m = g_i^m - (g_i^s + g_i^{up}\phi_i^{up} - g_i^{dn}\phi_i^{dn})$ models the deviation for generator i. The SO objective function in (1) can be split into two parts, (1a) is the Balancing Market Cost (BMC) while (1b) and (1c) represent the positive and negative Imbalance Settlement Cost (ISC), respectively. Observe that whilst ISCs are the charges incurred by the generators, they are also the recourse costs incurred by the SO in balancing, because of the non-firm regulating dispatch.

2) Constraints of proposed optimal-dispatch model:
Let $f(\phi, g^m) = \sum_i \delta_i^m = \sum_i (g_i^m - (g_i^s + g_i^{up}\phi_i^{up} - g_i^{dn}\phi_i^{dn}))$ be the deviation function, where ϕ and g^m are vector valued variables corresponding to ϕ_i and g_i^m. Fig. 2a depicts the time series for total scheduled generation and the mean of the total metered generation. The blue color band highlights the uncertainty in the total metered generation. At a given time instant, the red colored line represents the total scheduled generation at that time, orange colored line gives the regulation volume, and green colored line shows the total imbalance (deviation) at that time. Our paper discusses one such specific time instant. In Fig. 2b, the probability density of the total imbalance (deviation) is demonstrated, the details about which are discussed further in this section. In our formulation, we allow surplus generation ($f(\phi, g^m) > 0$) or unmet demand ($f(\phi, g^m) < 0$) with an appropriate small probability. The probability of $f(\phi, g^m)$ not exceeding a given α is given by:

$$\overline{\Psi}(\phi, \alpha) = \int_{f(\phi, g^m) \leq \alpha} p(g^m) dg^m \quad (2)$$

where $p(g^m)$ is the joint distribution function of g^m. Similarly, the probability of $-f$ not exceeding a given α (f exceeding $-\alpha$) is given by:

$$\underline{\Psi}(\phi,\alpha) = \int_{-f(\phi,g^m)\leq\alpha} p(g^m)dg^m \qquad (3)$$

For a specified probability level β, let the surplus generation be defined by:

$$\overline{\alpha}_\beta(\phi) := \min\{\alpha \in \mathbf{R}_+, \overline{\Psi}(\phi,\alpha) \geq \beta\} \qquad (4)$$

and the unmet level of demand is defined by:

$$\underline{\alpha}_\beta(\phi) := \min\{\alpha \in \mathbf{R}_+, \underline{\Psi}(\phi,\alpha) \geq \beta\} \qquad (5)$$

Thus, given a probability level β, the surplus generation does not exceed $\bar{\alpha}$ with probability β. In other words, $\bar{\alpha}$ is one endpoint of the non-empty interval of values α such that $\overline{\Psi}(\phi,\alpha) = \beta$. Similarly, unmet demand does not exceed $\underline{\alpha}$ with probability β. We then define the average surplus generation if it already exceeds $\overline{\alpha}$ as:

$$\overline{\mathcal{L}}_\beta(\phi) = (1-\beta)^{-1} \int_{f(\phi,g^m)\geq\overline{\alpha}_\beta(\phi)} f(\phi,g^m)p(g^m)dg^m \quad (6)$$

Similarly, we define the average unmet demand if it already exceeds $\underline{\alpha}$ as:

$$\underline{\mathcal{L}}_\beta(\phi) = (1-\beta)^{-1} \int_{-f(\phi,g^m)\geq\underline{\alpha}_\beta(\phi)} f(\phi,g^m)p(g^m)dg^m \qquad (7)$$

Note that the probability of $f(\phi,g^m)$ exceeding $\overline{\alpha}$ is $1-\beta$, so that $\overline{L}_\beta(\phi)$ gives conditional expectation, i.e., the average surplus generation, when it exceeds $\bar{\alpha}$. Similar comments hold for $\underline{L}_\beta(\phi)$. $\overline{L}_\beta(\phi)$ and $\underline{L}_\beta(\phi)$ can be incorporated into our optimization model through appropriately relaxed versions of these functions. These are defined as follows:

$$\overline{F}_\beta(\phi,\overline{\alpha}) = \overline{\alpha} + (1-\beta)^{-1} \int_{g^m} (f(\phi,g^m) - \overline{\alpha})_+ dg^m$$

$$\underline{F}_\beta(\phi,\underline{\alpha}) = \underline{\alpha} + (1-\beta)^{-1} \int_{g^m} (-f(\phi,g^m) - \underline{\alpha})_+ dg^m$$

where $(x)_+ = \max(x,0)$. Then $\overline{F}_\beta(\phi,\overline{\alpha})$ (respectively $\underline{F}_\beta(\phi,\underline{\alpha})$) is convex and continuously differentiable in $\overline{\alpha}$ (respectively $\underline{\alpha}$). Further, $\overline{\mathcal{L}}_\beta(\phi) = \min_{\overline{\alpha}\in\mathbf{R}} \overline{F}_\beta(\phi,\overline{\alpha})$ and a similar result holds for $\underline{\mathcal{L}}_\beta(\phi)$ [35]. Now, if we generate samples of g^m, we can write an approximation to $\overline{F}_\beta(\phi,\overline{\alpha})$ as:

$$\overline{\mathcal{F}}_\beta(\phi,\overline{\alpha}) = \overline{\alpha} + \frac{1}{N(1-\beta)} \sum_j (f(\phi,vec(g_{i,\omega}^m)) - \overline{\alpha})_+ \quad (9)$$

where $vec(x_i)$ represents a vector with x_i as its ith element, with its dimension determined by the context. $\underline{\mathcal{F}}_\beta(\phi,\underline{\alpha})$ is defined similarly. For a given System Imbalance Level (SIL), $\overline{\mathcal{F}}_\beta(\phi,\overline{\alpha}) \leq SIL$ and $\underline{\mathcal{F}}_\beta(\phi,\underline{\alpha}) \leq SIL$ can be written as in (10).

$$\overline{\alpha} + \frac{1}{N(1-\beta)}(f(\phi,vec(g_{i,\omega}^m)) - \overline{\alpha})_+ \leq SIL \qquad (10a)$$

$$\underline{\alpha} + \frac{1}{N(1-\beta)}(-f(\phi,vec(g_{i,\omega}^m)) - \underline{\alpha})_+ \leq SIL \qquad (10b)$$

where in (10), SIL in MWh is a behavioural parameter to reflect SO aversion to large real-time imbalances. It represents the total imbalance in the system tolerated by the SO and risk constraints (2) represent the probabilistic tolerance of this large imbalance.

3) The proposed optimal-dispatch model: As we consider the measured generation $g_{i,\omega}^m$ to be an uncertain parameter, it can be modeled using a set of scenarios. We assume that scenarios of $g_{i,\omega}$ ($\omega = 1, 2, \cdots, \mathcal{N}$) are available, along with the associated probability weights π_ω. The discussion of how to generate scenarios is explained in Section V. Explicit modeling of the ISC in the optimization model (12) exposes the intermittent-renewable generators participating in the balancing market to their imbalance penalties (because of the deviation from their nominated outputs). The deviation $\delta_{i,\omega}^m$ depends on the stochastic measured generation $g_{i,\omega}^m$ and the expression of deviation can be given as: $\delta_{i,\omega}^m = g_{i,\omega}^m - (g_i^s + g_i^{up}\phi_i^{up} - g_i^{dn}\phi_i^{dn})$. This term of $\delta_{i,\omega}^m$ when introduced in (1b) and (1c) motivates us to take the expected value of the expression in (1b) and (1c) due to the stochasticity. Therefore, the expression of BSC becomes (11):

$$\text{BSC} = \sum_i (\lambda_i^{up}g_i^{up}\phi_i^{up} - \lambda_i^{dn}g_i^{dn}\phi_i^{dn}) + \qquad (11a)$$

$$\mathbb{E}(\max(\sum_i \delta_{i,\omega}^m, 0) \frac{\sum_i \lambda_i^{dn}g_i^{dn}\phi_i^{dn}}{\sum_i g_i^{dn}\phi_i^{dn}} \mathbb{I}(\sum_i \phi_i^{dn} \geq 1)) + \quad (11b)$$

$$\mathbb{E}(\min(\sum_i \delta_{i,\omega}^m, 0) \frac{\sum_i \lambda_i^{up}g_i^{up}\phi_i^{up}}{\sum_i g_i^{up}\phi_i^{up}} \mathbb{I}(\sum_i \phi_i^{up} \geq 1)) \quad (11c)$$

Here it can be observed that (11b) and (11c) together comprise the expected value of ISC.

Hence, the proposed optimization model for SO considering both BMC and ISC can be presented as in (12).

$$\underset{\Omega_1=\{\phi_i^{up},\phi_i^{dn},\underline{\alpha},\overline{\alpha}\}}{\text{Minimize}} \quad \text{BSC} = \text{BMC} + \mathbb{E}(\text{ISC})$$

$$\text{subject to: } (10), \{\phi_i^{up}, \phi_i^{dn}\} \in \{0,1\}, \quad \underline{\alpha}, \overline{\alpha} \in \mathbf{R}, \forall i \quad (12)$$

4) The MILP formulation of the proposed optimal-dispatch model: We now present a MILP model of the deterministic equivalent problem for (12). The total deviation in a scenario w can be given as $\Delta_\omega = \sum_i \delta_{i,\omega}^m$ where $\delta_{i,\omega}^m = g_{i,\omega}^m - (g_i^s + g_i^{up}\phi_i^{up} - g_i^{dn}\phi_i^{dn})$. The $\Delta_\omega^+ = \text{Max}(\Delta_\omega, 0)$ can be linearized as shown in (13).

$$0 \leq \Delta_\omega^+ \leq b_\omega^+\Xi^{(1)} \qquad (13a)$$

$$\Delta_\omega \leq \Delta_\omega^+ \leq \Delta_\omega + (1 - b_\omega^+)\Xi^{(1)} \qquad (13b)$$

where $\Xi^{(1)}$ is a suitable large constant. The $\Delta_\omega^- = \text{Max}(-\Delta_\omega, 0)$ can be linearized similarly as in (13). Referring to the positive deviation part of (11), we denote $c_\omega = \frac{\sum_i \lambda_i^{dn}g_i^{dn}\phi_i^{dn}\Delta_\omega^+}{\sum_i g_i^{dn}\phi_i^{dn}}$ which can be written as (14):

$$c_\omega \sum_i g_i^{dn}\phi_i^{dn} = \sum_i \lambda_i^{dn}g_i^{dn}\phi_i^{dn}\Delta_\omega^+ \qquad (14)$$

Using change of variable $I_{i,w}^{dn} = c_\omega\phi_i^{dn}$ in (14), the non-linear term can be linearized by implementing (15):

$$0 \leq I_{i,w}^{dn} \leq \phi_i^{dn}\Xi^{(2)} \qquad (15a)$$

$$c_\omega - (1 - \phi_i^{dn})\Xi^{(2)} \leq I_{i,w}^{dn} \leq c_\omega \qquad (15b)$$

where $\Xi^{(2)}$ is a suitably large constant. Also, substituting $I_{i,w}^+ = \phi_i^{dn}\Delta_\omega^+$ in (14), it can be linearized as in (15). The linearized form of (14), after implementing the change of variable, can be written as in (16):

$$\sum_i g_i^{dn} I_{i,w}^{dn} = \sum_i \lambda_i^{dn} g_i^{dn} I_{i,w}^+ \qquad (16)$$

Consider the negative deviation part of (1), we define $s_\omega = \frac{\sum_i \lambda_i^{up} g_i^{up}\phi_i^{up}\Delta_\omega^-}{\sum_i g_i^{up}\phi_i^{up}}$ which means:

$$s_\omega \sum_i g_i^{up}\phi_i^{up} = \sum_i \lambda_i^{up} g_i^{up}\phi_i^{up}\Delta_\omega^- \qquad (17)$$

By implementing the substitutions, $I_{i,w}^{up} = s_\omega \phi_i^{up}$ and $I_{i,w}^- = \phi_i^{up}\Delta_\omega^-$, the linearized form of (17) can be written as $\sum_i g_i^{up} I_{i,w}^{up} = \sum_i \lambda_i^{up} g_i^{up} I_{i,w}^-$ by following the similar equations as in (15). The If condition in (1b) is written as linear constraints $\sum_i \phi_i^{dn} \geq 1 - \Xi^{(3)}(1-\underline{\phi})$ and $\sum_i \phi_i^{dn} \leq 1 + \Xi^{(3)}(\underline{\phi})$. A similar linearization can be adopted for (1c) and replaced with $\overline{\phi}$. Then the objective function becomes:

$$\underset{\Omega_2}{\text{Minimize}} \sum_i (\lambda_i^{up} g_i^{up}\phi_i^{up} - \lambda_i^{dn} g_i^{dn}\phi_i^{dn}) + \\ \sum_\omega \pi_\omega(\underline{\phi}c_\omega - \overline{\phi}s_\omega) \qquad (18)$$

where $\Omega_2 = \Omega_1 \cup \{\overline{\phi}, \underline{\phi}, b_\omega^+, b_\omega^-, c_\omega, s_\omega, \Delta_\omega^+, \Delta_\omega^-, I_{i,w}^{dn}, I_{i,w}^{up}, I_{i,w}^-, I_{i,w}^+\}$. If we use $z_\omega^{dn} = \underline{\phi}c_\omega$ and $z_\omega^{up} = \overline{\phi}s_\omega$, then the nonlinear expressions ($\underline{\phi}c_\omega$ and $\overline{\phi}s_\omega$) can be linearized as in (15). The probabilistic constraint can be written as:

$$\overline{\alpha} + \frac{1}{(1-\beta)}\sum_\omega^N \pi_\omega u_\omega^+ \leq SIL \qquad (19)$$

where $u_\omega^+ = Max(\Delta_\omega^+ - \overline{\alpha}, 0)$. Also,

$$\underline{\alpha} + \frac{1}{(1-\beta)}\sum_\omega^N \pi_\omega u_\omega^- \leq SIL \qquad (20)$$

where $u_\omega^- = Max(\Delta_\omega^- - \underline{\alpha}, 0)$.
Linearization of $Max(\cdot)$ operator is possible in similar way as in (13). Accordingly, the optimization problem (12) can be re-formulated as the stochastic mixed-integer linear program (MILP) in (21):

$$\underset{\Omega_A}{\text{Minimize}} \sum_i (\lambda_i^{up} g_i^{up}\phi_i^{up} - \lambda_i^{dn} g_i^{dn}\phi_i^{dn}) + \sum_\omega \pi_\omega(z_\omega^{dn} - z_\omega^{up})$$

$$\qquad (21a)$$

$$subject~to: ~(13),(15),(16),(19),(20) \qquad (21b)$$

$$\{\overline{\phi}, \underline{\phi}, \phi_i^{up}, \phi_i^{dn}, b_\omega^+, b_\omega^-\} \in \{0,1\} ~\forall~ i,\omega \qquad (21c)$$

where $\Omega_A = \Omega_2 \cup \{z_\omega^{up}, z_\omega^{dn}, u_\omega^+, u_\omega^-\}$ is the set of decision variables in (21).

B. Benchmark model for the DIP model

An illustration of the benchmark model for the DIP model is given in Fig. 3. In the benchmark model, the decision of the SO to activate up- or down-regulation bids is taken solely based on the marginal cost of the generators. This is achieved by minimizing expression (11a). The output of the optimization problem would then give the values for ϕ_i^{up} and ϕ_i^{dn}. These
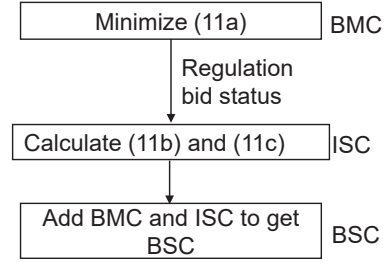

Fig. 3. Illustration of Benchmark model for the DIP model

values serve as inputs to the calculation of (11b) and (11c), which collectively give the value for ISC. The optimized value for BMC and the calculated value of ISC can be added together to find the BSC.

C. Single Imbalance Pricing with Spot Reversion (SIP-SR) Model

The SIP-SR mechanism differs in two respects from above. It is based on marginal rather than average pricing, i.e. the imbalance price is set according to the most expensive generator that is cleared during up-regulation hour, or the cheapest generator cleared during down-regulation hour. Further, as shown in Table II, if the deviations contribute to the system imbalance then the balance provider is charged according to the regulation price. However, if the deviations help to maintain the system in balance then it should not be penalized and accordingly the spot market price is applied [29]. This is the current Nordic model.

TABLE II
PRICES IN BALANCING MARKET FOR SIP-SR MODEL

	Up-regulation hour	Down-regulation hour
Δ_ω^-	p=λ_{reg}^{up}	p=λ^{spot}
Δ_ω^+	p=λ^{spot}	p=λ_{reg}^{dn}

The objective function of SIP-SR optimal dispatch model can be written as:

$$\underset{\Omega_3}{\text{Minimize}} \sum_i (\lambda_i^{up} g_i^{up}\phi_i^{up} - \lambda_i^{dn} g_i^{dn}\phi_i^{dn}) + \qquad (22a)$$

$$\sum_\omega \pi_\omega[\Delta_\omega^+ \lambda^{spot}\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} > \sum_i \phi_i^{dn} g_i^{dn}) + \qquad (22b)$$

$$\Delta_\omega^+ \lambda^{spot}\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} = \sum_i \phi_i^{dn} g_i^{dn}) + \qquad (22c)$$

$$\Delta_\omega^+ \lambda_{reg}^{dn}\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} < \sum_i \phi_i^{dn} g_i^{dn}) - \qquad (22d)$$

$$\Delta_\omega^- \lambda_{reg}^{up}\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} > \sum_i \phi_i^{dn} g_i^{dn}) - \qquad (22e)$$

$$\Delta_\omega^- \lambda^{spot}\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} < \sum_i \phi_i^{dn} g_i^{dn}) - \qquad (22f)$$

$$\Delta_\omega^- \lambda^{spot}\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} = \sum_i \phi_i^{dn} g_i^{dn})] \qquad (22g)$$

$\Omega_3 = \{\phi_i^{up}, \phi_i^{dn}, \lambda_{reg}^{dn}, \lambda_{reg}^{up}, \Delta_\omega^+, \Delta_\omega^-\}$ where $\lambda_{reg}^{dn} = Min(\lambda_i^{dn}\phi_i^{dn})$ and $\lambda_{reg}^{up} = Max(\lambda_i^{up}\phi_i^{up})$. Hence, the objective function (22) becomes:

$$\underset{\Omega_3}{\text{Minimize}} \left(\sum_i (\lambda_i^{up} g_i^{up} \phi_i^{up} - \lambda_i^{dn} g_i^{dn} \phi_i^{dn}) + \right.$$

$$\sum_\omega \pi_\omega [\Delta_\omega^+ \lambda^{spot} \mathbb{I}(\sum_i \phi_i^{up} g_i^{up} > \sum_i \phi_i^{dn} g_i^{dn}) +$$

$$\Delta_\omega^+ \lambda^{spot} \mathbb{I}(\sum_i \phi_i^{up} g_i^{up} = \sum_i \phi_i^{dn} g_i^{dn}) + \tag{23}$$

$$\text{Min}(\lambda_i^{dn} \phi_i^{dn} \Delta_\omega^+) - \text{Max}(\lambda_i^{up} \phi_i^{up} \Delta_\omega^-)$$

$$- \Delta_\omega^- \lambda^{spot} \mathbb{I}(\sum_i \phi_i^{up} g_i^{up} < \sum_i \phi_i^{dn} g_i^{dn}) -$$

$$\left. \Delta_\omega^- \lambda^{spot} \mathbb{I}(\sum_i \phi_i^{up} g_i^{up} = \sum_i \phi_i^{dn} g_i^{dn})] \right)$$

If $N_{i,\omega}^{dn} = \phi_i^{dn} \Delta_\omega^+$ and $N_{i,\omega}^{up} = \phi_i^{up} \Delta_\omega^-$, the non-linearity introduced (bilinear term $\phi_i^{dn} \Delta_\omega^+$ and $\phi_i^{up} \Delta_\omega^-$) can be linearized in the same way as (15). The expression $c_\omega^{dn} = \text{Min}(\lambda_i^{dn} N_{i,\omega}^{dn})$ is linearized in (24) (the term $c_\omega^{up} = \text{Max}(\lambda_i^{up} N_{i,\omega}^{up})$ can be linearized in the same way as shown in the Appendix):

$$-q_{i\omega}\Xi^{(5)} \leq c_\omega^{dn} - \lambda_i^{dn} N_{i,\omega}^{dn} \leq q_{i\omega}\Xi^{(5)} \tag{24a}$$

$$\sum_i q_{i\omega} = \mathcal{G} - 1 \tag{24b}$$

where $\mathcal{G}$ is the total number of generators in the electricity market and $q_{i\omega}$ is a binary variable. The If condition, $\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} > \sum_i \phi_i^{dn} g_i^{dn})$ in (23) is replaced by binary variable $\overline{\phi}$ where:

$$\sum_i \phi_i^{up} g_i^{up} \geq \sum_i \phi_i^{dn} g_i^{dn} - \Xi^{(6)}(1 - \overline{\phi}) + \epsilon \tag{25a}$$

$$\sum_i \phi_i^{up} g_i^{up} \leq \sum_i \phi_i^{dn} g_i^{dn} + \Xi^{(6)}\overline{\phi} - \epsilon \tag{25b}$$

where ϵ is a non-zero number. The If condition, $\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} < \sum_i \phi_i^{dn} g_i^{dn})$ in (23) can be linearized the same way as in (38) using binary variable $\underline{\phi}$. The If condition, $\mathbb{I}(\sum_i \phi_i^{up} g_i^{up} = \sum_i \phi_i^{dn} g_i^{dn})$ in (23) is linearized by replacing it by binary variable ϕ and rewriting the If condition as $\sum_i k_i^{up} g_i^{up} = \sum_i k_i^{dn} g_i^{dn}$ where $k_i^{up}(= \phi \phi_i^{up})$ is linearized using $\phi + \phi_i^{up} - 1 \leq k_i^{up} \leq \phi$ and $k_i^{up} \leq \phi_i^{up}$. The $k_i^{dn} = \phi \phi_i^{dn}$ can be linearized in the same way as k_i^{up}. The objective function can now be written as:

$$\underset{\Omega_4}{\text{Minimize}} \sum_i (\lambda_i^{up} g_i^{up} \phi_i^{up} - \lambda_i^{dn} g_i^{dn} \phi_i^{dn}) + \sum_\omega \pi_\omega [\Delta_\omega^+ \lambda^{spot} \overline{\phi}$$

$$+ \Delta_\omega^+ \lambda^{spot} \phi + c_\omega^{dn} - c_\omega^{up} - \Delta_\omega^- \lambda^{spot} \underline{\phi} - \Delta_\omega^- \lambda^{spot} \phi]. \tag{26}$$

$\Omega_4 = \Omega_3 \cup \{\phi, \overline{\phi}, \underline{\phi}, b_\omega^+, b_\omega^-, N_{i,\omega}^{dn}, N_{i,\omega}^{up}, c_\omega^{dn}, c_\omega^{up}, k_i^{dn}, k_i^{up}\}$ If we use $t_\omega^{up} = \Delta_\omega^+ \overline{\phi}, h^+ = \Delta_\omega^+ \phi, t_\omega^{dn} = \Delta_\omega^- \underline{\phi}$, and $h^- = \Delta_\omega^- \phi$ then the non-linear expressions can be linearized similar to (15). Accordingly, the final stochastic MILP model is given in (27).

$$\underset{\Omega_B}{\text{Minimize}} \sum_i (\lambda_i^{up} g_i^{up} \phi_i^{up} - \lambda_i^{dn} g_i^{dn} \phi_i^{dn}) + \tag{27a}$$

$$\sum_\omega \pi_\omega [\lambda^{spot}(t_\omega^{up} + h^+) + c_\omega^{dn} - c_\omega^{up} - \lambda^{spot}(t_\omega^{dn} + h^-)]$$

$$subject\ to: (13), (19), (20), (24) \tag{27b}$$

$$\{\phi, \overline{\phi}, \underline{\phi}, \phi_i^{up}, \phi_i^{dn}, b_\omega^+, b_\omega^-\} \in \{0, 1\} \ \forall \ i, \omega \tag{27c}$$

Where $\Omega_B = \{\Omega_4 \cup t_\omega^{up}, t_\omega^{dn}, h^+, h^-, u_\omega^+, \overline{\alpha}, u_\omega^-, \underline{\alpha}\}$ is the set of decision variables in (27).

D. Benchmark model for the SIP-SR model

As explained in Subsection IV-B, the benchmark model for the SIP-SR model works on the similar principle. Fig. 4 illustrates the functioning of the benchmark model. The SO ignores the deviations of the generators while choosing them for up- or down-regulation. The decision of ϕ_i^{up} and ϕ_i^{dn} is taken by minimizing (22a), which gives the BMC. Based on the output of the optimization problem, (22b), (22c), (22d), (22e), (22f), and (22g) are calculated to find the ISC. Thus, BMC and ISC are added together to get the BSC value for the benchmark model.
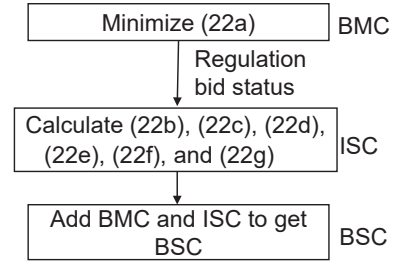

Fig. 4. Illustration of Benchmark model for the SIP-SR model

E. Single Imbalance Pricing (SIP) Model

The SIP model here, based on the imbalance pricing mechanism introduced in Great Britain (GB) in 2015, is summarized in Table III.

TABLE III
PRICES IN BALANCING MARKET FOR SIP MODEL

	Up-regulation hour	Down-regulation hour
Δ_ω^-	p=λ_{reg}^{up}	p=λ_{reg}^{dn}
Δ_ω^+	p=λ_{reg}^{up}	p=λ_{reg}^{dn}

The model is quite similar to SIP-SR in that the SO computes the Net Imbalance Volume (NIV), i.e. the net amount of up- and down- regulation called in the trading period, then, for that net amount, it computes the marginal offer price or bid price, but, in this case, without any reversions to the spot price. The motivation is to make it easier to hedge imbalance prices by market participants, especially vertically integrated companies that may be out of balance in opposite directions in their generation and retail accounts, and to go further than SIP-SR by actually encouraging participants to help to balance the system. Apart from GB, single imbalance pricing exists in Belgium, Austria and Germany subject to various conditions. In this section, we have presented the mathematical formulation of our proposed concept applied to three imbalance pricing models. We explain and discuss the dual imbalance pricing (DIP) model, a benchmark model corresponding to the DIP model, single imbalance pricing with spot reversion (SIP-SR) model, a benchmark model for the SIP-SR model, and the single imbalance pricing (SIP) model. Our concept of SO clearing the balancing market by considering the imbalances

TABLE IV
INPUT PARAMETER DATA FOR FIVE-GENERATOR SYSTEM IN ALL MODELS

gen no.	λ^{up} ($/MWh)	λ^{dn} ($/MWh)	g^{up} (MWh)	g^{dn} (MWh)	g^s (MWh)
G_1	10	8	30	20	400
G_2	11	7	40	40	500
G_3	12	6	25	15	350
G_4	13	5	35	25	450
G_5	14	4	45	35	550

of the generators has been applied to different imbalance pricing models. A CVaR constraint has been introduced to the optimization model in order to account for surplus generation or unmet demand, with an appropriate small probability. The resulting non-linear optimization models have been linearized through Big-M method.

V. PROPOSED DYNAMIC DENSITY ESTIMATION AND FORECASTING USING GAMLSS

To be implemented in practice, the above formulations require probabilistic forecasting of imbalances and we adopt a Generalized Additive Model for Location, Scale and Shape (GAMLSS) as effectively used in this context by [36]. In GAMLSS, one of a variety of density functions can be selected in which the mean and higher moments can be specified dynamically as conditional upon exogenous driving variables, [37]. In our proposed stochastic MILP models (21) and (27), the metered generation is the uncertain variable and the estimation flowchart is shown in Fig. 5. First, we fit the best distribution to the uncertain metered generation using the Akaike Information Criterion (AIC). The moments of the selected distribution are linearly regressed on selected explanatory variables (see Section VII for more details). With forecasts for the explanatory variables, the moments of the selected distribution are estimated. This distribution is then approximated using a set of scenarios [38].

VI. ILLUSTRATIVE EXAMPLES

We assume five generators G_1 to G_5 participate in our proposed balancing market to meet the system demand. Table IV shows the values of input parameter for the five-generator system in all the three proposed models. The simulations were performed by setting the values of $\Xi^{(1)}, \Xi^{(2)}, \Xi^{(3)}, \Xi^{(4)}, \Xi^{(5)}$, and $\Xi^{(6)}$ equal to 2000 in optimization models (21) and (27). The generation of G_1, G_2, G_3, G_4, and G_5 are distributed according to normal distribution[4] N(400, 0), N(500, 4), N(350, 8), N(450, 12) and N(550, 16), respectively. Using these normal distributions, 1000 scenarios are sampled and then reduced to 10 scenarios presented in Fig. 6. The MILPs for all the three proposed models are coded in GAMS and solved using CPLEX solver [39]. Simulations are carried out on a computer with 16 GB of RAM and 1.2GHz CPU. The computational times for the DIP, SIP-SR and SIP models were 0.25 seconds, 1.69 seconds and 67.89 seconds, respectively. As observed,

[4]The notation $N(\mu, \sigma)$ represents the normal probability distribution function (PDF) with mean μ and standard deviation σ.
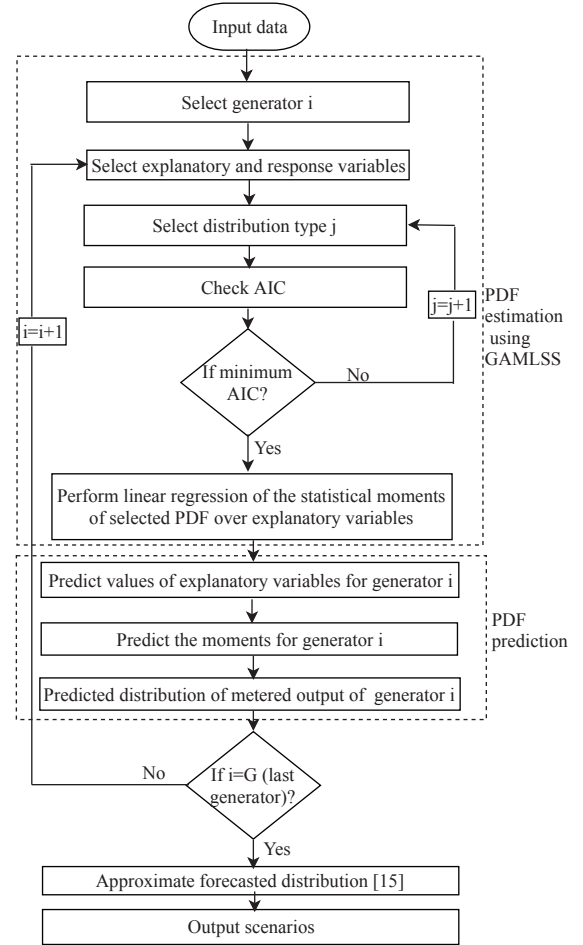

Fig. 5. Estimation and forecasting the dynamic distribution of metered generation g_i^m using GAMLSS

the computational time for the SIP model is the highest due to the several binary variables introduced for the linearization process [5]. We can reduce the computational time in our MILP models in two ways: first, we can employ decomposition algorithms to break the MILP model into smaller and easier-to-solve optimization problems [40], [41]. Second, we can use scenario generation and reduction algorithm to reduce the size of our MILP models [42], [43]. Using the reduced scenarios with updated probabilities that can represent the entire data set enables us to reduce the computational time significantly, which is desirable for real-time dispatch decisions. We employ the second approach in our paper and through a proposed scenario generation and reduction algorithm, the number of scenarios in our MILP models is reduced to 10 scenarios with updated probabilities. We select generator G_1 (the first generator in the merit-order dispatch) as the (renewable-) intermittent generator participating in the balancing market. The standard deviation of output generation for G_1 is changed from $\sigma_{\mathbf{G1}} = 0 \ MWh$ (firm output) to $\sigma_{\mathbf{G_1}} = 70 \ MWh$ (intermittent output). As seen in Fig. 7, the BSC increases with increasing output intermittency of G_1 in SIP and SIP-SR

[5]It is not the primary focus of the paper to reduce the computation time. However, it could be an interesting future work to look at possible ways to reduce the computation time for the SIP model.
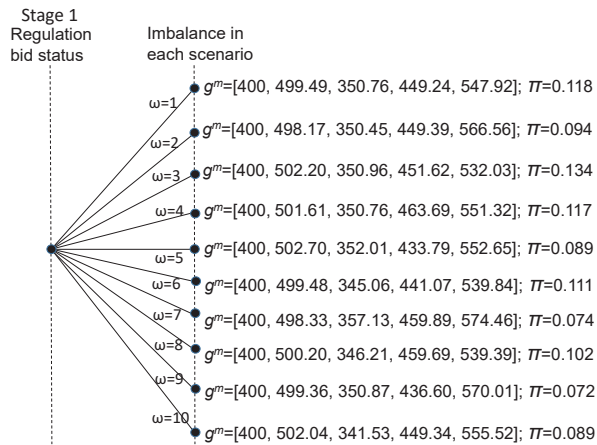
Fig. 6. Sample scenario tree for one-stage stochastic MILPs for all the three models
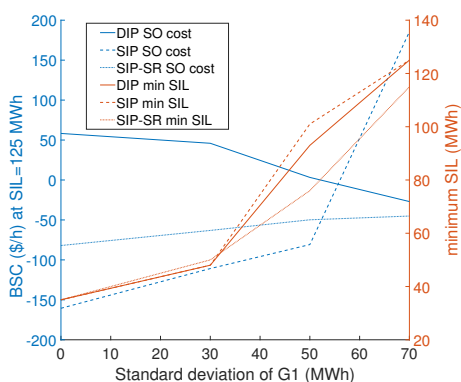

Fig. 7. Effect of varying standard deviation of G_1 metered generation on balancing and settlement cost (BSC) and minimum SIL for five-generator system (BSC axis is scaled down by 10 for SIP model)

models. This indicates that these models do not prefer to stay with the cheaper generator G_1 as it becomes more intermittent. Both SIP and SIP-SR use marginal cost pricing, and in this case they provide strong signals on the cost of intermittency. The DIP model however, being based on average pricing, is not able to give this signal as the BSC reduces in its case with the increase in intermittency of G_1. This is one of the reasons why the GB market moved from DIP to SIP model.

The effect of probabilistic constraint (10) is also shown in Fig. 7. The SO can satisfy the energy-balance constraint in 90% of scenarios ($\beta = 0.9$) with lower SIL level when the $\sigma_{\mathbf{G1}}$ is low. However, when generator $G1$ becomes more intermittent (higher $\sigma_{\mathbf{G1}}$), the SO needs to tolerate higher SIL levels in order to satisfy the probabilistic constraint (10) in 90% of scenarios.

A. Comparison of the proposed optimal-dispatch models with the benchmark models

In the benchmark model, the recourse costs incurred by the SO decision in the BMC (term $\mathbb{E}$(ISC) in optimization model (12)) are ignored. This benchmark optimal-dispatch model is solved and the BMC is calculated. The ISC for each scenario is also calculated and weighted average of the imbalance costs $\mathbb{E}$(ISC) is obtained. Results in Table V show the predictive nature of our optimal-dispatch model. Under $\sigma_{\mathbf{G1}} = 30\ MWh$, the Bench-DIP model leads to BSC=226.6 $/h. This is while

TABLE V
COMPARISON OF BMC, ISC AND BSC (ALL VALUES ARE IN $/h) FOR THE BENCHMARK (BENCH.) AND PROPOSED (PROP.) MODELS FOR $\beta = 0.9$ AND SIL=$125MWh$ AT DIFFERENT STANDARD DEVIATION (STD. DEV.) OF G_1

std. dev		DIP		SIP-SR		SIP	
		Bench.	Prop.	Bench.	Prop.	Bench.	Prop.
0	BMC	-55	210	-55	230	-55	1365
	ISC	216.2	-151.7	274.0	-1390.5	274.0	-2185
	BSC	161.1	58.2	219.0	-160.5	219.0	-820
30	BMC	-75	210	-75	930	-75	1035
	ISC	301.6	-164	445.2	-1040.8	445.2	-1666
	BSC	226.6	46	370.2	-110.8	370.2	-631
50	BMC	-195	210	-195	600	-195	735
	ISC	295.8	-206.7	338.8	-680.8	342.4	-1233
	BSC	100.8	3.2	143.8	-80.8	147.4	-498

under our Prop-DIP model, the BSC cost is reduced to 46 $/h. Under benchmark model, the SO has a revenue of BMC=75 $/h in the balancing market, however, the recourse cost of its decision amounts to ISC=301.6 $/h. This is while in our proposed optimal-dispatch model, the SO takes into account the ISC in the BM decision making process. The BMC is increased to 140 $/MWh$ at the benefit of reduced ISC to 164 $/MWh$ revenue. Once we increase the intermittency level of $G1$ ($\sigma_{\mathbf{G1}} = 50\ MWh$), the same pattern is observed. The benchmark optimal-dispatch model ignores the ISC leading to 100.8 $/h for total system cost (BSC). But in our proposed model, the BSC is reduced in order of five to BSC=3.2 $/h at SIL=125 MWh. The comparative analysis of prop-SIP-SR and prop-SIP models with their corresponding benchmark models show similar pattern for the costs. For our case studies, the Prop-SIP model has the lowest BSC as compared to the Prop-DIP and Prop-SIP-SR models.

VII. NUMERICAL RESULTS

In this section, we assume eight generators[6] participating in the BM, where each generator represents one area in the Nordic electricity market[7]. The metered generation of these generators are taken from Nord Pool website [46].

A. Proposed GAMLSS-based scenario generation algorithm in Fig. 5

To estimate and forecast, the dynamic distribution of metered generations, the GAMLSS packages in R is used [47] and this linearly regresses the parameters of the assumed distribution on lagged values, wind generation, hydropower and demand. Table VI gives the value of Akaike Information Criterion (AIC) for different distributions for the data of G_1 metered generation. The skew t distribution was found to have minimum AIC. Similarly, the analysis was performed for other generators in the system. Once the skew t distribution

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[6]With eight generators it was easier to interpret the choice of generators by the system operator (SO) for balancing market considering their intermittency, which is reflected by the ISC. Therefore, the choice of five- and eight-generator systems was for the ease of demonstration. As all our proposed models have been linearized, decomposition methods can be implemented to study a real system [38], [40], [41], [44], [45].

[7]Out of the eight generators, three of them are located in Sweden, one in Denmark and four in Norway.

TABLE VI
AIC VALUES FOR DIFFERENT DISTRIBUTIONS TESTED ON G_1 METERED GENERATION

Distribution	Skew t	Log normal	Negative binomial	Normal	Poisson
AIC values	4000	4894.05	4901.03	4925.47	23262.09

TABLE VII
INTERCEPTS AND COEFFICIENTS OBTAINED FROM GAMLSS FOR THE GENERATORS G_1 TO G_4

	α_1	γ_1	γ_2	γ_3	β_1	β_2	β_3
G_1	35.6	0.072	0.091	0.424	0.451	0.194	5.81
G_2	661.1	0.057	0.061	0.338	0.551	0.468	11.58
G_3	716.6	0.045	0.009	0.431	1.976	0.450	-33.05
G_4	-774.8	0.115	0.052	0.380	0.273	0.828	

is selected for G_1 metered generation, different parameters of this distribution (mean, standard deviation, skewness and kurtosis) are linearly regressed on those explanatory variables which were significant at 5%. The linear expression (28) shows the regression of the mean value ($\mu_t^{G_1}$) for G_1 metered generation over its explanatory variables. In (28), α_1 indicates the intercept of the response variable μ^{G_1}. The γ_1, γ_2 and γ_3 are the coefficients of the lag values of $\mu_t^{G_1}$. The coefficients corresponding to wind generation ($wind_t$), demand (dem_t) and hydropower (hyd_t) are β_1, β_2 and β_3, respectively.

$$\mu_t^{G_1} = \alpha_1 + \gamma_1\mu_{t-1}^{G_1} + \gamma_2\mu_{t-2}^{G_1} + \gamma_3\mu_{t-3}^{G_1} + \beta_1 wind_t^{G_1} + \beta_2 dem_t^{G_1} + \beta_3 hyd_t^{G_1} \quad (28)$$

Tables VII and VIII give the values of intercepts and coefficients for the mean value of metered generation as obtained from GAMLSS. Table IX shows the values of mean (μ), standard deviation (σ), skewness (ν) and kurtosis (τ) of the metered generation calculated by GAMLSS. These values are used for scenario generation from the skew t distribution. Each

TABLE VIII
INTERCEPTS AND COEFFICIENTS OBTAINED FROM GAMLSS FOR THE GENERATORS G_5 TO G_8

	α_1	γ_1	γ_2	γ_3	γ_4	β_2	β_3
G_5	-21.5	0.003	0	-0.006	1.086	-0.034	-1.38
G_6	687.7	0	0	-0.004	0.816	0.199	-7.83
G_7	6287	-0.003	-0.015	-0.004	-0.981	0.5346	-39.13
G_8	11090	0.003	-0.722	-0.254	-0.564	-2.341	41.35

TABLE IX
OBTAINED VALUES OF MEAN, STANDARD DEVIATION, SKEWNESS AND KURTOSIS FOR THE GENERATORS FROM GAMLSS

	μ(MWh)	σ(MWh)	ν	τ
G_1	1269.2	266.24	0.6082	5.6916
G_2	2560.7	494.664	0.8782	144.459
G_3	10988	236.78	-0.1696	3.309
G_4	1488.8	209.22	0.6236	12.114
G_5	1523.8	0	1.8538	8.6119
G_6	6749.8	0.0433	-0.2543	0.072
G_7	3214.1	0	0.8149	3.6506
G_8	3562.2	0	1.3201	3.4879

TABLE X
INPUT PARAMETER DATA FOR EIGHT-GENERATOR SYSTEM IN DIP, SIP-SR AND SIP MODELS

Gen	λ^{up} ($/MWh)	λ^{dn} ($/MWh)	g^{up} (MWh)	g^{dn} (MWh)	g^s (MWh)
G_1	15	4	300	300	1404.9
G_2	16	5	200	200	2700.2
G_3	17	6	1100	1100	11405
G_4	18	7	100	100	1604.7
G_5	19	8	100	100	1523.8
G_6	20	9	700	700	7122
G_7	21	10	200	200	3214.1
G_8	22	11	200	200	3562.2

TABLE XI
COMPARISON OF STATUS OF UP- AND DOWN-REGULATION BIDS FOR THE EIGHT-GENERATOR SYSTEM

Gen		G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
DIP	ϕ^{up}	1(0)	0(0)	0(0)	0(1)	0(0)	0(0)	0(0)	0(0)
Prop(Ben)	ϕ^{dn}	0(1)	0(0)	0(0)	0(0)	0(1)	1(0)	0(1)	0(1)
SIP-SR	ϕ^{up}	0(0)	1(0)	0(0)	0(1)	0(0)	0(0)	0(0)	0(0)
Prop(Ben)	ϕ^{dn}	0(1)	0(0)	0(0)	1(0)	1(1)	0(0)	1(1)	1(1)
SIP	ϕ^{up}	0(0)	0(0)	0(0)	0(1)	0(0)	0(0)	0(0)	0(0)
Prop(Ben)	ϕ^{dn}	0(1)	0(0)	0(0)	0(0)	1(1)	0(0)	1(1)	1(1)

skew t distribution is sampled by 1000 scenarios which are given as input to the fast forward selection reduction algorithm [38]. As a result of the scenario reduction algorithm, ten scenarios are obtained along with their updated probabilities, which are then used in the proposed and benchmark models[8]. The computational time for eight-generator system turned out to be 0.11 seconds, 1.5 seconds and 840.41 seconds for the DIP, SIP-SR and SIP models respectively. Again, the computational time for the SIP model was the highest even for this case study due to the several binary variables introduced for the linearization process.

B. Proposed economic-dispatch models

Table XI shows the status of up- and down-regulation offers and bids for the eight generators when the same input is given to all the proposed and benchmark models. The minimum SIL for DIP and SIP-SR was 603 MWh while for SIP it was 435 MWh to satisfy the probabilistic energy-balance constraint (10) at $\beta = 0.9$. It can be seen that some of the generators up-regulate while none of them down-regulate for

TABLE XII
COMPARISON OF BSC, BMC AND ISC (ALL VALUES ARE IN $/h$) FOR THE BENCHMARK (BENCH.) AND PROPOSED (PROP.) MODELS (POSITIVE VALUE INDICATES COST AND NEGATIVE VALUE IS REVENUE)

	DIP model		SIP-SR model		SIP model	
	Bench.	Prop.	Bench.	Prop.	Bench.	Prop.
BMC	-4400	-1800	-4400	-2500	-4400	-4200
ISC	69.4	-3671.3	656.7	-3809.4	745.7	-3018.6
BSC	-4330.5	-5471.3	-3743.2	-6309.4	-3654.2	-7218.6

[8]We use the same data for all the three dispatch models, since the generator characteristics can be taken as representative across both the regions (Nordic countries and Great Britain).

the given set of inputs. As shown in Table X, G_1 is the most expensive for down-regulation and has a high standard deviation (see Table IX), hence it was not chosen in any of the proposed models due to its high intermittency level. However, it was chosen by all benchmark models which ignore the intermittency cost of G_1. Table XII clearly shows the predictive nature of our optimal-dispatch model. The Bench-DIP model results in BSC=-4330.5 \$/h while our Prop-DIP model gives BSC=-5471.3 \$/h. This indicates a higher revenue in our Prop-DIP model as compared to the Bench-DIP model. The results followed the trend as observed in the Illustrative examples (see Table V). Even though the SO earns a higher revenue (BMC=4400 \$/h) in Bench-DIP model, the recourse cost of its decision leads to ISC of 69.4 \$/h. In our Prop-DIP model however, the ISC=-3671.3 \$/h. This results in a net reduction in BSC by 1140.8 \$/h in our Prop-DIP model as compared to Bench-DIP model. This reduction in BSC can be accounted to the fact that in our Prop-DIP model, the objective function of the optimization problem comprises of both the BMC and ISC (as given in equation (11)) that are minimized. However, in the benchmark model, only the BMC is minimized and the output for binary variables ϕ_i^{dn} and ϕ_i^{up} from minimizing BMC serves as inputs to (11b) and (11c) respectively for finding out the ISC. This value of ISC for the benchmark model turns out to be higher than that in our proposed model as the intermittency of the generators do not play any role in selecting the generators to be operated in the balancing market. Another interesting point to explain the large differences in the ISC values of Table XII as compared to those in Table V would be that it has to do with the differences in the input values of λ^{up}, λ^{dn}, g^{up}, g^{dn} and g^s in the five- and eight-generator systems. Table IV and Table X give the values of the inputs considered for the λ^{up}, λ^{dn}, g^{up}, g^{dn} and g^s in our simulations for five- and eight-generator systems, respectively. In order to obtain values for the scheduled output of the generation, g^s, we take average of the 10 values of $g_{i,\omega}^m$ that were obtained from the proposed scenario generation algorithm based on predictive conditional dynamic density distributions. The study has been performed on real market data acquired from the Nord Pool website to find the values for $g_{i,\omega}^m$. As the magnitudes of all these input values for the eight-generator system in the Table X are larger as compared to those assumed for the five-generator system in Table IV, the results of the ISCs have larger differences in Table XII as compared to those in Table V.

The reduction in BSC for the Prop-SIP-SR and Prop-SIP models were 2566.2 \$/h and 3564.4 \$/h respectively as compared to the corresponding benchmark models. A similar trend of reduction in BSC in the proposed models of Numerical examples has reaffirmed our observations from the Illustrative examples. The less reduction in BSC for the Prop-DIP-model can be accounted for in Table X. It can be seen that G_1 was chosen by the Prop-DIP model for up-regulation as it was the cheapest for up-regulation but also more variable than G_4, which was chosen by the corresponding Bench-DIP model.

VIII. Conclusion

This paper proposes three stochastic optimization models in order to enable the SO to dispatch into the balancing market not just based on the marginal cost but also on the intermittency of the generators. This capability will become increasingly important as more intermittent and stochastic generators come on to the system and indeed wish to take part in balancing actions. The three one-stage stochastic optimization models are based on different imbalance pricing methods. Out of the three proposed models, the single imbalance price models are able to give a clear signal to the stochastic (cheaper but more uncertain) generators to invest in technologies or hedge efficiently if they want to be chosen in the balancing market by the SO. For the same inputs to the MILP models, SIP model gives more saving for the SO as compared to the DIP model. Therefore, we observe why it was beneficial for the GB electricity market to move from DIP model to SIP model in 2015.

IX. Further discussions and Future work

In Nordic market, the balancing market is a two-stage market. In the first stage, the transmission constraints are ignored and the dispatching is done only based on the energy imbalances, then if the re-dispatch in the first stage is violating some transmission constraints, in the second stage, the counter trading will be performed [28]. The balancing market in GB is explicitly designed to deal with energy imbalances and any actions, which the system operator takes for locational or voltage reasons are recorded separately and included in the recovery of overall system costs by the SO. They are not part of the balancing market [30]. Thus, from a practical view-point while activating the bids in the balancing market, transmission constraints are not considered in the first stage of Nordic balancing market and GB electricity market. As our paper primarily focuses on these markets, we do not model transmission constraints in our paper. However, the explicit modeling of transmission constraints in our proposed model is a good future extension of our work, which adjusts our model for other electricity markets too.

Reference [48] explains how the SO in Sweden procures losses before the DA market clearing. In the balancing market, the SO activates the bids based on the merit order list independent of how large losses might occur due to activating a particular bid [29]. Later, the SO pays for the transmission grid losses, which it has to buy before the DA market clearing. Losses in GB are included in the settlement system, not the balancing market. All contracts are settled in comparison with metered volumes and imbalance volumes after the delivery periods and zonal loss factors are applied at that stage [49]. But in some other electricity markets, if it is important to model losses then the modeling of ohmic losses in our proposed formulation can be considered as a good future extension of our work.

The tests performed in Sections VI and VII serve the purpose of demonstrating the functionality of the proposed models. While the existing work tests the proposed models with five- and eight-generator systems, it is desirable to apply these models to a full system [38], [40], [41], [44], [45]. This will

be a further extension of the existing work.

The issue of fairness and openness in scenario generation and reduction, as employed in our method, is not addressed in the literature. This is one promising direction of future research.

X. Acknowledgement

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XI. Appendix

A. Linearizations related to DIP model

The non-liear expression, $\Delta_\omega^- = \text{Max}(-\Delta_\omega, 0)$, can be linearized by replacing it with the following two equations:

$$0 \le \Delta_\omega^- \le b_\omega^- \Xi^{(1)} \tag{29a}$$

$$-\Delta_\omega \le \Delta_\omega^- \le -\Delta_\omega + (1 - b_\omega^-)\Xi^{(1)} \tag{29b}$$

A similar linearization has been performed for $I_{i,w}^+ = \phi_i^{dn}\Delta_\omega^+$ by using the following equations:

$$0 \le I_{i,w}^+ \le \phi_i^{dn}\Xi^{(1)} \tag{30a}$$

$$\Delta_\omega^+ - (1 - \phi_i^{dn})\Xi^{(1)} \le I_{i,w}^+ \le \Delta_\omega^+ \tag{30b}$$

Again, for the non-linear expression, $I_{i,w}^{up} = s_\omega \phi_i^{up}$, it can be replaced by the equations given below:

$$0 \le I_{i,w}^{up} \le \phi_i^{up}\Xi^{(1)} \tag{31a}$$

$$s_\omega - (1 - \phi_i^{up})\Xi^{(1)} \le I_{i,w}^{up} \le s_\omega \tag{31b}$$

Similarly, for $I_{i,w}^- = \phi_i^{up}\Delta_\omega^-$, the following equations can be used for linearizaion:

$$0 \le I_{i,w}^- \le \phi_i^{up}\Xi^{(1)} \tag{32a}$$

$$\Delta_\omega^- - (1 - \phi_i^{up})\Xi^{(1)} \le I_{i,w}^- \le \Delta_\omega^- \tag{32b}$$

The 'If' condition in (1c) is written as linear constraints $\sum_i \phi_i^{up} \ge 1 - \Xi^{(1)}(1 - \overline{\phi})$ and $\sum_i \phi_i^{up} \le 1 + \Xi^{(1)}(\overline{\phi})$.
The bi-linear term, $z_\omega^{dn} = \underline{\phi}c_\omega$, corresponding to (18), can be linearized as:

$$0 \le z_\omega^{dn} \le \underline{\phi}\Xi^{(1)} \tag{33a}$$

$$c_\omega - (1 - \underline{\phi})\Xi^{(2)} \le z_\omega^{dn} \le c_\omega \tag{33b}$$

Similarly, another bi-linear expression, $z_\omega^{up} = \overline{\phi}s_\omega$, in (18) can be replaced by the following equations to make them linear:

$$0 \le z_\omega^{up} \le \overline{\phi}\Xi^{(1)} \tag{34a}$$

$$s_\omega - (1 - \overline{\phi})\Xi^{(1)} \le z_\omega^{up} \le s_\omega \tag{34b}$$

The Max operator, $u_\omega^+ = Max(\Delta_\omega^+ - \overline{\alpha}, 0)$, in the probabilistic constraint can be linearized as given by:

$$0 \le u_\omega^+ \le b_\omega^1 \Xi^{(1)} \tag{35a}$$

$$\Delta_\omega^+ - \overline{\alpha} \le u_\omega^+ \le \Delta_\omega^+ - \overline{\alpha} + (1 - b_\omega^1)\Xi^{(1)} \tag{35b}$$

Again, the other Max operator, $u_\omega^- = Max(\Delta_\omega^- - \underline{\alpha}, 0)$, can be replaced by:

$$0 \le u_\omega^- \le b_\omega^2 \Xi^{(1)} \tag{36a}$$

$$\Delta_\omega^- - \underline{\alpha} \le u_\omega^- \le \Delta_\omega^- - \underline{\alpha} + (1 - b_\omega^2)\Xi^{(1)} \tag{36b}$$

B. Linearizations related to SIP-SR model

The non-linearity introduced because of up-regulation price and total negative deviation in the SIP-SR model is given by $c_\omega^{dn} = \text{Max}(\lambda_i^{up}N_{i,\omega}^{up})$, in order to linearize it, it can be replaced with the following equations:

$$-r_{i\omega}\Xi^{(1)} \le c_\omega^{up} - \lambda_i^{up}N_{i,\omega}^{up} \le r_{i\omega}\Xi^{(1)} \tag{37a}$$

$$\sum_i r_{i\omega} = \mathcal{G} - 1 \tag{37b}$$

where $r_{i\omega}$ is a binary variable.

For the 'If' condition, $\mathbb{I}(\sum_i \phi_i^{up}g_i^{up} < \sum_i \phi_i^{dn}g_i^{dn})$, the below equations can be used:

$$\sum_i \phi_i^{up}g_i^{up} \ge \sum_i \phi_i^{dn}g_i^{dn} - \Xi^{(1)}(1 - \underline{\phi}) + \epsilon \tag{38a}$$

$$\sum_i \phi_i^{up}g_i^{up} \le \sum_i \phi_i^{dn}g_i^{dn} + \Xi^{(1)}\underline{\phi} - \epsilon \tag{38b}$$

The non-linear expression, $k_i^{dn} = \phi\phi_i^{dn}$, can be linearized as $\phi + \phi_i^{dn} - 1 \le k_i^{dn} \le \phi$ and $k_i^{dn} \le \phi_i^{dn}$

From the objective function (26), $t_\omega^{up} = \Delta_\omega^+ \overline{\phi}$ can be replaced by the following equations:

$$0 \le t_\omega^{up} \le \overline{\phi}\Xi^{(1)} \tag{39a}$$

$$\Delta_\omega^+ - (1 - \overline{\phi})\Xi^{(1)} \le t_\omega^{up} \le \Delta_\omega^+ \tag{39b}$$

Another non-linear term, $h^+ = \Delta_\omega^+ \phi$, from (26) can be written as:

$$0 \le h^+ \le \phi\Xi^{(1)} \tag{40a}$$

$$\Delta_\omega^+ - (1 - \phi)\Xi^{(1)} \le h^+ \le \Delta_\omega^+ \tag{40b}$$

On similar lines, the term $\Delta_\omega^-\underline{\phi}$, in (26), can be denoted by t_ω^{dn} and replaced with:

$$0 \le t_\omega^{dn} \le \underline{\phi}\Xi^{(1)} \tag{41a}$$

$$\Delta_\omega^- - (1 - \underline{\phi})\Xi^{(1)} \le t_\omega^{dn} \le \Delta_\omega^- \tag{41b}$$

Finally, the non-linearity introduced in (26), by $h^- = \Delta_\omega^- \phi$ can be tackled by replacing it with the following equations:

$$0 \le h^- \le \phi\Xi^{(1)} \tag{42a}$$

$$\Delta_\omega^- - (1 - \phi)\Xi^{(1)} \le h^- \le \Delta_\omega^- \tag{42b}$$

Please note that, in our case, the same value of Ξ worked for all the linearization equations. But it can also be different corresponding to each specific linearization, even though we have written $\Xi^{(1)}$ throughout the Appendix section.