POLICY RESEARCH WORKING PAPER 5943

Optimal Food Price Stabilization in a Small Open Developing Country

Christophe Gouel
Sébastien Jean

The World Bank
Development Research Group
Agriculture and Rural Development Team
January 2012

Policy Research Working Paper 5943

Abstract

In poor countries, most governments implement policies aiming to stabilize the prices of staple foods, which often include storage and trade measures insulating their domestic market from the world market. It is of crucial importance to understand the precise motivations and efficiency of those interventions, because they can have consequences worldwide. This paper addresses those issues by analyzing the case of a small, open developing country confronted by shocks to both the crop yield and foreign price. In this model, government interventions may be justified by the lack of an insurance market for food prices. Considering this market imperfection, the authors design optimal public interventions through trade and storage policies. They show that an optimal trade policy largely consists of subsidizing imports and taxing exports, which benefits consumers at the expense of producers. Import subsidies alleviate the non-negativity of food storage. In other words, when stocks are exhausted, subsidizing imports prevents domestic price spikes. One striking result: an optimal storage policy on its own is detrimental to consumers, since its stabilizing benefits leak into the world market and it raises the average domestic price. By contrast, an optimal combination of storage and trade policies results in a powerful stabilizing effect for domestic food prices.

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Produced by the Research Support Team

Optimal food price stabilization in a small open developing country*

Christophe Gouel† Sébastien Jean‡

Keywords: food security, incomplete markets, storage, trade policy, export restrictions.

JEL classification: D52, F13, Q11, Q17, Q18.

Sector Board: Agriculture and Rural Development (ARD).

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*The authors thank Jean-Marc Bourgeon, Édouard Challe, Christopher Gilbert, Michel Juillard, Will Martin and Brian Wright for helpful comments. This research was generously supported by the Knowledge for Change (KCP) Trust Fund and by the AgFoodTrade project, funded under the 7th Framework Programme for Research and Development, DG-Research, European Commission. Much of this work was done while Christophe Gouel was a PhD student at INRA and École Polytechnique. The views expressed here are the sole responsibility of the authors and do not reflect those of the European Commission or of the World Bank.

†The World Bank and CEPII (`cgouel@worldbank.org`)

‡INRA and CEPII (`sebastien.jean@inra.grignon.fr`)

1 Introduction

In developing countries, staple foods frequently account for a significant share of poor households' budgets. Many poor people have limited possibilities to insure against adverse price shocks. Price spikes are very problematic for poor households that are not self-sufficient, and often jeopardize their capacity to feed themselves. An important response of developing country governments to expressions of this concern is to implement food price stabilization policies. Yet, the study of these policies has been confined mainly to closed economy contexts, with an emphasis on the role of storage (see Wright, 2001, for a survey). From a theoretical standpoint, little is known about the role of trade policy in price stabilization programs, despite their widespread use. Recourse to trade policy to counter price volatility has been common in most Asian countries where stabilizing the domestic price of rice is a central objective (Timmer, 1989, Islam and Thomas, 1996, Dorosh, 2008), and also in Middle East and African countries in the case of wheat and to a lesser extent maize as well as rice (Dorosh, 2009, Wright and Cafiero, 2011). The question is probably less acute in Latin America, where most countries are net exporters of grains, but it is not irrelevant, as witnessed by the use by Chile of a price-band system for wheat and a few other food products (Bagwell and Sykes, 2004). More generally, based on a large-scale database on agricultural price distortions, Anderson and Nelgen (2012) show that countries tend to vary their nominal rate of assistance to agriculture so as to limit the effects of variations in world prices on domestic prices.

Trade and trade policies are key aspects that need to be taken into account in considerations of food security and price stabilization in developing countries. They raise numerous questions, the most important perhaps being: How should storage and trade policies be combined to achieve price stabilization? Increased reliance on national buffer stocks is frequently suggested as a remedy for developing countries faced with significant volatility in world prices. But is this a consistent policy *per se*, independent of trade policy interventions? Dorosh (2008) suggests that greater reliance on the world market allowed much more cost-effective price stabilization in Bangladesh than in India; the latter relied almost exclusively on huge public stocks and severe restrictions on imports. Can it be taken for granted that greater trade openness, or more reactive trade policy, would reduce the amounts of stocks needed to achieve a given stabilization target, and to what extent?

Export restrictions raise a number of additional questions. Most analysts of the 2007–08 food crisis agree that trade policies played a significant role in fueling international price spikes (von Braun, 2008, Mitra and Josling, 2009, Headey, 2011). In particular, export bans enforced by several rice exporters seem to have contributed greatly to the astonishing price levels reached (Slayton, 2009). Noting the similar situation in the 1973–74 crisis, Martin and Anderson (2012) emphasize the collective action problem created by export restrictions: their use by some countries to provide shelter from price spikes aggravates the problem for others (see also Bouët and Laborde Debucquet, forthcoming). The restrictions imposed by Russia on its cereal exports following a drought in 2010 can only add to this concern. A first step towards coping with this problem is to achieve a better understanding of the motivations and consequences of export restrictions. Based on Marshallian surplus analysis, many authors conclude that such policies are harmful to the countries enacting them. Is this really the case, or do export restrictions make economic sense for a small open economy? And, in this case, is refraining from the imposition of export restrictions an important sacrifice for the country concerned? Would specific flanking policies be preferable?

The way that uncertainty affects trade theory results has been widely studied, as discussed in the next section. However, a specificity of staple food products is that they are storable, and this is not taken into account in most of these works. Storage and its consequences are the subject of a separate strand in the literature, which includes some analyzes of its relationships with trade and trade policy. Although several cases are studied, these studies do not identify optimal policies. In contrast, the present paper proposes a design of optimal stabilization policies suitable for a small open economy, within a rational expectations storage model using tools developed for the analysis of optimal dynamic policies. The focus is on food security concerns in developing countries, assuming consumers to be risk averse with no insurance possibilities and a country that is self-sufficient on average.

This is challenging because the combination of rational expectations and non-negative storage and trade constraints renders the model (which does not admit closed-form solutions) problematic to solve, and even more difficult to optimize along a dynamic path. To achieve model tractability and identify stylized results requires some simplifications. For this reason, we focus on consumers' risk aversion—most directly linked to food security concerns—but overlook producers' risk aversion, despite the significant proportion of poor farmers in developing countries. We disregard also supply reaction which, while being a potentially important mechanism, is usually of limited quantitative importance in the time frame of a price surge. We work with a single-country model also in the interests of simplicity and, to obtain initial insights about the issue of export restrictions, we assess the consequences for developing countries of refraining from imposing such restrictions.

2 Trade, uncertainty and storage: Related literature

Uncertainty is widely seen as potentially affecting the main conclusions of trade theory. David Ricardo (1821, Ch. 19) concluded that temporary tariffs on cereals might be justified to avoid large losses to farmers who after increasing their production, and so the required capital, to face a sudden change in trade, such as wars, would suffer a lot from an immediate return to the situation prevailing before the crisis. The first formalization of this issue was achieved by Brainard and Cooper (1968). Based on a portfolio approach, they showed that diversification in a primary producing country decreases fluctuations in national income, which increases national welfare if the country is risk averse. Based on a comparable framework, including risk aversion in a context where productive choices are made before uncertainty is resolved, several other papers challenge the idea of the optimality of free trade under uncertainty (Batra and Russell, 1974, Turnovsky, 1974, Anderson and Riley, 1976).

Helpman and Razin (1978) point out that this result hinges crucially on the assumption of incomplete risk-sharing markets. They show that the main results of Ricardian and Heckscher-Ohlin theories of international trade, including the optimality of free trade, carry over to uncertain environments if risk can be shared appropriately. In their model, this is the case because the stock market allows households to diversify their capital, and cross-border trade in financial assets opens the possibility for international risk-sharing arrangements.

Helpman and Razin's seminal contributions clarify decisively the conditions underlying potential deviations from standard results and pave the way to numerous insightful elaborations. Yet there is a variety of reasons why the conditions required for their results might not hold. For instance, in the case that households need to invest their capital in a particular activity, without any possibility to diversify, to insure, or to trade the corresponding risk. In this context, which is plausible especially for rural households in developing countries, Eaton and Grossman (1985) show that optimal trade policy for a small open economy is not free trade. On average, the optimal policy entails an anti-trade bias. Similar conclusions emerge if market incompleteness is the result of lack of international trade in financial assets (Feenstra, 1987). In a specific-factor model with risk-averse factor owners, Cassing et al. (1986) also show that a state-contingent tariff policy can increase the expected utility of all agents. Newbery and Stiglitz (1984) provide another illustration of this potential insurance role of trade restrictions, extending the analysis to a two-country model. Without insurance markets, they show that free trade may be Pareto-inferior to no trade. Indeed, autarky directly links domestic prices to domestic output, thus providing, for a unitary price elasticity of demand, a perfect income insurance for farmers.

These different cases show that a departure from free trade may be motivated by risk-sharing objectives, when other arrangements are not available. When dealing with food security in developing countries, assuming incomplete insurance markets seems reasonable. Poor households have little opportunity to insure against the real income risk associated with variable food prices, and poor farmers (as well as many other poor workers) cannot diversify their income source, at least in the short run. Since we focus on food security in a developing country, we adopt this assumption of market incompleteness and assume consumers to be risk averse, with no insurance schemes available.

Dealing with food security requires accounting for the fact that staple food products are storable. This is especially important because storage is a central feature of these markets and can be thought of as an intertemporal risk-sharing arrangement. Studies of food security have long regarded storage as a key feature. Early analyzes of storage-trade interactions relied upon idealized or arbitrary storage technologies (Feder et al., 1977, Pelcovits, 1979, Bigman and Reutlinger, 1979, Reutlinger and Knapp, 1980). They tend to emphasize that trade is more cost-effective than storage at promoting price stability. While useful to ensure tractability, such simplified representations, not rooted in a consistent description of agent behaviors, do not accurately reflect the risk-sharing properties of storage. A consistent modeling of these properties requires a proper accounting for agents' expectations: the best way to do this is to work in a rational expectations, infinite-horizon framework.

Apart from specific analyzes of oil-related problems, where world prices are the main source of uncertainty (Teisberg, 1981, Wright and Williams, 1982b), storage-trade interactions were first studied, in a rational expectations, infinite-horizon framework, by Williams and Wright (1991, Ch. 9), where a small open market is considered as the extreme case in a two-country model.[1] In a series of papers, Jha and Srinivasan (1999, 2001, Srinivasan and Jha, 2001) model Indian agricultural markets in relation to the world. They consider the rice market, in which India is a large country, and the wheat market, where India is a small country. In both markets, there are competitive private storers. World prices are randomly generated without accounting for serial correlation. The authors find international trade to be stabilizing even when the international price is more volatile than the domestic price. Brennan (2003) considers the Bangladesh rice market. She shows that opening the market to trade is as stabilizing as some public policies (such as subsidies to private storage or price ceiling), and is without any fiscal cost. In all these studies, welfare is measured by changes in surpluses.

In contrast to the theoretical analyzes of trade under uncertainty mentioned above, storage-trade models all focus on the assessment of given exogenous policies. They provide no hints as to what the optimal policy would be. The present paper extends normative analyzes of trade theory in an uncertain environment to an intertemporal framework with storage under rational expectations. Since it is very difficult to design optimal policies in a dynamic setting, we consider a single-country model. We follow Williams and Wright's insight and represent the world price as generated by a storage model and considered as exogenous to the country.

3 The model

We consider the market for a storable commodity in a small open economy. The world price is taken as given and the per-unit transport cost is constant. Consumers are risk averse and domestic food price volatility is driven by random output and a stochastic world price. Production is defined by exogenous stochastic shocks, so producers are not represented explicitly, but introduced later when we account for the effect of the policies on their welfare.

3.1 Consumers

The economy is populated with risk-averse consumers whose final demand for food has an isoelastic specification: $D(P_t) = dP_t^{\alpha} Y^{\eta}$, where $d > 0$ is a parameter of normalization; P_t is period t price; and α, with $\alpha < 0$ and $\alpha \neq -1$, and $\eta \neq 1$ are the price and income elasticities. Income, Y, is assumed to be constant over time, which limits the number of state variables (see Section 3.4) and allows a diagrammatic exposition of the results. Assuming there are only two goods and the second good is the numeraire, integration of this demand function gives the following instantaneous indirect

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[1] Miranda and Glauber (1995) confirm Williams and Wright's results and propose an improved numerical method. Makki et al. (1996, 2001) present a policy application of this model; based on a three-country model including the EU, the USA and the Rest of the World, they analyze the effects of removing current policy distortions such as export subsidies. Coleman (2009) extends Williams and Wright's work by considering that trade takes time. The time to ship then brings a new motive for stockpiling.

utility function (Hausman, 1981)

$$\hat{v}\left(P_t, Y\right) = \frac{Y^{1-\eta}}{1-\eta} - d\frac{P_t^{1+\alpha}}{1+\alpha}. \tag{1}$$

This utility function has relative risk aversion equal to the income elasticity of demand. To distinguish income elasticity from risk aversion, we follow Helms (1985a): we assume $\hat{v}\left(P_t, Y\right)$ to be positive and apply a monotone transformation to the indirect utility function,

$$v\left(P_t, Y\right) = \frac{\left[\hat{v}\left(P_t, Y\right)\right]^{1+\theta}}{1+\theta}, \tag{2}$$

with $v\left(P_t, Y\right) \to \ln \hat{v}\left(P_t, Y\right)$ as $\theta \to -1$. This specification is still consistent with the isoelastic demand function, but its coefficient of relative risk aversion is

$$\rho\left(P_t, Y\right) = \eta - \theta\frac{Y^{1-\eta}}{\hat{v}\left(P_t, Y\right)}, \tag{3}$$

with θ indexing the degree of risk aversion.

For simplicity, the representative consumer is assumed to adopt hand-to-mouth behavior: he consumes current income and does not save to smooth out fluctuations. The dynamics are thus simplified, since consumer's "cash on hand" does not have to be included as a state variable. This assumption overlooks the role of self-insurance through saving. However, such self-insurance remains limited in practice and falls short of providing protection comparable to what a complete market delivers, due *inter alia* to borrowing constraints and to the rather large share of the budget accounted for by staple food in many developing countries, especially for poor households.

Given the absence of saving, the consumer does not solve an intertemporal problem. At each period, he is concerned only with current-period demand, which is not affected by the degree of risk aversion. Spatial and intertemporal arbitrages are thus independent from consumer risk aversion, which creates the need for public intervention.

3.2 Storers

The single representative speculative storer is assumed to be risk neutral and acts competitively. Storage transfers a commodity from one period to the next. Storing the quantity S_t from period t to period $t+1$ entails a purchasing cost, $P_t S_t$, and a storage cost, kS_t, with k the unit physical cost of storage. A (positive or negative) per-unit subsidy ζ_t for private storage is also considered. The benefits in period t are the proceeds from the sale of previous stocks: $P_t S_{t-1}$. The storer maximizes his expected profit as stated by the following sum of cash flows

$$V^S\left(S_{t-1}, P_t, \zeta_t\right) = \max_{\left\{S_{t+i}\geq 0\right\}_{i=0}^{\infty}} \mathrm{E}_t\left\{\sum_{i=0}^{\infty}\beta^i\left[P_{t+i}S_{t+i-1} - \left(P_{t+i} + k - \zeta_{t+i}\right)S_{t+i}\right]\right\}, \tag{4}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, and β is the discount factor. The storer's problem can be expressed in a recursive form using the following Bellman equation:

$$V^S\left(S_{t-1}, P_t, \zeta_t\right) = \max_{S_t \geq 0} \left\{ P_t S_{t-1} - \left(P_t + k - \zeta_t\right) S_t + \beta\, \mathrm{E}_t \left[V^S\left(S_t, P_{t+1}, \zeta_{t+1}\right) \right] \right\}. \tag{5}$$

This equation has three state variables: the price and the subsidy, whose dynamics are considered exogenous by the storer, and the stock carried over from the previous year. Using the first-order condition on S_t and the envelope theorem, and taking into account the possibility of a corner solution (i.e., the non-negativity constraint of storage), this problem yields the following complementary condition:[2]

$$S_t \geq 0 \quad \perp \quad \beta\, \mathrm{E}_t\left(P_{t+1}\right) + \zeta_t - P_t - k \leq 0, \tag{6}$$

which means that inventories are null when the marginal cost of storage is not covered by expected marginal benefits; for positive inventories, the arbitrage equation holds with equality. The storer thus buys when present prices are low enough compared to their expected future level.

3.3 International trade

Since the model describes a homogeneous product for a small open economy, international trade modeling collapses to two arbitrage conditions, between the domestic price on the one hand, and export or import parity price on the other hand. Expressed in complementarity form,

$$M_t \geq 0 \quad \perp \quad P_t - \nu_t^M - \left(P_t^w + \tau\right) \leq 0, \tag{7}$$

$$X_t \geq 0 \quad \perp \quad \left(P_t^w - \tau\right) - P_t - \nu_t^X \leq 0, \tag{8}$$

where M_t and X_t are imports and exports; P_t^w is world price; and τ represents per-unit import and export costs, assumed to be constant and identical. ν_t^M and ν_t^X denote (positive or negative) per-unit taxes on imports and exports. The complementarity equations for trade (7)–(8) imply that the domestic price is restricted to evolving in a moving band defined by the world price, trade costs, and trade taxes if any:

$$P_t^w - \tau - \nu_t^X \leq P_t \leq P_t^w + \tau + \nu_t^M. \tag{9}$$

3.4 Recursive equilibrium

Period t harvest is denoted ϵ_t^H and is an i.i.d. random variable. The model has three state variables: S_{t-1}, ϵ_t^H and P_t^w. In any time period, the first two can be combined into one variable, availability

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[2] Complementarity conditions in what follows are written using the "perp" notation ($\perp$). This means that both inequalities must hold, and at least one must hold with equality. So $a \leq X \perp F(X) \geq 0$ is a compact formulation for $X = a \Rightarrow F(X) \geq 0$ and $X > a \Rightarrow F(X) = 0$.

(A_t), the sum of production and private carry-over:

$$A_t = S_{t-1} + \epsilon_t^H. \tag{10}$$

The other state variable, the world price, follows a continuous Markov chain, defined in the next section and characterized by the following transition function:

$$P_{t+1}^w = f\left(P_t^w, \epsilon_{t+1}^w\right), \tag{11}$$

where ϵ^w is the random production in the world market, which is assumed to be uncorrelated with domestic production shocks.[3]

Market equilibrium can be written as

$$A_t + M_t = D\left(P_t\right) + S_t + X_t, \tag{12}$$

Based on the above, we can define the recursive equilibrium of the problem without public intervention:

Definition. In the absence of a stabilization policy (i.e., $\zeta_t = \nu_t^M = \nu_t^X = 0$), a recursive equilibrium is a set of functions, $S\left(A, P^w\right)$, $P\left(A, P^w\right)$, $M\left(A, P^w\right)$ and $X\left(A, P^w\right)$ defining private storage, price, import and export over the state $\{A, P^w\}$ and transition equations (10)–(11) such that (i) the storer solves (4), (ii) trade obeys the arbitrage equations (7)–(8), and (iii) the market clears.

3.5 World price

Modeling world price dynamics is a crucial issue here. The level of the world price directly influences domestic price through arbitrage with export and import parity prices, and may also matter for expectations about its future level, by influencing storage decisions and domestic price expectations, central to the issues considered here.

In single-country models, the world price is generally represented as a stochastic process following a standard distribution, including in some cases first-order autocorrelation (Srinivasan and Jha, 2001, Brennan, 2003). Such simplifications are not consistent with the stylized facts on agricultural prices (Deaton and Laroque, 1992), which appear to be correctly represented by a storage model (Cafiero et al., 2011). Taking full account of this storage-based representation of world prices would thus require a two-country model, in which the rest of the world is modeled alongside the economy being studied. The complexity cost of such an option would be high, and difficult to reconcile with our objective of identifying optimal policies. Following Williams and Wright (1991, Ch. 9), a way out of this dilemma is to consider the small-open economy model as a limit case of a two-country model as the size difference between the two countries increases. In this limit case, the small economy is negligible compared to the big one, meaning that the rest of the world can be modeled without

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[3] It would not be hard to allow for correlated production shocks if needed.

paying attention to the small economy under study. In other words, overlooking the influence of the economy on the world market allows world prices to be modeled as a separate process, which is exogenous from the economy's point of view. This assumption of a price-taker economy leads to a disregard for motivations potentially linked to the influence of government decisions on world markets, and greatly simplifies the analysis.

World prices are thus assumed to result from a storage model with random inelastic production; they are set as a result of a system of three equations equivalent to (6), (10) and (12), without import and export variables, and without storage subsidy. This system has one state variable: availability. All variables and functions corresponding to the world market are indicated with the superscript w. Given the model's structure, the observation of price allows us to define the state of the system; price dynamics can thus be defined as a continuous state Markov chain. Its expression can be derived using equation (10) and the decision rules, $P^w\left(A^w\right)$ and $S^w\left(A^w\right)$. It gives

$$\begin{aligned}
P_{t+1}^{w} &= P^{w}\left(A_{t+1}^{w}\right) && (13)\\
&= P^{w}\left(S^{w}\left(A_{t}^{w}\right)+\mu\epsilon_{t+1}^{w}\right) && (14)\\
&= P^{w}\left(S^{w}\left(\left(P^{w}\right)^{-1}\left(P_{t}^{w}\right)\right)+\mu\epsilon_{t+1}^{w}\right), && (15)
\end{aligned}$$

from which equation (11) follows. μ is a scale parameter that allows a shift in world yield distribution.

In the following, we want to focus on economic mechanisms independent of structural differences between the economy and the rest of the world, so we consider a perfectly symmetrical situation in which the world market is calibrated with the same parameter values as the domestic country (so in the benchmark calibration $\mu = 1$). Given that we consider an asymptotic situation in which the rest of the world is infinitely larger than the country, trade does not affect the world equilibrium and the calibration does not need to account for any size difference. The symmetry between the rest of the world and the economy may seem arbitrary. For instance, the diversification of risks across the many countries that are part of the rest of the world could be assumed to lead to a lower variability of output. If the rest of the world is richer on average than the country considered, it could be argued also that price elasticity should be assumed to be smaller in the rest of the world. This would make sense, but this symmetry assumption means that international trade is not motivated by any structural difference, but only by the existence of country-specific production shocks uncorrelated to worldwide shocks. The sensitivity of the results to this assumption is analyzed in Section 6.6.

3.6 Calibration

The rational expectations storage model does not allow a closed-form solution. It has to be approximated numerically. The numerical algorithm that we use is based on a projection method and is described in detail in Appendix D.

The parameters are set such that, at the non-stochastic steady-state equilibrium, price, production, consumption and availability are equal to 1, and imports and exports are equal to 0 (see Table 1 for parameter values). As a result, the country is self-sufficient in the steady state and no trade takes place.

Table 1. Parameterization

Parameter	Economic interpretation	Assigned value
β	Annual discount factor	0.95
η	Income elasticity	0.5
α	Own-price demand elasticity	-0.4
γ	Commodity budget share	0.15
Y	Income	6.67
d	Normalization parameter of demand function	0.39
θ	Parameter defining risk aversion	-2.62
k	Physical storage cost	0.06
τ	Trade cost	0.2
μ	Normalization parameter of world yield distribution	1
ϵ^H, ϵ^w	Probability distribution of yield	$B\,(2,2)\cdot 0.5+0.75$

An annual interest rate of 5% is used for discounting. Based *inter alia* on Korinek and Sourdin (2010), we set trade costs to 20%. This is more than the average cost cited in this study for agricultural products, reflecting our focus on grains for poor countries.[4]

Seale and Regmi (2006) estimate elasticities for food consumption for 144 countries. From their research, we choose cereal elasticities typical of low-income countries: -0.4 for price elasticity and 0.5 for income elasticity. We assume that consumers spend, at the steady state, $\gamma = 15\%$ of their income on the staple, a value intermediate between what is observed for rice consumption in poor and affluent households in Asia (Asian Development Bank, 2008). Since steady-state consumption and price are equal to 1, income, which is assumed to be constant, is equal to the inverse of the commodity budget share, $1/\gamma$. We assume at the steady state a relative risk aversion parameter of 2, implying $\theta = -2.62$.

We follow Brennan (2003) and assume a per-unit storage cost of 6% of the steady-state price (i.e., $k = 0.06$). Combined with the opportunity cost, this physical storage cost entails an overall storage cost at steady state equal to 11.3% of the steady-state price. There is no evidence on which to base the choice of a particular probability distribution for yield at a country level. Thus, we assume that the random productions, ϵ^H and ϵ^w, follow a beta distribution. The beta distribution has the advantage of being empirically supported and popular in stochastic yield modeling at a local level (see, among others, Nelson and Preckel, 1989, Babcock and Hennessy, 1996), and of having a bounded support, which is computationally convenient. We assume the distribution to have shape parameters 2 and 2, which makes it unimodal at 0.5, and symmetric. The distribution is translated and rescaled to vary between 0.75 and 1.25, implying a coefficient of variation of 11.2%.

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[4] Noteworthily, international trade also frequently entails beyond-average domestic transport costs.

4 Dynamics without public policy

To understand the consequences of public policies, the situation without public intervention is a natural and useful benchmark. Since the model used here differs significantly from those discussed in the literature so far, we analyze this benchmark case in some detail.

4.1 Price, trade and storage behavior

For a small open economy, without any trade taxes, domestic prices necessarily lie in a moving band defined by world prices plus or minus trade costs. Compared to a closed economy, this context radically modifies storage behavior and its consequences. Abundant availability usually favors storage, but here exporting is another potential profitable outlet; when scarcity prevails, the stabilizing effect of selling inventories may be redundant in the face of the price ceiling imposed by import competition.

A first salient feature is that there is no storage when the country imports (see Figure 1(a)). Indeed, in this case, the domestic price is exactly equal to the world price plus trade costs, which allows rewriting equation (6) as follows

$$\beta \,\mathrm{E}_t \left(P_{t+1}\right) - P_t - k = \beta \,\mathrm{E}_t \left(P_{t+1}\right) - P_t^w - \tau - k. \tag{16}$$

Since the storage arbitrage condition (6) holds in the rest of the world (as assumed here given the way world prices are determined),

$$-P_t^w - k \leq -\beta \,\mathrm{E}_t \left(P_{t+1}^w\right), \tag{17}$$

which combined with (16) gives

$$\beta \,\mathrm{E}_t \left(P_{t+1}\right) - P_t - k \leq \beta \,\mathrm{E}_t \left(P_{t+1} - P_{t+1}^w\right) - \tau. \tag{18}$$

Given (9), the domestic price is always inferior to the import parity price, which also holds in expectations terms. As a result,

$$\beta \,\mathrm{E}_t \left(P_{t+1}\right) - P_t - k \leq \left(\beta - 1\right) \tau < 0. \tag{19}$$

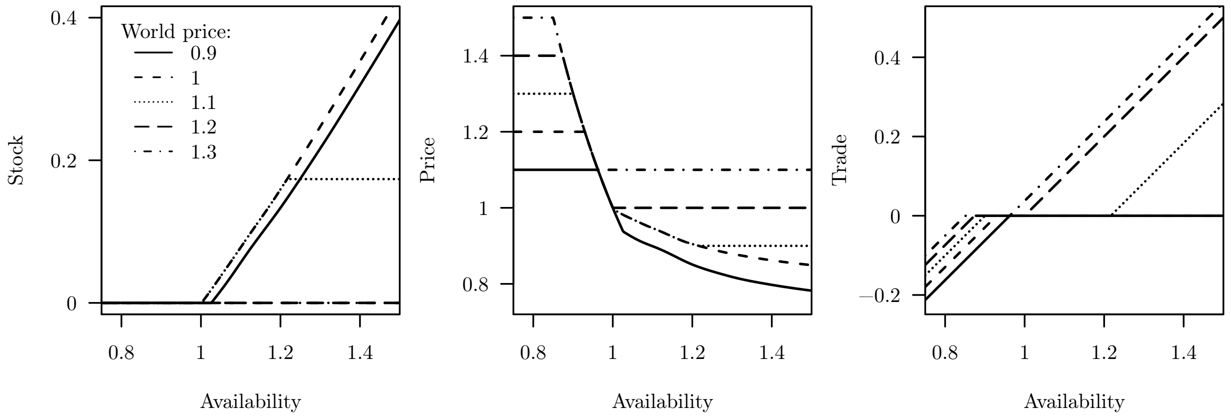
This implies that domestic storage is not profitable while importing, because the expected value of next year's difference between domestic and world prices cannot exceed trade costs.

This feature, emphasized *inter alia* by Williams and Wright (1991) in a two-country framework, reflects the fact that importing for storage never makes economic sense when intertemporal arbitrage is the same at home and abroad: deferring until the next year the decision about whether importing will be necessary or not is always preferable.

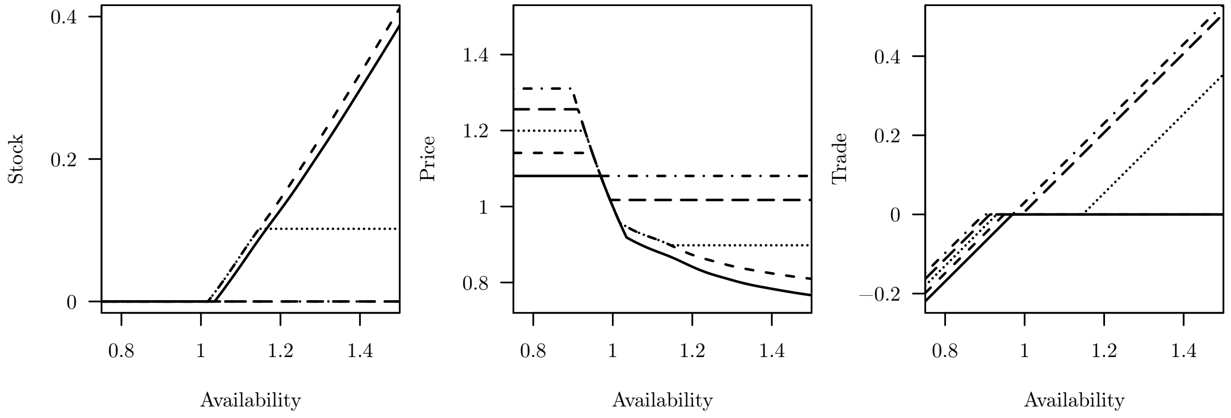
Storage and exports may coexist in cases where availability is relatively abundant. When the country exports, the domestic price equals the export parity price and the storage arbitrage equation becomes

$$S_t \geq 0 \quad \perp \quad \beta\, \mathrm{E}_t\left(P_{t+1}\right) - P_t^w + \tau - k \leq 0. \tag{20}$$

For a high enough world price, the expected domestic price cannot be so high as to make speculation profitable: exporting is more profitable than storing, and no storage takes place. The coexistence of storage and exports is only observed for intermediate world price levels: high enough compared to domestic prices as to make exporting profitable, but not so high as to make the first unit of storage less profitable than exporting.[5] The interrelations between storage and exports are illustrated also by the storage rule, where exports are reflected in flat storage curves for relatively large availabilities. In the left panel of Figure 1(a), this situation of non-zero storage is observed only with a world price equal to 1.1. As soon as the world price reaches 1.2, exporting is always more profitable than storing, so that the storage rule is flat throughout.



(a) Without public intervention



(b) Optimal trade policy

Figure 1. Stock, price and trade behavior. Negative trade values refer to imports.

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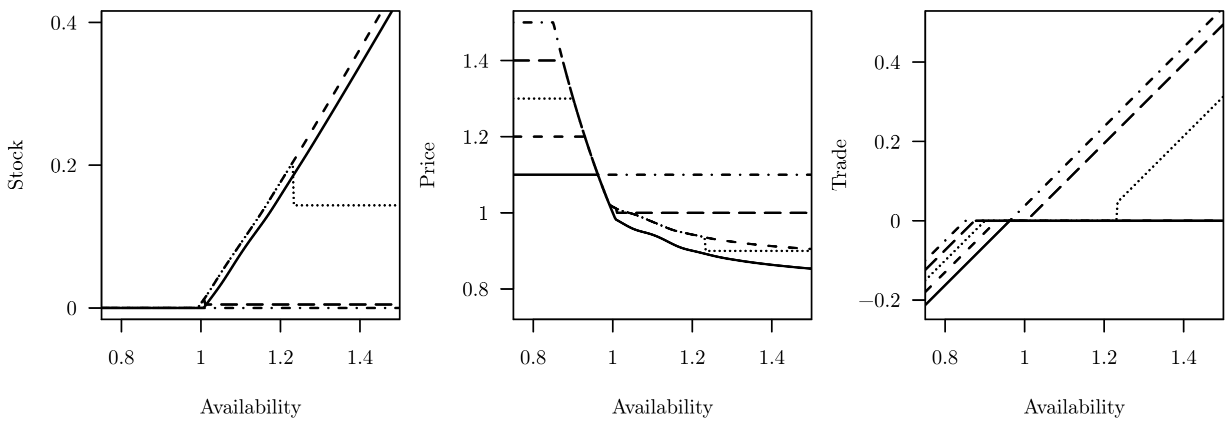
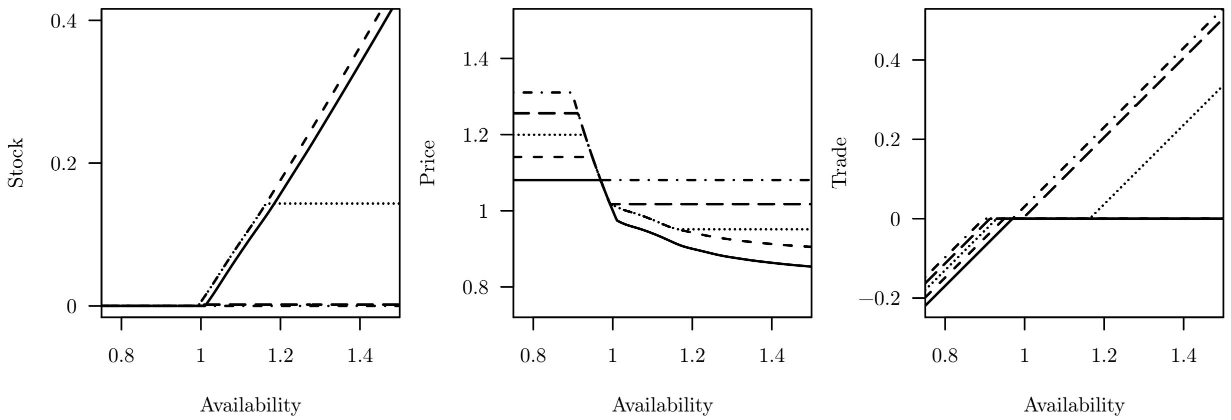
[5] The returns to storage are declining due to its negative influence on expected future domestic prices



(c) Optimal storage subsidy



(d) Optimal policy with both instruments

Figure 1. (*continued*)

Notably, the current world price affects domestic storage even in the absence of trade. Indeed, world prices are positively autocorrelated since they are generated by a storage model. As a result, a higher current world price entails higher world price expectations for the next period. Accordingly, storage outside trade situations moves slightly upward for higher world prices.

Under the assumption of a small economy, exporting implies a complete disconnect between domestic prices and availability, as reflected in the flat segments of the price curves observed for large enough availabilities for world prices equal to 1.1 and above, in the central panel of Figure 1(a). Similarly, for limited availabilities, the domestic price is disconnected from availability when it reaches the import trigger price, equal to the world price plus trade costs. In between these two cases, the price curve takes a standard form in the presence of storage, with a strongly downward sloping curve for availabilities below a given threshold under which no storage takes place, and a smoother curve thereafter. As far as trade is concerned, assuming exogenous world prices implies that the net trade curve has a unitary slope whenever trade is not zero.

A sample simulation of world and domestic prices illustrates the link between them (Figure 2). The domestic price tends to be set to the import parity price when the world price is low, and to the export parity price when the world price is high. Most world price spikes are imported through trade to the domestic market.

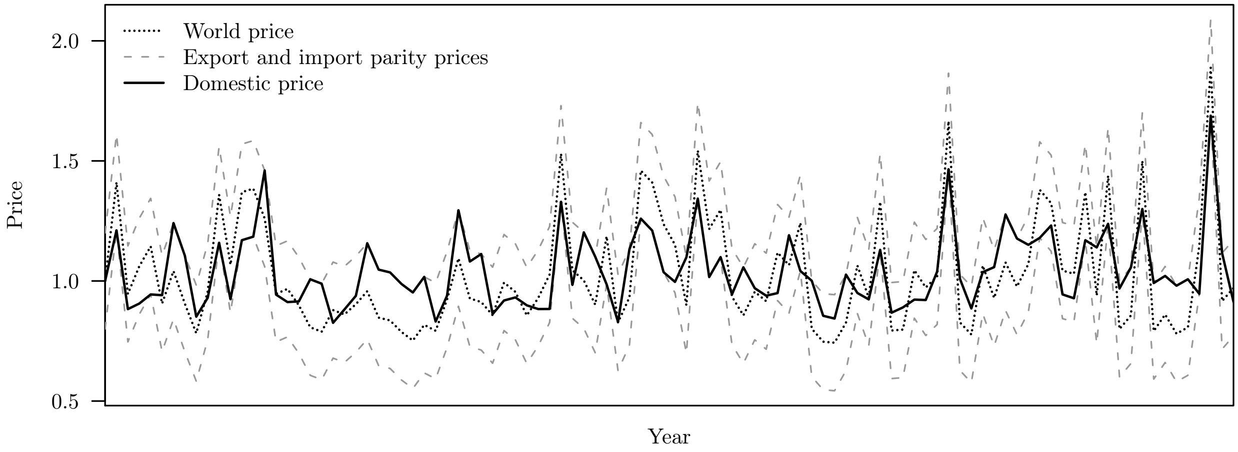


Figure 2. Simulated history of prices without public intervention

4.2 Effects of trade and storage costs

While commodity trade is usually seen as a way of smoothing production shocks in different markets, in the present case, trade does not necessarily decrease price volatility. Actually, price instability is the same when trade costs are zero and when they are prohibitive: in the first case, the domestic price is exactly equal to the world price, while in the second case the economy behaves as if it were a closed economy. Both situations are equivalent since we assume that world prices result from a closed economy behavior, with the same parameters as those used for the economy studied.

When trade is costless, however, the strict link with world prices means that only storage in the world market matters, and that storage in the small economy does not matter. Under prohibitive trade costs, in contrast, only domestic storage matters. Intermediate cases exhibit less volatility, with domestic and foreign storage combining to smooth price variability: domestic shortages can be alleviated by imports while surpluses can be exported in the world market, but the country remains partly shielded from world prices by trade costs. The result is a U-shaped relationship between trade costs and volatility (Figure 3, left panel). Volatility is lowest for trade costs close to 0.23.[6]

This result contrasts with the influence of the domestic physical storage cost, which always increases domestic price instability (Figure 3, right panel). For prohibitive storage costs, instability is slightly lower than in the world market since trade costs partly insulate the country from worldwide instability.

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[6] Numerical results here and in the following sections are calculated over 1,000,000 sample observations from the asymptotic distribution.

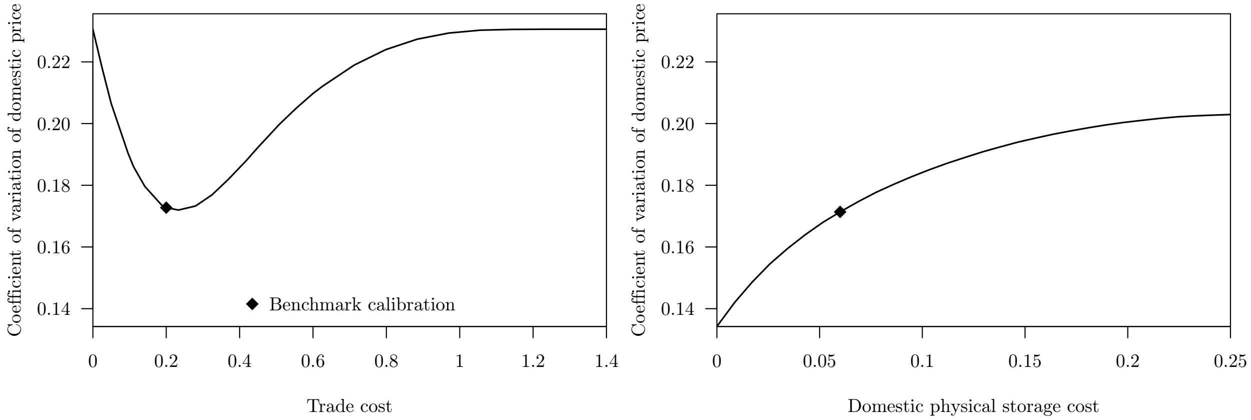

Figure 3. Dependence of price instability on trade and storage costs. The value of storage cost is maintained equal to the benchmark when analyzing the influence of the trade cost, and reciprocally.

For zero physical storage costs, instability is still significant for various reasons: storage is profit seeking and entails opportunity costs, so it will not provide for complete stabilization; and there is some imported instability.

5 Optimal stabilization policy

In modeling the relationships between storage and trade policies, we do not want only to analyze specific cases, we want also to assess what would be the optimal use of these policies. To do this we assume that government cannot commit to future policies and has to follow time-consistent policies. Dynamic programming is used to derive the Markov-perfect equilibrium associated with discretionary policy. To design such optimal stabilization policy, we need to formulate a meaningful objective. The optimization problem of the discretionary optimal policy can then be stated.

5.1 Social welfare function

Policy design is usually based on maximizing the sum of all agents' surpluses. This standard practice is not valid here, since expected surplus is not a suitable measure of risk-averse consumers' welfare. Helms (1985b) shows that, in contrast, *ex ante* equivalent variations are meaningful welfare indicators. This corresponds to the amount of income that, in the price regime without intervention, would bring the same change in expected utility as the intervention considered here. In the intertemporal framework considered here, where savings are not taken into account, the temporal allocation of this equivalent income is not neutral.[7] Assuming equivalent variation to take the form of a constant

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[7] Gollier (2010) shows, in a different context, how savings behavior ensures equivalence between alternative patterns of allocation of replacement income over time.

income flow, the corresponding amount EV_t is implicitly defined, at period t, by

$$\mathrm{E}_t\left\{\sum_{i=0}^{\infty}\beta^i\left[v\left(\tilde{P}^t_{t+i},Y+EV_t\right)-v\left(P_{t+i},Y\right)\right]\right\}=0, \tag{21}$$

where $\tilde{P}^t_{t+i}$ refers to price in period $t+i$ when policy intervention is stopped from t onward. A first-order Taylor series expansion of the first term around the path followed without further intervention then gives

$$EV_t\approx\frac{1}{w_t}\,\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[v\left(P_{t+i},Y\right)-v\left(\tilde{P}^t_{t+i},Y\right)\right], \tag{22}$$

where $w_t=\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i v_Y\left(\tilde{P}^t_{t+i},Y\right)/\left(1-\beta\right)$ is the discounted average of the future marginal utility of income, in expected terms. Social welfare can be measured combining this *ex ante* equivalent variation for consumers with the surplus of other agents:[8]

$$\begin{aligned}W_t=&\frac{1}{w_t}\,\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[v\left(P_{t+i},Y\right)-v\left(\tilde{P}^t_{t+i},Y\right)\right]\\&+\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[P_{t+i}\epsilon^H_{t+i}+P_{t+i}S_{t+i-1}-\left(P_{t+i}+k-\zeta_{t+i}\right)S_{t+i}-Cost_{t+i}\right],\end{aligned} \tag{23}$$

where $Cost_{t+i}$ denotes period $t+i$ fiscal cost of public policies.

We neglect the distortionary cost caused by revenue collection, so that the fiscal cost of policy intervention is the cost of subsidizing private storage plus the net tax cost of trade policy:

$$Cost_t=\zeta_t S_t-\nu^M_t M_t-\nu^X_t X_t. \tag{24}$$

Using (10) and (24), the social welfare can be simplified to

$$W_t=\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left\{\frac{v\left(P_{t+i},Y\right)-v\left(\tilde{P}^t_{t+i},Y\right)}{w_t}+P_{t+i}A_{t+i}-\left(P_{t+i}+k\right)S_{t+i}+\nu^M_{t+i}M_{t+i}+\nu^X_{t+i}X_{t+i}\right\}. \tag{25}$$

The per-unit storage subsidy enters positively in private agents' profit and negatively in public cost, so it does not feature directly in the social welfare function.

While theoretically exact, this formulation of social welfare is not easily tractable in a dynamic framework. In particular, it is not amenable to a standard recursive dynamic specification, due to the time variability of w_t. However, this time variability of the expected marginal utility of income is limited in practice, and remains of second order.[9] Numerical simulations can thus

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[8] Note that, were income elasticity and relative risk aversion equal to zero, this definition would make w_t equal to 1, so that the social welfare function below would actually be the classic sum of the surpluses.

[9] In the simulations presented below, w_t does not vary by more than one percent over time.

be greatly simplified by assuming w_t to be constant over time and equal to its value at $t = 0$: $w_t \approx \mathrm{E}_0 \sum_{i=0}^{\infty} \beta^i v_Y \left(\tilde{P}_i^0, Y\right) / (1-\beta) = w$. This assumption is made throughout the simulations presented below.

5.2 Optimization problem

The social welfare function is a natural objective for policy optimization. In stating this problem, policy is assumed to start at period 0 and to be unanticipated. Commitment is unlikely in most countries and especially in developing countries; the policy is thus assumed to be discretionary. Three state-contingent instruments can be used to stabilize prices: a tax on, or a subsidy for private storage, and a trade policy (import and/or export tax or subsidy). Initial state variables are taken as being at their non-stochastic steady-state level (i.e., $A_0 = 1$ and $P_0^w = 1$) and initial stocks are assumed to be null.

Although subsidies to private storage have been used (often in the form of interest rate subsidies, see Gardner and López, 1996), storage policies usually take the form of public storage. In many countries, public storage were undertaken by parastatals that were given a monopoly on trade and storage of grains (Rashid et al., 2008). An optimal subsidy to private storage can be interpreted as the losses incurred by an efficient monopolistic public agency following an optimal storage rule. In addition, our focus on storage subsidy removes the need to study the interaction between private and public storage, which removes a numerical burden.

At each period, optimization entails maximizing the expected sum of the discounted social welfare function subject to the constraints imposed by private agents' behavior and the market equilibrium. The optimization is carried out over current endogenous and control variables, taking future variables as given:[10]

$$\max_{\substack{S_t \geq 0, P_t, M_t \geq 0, \\ X_t \geq 0, A_{t+1}, \zeta_t, \nu_t^M, \nu_t^X}} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ v\left(P_{t+i}, Y\right) + w\left[P_{t+i} A_{t+i} - \left(P_{t+i} + k\right) S_{t+i} + \nu_{t+i}^M M_{t+i} + \nu_{t+i}^X X_{t+i}\right]\right\} \tag{26}$$

subject to equations (6)–(8) and (10)–(12), A_t and P_t^w given, and anticipating $\left\{S_{t+i}, P_{t+i}, M_{t+i}, X_{t+i}, A_{t+i+1}, \zeta_{t+i}, \nu_{t+i}^M, \nu_{t+i}^X, P_{t+i}^w\right\}$ for $i \geq 1$.

The above problem defines an optimal stabilization policy using both storage and trade taxes or subsidies. A policy using only one of the two instruments can be defined easily by removing from the objective and constraints all occurrences of ζ to define a trade policy, or of ν^M and ν^X to define a storage policy. In each case, solving the problem requires reformulating the constraints defined as complementary equations and writing the dynamic programming problem. See appendix for details.

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[10] This objective function is obtained, from (25), by multiplying the social welfare function by w, and subtracting the term independent from policy choice reflecting consumer's utility in the situation without intervention, $v\left(\tilde{P}_{t+i}^t, Y\right)$.

6 Characterization and consequences of optimal public interventions

The analytical framework laid out above allows optimal public interventions to be characterized. Three cases are considered here, corresponding to: optimal use of trade policy alone, of storage policy alone, and of both policies jointly. We first describe the nature of these policy interventions, and analyze their consequences for decision rules. We then assess the resulting consequences for welfare.

6.1 Optimal trade policy

In the absence of any storage policy, the optimal use of trade policy can be characterized mainly as comprising two types of interventions. The first consists of subsidizing imports when availability is low. As illustrated in the central panel of Figure 1(b), the corresponding subsidies can be substantial, with powerful trimming impacts on the upper tail of the price distribution. Thus, for a small open economy, import subsidies are an efficient way to avoid very high prices: despite their cost, *ad valorem* subsidies higher than 15% are shown to be optimal when domestic scarcity coincides with high world prices. The use of import subsidies may raise questions, since this instrument is not commonly used in practice. They appear in our framework because our benchmark situation is one of free trade. In practice, even structural food importers tend to maintain positive barriers to trade, motivated by political economy reasons (Grossman and Helpman, 1994) or the need to collect fiscal revenue. Given this alternative benchmark of positive tariffs, import subsidies could be interpreted as a tariff reduction for which we have much evidence (Anderson and Nelgen, 2012).

The second type of intervention consists of taxing exports when availability is abundant and world prices are high. Such interventions decrease the export parity price and avoid importing price spikes in the world market, through exports. The corresponding tax levels remain moderate, typically less than 5% even for world prices as high as 1.4.

For high availability and a medium, although sufficiently high to make export profitable, world price, such as 1.2, optimal trade policy consists of subsidizing exports. The risk aversion of consumers does not compel to trim high prices, but to reduce the variability around the mean. For a world price equal to 1.2, the export parity price is 1, which is just below the mean price (Table 2). Slightly subsidizing exports in this situation raises the domestic price closer to its mean.

For the asymptotic distribution of prices, such an optimal trade policy results in a decline in the mean (by 2.4%) and a 20% decrease in the standard deviation. This reduced variability is strongly asymmetrical, as illustrated by changes in quantiles: it stems mainly from a heavy reduction in the high prices obtained thanks to import subsidies and export taxes (the top percentile of the price distribution is driven down from 1.63 without policy to 1.41, a cut of one-third in the deviation from the average price). The asymmetry typical of price distributions for storable commodities is greatly reduced, with a skewness half that of the benchmark.

Table 2. Descriptive statistics on the asymptotic distribution

	Benchmark	Trade policy	Storage subsidy	Both instruments
Price				
Mean	1.045	1.020	1.054	1.034
Coefficient of variation	0.173	0.141	0.159	0.121
Skewness	1.248	0.641	1.524	0.995
Correlation coefficient				
with domestic shocks	−0.474	−0.569	−0.414	−0.482
with world price	0.788	0.729	0.807	0.780
Quantiles				
1%	0.790	0.787	0.837	0.837
25%	0.915	0.900	0.940	0.942
50%	1.000	1.015	1.001	1.008
75%	1.131	1.107	1.123	1.100
99%	1.628	1.410	1.625	1.406
Mean stocks	0.033	0.027	0.048	0.047
Mean imports	0.018	0.021	0.016	0.018
Mean export	0.028	0.024	0.031	0.028

Not surprisingly, these trade policy interventions result in increased imports and decreased exports. Trimming the upper tail of the price distribution also reduces the profitability of storage. As a result, the storage rule is moved to the right, and the mean stock level is reduced by approximately 20% in the asymptotic distribution (Table 2).

6.2 Optimal storage policy

In a closed economy, an optimal storage policy motivated by consumer risk aversion under incomplete insurance markets increases the level of storage compared to the situation without intervention (Gouel, 2011). In the present context of a small open economy without any trade policy intervention, this is true when storage does not compete directly with exporting, i.e., when the domestic price remains above the export parity price, because availability is limited and/or the world price is not too high. In this case, the storage subsidy increases storage and domestic price, contributing to smoothing of intertemporal price variability in the domestic market.

When availability is abundant and the world price is high enough (e.g., $P^w \geq 1.3$), exporting is significantly more profitable than storage. This does not leave any room for a worthwhile storage policy.

The storage subsidy is positive when the current social marginal value of availability (i.e., the Lagrange multiplier in the market clearing equation (12), χ_t in appendix) is lower than its discounted, expected future value. For a world price equal to 1.1 and for intermediate availabilities (between 1 and 1.2), storage takes place when exports are not profitable (see Figure 1(c)). In this situation, the small excess of availability with respect to steady-state consumption leads to a present social marginal value of availability lower than its expected discounted value and thus to a positive subsidy for storage. However, as soon as the domestic price reaches the export parity price, the social marginal value of availability jumps to zero (when there is some trade, the market clearing equation is not binding, since foreign demand and supply are infinitely elastic). Since stock levels are already high (around 10% of steady-state consumption), the expected future marginal value of availability is negative in this situation of glut, it then becomes optimal to tax storage. Hence the discontinuity observed in the storage rule and in the export curve for an availability around 1.1. Beyond this level, storage coexists with trade, and the optimal policy remains a storage tax (worth approximately 3%). This outcome is paradoxical: while public intervention is motivated by consumer risk aversion, it tends to discourage storage. Intuitively, this is a context of overly abundant availability, where dispensing with it through exports is socially preferable to retaining it through storage, even if the storer would break even to do so.

When storage and export coexist, there is not always a storage tax. For high world prices and, therefore, relatively high domestic prices, storage profitability is quickly driven down to zero. However, the expected social marginal value of availability is positive in this situation, which justifies subsidizing storage.

The impact of such an optimal storage policy on the asymptotic price distribution is strongly constrained by foreign trade: as soon as the country exports or imports, its domestic price is determined by the world price, since no trade policy intervention is assumed to take place. The two main channels through which this optimal storage policy influences domestic prices are as follows: storage subsidies increase domestic prices in situations of stock accumulation, when above-average availability is combined with low to intermediate world price, with the exception of the above-mentioned case of storage tax, when storage coexists with exports; ensuing higher stock levels decrease prices when stocks are sold, but this situation often coincides with situations of international trade, thus limiting the price fall.

As a result, an optimal public storage policy increases the mean price of the asymptotic distribution (see Table 2). This seems paradoxical since the standard result obtained for a closed economy is that introducing private storage or increasing levels of storage through public intervention depresses average prices (Wright and Williams, 1982a, Miranda and Helmberger, 1988, Gouel, 2011). To interpret this puzzling result, it is useful to consider such a stabilization policy through storage as a sequence of transfers of demand from one period t_1 to another t_2, when the price is higher ($p_1 < p_2$), as suggested by Newbery and Stiglitz (1981, p. 251, theorem 2). Consumers' demand is reduced by any additional storage, and it is increased by the same amount (in the absence of spoilage) when the quantity stored is finally sold. To see how demand transfer influences prices, let us differentiate the market clearing equation (12) with regard to stocks:

$$D'\left(P_t\right)\frac{\partial P_t}{\partial S_t}+1+\frac{\partial (X_t-M_t)}{\partial S_t}=0. \tag{27}$$

In a closed economy, the last term is pointless, and this equation collapses to $\partial P_t/\partial S_t = -1/D'(P_t)$. As soon as the demand function is convex, as in case of a constant elasticity function, this partial derivative is an increasing function of prices, meaning that it is larger in t_2 than in t_1. As a result, modifying storage to operate a small transfer of consumption from t_1 to t_2 would cut the price in t_2 by more than it would increase it in t_1. This explains why stabilization through additional storage usually depresses mean prices in a closed economy.

In an open economy, in contrast, the last term on the left hand side of (27) is not zero. In particular, it may be the case that the country exports when the consumption transfer takes place. In this case, a small enough additional storage will not drive exports down to zero, meaning that they will not change the domestic price, which will remain equal to the world price minus transport cost. Conversely, when there is trade, selling additional stocks does not depress the domestic price. This latter effect dominates in practice, mainly because it means that the country cannot insulate its market from episodes of very high world prices. In this case, domestic stocks are sold on the world market (either directly or indirectly, when domestic consumption of domestic stocks displaces imports). This is profitable given the high price level, but it does not curb domestic prices. Hence, the limited efficiency of a storage policy for avoiding high-price episodes.

While this optimal storage policy reduces the standard deviation of prices by approximately 7%, stabilization comes only from the increase in low prices, a paradoxical result for a public intervention linked to consumer welfare. The upper quantiles of the asymptotic price distribution hardly change compared to the benchmark, while the first percentile increases by 6% (Table 2).

The impact on trade is not trivial. The significantly higher average stock reduces the frequency of scarce domestic availability and of the associated large imports. On average, imports are reduced by 11%. For the same reason, abundant availabilities are more frequent, and they increase exports. When storage coincides with exports, this export-enhancing effect is magnified by the storage tax mentioned above, which reduces the demand for storage, thus increasing the volumes of domestic output absorbed by the world market. On average, exports increase by 11%. This increased importance of exports is also a strong driver of domestic price variability, as illustrated by the increased correlation between domestic and world prices. On the whole, and despite the absence of trade policy, this policy could be called opportunistic in trade terms, to the extent that the country tends to favor storage when world prices are low, but to discourage it when they are high enough.

6.3 Optimal trade and storage policy

Combining optimally trade and storage policies allows a powerful stabilization policy to be devised, with the standard deviation of domestic prices cut by more than a quarter compared to the benchmark. This is not surprising given that trade policy is very efficient at preventing domestic prices from reaching very high levels, while storage is a powerful tool to avoid excessively low prices. The basics of an optimal policy mix thus consist of using import subsidies to trim the upper tail of the distribution of domestic prices, and storage subsidies to trim the lower tail. The reduced variability of domestic prices comes from both ends of the distribution, which get substantially closer to the mean than in the benchmark (see quantiles in Table 2). This outcome is comparable to the outcome obtained with storage policy for the lower quantiles, and with trade policy for upper quantiles.

The instruments are not independent though, and their interrelationships are especially strong when the country both exports and stores, as is the case of abundant availability under intermediate world prices (1.1 in Figure 1(d)). When both instruments are combined, the optimal policy consists of subsidizing exports while subsidizing storage (for a world price of 1.1, the per-unit subsidy ζ is 0.04, or 4% of the steady-state domestic price). Despite lower stocks, the result is a domestic price beyond the price that would prevail without intervention, i.e., closer to the distribution mean. This shift thus contributes to limiting price variability. For a higher world price, however, private storage is still subsidized, but exports are taxed, which prevents domestic prices from reaching excessive levels.

The mean price of the asymptotic distribution is slightly lower than in the benchmark, an outcome that is between those for trade and storage policies. It is intermediate also in terms of the correlation of domestic prices with world prices and with domestic shocks. The impact on foreign trade appears intermediate between the impacts for each instrument individually, with a slight increase in mean imports and a slight decrease in mean exports. While interventionist, and entailing potentially significant export taxes, such an optimal policy is thus not trade-restrictive on average.

This policy leads to an average price transmission elasticity of 0.63 (obtained by regressing the logarithm of domestic price on the logarithm of border price P^{T} for non zero trade, see Appendix A for P^{T} definition). This elasticity is consistent with econometric evidence. For instance, Anderson and Nelgen (2012) find that for 75 countries the average short-run price transmission elasticities are 0.52, 0.47, 0.57 and 0.72, respectively, for rice, wheat, maize and soybean. However, contrary to what happened during the recent food crisis, the optimal policy never involves a complete export ban.

6.4 Decomposition of welfare changes

By changing the distributions of prices, stabilization policies transfer both risk and resources across agents. To understand the distributive effects of these policies, welfare change can be decomposed for each agent into efficiency and transfer changes, where only the former matter for the economy as a whole.[11] This is done here for expected values at period 0.

Consumer gains are given by the *ex ante* per-period equivalent variation, EV_0, which includes two components: a transfer term corresponding to the change in mean expenditure, which exactly matches the opposite change in the average revenues of the other agents, and an efficiency term corresponding to risk benefits and changes in mean consumption. There is no closed-form solution for the efficiency term, which, following Newbery and Stiglitz (1981), is defined by the difference

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[11] See Newbery and Stiglitz (1981, Ch. 6 and 9) for a discussion of efficiency and transfer gains from price stabilization.

between the equivalent variation and the change in mean expenditures:

$$EV_0 + (1-\beta)\,\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \, \Delta \left[P_t D\left(P_t\right) \right], \tag{28}$$

where the operator Δ before an expression refers to the difference between its value after and before public stabilization.

Changes in storers' profit, defined by equation (4), contain an efficiency term representing changes in storage costs, $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t k \, \Delta \, S_t$, and two transfer terms: $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \, \Delta[P_t(S_{t-1} - S_t)]$, a transfer from other private agents, and $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \zeta_t S_t$, a transfer from the government.

Producers are not represented explicitly since production is defined by exogenous stochastic shocks but they are affected by policies, through price changes. Their welfare change is a transfer term defined by their average change in benefits: $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \, \Delta \left(P_t \epsilon_t^H \right)$.

International trade can be decomposed into four effects on an agent labeled "shipper" (in Table 3). The mean change in imports and exports valued at domestic price, $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \, \Delta[P_t(M_t - X_t)]$, is a transfer from private domestic agents; the mean change in trade costs, $\tau \, \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \, \Delta(M_t + X_t)$, is an efficiency term; the mean change of imports and exports valued at the world price, $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t$ $\Delta[P_t^w(X_t - M_t)]$, which we call trade balance, is an efficiency term accounting for the possibility to buy foreign goods allowed by an export surplus in the food commodity; and there is a transfer from government through the trade policy instrument.

Government is only involved in transfers to private storers through subsidies and to the shipper through trade policy.

This decomposition is illustrated for the three optimal policies in Table 3.[12] It is striking that total gains are small in comparison to transfers: to protect consumers from price fluctuations, public intervention induces comparatively large changes for other agents. These transfers stem mainly from two effects. The first one is the change in the mean price. A lower mean price, as a result of optimal use of trade policy or both instruments, benefits consumers at the expense of producers. The reverse is true under storage policy alone. Changes in the covariance between prices and production shocks also originate transfers between producers and consumers but their importance is more limited. Changes in the foreign trade balance valued at domestic prices are the second main source of transfers, from shipper to consumers when the balance increases. Once trade costs and possible taxes are taken into account, changes in the trade balance affect the consumer, not the shipper (whose profit is assumed to remain zero throughout). The fiscal cost of policies is an additional source of transfer: trade policy intervention generates fiscal revenue, while storage subsidies entail costs. While the magnitude of these fiscal effects is limited compared to other effects, the cost of a storage policy is not negligible.

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[12] Welfare terms are all discounted infinite sums. They are calculated by transformation to a recursive formulation and value function iteration.

Table 3. Decomposition of welfare impacts of optimal policies on transitional dynamics (as percentage of the steady-state commodity budget share.)

	Trade policy	Storage subsidy	Both instruments
Consumers	2.47	−0.94	1.05
Efficiency[a]	1.05	−0.35	0.52
Consumption expenditures[b]	1.42	−0.59	0.53
Producers[b]	−2.53	1.08	−0.92
Storers	0	0	0
Transfers[b]	−0.03	−0.04	−0.10
Storage costs[a]	0.03	−0.08	−0.08
Storage subsidy[b]	–	0.12	0.17
Shipper	0	0	0
Transfers[b]	1.15	−0.45	0.49
Trade costs[a]	0.03	−0.02	0.01
Trade balance[a]	−1.06	0.47	−0.35
Trade policy[b]	−0.12	–	−0.14
Government	0.12	−0.12	−0.03
Storage subsidy[b]	–	−0.12	−0.17
Trade policy[b]	0.12	–	0.14
Total	0.06	0.03	0.10

[a] Efficiency terms. Total gains are equal to their sum.

[b] Transfer terms. They sum to zero, except for the numerical approximations, and do not contribute to total gains.

Efficiency gains stem from four terms. Two of them, storage and trade costs, are comparatively small in all cases. Changes in storage costs are positive for trade policy, reflecting the decrease in storage resulting from trade policy intervention. For the other two policies, they are negative as storage increases. Most efficiency effects correspond to the remaining two effects, namely consumers' efficiency gains and trade balance changes. Consumers' efficiency gains are comprised of two components, not decomposed in Table 3: the gains originating in the change in mean consumption (i.e., the traditional welfare triangle in a surplus analysis); and the reduction in the risk premium. This latter is necessarily positive since instability decreases with all policies, while the former may be positive or negative depending on the mean price change, which reveals the mean consumption change. With respect to the distribution of these various gains, the policies considered stand in stark contrast.

An optimal storage subsidy without trade policy intervention has counter-intuitive impacts. While public intervention is motivated by consumers' risk aversion, it actually results in efficiency losses for consumers: decreased price volatility is more than compensated by reduced mean consumption due to higher prices. The policy is still beneficial socially, because the risk-premium decreases, but it does not increase consumers' welfare. In the absence of trade policy, a storage subsidy makes consumers worse off, since stocks are accumulated mostly when prices are affected by stock accumulation, but sold when the economy is connected to the world market (see the effect on quantiles in Table 2).

With an optimal trade policy, consumers enjoy significant efficiency gains, resulting from decreases in both the mean price and price variability, in particular from less frequent price spikes; but this result is at the cost of a significantly deteriorated trade balance. The order of magnitude of both effects is 1%, with a net gain to the economy worth 0.06%.

While intermediate between single-instrument policies, the impacts of an optimal combination of trade and storage policies are closer to those found for trade policy. Consumers enjoy efficiency gains, mainly reflecting the policy's effectiveness in preventing price spikes, and the cost is a deterioration in the trade balance. However, public intervention is more effective in this case: while consumers' efficiency gains do not reach 1%, total social gains reach 0.1%. With smaller transfers, this policy achieves more gains. This finding illustrates the strong complementarity between trade and storage policies.

6.5 Consequences of a discipline on export restrictions

The use of trade policies to cope with price volatility is potentially problematic because these are non-cooperative policies, with potential negative consequences for partner countries. In particular, when a country uses trade policy to insulate its domestic market from extreme world prices values, its action may further increase the level of world prices. If all trading countries apply this policy, the collective-action problem may reach the point where individual countries' efforts to insulate themselves from high world prices cancel each other out (Martin and Anderson, 2012). The use of export restrictions during the 2007–08 food crisis epitomizes this concern; it played a significant role in price upsurges and spurred calls for multilateral disciplines in the use of such instruments (Mitra and Josling, 2009, is an example).

The small-open economy setup adopted here does not allow explicit analysis of the problem in terms of international cooperation. Nonetheless, this problem is rooted in distorted excess supply curves in countries undertaking price-insulation policies, and these curves can be characterized here based on the expected value of net exports, for a given world price level. Based on the asymptotic distribution, these curves are plotted in Figure 4 for the benchmark case of no policy intervention and under different optimal policy settings. Since policy intervention is justified here by consumers' risk aversion, while assuming away producers' risk aversion, the most meaningful deviations correspond to high world price levels. An optimal trade policy alone decreases excess supply for all world prices, and the deviation is significant for world prices above 1.2, thus confirming the prior that the use of trade policy may increase high world price episodes. When trade policy is optimally combined with storage policy, though, deviations from the benchmark are less pronounced. Expected excess supply is significantly reduced only for world prices above 1.35, and it is slightly increased for intermediate world price levels, due to the positive effect of storage policy on asymptotic availability. The latter effect explains why excess supply is always increased when only storage policy is applied.

These results confirm that trade policy intervention may significantly distort excess supply curves. This is especially true for high prices, reflecting the use of export taxes. Considering multilateral disciplines on export restrictions thus makes sense. To evaluate what would be at stake in the case of such disciplines, we assess the consequences, for a developing country, of deviating from its optimal

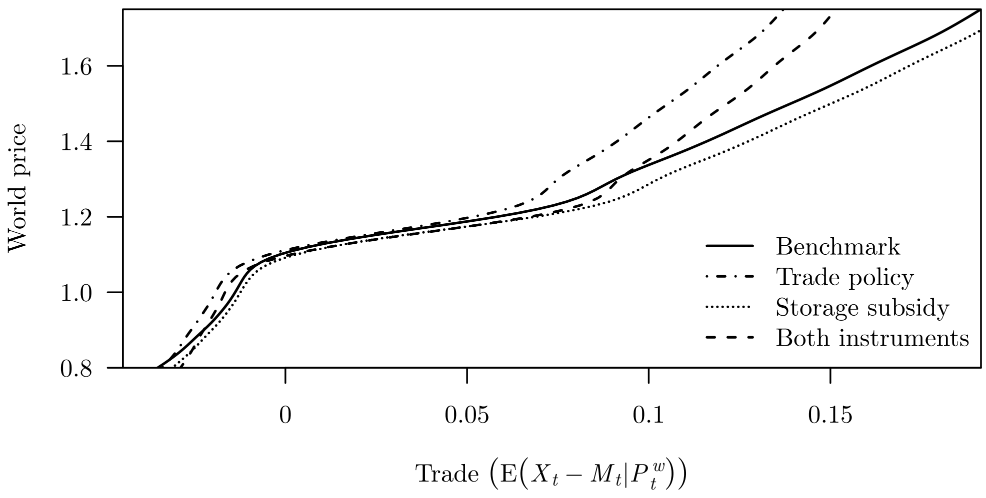

Figure 4. Asymptotic excess supply curves. Smoothing splines of the trade-world price relationship over 1,000,000 observations of the asymptotic distribution.

policy by committing not to use any export restrictions (given existing disciplines, we also assume export subsidies to be prohibited).

Banning export taxes prevents the country from insulating against high world prices when domestic availability is large enough. This substantially increases the occurrence of high domestic prices, with an upper tail of the price distribution closer to the no-intervention benchmark than to the undisciplined optimal policy (the 1, 25, 50 ,75 and 99 quantiles are respectively at 0.84, 0.94, 1.00, 1.11 and 1.61). Since the optimal storage subsidy decreases the occurrence of low prices by increasing stock levels, the overall effect is a slight increase in the mean price (up to 1.048), compared to the benchmark, rather than a decline resulting from an undisciplined optimal policy.

With a limited decrease in volatility (the coefficient of variation of domestic price is 0.15), welfare effects on consumers are dominated by the increased mean price, so that consumers lose as a result of this policy, as in the case of only a storage policy. Export disciplines thus entail significant transfers from consumers to producers. Overall, the gains for the whole economy are more than halved compared to the undisciplined optimal policy. In sum, export taxes appear to be key components of price-stabilizing policies that benefit consumers: the stakes of possible multilateral disciplines on export restrictions would be high for a number of poor countries.

6.6 Optimal policy when the country is not self-sufficient on average

So far, we have considered a situation where the country trades with the rest of the world because yield shocks are distinct in the two regions. Now we consider in addition that the mean price in autarky differs from the mean world price, so that trade will take place even in the absence of stochastic shocks. For convenience, the change is implemented through μ, which parameterizes the world yield distribution (see equation (14)). Its default value is 1 and we consider two alternatives, 0.9

and 1.1—a 10% increase or decrease in the world mean production—where the country is respectively a structural exporter or a structural importer.

For $\mu = 0.9$ the deterministic steady-state price in the world market is equal to 1.3, whereas in autarky the steady-state price is still 1 in the domestic market. Thus, when open to trade, the country exports and has a mean price higher than its mean autarkic price (see Table 4). Storage behavior is consistent with what was previously described for the trade/storage relationship: the mean stock level is very low when the country is an importer, and is much higher if it is an exporter. At the aggregate level, the parameter μ does not seem to affect the optimal policy pattern, which consistently increases stock levels and produces very limited changes in average trade. As mentioned above, the optimal policy's trade effect consists of distorting the distribution of trade, not in isolating the economy from the world market, which would be too costly. In the benchmark case, the distribution distortion is achieved mainly through the use of import subsidies and export taxes. With $\mu = 0.9$, export taxes and subsidies become the most important trade policy instruments. For prices higher than the mean price, export restrictions are used to avoid importing high prices from the world market. For prices below the mean price, exports are subsidized so as to move domestic price closer to its mean. For $\mu = 1.1$, the behavior is similar but applied to imports. Imports are subsidized when the domestic price is above its mean and taxed when it is below it. So the optimal behavior is to exploit the world market so as to stabilize the domestic price around its mean.

Table 4. Sensitivity to the assumption of self-sufficiency

	Structural exporter (μ =0.9)		Self-sufficient (μ =1)[a]		Structural importer (μ =1.1)	
	No Policy	Optimal Policy	No Policy	Optimal Policy	No Policy	Optimal Policy
Descriptive statistics						
Mean price	1.230	1.208	1.045	1.034	0.915	0.913
CV of price	0.216	0.134	0.173	0.121	0.156	0.109
Mean stocks	0.090	0.120	0.033	0.047	0.013	0.020
Mean imports	0.003	0.004	0.018	0.018	0.050	0.048
Mean exports	0.074	0.074	0.028	0.028	0.008	0.008
Welfare						
Consumers gains		1.34		1.05		0.25
Producers gains		−1.38		−0.92		−0.10
Government		0.27		−0.03		−0.09
Total gains		0.23		0.10		0.07

Notes: Columns No Policy and Optimal Policy display respectively results for the model without public intervention and for the optimal policy using the two instruments.
[a] Benchmark

The welfare analysis shows that policy intervention is more beneficial from a social point of view for structural exporters, while the gains to structural importers are sharply reduced in comparison to the benchmark. If anything, this only adds to the policy concerns surrounding the use of export restrictions, since structural exporters seem to have potentially serious motivations to persist in using them.

7 Conclusion

To our knowledge, this paper is the first attempt to design optimal dynamic food price stabilization policies in an open economy setting. The model can only be solved numerically, and tractability requires simple specification. Our analysis focuses on the optimal use of trade and storage policies on the food market of a small developing economy, where public intervention is justified by the lack of insurance against price volatility for risk-averse consumers. For the sake of tractability, the model overlooks supply reaction and producers' risk aversion. The framework developed here, however, could be applied to other cases, in terms of both its parameterization and specification.

Our results show that, for the case of a normally self-sufficient country, an optimal trade policy in the presence of risk aversion includes subsidizing imports and taxing exports in periods of high domestic prices. This policy truncates the upper half of the distribution and is not fiscally costly since the proceeds from export taxation cover the fiscal cost of import subsidies. Import subsidies alleviate the traditional limit of food storage: its non-negativity. When stocks are zero, subsidizing imports prevents price spikes.

When stabilization is pursued through storage subsidies only, it does not improve consumers' welfare. Additional storage increases low prices through additional demand for stockpiling, but it is not effective at preventing price spikes. In a small open economy, price spikes occur often when the world price is high, in which case any additional stock is sold on the world market. While domestic prices are stabilized to some extent, the potential benefits for consumers are wiped out by the increase in the mean price. Such a policy improves the country's trade balance by giving it more resources to export when the world price is relatively high, but it does not benefit consumers. Since storage policies are generally seen as a way to help consumers, these results sound a warning that storage policies designed without any flanking trade policy might be inconsistent: the limited insulation provided by trade costs—especially when they are relatively small—does not allow any independent food price policy to be pursued. In contrast, a well-designed combination of trade and storage taxes and subsidies can be a cost-efficient price-stabilizing policy.

These policies have an important common limitation. They produce distributive welfare effects that are much larger than total gains. Reducing consumers' risk bearing by manipulating prices in an open economy may thus face strong opposition. This result emphasizes the drawbacks of stabilization policies compared to policies targeting specifically poor households.

Our results show that an optimal combination of trade and storage policies trims both the lower and upper parts of the domestic prices distribution. The optimal policy identified here should thus remain welfare-increasing even when producers are risk averse, since they should value the trimming of low prices. The same goes for the issue of supply elasticity: since expected prices are not strongly modified, supply reaction should remain limited.

We find that export taxes are important for designing price-stabilizing policies that benefit consumers. This result should not be understood as a plea for export restrictions, the destabilizing effects of which are well documented and are a legitimate and serious source of concern at the multilateral level (Mitra and Josling, 2009). Nonetheless, this analysis may provide a better understanding of why export restrictions are such a frequent occurrence. It might also help to gauge the stakes of potential multilateral disciplines. For the poorest countries, in particular, banning export restrictions altogether may be politically difficult, at least if no substantial compensating measures are offered. The collective action problem created by export restrictions certainly deserves closer scrutiny. In any case these constraints, disturbing as they are, are better acknowledged than ignored.

A First-order conditions of the optimal policy problem

To solve the optimal policy problem presented in Section 5, we need to reformulate the complementarity equations (6)–(8) since they cannot enter directly as constraints in a maximization problem. We restate these equations as a combination of inequalities and equations. For equation (6), we introduce a positive slack variable, ϕ, with its associated complementarity slackness conditions

$$\phi_t = P_t + k - \beta \operatorname{E}_t \left(P_{t+1}\right) - \zeta_t, \tag{29}$$

$$S_t \phi_t = 0. \tag{30}$$

To limit the number of equations and variables, the two trade policy instruments are merged into one with $\nu_t = \nu_t^X - \nu_t^M$, which is equivalent since each instrument is redundant when the other is active. The equations governing trade, (7)–(8), can be restated as

$$X_t \left[P_t^{\mathrm{T}} - \left(P_t^w - \tau\right)\right] = 0, \tag{31}$$

$$M_t \left[\left(P_t^w + \tau\right) - P_t^{\mathrm{T}}\right] = 0, \tag{32}$$

$$P_t^{\mathrm{T}} = P_t + \nu_t, \tag{33}$$

where $X_t \geq 0$, $M_t \geq 0$, and $P_t^w - \tau \leq P_t^{\mathrm{T}} \leq P_t^w + \tau$.

Discretionary equilibria are known to be difficult to characterize since they involve functional equations, often called generalized Euler equations (Klein et al., 2008, Ambler and Pelgrin, 2010). A model setting with occasionally binding constraints is even more complex because first-order conditions cannot be derived (see appendix C). For optimal policies with both instruments and with storage subsidy alone, this difficulty can be sidestepped by noting that the policy maker acts identically under commitment and under discretion. This peculiarity makes it possible to use the solution to the problem under commitment, more easily computed, as a solution to the problem under discretion.

The equivalence between commitment and discretion for these two policies arises because the storage subsidy allows full control of the only intertemporal trade-off, namely the storage decision. Formally, following Marcet and Marimon (2011), the optimal policy problem under commitment can be expressed as a saddle-point functional equation problem:

$$\begin{aligned} J\left(A_t, P_t^w, \lambda_{t-1}\right) = \min_{\Phi_t} \max_{\Omega_t} \Big\{ & v\left(P_t, Y\right) + w\left[P_t A_t - \left(P_t + k\right) S_t + \nu_t \left(X_t - M_t\right)\right] \\ & + \chi_t \left[A_t + M_t - D\left(P_t\right) - S_t - X_t\right] \\ & + \lambda_t \left(\phi_t + \zeta_t - P_t - k\right) \\ & + \lambda_{t-1} P_t \\ & + \delta_t^S S_t \phi_t \\ & + \delta_t^M M_t \left(P_t^w + \tau - P_t^{\mathrm{T}}\right) \\ & + \delta_t^X X_t \left(P_t^{\mathrm{T}} - P_t^w + \tau\right) \\ & + \kappa_t \left(P_t^{\mathrm{T}} - P_t - \nu_t\right) \\ & + \beta \,\mathrm{E}_t \left[J\left(S_t + \epsilon_{t+1}^H, f\left(P_t^w, \epsilon_{t+1}^w\right), \lambda_t\right)\right] \Big\}, \end{aligned} \tag{34}$$

where $\Omega_t = \{S_t \geq 0, P_t, M_t \geq 0, X_t \geq 0, P_t^w - \tau \leq P_t^{\mathrm{T}} \leq P_t^w + \tau, \zeta_t, \phi_t \geq 0, \nu_t\}$ and $\Phi_t = \{\chi_t, \lambda_t, \delta_t^S, \delta_t^M, \delta_t^X, \kappa_t\}$. From the first-order conditions and the envelope theorem, and following some manipulation, the recursive equilibrium under an optimal stabilization policy can be characterized by the following system of complementarity conditions:[13]

$$S_t : S_t \geq 0 \quad \perp \quad -w\left[P_t + k - \beta \,\mathrm{E}_t\left(P_{t+1}\right)\right] - \chi_t + \beta \,\mathrm{E}_t\left(\chi_{t+1}\right) + \delta_t^S \phi_t \leq 0, \tag{35}$$

$$P_t : v_P\left(t\right) + wD\left(P_t\right) - \chi_t D'\left(P_t\right) - \lambda_t + \lambda_{t-1} = 0, \tag{36}$$

$$M_t : M_t \geq 0 \quad \perp \quad -w\nu_t + \chi_t + \delta_t^M \left(P_t^w + \tau - P_t^{\mathrm{T}}\right) \leq 0, \tag{37}$$

$$X_t : X_t \geq 0 \quad \perp \quad w\nu_t - \chi_t + \delta_t^X \left(P_t^{\mathrm{T}} - P_t^w + \tau\right) \leq 0, \tag{38}$$

$$P_t^{\mathrm{T}} : P_t^w - \tau \leq P_t^{\mathrm{T}} \leq P_t^w + \tau \quad \perp \quad -\delta_t^M M_t + \delta_t^X X_t \tag{39}$$

$$\zeta_t : \lambda_t = 0, \tag{40}$$

$$\phi_t : \phi_t \geq 0 \quad \perp \quad \lambda_t + \delta_t^S S_t \leq 0, \tag{41}$$

$$\nu_t : P_t^{\mathrm{T}} - P_t - \nu_t = 0, \tag{42}$$

$$\chi_t : A_t + M_t = D\left(P_t\right) + S_t + X_t, \tag{43}$$

$$\lambda_t : \phi_t + \zeta_t - P_t - k + \beta \,\mathrm{E}_t\left(P_{t+1}\right) = 0, \tag{44}$$

$$\delta_t^S : S_t \phi_t = 0, \tag{45}$$

$$\delta_t^M : M_t \left(P_t^w + \tau - P_t^{\mathrm{T}}\right) = 0, \tag{46}$$

$$\delta_t^X : X_t \left(P_t^{\mathrm{T}} - P_t^w + \tau\right) = 0. \tag{47}$$

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[13] Here the "perp" notation ($\perp$) is extended to situations with two complementarity constraints. The expression $a \leq X \leq b \perp F\left(X\right)$ is a compact formulation for $X = a \Rightarrow F\left(X\right) \geq 0, X \in]a, b[\Rightarrow F\left(X\right) = 0, X = b \Rightarrow F\left(X\right) \leq 0$.

The transition from one period to the next is still governed by equations (10)–(11). Combining equations (37)–(38) and (46)–(47) gives $\nu_t = \chi_t/w$ for positive trade. It means that the optimal trade policy is to base trade decisions on the total marginal value of the commodity (the sum of private marginal value, domestic price, and social marginal value, χ_t/w) instead of the price.

Usually, the time-inconsistency of policies under commitment shows up in the lagged Lagrange multipliers. Here, the multiplier λ is always null (equation (40)), which demonstrates that there is no commitment problem. The equilibrium, therefore, is recursive in its natural state variables and is time-consistent. It is identical under commitment and under discretion.

B First-order conditions of the optimal storage subsidy

The first-order conditions of the optimal storage subsidy are obtained similarly, except that variables ν and P^{T} have to be dropped from (34). From the first-order conditions with both instruments, equations (35), (40), (41), and (43)–(45) remain valid. The following first-order conditions must be considered in addition:

$$P_t : P_t^w - \tau \le P_t \le P_t^w + \tau \quad \perp \quad v_P(t) + w(A_t - S_t) - \chi_t D'(P_t) - \delta_t^M M_t + \delta_t^X X_t, \tag{48}$$

$$M_t : M_t \ge 0 \quad \perp \quad \chi_t + \delta_t^M (P_t^w + \tau - P_t) \le 0, \tag{49}$$

$$X_t : X_t \ge 0 \quad \perp \quad -\chi_t + \delta_t^X (P_t - P_t^w + \tau) \le 0, \tag{50}$$

$$\delta_t^M : M_t (P_t^w + \tau - P_t) = 0, \tag{51}$$

$$\delta_t^X : X_t (P_t - P_t^w + \tau) = 0. \tag{52}$$

C Characterization of the optimal trade policy

For the optimal trade policy, the solutions under commitment and under discretion being different, we have to characterize the discretionary solution directly. Since we focus on a Markovian equilibrium, price can be characterized by a function of the state variables: $P_t = \psi(A, P_t^w)$. Using this function ψ to characterize price expectations, the value function is defined by the following Bellman equation

$$\begin{aligned} J(A_t, P_t^w) = \min_{\Phi_t} \max_{\Omega_t} \big(& v(P_t, Y) + w\left[P_t A_t - (P_t + k) S_t + \nu_t (X_t - M_t)\right] \\ & + \chi_t \left[A_t + M_t - D(P_t) - S_t - X_t\right] \\ & + \lambda_t \left\{\beta \operatorname{E}_t \left[\psi\left(S_t + \epsilon_{t+1}^H, f\left(P_t^w, \epsilon_{t+1}^w\right)\right)\right] + \phi_t - P_t - k\right\} \\ & + \delta_t^S S_t \phi_t \\ & + \delta_t^M M_t \left(P_t^w + \tau - P_t^{\mathrm{T}}\right) \\ & + \delta_t^X X_t \left(P_t^{\mathrm{T}} - P_t^w + \tau\right) \\ & + \kappa_t \left(P_t^{\mathrm{T}} - P_t - \nu_t\right) \\ & + \beta \operatorname{E}_t \left[J\left(S_t + \epsilon_{t+1}^H, f\left(P_t^w, \epsilon_{t+1}^w\right)\right)\right] \big). \end{aligned} \tag{53}$$

In this setting with occasionally binding constraints, we cannot assume ψ to be differentiable everywhere. Thus, in theory, we cannot derive the first-order conditions of this problem since they would imply derivatives of ψ. But since, in practice, ψ is approximated by a spline, which is differentiable everywhere, we can solve the dynamic programming problem numerically by solving the first-order conditions of the approximated problem:

$$
\begin{aligned}
S_t &: S_t \geq 0 \perp -w\left(P_t+k\right)-\chi_t+\beta\lambda_t \operatorname{E}_t \psi_A\left(A_{t+1}, P^w_{t+1}\right)+\delta^S_t\phi_t+\beta \operatorname{E}_t\left(wP_{t+1}+\chi_{t+1}\right) \leq 0, && (54)\\
P_t &: v_P\left(t\right)+wD\left(P_t\right)-\chi_t D'\left(P_t\right)-\lambda_t=0, && (55)\\
M_t &: M_t \geq 0 \quad \perp \quad -w\nu_t+\chi_t+\delta^M_t\left(P^w_t+\tau-P^{\mathrm{T}}_t\right) \leq 0, && (56)\\
X_t &: X_t \geq 0 \quad \perp \quad w\nu_t-\chi_t+\delta^X_t\left(P^{\mathrm{T}}_t-P^w_t+\tau\right) \leq 0, && (57)\\
P^{\mathrm{T}}_t &: P^w_t-\tau \leq P^{\mathrm{T}}_t \leq P^w_t+\tau \quad \perp \quad -\delta^M_t M_t+\delta^X_t X_t, && (58)\\
\phi_t &: \phi_t \geq 0 \quad \perp \quad \lambda_t+\delta^S_t S_t \leq 0, && (59)\\
\chi_t &: A_t+M_t=D\left(P_t\right)+S_t+X_t, && (60)\\
\lambda_t &: \beta \operatorname{E}_t\left(P_{t+1}\right)+\phi_t-P_t-k=0, && (61)\\
\delta^S_t &: S_t\phi_t=0, && (62)\\
\delta^M_t &: M_t\left(P^w_t+\tau-P^{\mathrm{T}}_t\right)=0, && (63)\\
\delta^X_t &: X_t\left(P^{\mathrm{T}}_t-P^w_t+\tau\right)=0, && (64)\\
\nu_t &: P^{\mathrm{T}}_t-P_t-\nu_t=0. && (65)
\end{aligned}
$$

D Numerical algorithm

The numerical algorithm used here is inspired by Fackler (2005) and Miranda and Glauber (1995). It is a projection method with a collocation approach. Since several models are solved, we present here a general method that can be applied to all of them. Following Fackler (2005), rational expectations problems can be expressed using three groups of equations. State variables s are updated through a transition equation:

$$\dot{s} = g(s, x, \dot{e}), \tag{66}$$

where x are response variables, e are stochastic shocks, and next-period variables are indicated with a dot on top of the character. Response variables are defined by solving a system of complementarity equilibrium equations:

$$\underline{x}(s) \leq x \leq \overline{x}(s) \quad \perp \quad f(s, x, z). \tag{67}$$

Response variables can have lower and upper bounds, $\underline{x}$ and $\overline{x}$, which can themselves be function of state variables.[14] For generality, equilibrium equations have been expressed as complementarity problems. In cases where response variables have no lower and upper bounds, equation (67) simplifies

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[14] It is the case in equation (39) where lower and upper bounds on P^{T}_t are functions of P^w_t.

to a traditional equation:

$$f(s,x,z)=0. \tag{68}$$

z is a variable representing the expectations about next period and is defined by

$$z=\mathrm{E}_{\dot{e}}\left[h(s,x,\dot{e},\dot{s},\dot{x})\right]. \tag{69}$$

One way to solve this problem is to find a function that approximates well the behavior of response variables. We consider a spline approximation of response variables,

$$x\approx\Phi\left(s,\theta\right), \tag{70}$$

where θ are the parameters defining the spline approximation. To calculate this spline, we discretize the state space (using, for the storage-trade model, 41 points for availability and 23 for world price), and the spline has to hold exactly for all points of the grid.

The expectations operator in equation (69) is approximated through a Gaussian quadrature (with 5 points on each dimension), which defines a set of pairs $\{e_l,w_l\}$ in which e_l represents a possible realization of shocks and w_l the associated probability. Using this discretization, and equations (66) and (69)–(70), we can express the equilibrium equation (67) as

$$\underline{x}(s)\leq x\leq\overline{x}(s)\quad\perp\quad f\left(s,x,\sum_l w_l h\left(s,x,e_l,g\left(s,x,e_l\right),\Phi\left(g\left(s,x,e_l\right),\theta\right)\right)\right). \tag{71}$$

For a given spline approximation, θ, and a given s, equation (71) is a function of x and can be solved using a mixed complementarity solver.

Once all the above elements are defined, we can proceed to the algorithm, which runs as follows:

1. Initialize the spline approximation, θ_0, based on a first-guess, x^0.
2. For each point of the grid of state variables, s_i, solve for x_i equation (71) using the solver `PATH` (Dirkse and Ferris, 1995):

$$\underline{x}(s_i)\leq x_i\leq\overline{x}(s_i)\quad\perp\quad f\left(s_i,x_i,\sum_l w_l h\left(s_i,x_i,e_l,g\left(s_i,x_i,e_l\right),\Phi\left(g\left(s_i,x_i,e_l\right),\theta_n\right)\right)\right). \tag{72}$$

3. Update the spline approximation using the new values of response variables, $x=\Phi\left(s,\theta_{n+1}\right)$.
4. If $\|\theta_{n+1}-\theta_n\|_2\geq 10^{-8}$ then increment n to $n+1$ and go to step 2.

Once the rational expectations equilibrium is identified, the spline approximation of the decision rules is used to simulate the model.

The storage model representing the world market is solved using this algorithm and a 50-node spline approximation. This spline approximation defines the continuous Markov chain (11) representing the world price dynamics. It is subsequently used in the storage-trade model as a transition equation, which means that it is a part of equation (66).

For the optimal trade policy, the method requires a few modifications, since equation (54) includes a derivative of ψ, ψ being an approximation of the behavior of price with respect to state variables. The calculation of this approximation is actually done in the above method in step 3. Since derivatives of ψ appears only in expectations terms, to apply the method we have to make sure that the definition of the function h includes these terms, which leads to a new equation (69):

$$z = \mathrm{E}_{\dot{e}}\left[h(s, x, \dot{e}, \dot{s}, \dot{x}, \Phi_s\left(s, \theta\right))\right], \tag{73}$$

where $\Phi_s\left(s, \theta\right)$ is the derivative of Φ with respect to state variables.

This is only a sketch of the solution method. In fact, several methods are used in this paper to solve the models, depending on which one is the most efficient. For example, instead of using the simple updating rule in step 3, a Newton, or inexact Newton, updating is used when feasible. For more precisions, see the `RECS` solver (`https://github.com/christophe-gouel/recs`), with which the models are solved.

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