OPTIMAL FIXED-INTERVAL INTEGRATED GUIDANCE-CONTROL LAWS

FOR HIT-TO-KILL MISSILES

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ABSTRACT

Due to their potential for reducing the weapon size and efficiency, design methods for realizing hit-to-kill capabilities in missile systems are of significant research interest in the missile flight control community. As defined in this paper, hit-to-kill capability requires the missile to consistently achieve point-mass miss distances less than half the minimum dimension of the target. It has been noted in the literature that the chief contributors to the miss distance in homing missiles are the seeker errors, autopilot lag, target maneuvers, and target state estimation lag. Guidance laws for ameliorating the effects of each of these miss distance components have been discussed in several recent publications.

The present research addresses the hit-to-kill missile flight control problem by casting it as an integrated guidance-control problem. By including the complete dynamics of the missile, the integrated guidance-control formulation automatically compensates for the impact of the autopilot lag on the miss distance. The resulting finite-interval control problem is then solved using a transformation approach. Interception by a kinetic warhead is used as an example to illustrate the performance of the integrated guidance-control law.

1. INTRODUCTION

There has been an increasing interest in methods for integrated synthesis of missile guidance and control systems in recent literature[1-9]. These techniques have the potential to enhance missile performance by exploiting the synergism between guidance and control (autopilot) subsystems. Additional feedback paths established by integrated design methods in the flight control system allow the designer to realize beneficial interactions between these subsystems. Resulting improvements in target interception accuracy will allow the use of smaller warheads, leading to a more efficient weapon system.

The focus of the research presented in this paper is on the development of design methods for integrated guidance-autopilot systems for homing missiles. Infinite-time nonlinear regulator formulations of these problems have been previously presented[1-5]. The present paper will discuss finite-duration nonlinear optimal control formulations of the integrated guidance-control problem. Although linear, gain-scheduled approaches to the integrated guidance-control problem are feasible[6], present research will focus on nonlinear techniques for integrated system design. The benefits of nonlinear design methods are that they can use all the available information about the missile dynamics to provide better performance, and avoid the onerous gain scheduling process. Computational complexity introduced by the nonlinear approach can easily be handled using state-of-the-art, onboard processors.

The traditional approach to missile guidance and control system design has been to neglect interactions between these systems, and to treat individual missile subsystems separately. Missile dynamics are split into relative position dynamics and missile short-period dynamic components. Relative position dynamics are used to synthesize the guidance law, while the short period dynamics is employed to design an autopilot to stabilize the missile and to track guidance commands. Designs are generated for each subsystem and these designs are then assembled. If the overall system performance is unsatisfactory, individual subsystems are re-designed to improve the system performance. Due to its iterative nature, this latter part of the design process can be highly time-consuming and expensive.

Figure 1 illustrates the differences between traditional and integrated guidance-control systems. In the conventional approach, the guidance law does not employ the missile body rates or sensed acceleration components to generate autopilot commands. As a result, in engagement scenarios requiring agile maneuvers, the guidance commands can sometimes exceed the autopilot performance limits. If the autopilot employs high DC gain dynamic compensators for improved command tracking and disturbance rejection, these guidance commands can drive the flight control system unstable. Additionally, since the autopilot does not use target relative missile position and velocity components for feedback, it cannot adjust its

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response to accommodate for agile target maneuvers that may occur towards the end of the engagement.

Consequently, the traditional design approach requires the autopilot to have a small time constant when compared with the guidance system to assure the stability and performance of the overall flight control system. In fact, the autopilot time constant often dictates the achievable interception accuracy of missiles equipped with conventional flight control systems[7, 8]. While the autopilot time constant requirement can be easily met when the missile is far away from the target, it becomes increasingly difficult as it gets closer to the target. This is due to the fact that most guidance laws are functions of time-to-go, which make their responses faster as the missile gets close to the target. In fact, it has been shown that conventional flight control systems can sometimes become unstable as the missile approaches the target[9].
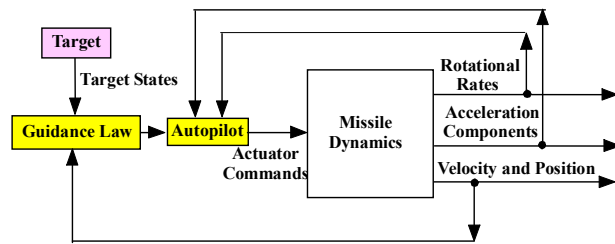
(a) Conventional Missile Guidance and Control System
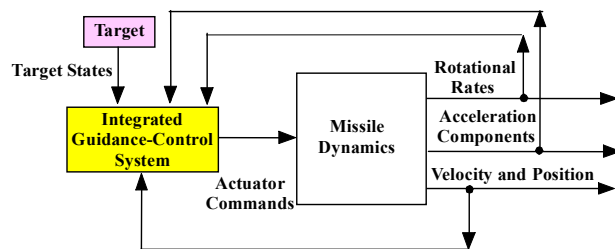
(b) Integrated Guidance and Control System

Figure 1. Conventional and Integrated Guidance-Control Systems.

On the other hand, in the integrated design approach, the guidance and control functions are realized using all the available measurements. This fact makes the system less likely to encounter saturation and stability problems. Moreover, the integrated design approach eliminates the iterative steps required to ensure the compatibility between the guidance and autopilot systems.

While there are definite operational advantages in employing integrated guidance-control systems, their design is complicated. This is due to the fact that the increased dimension of the nonlinear control problem makes it awkward to apply a conventional gain-scheduling[10] design methodology. These high-order designs will require gain scheduling not only with respect to the airframe performance variables, but also with respect to the engagement geometry. Although nonlinear control system design techniques[11-13] can make these problems more tractable, symbolic manipulations required for their development can be formidable. Recent advancements in computer-aided nonlinear control system design technology[14] offer more direct approaches for integrated system design, and avoid the need for any symbolic manipulations.

Another difficulty in integrated guidance-control system design arises from the fact the problem has to be posed as a finite-interval control problem. While it is awkward to adapt linear finite-time control system results[15] to formulate and solve the nonlinear integrated guidance-control problem, numerical nonlinear control techniques[14] can be readily employed for this purpose.

With the foregoing as the background, this paper will discuss two different formulations of the finite-interval integrated guidance and control system designs for a homing missile. Using the feedback linearization approach, the integrated guidance-control design problems will be formulated as finite-interval optimal control problems. The solutions will then be obtained as online solvable two-point boundary-value problems, specified in terms of time-to-go or range-to-go.

In each case, integrated guidance-control systems will be designed using a six degree-of-freedom nonlinear simulation model of an air-to-air missile in conjunction with computer-aided, nonlinear control system design software[14]. Sample engagement scenarios with no maneuvering and maneuvering targets will be given to illustrate integrated guidance-control system performance.

2. INTEGRATED GUIDANCE-CONTROL SYSTEM

Integrated guidance-control designs discussed in previous research efforts[1, 3-5] were formulated as infinite-duration control problems. Although time-to-go appears implicitly in the proportional navigation and the zero-effort miss formulations, the finite interval nature of the control problem is not explicitly recognized in the design process. This section will present two different formulations of the finite-interval integrated guidance-control problem.

The integrated guidance-control problem will be cast as a finite-interval optimal control problem that

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drives the miss distance to zero while achieving a specified terminal aspect angle. Desired terminal aspect angle is achieved by enforcing transversality conditions on target-relative flight path angle and the heading angle at the final time or range.

An additional coordinate frame is introduced in the system dynamics to simplify the problem formulation. This is a target-centered system, with the X_c-axis oriented in the direction of the desired intercept angle. The Z_c-axis direction points into the plane of the paper. Figure 2 illustrates the coordinate frame. In most cases, the X_c-axis would be parallel to the target's heading; i.e. the desired terminal aspect angle would then be either zero or 180 degrees ($\Delta\chi = 0, 180$).
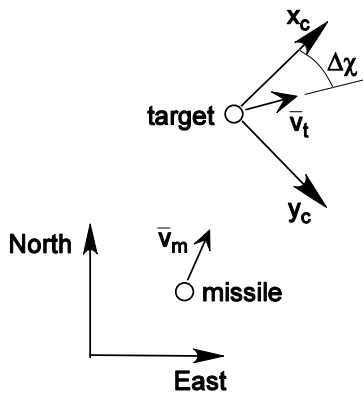

Figure 2. Guidance Frame Definition

The objective of the guidance-control problem is to drive the target relative missile position coordinates y_C and z_C to zero, while achieving the desired aspect angle at the final time.

The finite-interval integrated guidance-control problem is a trajectory optimization proble, which can be solved using iterative numerical algorithms[15]. However, since iterative algorithms are not desirable for onboard implementation due to the uncertainties in their convergence, a semi-numerical approach will be developed in this paper.

As in the formulations of the integrated guidance-control problems discussed in References 3 and 4, the system dynamics will be first transformed into linear, time-invariant form using the feedback linearization[11 - 13] methodology. The trajectory optimization problem is then formulated in terms of the feedback linearized dynamic system. The resulting control law is transformed back into the original space to obtain a nonlinear, finite-interval integrated guidance-control law. Note that this represents a new approach to on-line nonlinear trajectory optimization of flight vehicles using six degrees-of-freedom models.

Since the missile model is of high order and contains numerical tables, it is not practical to derive feedback-linearized models using algebraic manipulations. However, a recently developed software package[14] can be used to derive feedback-linearized models directly from computer simulations.

The first step in the numerical feedback linearization approach is that of defining the flow of the control variables through the system states. These dependencies can be symbolically represented using the following expressions:

$$\delta_P \rightarrow p \rightarrow \phi \qquad (1)$$

$$\delta_q \rightarrow q \rightarrow \alpha \rightarrow z_c \qquad (2)$$

$$\delta_r \rightarrow r \rightarrow \beta \rightarrow y_c \qquad (3)$$

Expressions (1) – (3) state that the roll, pitch, yaw fin deflections influence the body rates, which in turn influence roll attitude, angle of attack and angle of sideslip. The angle of attack and angle of sideslip then influence the target relative missile altitude and cross range. The nonlinear control system design software[14] uses the control flow definitions given in Equations (1) through (3) to develop the numerically feedback linearized model. The feedback-linearized model is then used to derive finite-interval integrated guidance-control laws.

The state variables in the feedback-linearized form of the missile dynamics are altitude, cross range, roll attitude and their derivatives. In the case of maneuvering targets, it is assumed that the target velocity and acceleration components will be available from an estimator. The feedback linearization process transforms the fin deflections into new pseudo-control variables. The system dynamics are in a linear, time-invariant form with respect to the pseudo-control variables. If the instantaneous values of the state variables are known, fin deflections can be extracted using a nonlinear, state-dependent transformation.

The finite-interval optimal control problem can be formulated in terms of the transformed state and control variables. If the performance index is specified as a quadratic function of transformed state and control variables, closed-form solutions can be obtained using linear-quadratic optimal control theory[15]. Boundary conditions can be imposed in this formulation to meet the terminal aspect angle constraints. This process will be illustrated in the following sections.

2.1. Finite-Time Integrated Guidance-Control

The feedback-linearized missile dynamics are in the form of a linear, time-invariant dynamic system:

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad (4)$$

The state vector x is of dimension 8 and the pseudo-control vector u has a dimension of 3. The states in the feedback-linearized missile dynamics are the target relative position of the missile z_c, y_c and their first and second derivatives. The roll channel states are the roll attitude and its first derivative. Control variables are complex functions of the transformed state variables and the fin deflections.

A quadratic performance index is next defined as:

$$J = \frac{1}{2}x_f^T S_f x_f + \frac{1}{2}\int_{to}^{tf}\left(x^T Qx + u^T Ru\right)d\tau \quad (5)$$

where $x_f = x(t_f)$ is the state vector at the final time t_f, and the lower limit of integration t_o represents the current time t. Time-to-go is defined as: $t_f - t_o$. S_f is a positive semi-definite terminal state-weighting matrix. Q is a positive semi-definite state-weighting matrix and R is a positive definite pseudo-control weighting matrix.

Using Optimal Control Theory[15], it can be shown that the optimal control at a time instant t is:

$$u(t) = -R^{-1}B^T S(t_o)x(t) \quad (6)$$

where the positive definite matrix $S(t_o)$ is found by integrating the Matrix Riccati equation:

$$\dot{S} = -A^T S - SA - Q + SBR^{-1}B^T S, \quad S(t_f) = S_f \quad (7)$$

backward from the final time to the current time. In this formulation, desired components of the state vector at the final time can be driven to zero by introducing weighting terms in the terminal state weighting matrix S_f. Larger values of the weights will enable a tighter control over the terminal state variables.

A more direct approach for satisfying the terminal boundary conditions is to introduce them into the optimal control problem through transversality conditions[15]. This approach leads to a slightly different form of the solution. Following Reference 15, let ψ be a q-dimensional vector of the desired final values of the states, where $1 \leq q \leq n$. It will be assumed that the states are ordered so that the ones with terminal constraints are the initial entries in the state vector. In order to be consistent, it is important that no quadratic terminal state weightings be placed on those states that have to meet terminal transversality conditions.

The optimal control at time t for the linear-quadratic optimization problem with terminal boundary conditions can be shown to be[15]:

$$u(t) = -R^{-1}B^T\left[\left(S(t_o) - T(t_o)V^{-1}(t_o)T^T(t_o)\right)x(t)\right.$$
$$\left. + T(t_o)V^{-1}(t_o)\psi\right]$$
$$(8)$$

where $S(t_o)$ is found using the Riccati equation (7). The matrices $T(t_o)$ and $V(t_o)$ are found by integrating the differential equations:

$$\dot{T} = \left(SBR^{-1}B^T - A^T\right)T, \quad T(t_f) = \begin{bmatrix} I_q \\ [0]_{(n-q)\times q} \end{bmatrix} (9)$$

$$\dot{V} = T^T BR^{-1}B^T T, \quad V(t_f) = [0]_{q\times q} \quad (10)$$

backward from the final time, with the specified boundary conditions. When compared with the case without the terminal boundary conditions, it can be seen that the expression for optimal control contains additional terms. These terms provide a balance between minimizing the performance index and satisfying the specified terminal values. It is important to note that the solutions of T and V in (9) and (10) are not guaranteed for all t, nor is the existence of the inverse of V. Even when the inverse exists for all $t < t_f$, the control will become unbounded as $t \rightarrow t_f$, since $V \rightarrow 0$. This fact has important implications on the robustness of the system in the presence of modeling uncertainties and noise, as noted in Reference 15. The optimal control can be defined in terms of time-to-go by letting $t_{go} = t_f - t$ in the equations (6), (9) and (10).

The optimal pseudo-control vector u can be inverse transformed using the nonlinear control system design software[14] to obtain the fin deflections. Although the finite-duration formulation can be applied to all the three channels, in the present research, the finite-duration integrated guidance-control problem was solved for the pitch and yaw channels, and an infinite-time LQR controller was designed for the roll channel. The state and control weights used for the pitch and yaw channels are:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R = 0.01, S_f = [0]_{4\times4} \quad (11)$$

The state and control weights in the roll channel are:

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}, R = 0.01, S_f = 0 \quad (12)$$

Terminal constraints are placed on z_c and its first derivative, which is related to flight path angle, and on y_c and its first derivative, which is related to the heading angle.

The guidance-control system is required to drive the target relative missile altitude and target referenced cross range to zero, and altitude rate and cross range rate to zero at the final time. The two latter constraints will indirectly constrain the flight path angle and heading angle. The constraint values

on the derivatives could be set to non-zero values if it were desired to have non-parallel trajectories at interception. Limits of 30 degrees were placed on the fin deflections, but no limits were imposed on the normal or lateral accelerations.

The performance of this finite-time integrated guidance-control law will be illustrated in two engagement scenarios in this section. In the first engagement, the missile and target are both at an altitude of 10,000 ft. The target is flying a heading of 90 degrees, at a constant velocity of 1,100 ft/s. The missile is initially at 10,000 ft west and 20,000 ft south of the target and has a heading of 0 degrees. The initial velocity of the missile is Mach 4.5. The time step for both forward and backward integration is chosen to be 0.002 seconds. The results are shown in Figures 3 through 7.
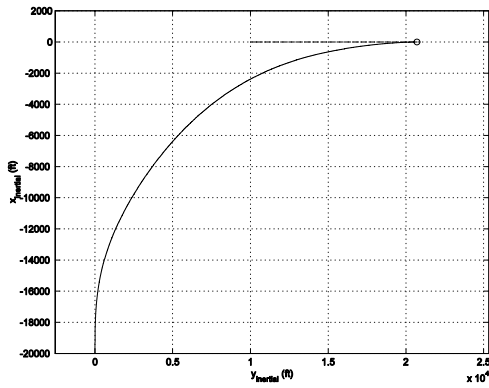
Figure 3. Horizontal Plane Trajectories of the Missile and the Target,
Solid Line: Missile, Dashed Line: Target
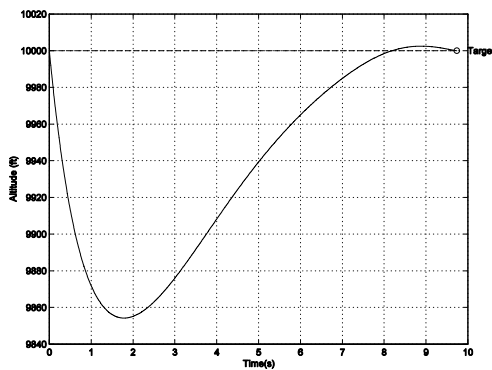
Figure 4. Missile and Target Altitude Time Histories,
Solid Line: Missile, Dashed Line: Target

The minimum distance to the target is 0.00034 ft, the final difference in the headings is 0.25 degrees,

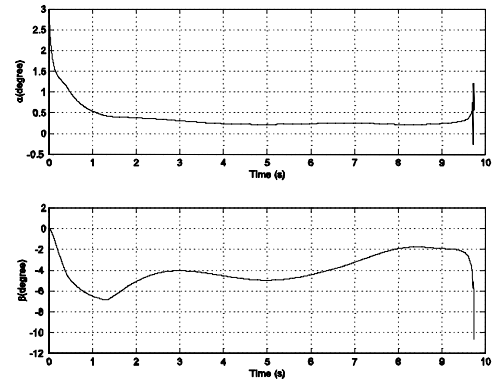and the final value of the flight path angle is 0.07 degrees.

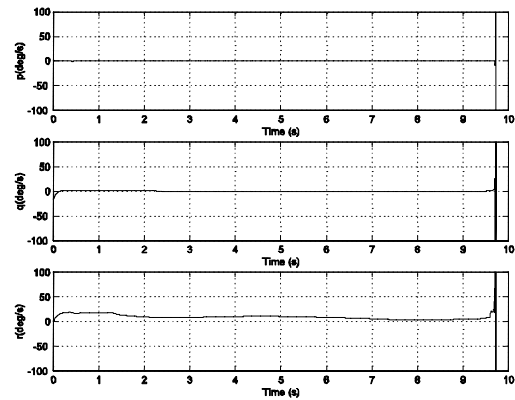**Figure 5. Angles of Attack and Sideslip vs. Time**

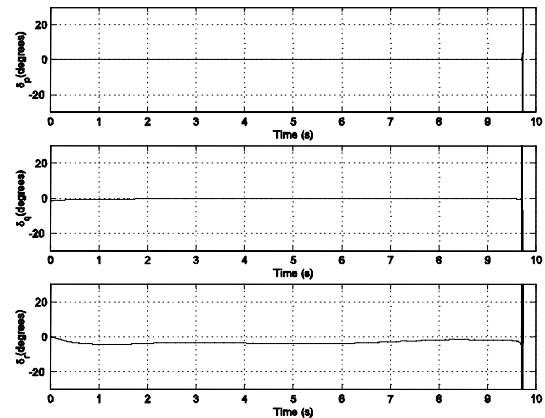**Figure 6. Missile Body Angular Rate Time Histories**

Figure 7. Time History of Fin Deflections

As expected, the missile angle of attack, angle of sideslip and the body rates exhibit large amplitude motions at the end of the engagement to satisfy the specified boundary conditions. The fin deflection

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requirements in the finite-interval integrated guidance-control law remain modest until the very end of the maneuver. They then approach and remain at specified fin deflection limits.

The second engagement scenario is similar to the one presented in Reference 16, but in this case the missile velocity is not constant. The initial heading of the missile is 10 degrees and the initial heading of the target is 198.4 degrees, and the engagement begins with missile at 16,400 ft south of the target. The target flies at 10,000 ft. altitude, and the initial altitude of the missile is 1,640 ft higher than the target. The target's velocity is 1,037 ft/s and the missile's initial velocity is Mach 3.5. The target is turning with an acceleration of 3g's. The step size for both forward and backward integration was 0.001 seconds. The results for the engagement are shown in Figures 8 through 12.
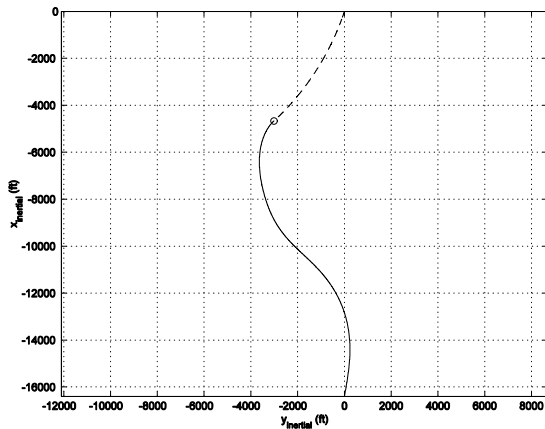

Figure 10. Missile Angles of Attack and Sideslip Time Histories

Figure 8. Horizontal Plane Trajectories of the Missile and the Target,
Solid Line: Missile, Dashed Line: Target

Figure 11. Missile Body Angular Rates vs. Time

Figure 12. Time Histories of Fin Deflections

The minimum distance between the missile and the target is 0.035 ft., the final difference in headings is 179.47 degrees, and the final flight path angle is -0.63 degrees.

Figure 9. Missile and Target Altitude Time Histories
Solid Line: Missile, Dashed Line: Target

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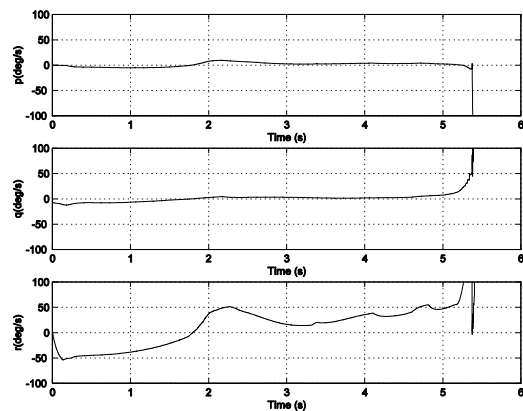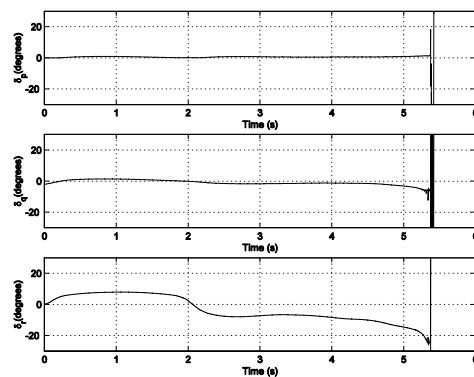## 2.2. Finite-Range Integrated Guidance-Control

The finite-time formulation of the integrated guidance-control problem discussed in the previous section requires the knowledge of time-to-go. Since this quantity cannot be measured, it is normally estimated as the negative of the ratio of the range and range rate.

The formulation presented in this section avoids the need for a time-to-go estimate by recasting the integrated guidance-control problem with range as the independent variable. Resulting integrated guidance-control law computations are based on range-to-go, which may be a directly measurable quantity.

The independent variable in the missile dynamics can be changed from time to range using the identity:

$$\frac{d(.)}{dR} = \frac{d(.)}{dt} \Big/ \frac{dR}{dt},$$

with

$$\frac{dR}{dt} = \frac{\left(\dot{x}_M - \dot{x}_T\right) + \left(\dot{y}_M - \dot{y}_T\right) + \left(\dot{h}_M - \dot{h}_T\right)}{\sqrt{\left(x_M - x_T\right)^2 + \left(y_M - y_T\right)^2 + \left(h_M - h_T\right)^2}} \quad (13)$$

Here, x_M, y_M, h_M, x_T, y_T, h_T define the position of the missile and the target in an inertial frame. Dots over the variables denote velocity components.

Missile dynamics with range as the independent variable are next feedback-linearized using the nonlinear control system design software[14] package. Since the change of the independent variable does not alter the manner in which the system dynamics are affected by the fin deflections, the symbolic expressions (1) through (3) can be used to set up the feedback linearization process. Feedback linearized missile dynamics will be of the form:

$$\phi'' = v_1, \ h''' = v_2, \ y_c''' = v_3 \quad (14)$$

A prime denotes differentiation with respect to range, and the variables v_1, v_2, v_3 are the transformed pseudo-control variables. Since the range rate can be numerically large, changing the independent variable will produce a dynamic system with small right-hand-sides. In order to improve the conditioning of the problem, the system dynamics are scaled by a factor of 1000. In this model, time is given by the solution to the differential equation:

$$\frac{dt}{dR} = \frac{\sqrt{\left(x_M - x_T\right)^2 + \left(y_M - y_T\right)^2 + \left(h_M - h_T\right)^2}}{\left(\dot{x}_M - \dot{x}_T\right) + \left(\dot{y}_M - \dot{y}_T\right) + \left(\dot{h}_M - \dot{h}_T\right)} \quad (15)$$

with

$$t = 0 \quad at \quad R = R_0$$

The linear-quadratic optimization problem formulation in terms of the transformed system dynamics remains the same as in Section 2.1, with the only change being the limits of integration and the integration time step. In both examples, the step size for forward and backward integration is 5 ft, although a larger step size could have been used without a significant increase in terminal error. The range-to-go integrated guidance-control system is next evaluated in the two scenarios described in Section 2.1.

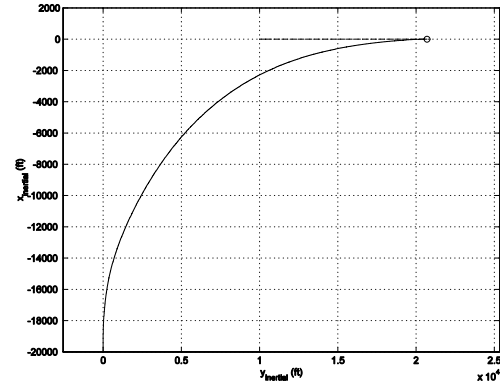The results for the first engagement are shown in Figure 13 through 17.

**Figure 13. Horizontal Plane Trajectories of the Missile and the Target,
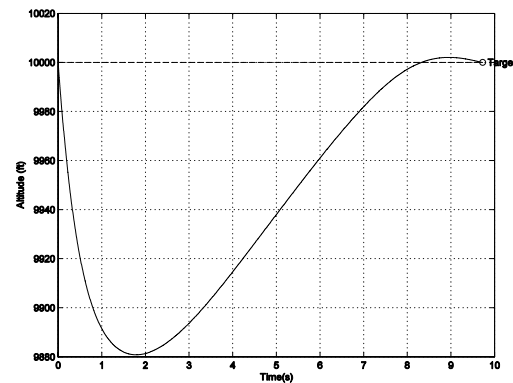Solid Line: Missile, Dashed Line: Target**

**Figure 14. Missile and Target Altitudes vs. Time
Solid Line: Missile, Dashed Line: Target**

The minimum distance between the missile and the target at interception is 0.00038 ft. The difference in headings is -0.38 degrees, and the final flight path angle is -0.019 degrees. As expected, the results are nearly identical to those given in Section 2.1. Variations arise due to the differences in numerical conditioning between the two formulations.
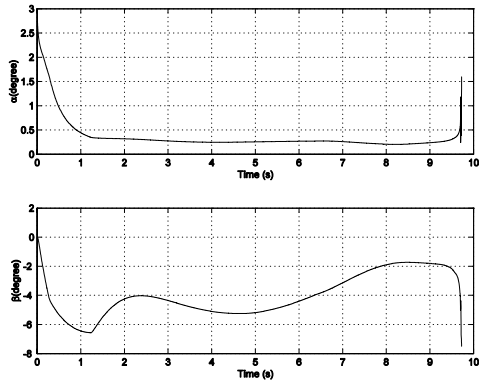
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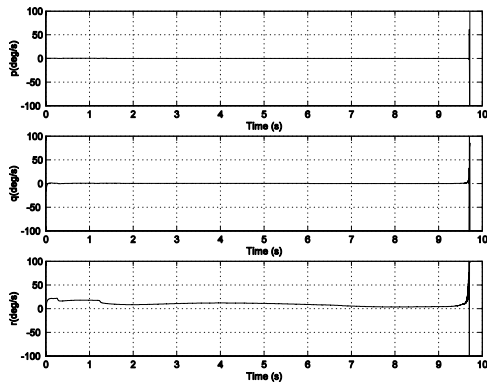**Figure 15. Missile Angles of Attack and Sideslip Time Histories**

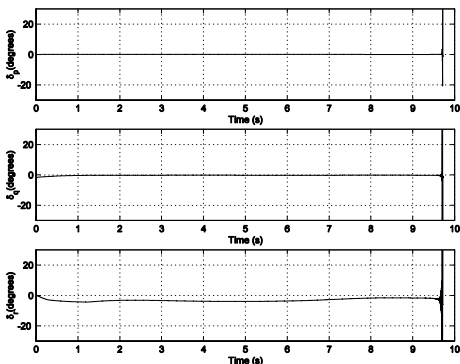**Figure 16. Missile Body Rates vs. Time**

Figure 17. Fin Deflection Time Histories

The results for the second scenario from Reference 22 are shown in Figures 18 and 19. The minimum distance between the target and the missile is 0.0011 ft., with the final difference in headings being 179.59 degrees, and the final flight path angle being -0.85 degrees. As in the previous engagement scenario, the results are nearly identical to the case where time was used as the independent variable.

Since the angle of attack, angle of sideslip, body rates and fin deflection time histories are practically same as those in Figures 10 through 12, they are not given here.
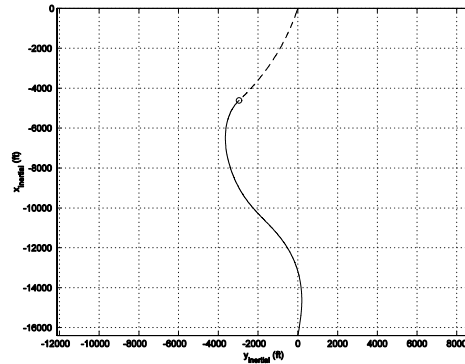

**Figure 18. Horizontal Plane Trajectories of the Missile and the Target,
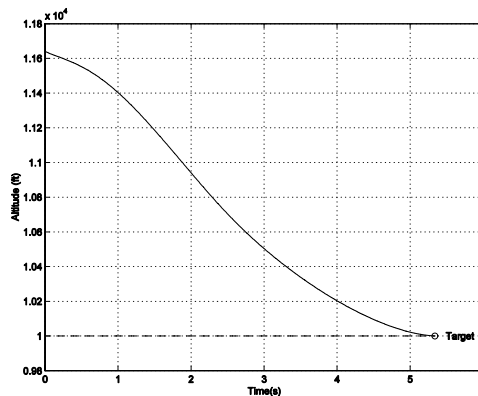Solid Line: Missile, Dashed Line: Target**

**Figure 19. Missile and Target Altitude Time Histories,
Solid Line: Missile, Dashed Line: Target**

The finite-interval integrated guidance-control laws developed in this section can deliver very small miss distances while satisfying terminal aspect angle requirements. However, these formulations require the online solution of a matrix Riccati equation and the two related linear differential equations. Consequently, implementation of finite-interval integrated guidance-control techniques will require the development of more sophisticated computational capability on board the missile. Recent research on fast numerical methods for solving guidance-control problems[17] can form the starting point for this research. Additional research issues that need to be addressed in future work include the assessment of

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the effects of target maneuvers and sensor errors on these guidance-control laws.

3. CONCLUSIONS

This paper described two different formulations of finite-interval integrated guidance-control problem. The first formulation was in terms of time-to-go, while the second formulation was in terms of range-to-go. The design methodology was based on numerically transforming the missile-target dynamics using nonlinear control system design software. Transformed dynamics were then used to formulate finite-interval linear-quadratic trajectory optimization problems with terminal boundary conditions. Terminal transversality conditions were used to satisfy aspect angle constraints at target interception. Solution of the trajectory optimization problem was then obtained by integrating three matrix equations with time-to-go or range-to-go as the independent variable. Fin deflection commands were then obtained from this solution using the inverse of the feedback-linearizing transformation. Performance of the finite interval guidance-control laws was illustrated in two engagement scenarios.

ACKNOWLEDGEMENT

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