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MARKET INTERVENTION

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Not for Quotation  
Comments Welcomed

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Abstract

The problem of exchange rate stabilization is posed within the Brainard-Poole framework. The optimal policy depends upon the source and correlation of disturbances to the economy.

For most economies the optimal policy of foreign exchange intervention is one of "dirty floating" in which the value of the currency in terms of foreign money is permitted to move in the same direction as the change in the money supply. The movement of the exchange rate should be greater than that of the money supply when expressed in percentage terms.

## Optimal Foreign Exchange

### Market Intervention

#### I. Introduction

The discussion of the appropriate exchange rate policy for an open economy has been extensive and filled with controversy (Friedman (1953), Johnson (1969), Kindleberger (1969), Mundell (1969), Parkin (1973), Fischer (1974)). It has been agreed by all that the ultimate goal of exchange rate policy is the stabilization of domestic variables such as the price level, employment and output. The dispute has been about which of the two extremes of exchange market intervention, fixed or flexible exchange rates, can better attain this goal.

In fact central banks do not choose their policies only from these extremes, but also consider intermediate regimes, which are appropriately called "managed floating".<sup>1</sup> There was some analysis of such an exchange rate system, under the title of "crawling peg", during the pre-Smithsonian period.<sup>2</sup> That analysis was a pragmatic approach to increasing the long-run flexibility of the international monetary system. This paper seeks the optimal, short-run foreign-exchange policy for a small, open economy, with given fiscal and commercial policies. A model is formulated so that we can compare, in a single consistent framework, the desirability of fixed rates, freely fluctuating rates, and managed floating.

In the absence of stochastic factors, the choice of an exchange rate regime is irrelevant. The same equilibrium can be attained under any regime, so long as devaluation is permitted under fixed exchange rates.<sup>3</sup> Differences

arise only if the economy is subjected to stochastic disturbances which cannot be offset contemporaneously.

The problem of optimal exchange rate policy, posed in these terms, is precisely analogous to that set by Brainard (1967) and solved for closed economy monetary policy by Poole (1970). The conclusion reached by that literature is that, in general, the optimal stabilization policy involves the use of all the pure policies in certain proportions, the appropriate proportions depending upon the structure of the economy and of the stochastic disturbances to which it is subjected.<sup>4</sup>

A model of an open economy is set out in Section II of this paper. Section III demonstrates diagrammatically the similarity between the structure of the Brainard-Poole problem and that of the optimal foreign exchange market policy. Section IV states this problem and its solution in general terms with a formal derivation being provided in an appendix. Section V applies the general results to the specific case of intervention by the central bank to stabilize the exchange rate. Section VI provides a summary and conclusion of the paper.

## II. A Model of an Open Economy

A small economy, which takes as given the foreign-currency price of the traded good and the rate of return on traded assets,<sup>5</sup> wishes to minimize the effect of exogenous shocks on domestic variables such as the price level, output and employment. The supply of labor is perfectly inelastic and full employment obtains because of rapid clearing of the labor market.<sup>6</sup> Thus real income can be treated as constant.<sup>7</sup> In light of these restrictions the

goals of the authorities reduce to that of minimizing the variability of the price index.<sup>8</sup> In their concern with the movement of domestic variables, the authorities place no weight on the behavior of the balance of payments accounts or of the exchange rate.<sup>9</sup>

The problem of the authorities can be set out in terms of the following framework. The endogenous variables of the system are

- p the relative price of the nontraded good ✓
- w the real value of wealth ✓
- P the domestic-currency price index ✓
- $P_d$  the domestic-currency price of the nontraded good ✓

The domestic-currency value of nominal wealth is taken as given in this short-run analysis because of the absence of foreign-currency-denominated market instruments. The single parameter of the system is  $\alpha$ , the proportion of output produced in the nontraded goods sector, so that  $1 - \alpha$  is the proportion of output produced in the traded goods sector. The money supply,  $M$ , is endogenous under fixed exchange rates, and exogenous under flexible rates. Conversely the exchange rate,  $e$  (defined here as the foreign-currency price of a unit of domestic currency), is exogenous under fixed exchange rates and endogenous under flexible rates.<sup>10</sup>

The model consists of five equations: First three market-clearing conditions:

$$\begin{aligned} X(p,w) &= 0 & (1) \\ 0 > X_1, 0 < X_2 \end{aligned}$$

$$\begin{aligned} P \cdot l(w) - M &= 0 & (2) \\ 0 < l', l'w < 1 \end{aligned}$$

$$G(e,M) = 0 \quad (3)$$

where  $X(\cdot)$  is the excess demand function for the nontraded good,  $l(\cdot)$  is the demand for real balances, and  $G(\cdot)$  is a reaction function determining the nature of the exchange rate regime. (A reaction function of this form has been considered by Mussa (1976).) Second, three identities,

$$p = P_d \cdot e \quad (4)$$

$$P = P_d^\alpha \cdot e^{\alpha-1} \quad (5)$$

$$w = W/P, \quad (6)$$

further define the endogenous variables on the left-hand side of the equations. Units are chosen so that the equilibrium value of each nominal price variable and the money supply equal one initially.

The model can be simplified by substituting the definitions of the endogenous variables into the market-clearing conditions so that they solve for the key central variables:  $P$ , and  $M$  or  $e$ . The reduced form system then becomes:

$$X(P^{1/\alpha} \cdot e^{1/\alpha}, W/P) = 0$$

$$P \cdot l(W/P) - M = 0$$

$$G(e, M) = 0$$

Differentiating this system yields

$$\left(\frac{X_1}{\alpha} - X_2 w\right) \cdot dP + \frac{X_1}{\alpha} de = 0 \quad (7)$$

$$(1 - l'w) \cdot dP - dM = 0 \quad (8)$$

$$de - \gamma \cdot dM = 0 \quad (9)$$

where  $\gamma$  is defined as the ratio of partial derivatives



$$\gamma = - \frac{G_2}{G_1}$$

and is equal to the ratio of the percentage changes in the exchange rate to the money supply. The slope of the LM curve is equal to

$$\left. \frac{de}{dP} \right|_{LM} = + (1 - b'w)\gamma . \quad (10)$$

Figure I gives a diagrammatic representation of the system under fixed exchange rates, and Figure II under flexible exchange rates. (Ignore all dotted lines for the moment.) The IS curve is the locus of equilibria in the non traded goods market, and its form is independent of the exchange rate regime, so that it has the same slope in both figures. Its negative slope can be attributed to the fact that, when the exchange rate rises, this causes an excess demand for the non-traded good since its relative price has fallen. A rise in the price level, keeping the exchange rate constant, causes an excess supply of the nontraded good because of the opposite movement of the relative price. If these changes are commensurate then the nontraded goods market remains in equilibrium.

Expressed algebraically, the slope of this locus equals:

$$\left. \frac{de}{dP} \right|_{IS} = - \frac{\frac{X_1}{\alpha} - X_2 w}{\frac{X_1}{\alpha}} < 0 \quad (11)$$

When pursuing fixed exchange rates the central bank keeps the exchange rate equal to one by altering the supply of money. This process is captured mathematically by setting  $\gamma$  equal to zero in equation (9). Thus, the points of equilibrium in the money market constitute the horizontal LM line drawn at  $e = 1$  in Figure I, reflecting the complete elasticity of the money supply with respect to the exchange rate.

When the central bank pursues a flexible exchange rate policy the LM locus is vertical as in Figure II. This exchange rate regime is represented mathematically by setting  $\gamma$  equal to an indefinitely large number. The reason for this slope for such an exchange rate regime is that the supply of money

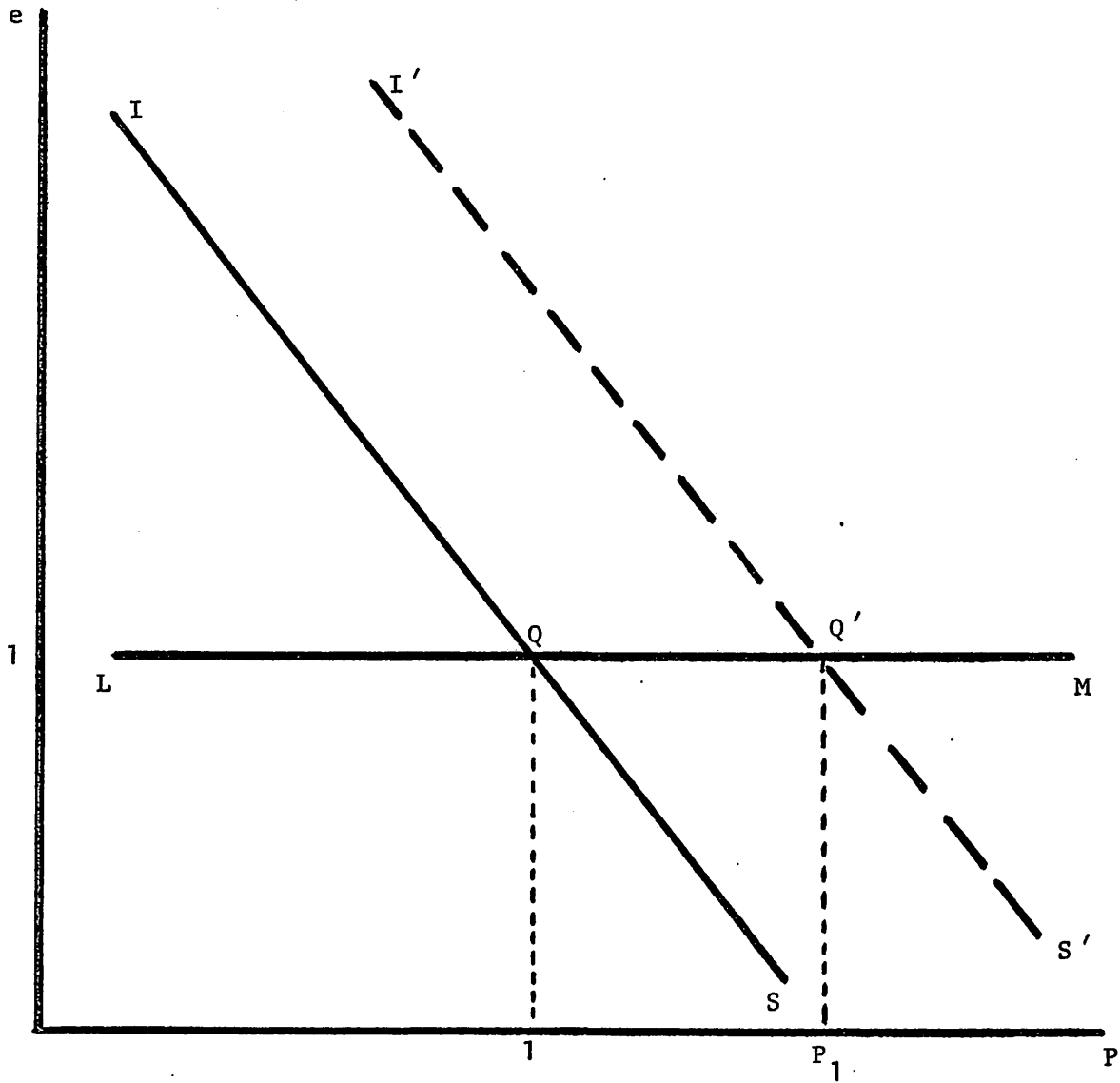


Figure I: Fixed Exchange Rates

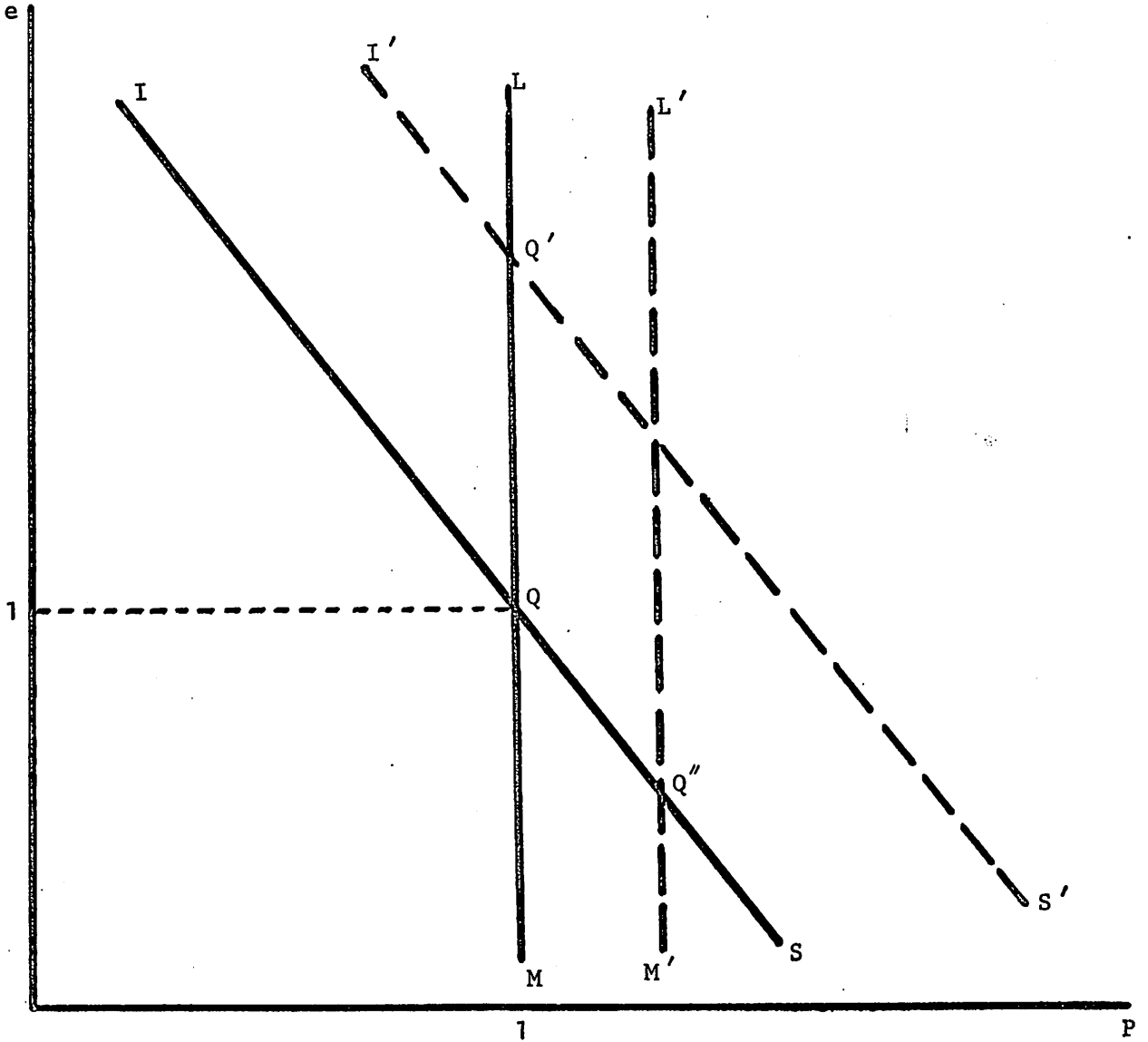


Figure II: Flexible Exchange Rates

is constant, and the demand for money is independent of the exchange rate. This independence of the asset market equilibrium condition from the exchange rate manifests itself as a complete inelasticity of the LM locus in Figure II.

As the parameter  $\gamma$  can take on any value, so the LM curve can have any orientation. While the expression "exchange rate management" is applied to all values of this parameters, it is found that positive values are likely to be optimal. This causes the LM curve to be positively sloped, a situation here called "dirty floating." This description of the process of "dirty floating" is different from that found in some parts of the literature.<sup>11</sup> In that discussion the exchange rate could be affected by changing the level of reserves without changing the money supply. This process could be analyzed in the present framework if there were a separate demand function for reserves. However, in our aggregation of the central bank with the private sector we are assuming that no such demand curve exists, and therefore that the only way that the central bank can alter the exchange rate is to change the money supply.

### III. Pure Exchange Rate Regimes

Assume that this economy is subjected to shocks which arise exogenously and can be characterized as creating autonomous changes in the excess demands in various markets. For the moment consider an economy which is subjected to shocks in only one market: either the goods market or the money market. Clearly the influence of these shocks on the domestic price variable depends upon the pure exchange rate regime pursued.

Consider first the case where shocks occur in the goods market shifting that market's equilibrium locus from its original position IS to a new position I'S' as in Figures I and II.<sup>12</sup> (The goods market locus takes on a continuum of positions as it shifts both to the right and the left. The I'S' line is shown merely as a typical location of that locus away from its original position.) If the economy pursues a fixed exchange rate then the movement of the price

level is from  $l$  to  $P_1$  as in Figure I as the equilibrium shifts from point  $Q$  to point  $Q'$ .

If the central bank pursues a regime of flexible exchange rates in the face of these shocks, as in Figure II, the price level does not change in value. The reason is that then the money market locus is vertical. A shift in the IS locus moves the equilibrium point from  $Q$  to  $Q'$  but does not affect the value of the price index at their intersection. Instead, the exchange rate adjusts by a substantial amount to permit clearing of both these markets.

The conclusion of this simple analysis for the case of shocks to the goods market is straightforward. If the main shocks to the economy occur in this market, with money being the intervention market, then the central bank authorities should follow a policy of flexible exchange rates in order to minimize fluctuations in the price level. This conclusion is hardly surprising. It is the converse of Mundell's theorem that fiscal policy under fixed rates has some effectiveness, whereas such policy under flexible exchange rates is impotent.<sup>13</sup>

If we assume that shocks occur in the money market, the market of intervention, we get quite different results.<sup>14</sup> Such shocks shift the IM locus to the left and right with some variance around the initial equilibrium at  $Q$ . If the authorities maintain fixed exchange rates, as in Figure I, the IM curve is horizontal and its intersection with a stationary IS locus remains at the same point. Therefore the resulting variance of the price level under a fixed exchange rate regime is zero (since these shocks affect only quantities of assets held).

In contrast, if the central bank maintains an exogenous supply of money and permits the exchange rate to vary, these shocks do affect the domestic price level, as Figure II shows. These shocks shift the IM curve to the left and right about the mean point  $Q$  and this sweeps out intersections, of which  $Q''$  is an example, along an unshifting IS locus.

This conclusion is again consistent with a well-known theorem: Monetary policy is impotent under fixed exchange rates (since the money supply is endogenous), whereas it is effective under flexible exchange rates because then the exchange rate does the adjusting.<sup>15</sup> The converse of this theorem has been developed here: The effects of shocks to the money market upon domestic variables are minimal under fixed exchange rates, whereas they are substantial under flexible rates.

The similarity between the argument here and that presented by Poole (1970) can be easily grasped. He argued, in a closed economy context, that the optimal slope of the IM curve depends upon the source of shocks to the system. Although he placed the interest rate on the vertical axis where the present analysis places the exchange rate, the diagrammatic argument is the same.<sup>16</sup>

When disturbances arise only in the money market and the authorities seek only to control variation in the variable viewed on the horizontal axis, the optimal slope of the IM curve is zero no matter what the endogenous variable on the vertical axis. In the closed economy case where the rate of interest is the relevant variable this is justification for a fixed interest rate policy. For the open economy case, where the exchange rate is more important, this argues for a fixed exchange rate regime.

When shocks arise in the goods market, optimally set the IM curve to be vertical. Of course, a positively-sloped IM curve is better than one which is flat, so that a constant money supply policy is preferable to a constant interest rate policy in a closed economy analysis. But the optimal policy is one which adjusts the money supply so that the effective IM curve is vertical. This is precisely analogous to what happens in an open economy with flexible exchange rates. Because the demand for money is independent of the exchange rate, when

the money supply is kept fixed the money market equilibrium does not depend upon the price of foreign currency.

It should be noted that these results depend entirely on the assumption that the money market is the one in which intervention takes place. If the goods market were used for intervention through commercial or fiscal policies these conclusions would have to be changed completely. If the goods market were the location of intervention, then a fixed exchange rate (or interest rate) policy would be represented by a horizontal IS locus in Figures I and II. Under these circumstances shocks to the goods market do not affect domestic price variables and such an intervention policy is appropriate if this is the source of shocks. This counter-intuitive result is consistent with the effects of financial policies under a fixed exchange rate: fiscal policy has no influence under fixed exchange rates since budgetary expenditures are the endogenous "pegging" variable. However, monetary policy has an influence under this regime since it is freed from fixing the exchange rate. Conversely, if shocks arise in the asset markets the authorities should pursue a flexible exchange rate policy with the effective IS curve optimally vertical. With a vertical IS curve, under flexible exchange rates, fiscal policy now has an influence on the price level since it does not fix the exchange rate.<sup>17</sup> It is monetary policy which now has no influence on domestic variables since the point of equilibrium merely moves up and down a vertical IS curve altering only the exchange rate.

#### IV. A General Model of Stabilization

(refer to Appendix for a formal derivation of these results)

The stochastic portion of the model presented so far is excessively stylized in keeping with the limits imposed by diagrammatic exposition. The task now is to generalize the problem by introducing a richer stochastic

structure which permits shocks to arise in both markets simultaneously with due allowance being made for contemporaneous correlation between them. This analysis can indicate the general circumstances under which the pure exchange rate regimes are optimal. It can also delineate the characteristics of economies for which some intermediate exchange rate regime is appropriate.

Consider an economy which is stationary in both its structure and its formation of expectations, but which is subjected to random shocks. These shocks take the form of serially uncorrelated autonomous excess demands which enter the private sector's behavioral functions, with mean of zero and known variance.

Assume that the markets of the economy can be aggregated in keeping with the intervention policy pursued by the authorities. The aggregated market in which the authorities intervene is denoted by the subscript "a"; the market free from intervention is denoted by a "p" subscript (indicating private sector).<sup>18</sup> In the same way there is a single aggregate target variable entering a quadratic loss function<sup>19</sup> which the authorities wish to stabilize, and a single indicator variable which can be observed and adjusted contemporaneously so as to provide stabilization.<sup>20</sup> The slope of the equilibrium locus for the intervention market is a choice variable which the authorities can set at any particular value by following the appropriate reaction function. In contrast, the slope of the private sector equilibrium locus is an exogenous quantity since the authorities do not seek to influence that market. This slope, expressed as the ratio of the change in the indicator variable to the change in the target variable such that the market continues to clear, is denoted by  $s_p$ . The variances of the equilibrium loci with respect to the target variable for a constant value of the indicator variable are denoted by  $\sigma_p$  and  $\sigma_a$  for the private



sector and the authorities sector respectively.<sup>21</sup> The coefficient of contemporaneous correlation between the shocks in these markets is denoted by  $\rho$ .

It can be shown (see the Appendix) that the optimal slope of the authorities' equilibrium locus,  $s_a^*$ , is proportional to the private sector slope:

$$s_a^* = -s_p \left( \frac{\rho - \frac{\sigma_p}{\sigma_a}}{\rho - \frac{\sigma_a}{\sigma_p}} \right) . \quad (12)$$

The factor of proportionality is a function only of the ratio of the variances in the two markets and of the coefficient of contemporaneous correlation between them.

Table I identifies the values of this stochastic proportionality factor when the ratio of these variances is very small and when it is very large. Table II describes this proportionality factor for general values of the stochastic parameters. The third column of that table identifies the areas of Figure III which satisfy the stochastic parameter values of any row. Figure III graphs this function in an implicit fashion. The locus dividing Area A from Area B, denoted in that figure by NN, is a 45° line along which

$$\rho = \frac{\sigma_p}{\sigma_a} .$$

The locus dividing Area C from Area D, denoted by FF, is a rectangular hyperbola along which

$$\rho = \frac{\sigma_a}{\sigma_p} .$$

Table I

$$\frac{s_a^*}{s_p} = \frac{\rho - \frac{\sigma_p}{\sigma_a}}{\rho - \frac{\sigma_a}{\sigma_p}}$$

If  $\frac{\sigma_p}{\sigma_a}$  equals

then  $\frac{s_a^*}{s_p}$  equals

0

0

∞

∞

Table II

$$\frac{s^*}{s} = \frac{\rho - \frac{\sigma_p}{\sigma_a}}{\rho - \frac{\sigma_a}{\sigma_p}}$$

If  $\frac{\sigma_p}{\sigma_a}$  has  
a value

and  $\rho$  has  
a value

which is true  
of points in  
Figure III area

then  $\frac{s^*}{s}$  lies in  
the interval

&lt; 1

 $> \frac{\sigma_p}{\sigma_a}$ 

A

(0, 1)

&lt; 1

 $< \frac{\sigma_p}{\sigma_a}$ 

B

(-1, 0)

&gt; 1

 $< \frac{\sigma_a}{\sigma_p}$ 

C

(-∞, -1)

&gt; 1

 $> \frac{\sigma_a}{\sigma_p}$ 

D

(1, ∞)

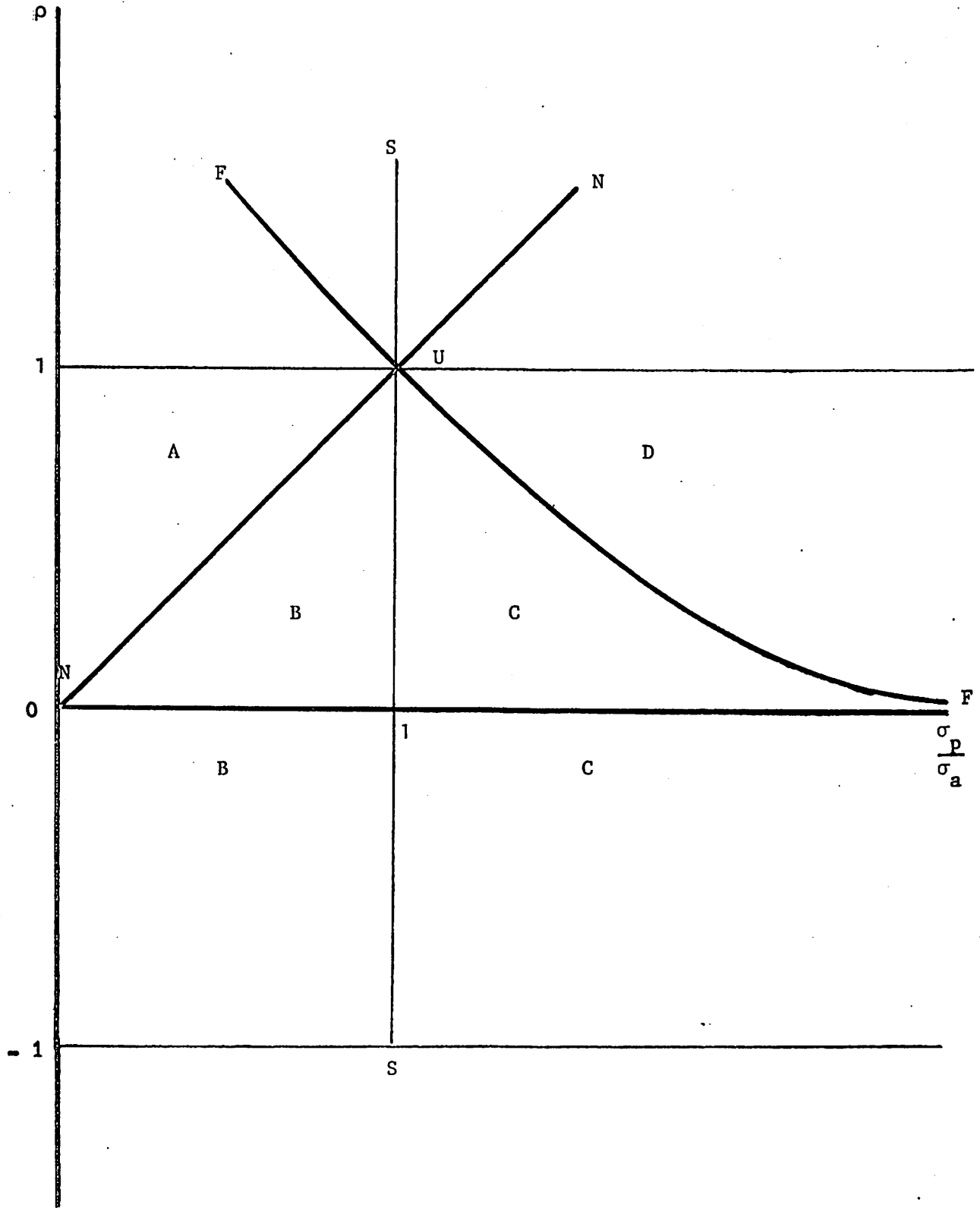


Figure III: The Ratio  $\frac{s^*}{s_p}$

## V. Optimal Exchange Rate Management

The structure of the general framework of Section IV is clearly relevant to the problem of optimal exchange rate management. Its immediate applicability is contingent upon the identification of the general variables of intervention policy of that section with the typical exchange rate pegging operation described in Section II.

A central bank intervenes in an economy's asset markets, adjusting the supply of money and foreign exchange as policy instruments in order to influence the exchange rate.<sup>22</sup> The slope of the LM curve is a choice variable, dependent upon the reaction function chosen by the central bank.<sup>23</sup> The flow markets are free from intervention, so the IS curve has an exogenously given slope. The target of policy action is a domestic variable, identified here only as the price level, and the exchange rate is an indicator of policy. With this classification of variables in mind, let us use the tables and figure of the previous section to determine the optimal policy for the authorities.

Table I confirms the two results of our earlier investigations. It shows that when shocks in the money (authorities) market predominate, so that  $\sigma_p/\sigma_a$  is approximately equal to zero, the optimal slope of the LM curve is zero. Clearly, this implies that the authorities should use their control of the money supply so as to keep the exchange rate exogenous. At the other extreme, when shocks in the money market are much smaller than those in the goods market so that  $\sigma_p/\sigma_a$  is indefinitely large, the optimal LM curve is vertical. This implies that the central bank should pursue a policy of completely flexible exchange rates by keeping the money supply constant.<sup>24</sup>

In order to determine the optimal policy for the authorities in the general case we need to refer to Table II and Figure III. They show that every slope for LM is optimal for certain values of the stochastic

parameters, except for that parallel to (or coincident with) the IS curve.<sup>25</sup> Nowhere in Figure III is the stochastic proportionality factor equal to plus one. In Area A this factor is positive and less than one: the optimal LM curve is negatively sloped and flatter than IS. In Area B the factor is negative and less than one in absolute value: the optimal LM is positively sloped and flatter than IS. In Area C the optimal LM has a positive slope, steeper than IS. And in Area D, optimally LM is negatively sloped and steeper than IS. This shows that the optimal policy of the authorities may require the LM curve to have a slope of the same sign as the IS curve if the coefficient of contemporary correlation is sufficiently large relative to either the ratio of the variances or the inverse of that ratio.<sup>26</sup> A description in economic terms of the response patterns in these cases is appropriate.

Figure III identifies the parameter values for which a fixed exchange rate regime is optimal as those along the locus NN. In this exchange rate regime the money supply is adjusted by just that amount so that the exchange rate stays constant. If the money supply were adjusted by less, then the exchange rate would be inclined to move in the direction that it would under flexible exchange rates only by less; this is what we have termed "dirty floating." In contrast, if the money supply would be adjusted by more, as would be optimal for an economy in Area A of Figure III, the exchange rate would adjust in the opposite direction from that which it moves under flexible exchange rates. Because the movement of the exchange rate is opposed to that under flexible exchange rates this could be called "anti-dirty floating." A similar examination of the behavior of economic variables in Area D of Figure III shows that the exchange rate there moves in the same direction that it does under flexible exchange rates but further (and the money supply moves in the opposite direction from the way it would under fixed rates). Because of the

excessive movement of the exchange rate, this process is called "super-dirty floating."

Two points should be made about the circumstances in which the optimal slope of the LM curve is of the same sign as that of the IS curve. First, the a priori probability that this is the optimal strategy for a typical economy is quite low. Areas A and D constitute a rather small portion of the total surface of Figure III.<sup>27</sup> Second, a policy of making the LM curve of opposite slope from IS has much smaller expected losses when the optimal policy is one of having LM and IS of the same slope, than those for the opposite situation in which a same slope policy is pursued when an opposite slope policy is optimal.<sup>28</sup> These two observations suggest that the appropriate policy is one of making the LM curve of opposite slope from the IS curve. This suggestion is one whose implications we will investigate further.

Excluding those situations in which the slope of the optimal LM curve is negative, we see that usually the appropriate policy of the central bank is to "dirty float." The optimal LM curve, except in the unusual circumstances outlined in Table I, is neither vertical nor horizontal but rather upward sloping. This implies that an exogenous shock is met by the authorities adjusting the money supply in a stabilizing manner so that the exchange rate moves in the same direction which it would under flexible exchange rates but not by so much. It is possible to identify the slope of the optimal LM curve and the size of the optimal reaction function coefficient more precisely by making some plausible assumption about the relative sizes of shocks.<sup>29</sup>

If we assume that shocks which arise from the private portion of the economy and from the rest of the world tend to be of real origin,<sup>30</sup> such that  $\frac{\sigma_p}{\sigma_a} > 1$ , then the ratio of the optimal slope of LM to the slope of IS is greater than minus one. This can be established from Table II. This

assumption clearly biases the optimal exchange rate policy towards a flexible regime.

The slope of the IS curve is given by expression (11). That expression can be written as

$$s_p = \frac{de}{dP} \Big|_{IS} = -1 + \frac{X_2 w\alpha}{X_1}$$

showing that the slope is negative and larger in absolute value than one.

The reason for this is that a rise in the price level with the exchange rate constant has two reinforcing effects: a rise in the relative price of the non traded good, and a reduction in the real value of nominal wealth. This causes an excess supply of the non traded good. A rise in the exchange rate has only one of these effects: a reduction in the relative price. Therefore, it requires a greater increase in the exchange rate than the price level to keep the non traded goods market in equilibrium. The slope of IS is steeper than  $45^\circ$  because of these effects.

The optimal slope of the LM curve is a product of the private sector slope and the stochastic proportionality factor, as equation (12) shows. Since each of these factors is negative and greater than one in absolute value the optimal slope of the LM curve is greater than plus one. This means that that curve should be steeper than a unitary elastic ( $45^\circ$ ) line.

The slope of the optimal LM curve is related to the reaction function coefficient (equation (10)):

$$\gamma^* = \frac{1}{(1 - b'w)} \cdot s_a^*$$

where  $(1 - b'w)$  is positive and less than one (so that its inverse is greater than plus one). The reason for this inequality is that  $(1 - b'w)$  measures the relative influence of a rise in the money supply versus a rise in the price level on the state of the money market. A rise in the money supply of one



percent causes an equivalent excess supply in this market. In contrast, a rise in the price level of 1% causes a smaller excess demand because the real value of wealth is reduced somewhat, thereby depressing the demand for real balances.

Equation (13) shows that the optimal reaction coefficient, being the product of two factors, both positive and greater than one, must itself be greater than one. The complete equation for this coefficient in terms of the system parameters is:

$$\gamma^* = - \frac{-1 + \frac{X_2 w \alpha}{X_1}}{(1 - h'w)} \left( \frac{\rho - \frac{\sigma_p}{\sigma_a}}{\rho - \frac{\sigma_a}{\sigma_p}} \right).$$

Equation (10) showed that the simplest interpretation of this coefficient is as the ratio of the change in the exchange rate to the change in the money supply. Since the optimal value for this coefficient is greater than one, the authorities should cause the movement of the exchange rate to be greater than the movement of the money supply.

Our conclusions may be summarized as follows: For many open economies the optimal exchange rate policy of the central bank is one of "dirty floating," in which the change in the money supply and the change in the value of the currency are both in the same direction. In addition, the change in the money supply should be sufficiently small so as to permit a larger change in the exchange rate when both are expressed in percentage terms.

A numerical example helps to make this policy prescription understandable. Assume that a shock impinges on an open economy such that there are inflationary pressures to which the authorities must react.<sup>31</sup> Assume further that the structure of the private sector markets is such that either

holding the money supply constant the exchange rate must rise by 6% or keeping the exchange rate constant the money supply must rise by 3%. Of course, any combination of movements in these variables such that the percentage change of the exchange rate plus twice that of the money supply sum to 6% also yields a sustainable equilibrium. For example, the exchange rate could be permitted to rise by 2% while the money supply increases by the same amount; or the exchange rate could be increased by 4% while the money supply only rises by 1%. The rule stated in the previous paragraph indicates that this last reaction is the optimum for many economies. In that response, both the money supply and the exchange rate are permitted to move and in the same direction, however the movement of the exchange rate is greater than the movement of the money supply in percentage terms.

#### VI. Conclusions

An exchange rate regime can be evaluated on the basis of the economy's response to shocks of various kinds, in the same manner that other policy systems, such as a closed economy financial policy regime, can be assessed. In all cases the optimal policy system pegs the value of the indicator variable if the main shocks to the system occur in the markets in which the authorities operate, whereas they operate indicator-independent if the shocks occur mainly in other markets. In terms specifically of intervention in the money market, this argues for pegging the exchange rate or the interest rate if shocks occur there, and freeing the exchange rate or the interest rate if they occur in the goods market.

The process of "dirty floating," whether of the exchange rate or of the interest rate, is optimal for a wide range of circumstances. While one can conceive of situations in which it is appropriate to do otherwise, these are cases in which the losses from nonoptimal policy tend to be small.

The appropriate reaction function for the foreign exchange authorities is to allow the international value of the domestic currency to move in the same direction as the money supply, permitting the movement of the exchange rate in percentage terms to be the larger one.

Footnotes

<sup>1</sup>The concern typical of a flexible exchange rate country can be exemplified by the following quotation:

"The exchange rate is a very important price in a country that trades with the outside world on the scale that Canada does... It is not therefore to ignore it even when it floats... ."

From the Annual Report of 1970 from the Governor of the Bank of Canada to the Minister of Finance.

Fixed exchange rate economies can alter the value of the currency within the band as well as change its parity value.

<sup>2</sup>For interesting discussions of this proposal see Williamson (1965), Halm (1970), and Kenen (1975).

<sup>3</sup>That is, the same values of all variables can be attained by the appropriate level of the money supply under flexible exchange rates or the equivalent level of the (exogenous) exchange rate under fixed exchange rates.

<sup>4</sup>See Brainard (1967, pp. 418-21) and Poole (1970, pp. 208-9). For more recent work in this spirit see Turnovsky (1975), Friedman (1975), and Parkin (1976).

<sup>5</sup>The foreign-currency price of traded goods is set equal to one by appropriate choice of units. Hicks' composite good theorem permits the analysis of both imports and exports as a single good.

If nontraded bonds exist then the framework presented here must be considered a reduced form in which the interest rate on that bond has been "solved out". This reduction of the model does not permit the analysis of shocks which impinge on the nontraded bond market.

<sup>6</sup>These assumptions are made for analytical convenience but are not necessary for the conclusions. It is precisely because domestic prices and wages do not move rapidly that fluctuations in the price level (and concomitant movements in the level of employment) are to be avoided. See Laidler (1975) for an open economy model with unemployment.

<sup>7</sup>The price index has been so defined that the real value of output along the transformation curve is independent of the composition of output.

<sup>8</sup>The variable called "price index" can be thought of as a measure of nominal income. In this way the analysis can be applied to the case of variable employment when prices are rigid.

<sup>9</sup>The authorities may include movements in the exchange rate in their loss function. This seems to be precisely what recent proponents of fixed exchange rates are suggesting. Furthermore, if citizens hold foreign-currency-denominated assets (as in Boyer (1976)) variability of the exchange rate may be considered bad because of its effects on the value of nominal wealth. The author has started an analysis of this more general loss function.

<sup>10</sup>This definition of the exchange rate is employed in order to provide similarity between the diagrammatic techniques developed here and those used by Poole (1970).

<sup>11</sup>This method of intervening, in the foreign exchange market with reserves rather than in the asset market with money, in order to prevent large fluctuations in the exchange rate is discussed by Mikesell and Goldstein (1975).

<sup>12</sup> On this definition a rise in foreign-currency prices abroad is a good example of a real shock. This shifts only the IS curve since prices of foreign goods are already built into the price index. See Boyer (1976).

<sup>13</sup> This theorem has been proven by Mundell (1963), McKinnon and Oates (1966), Sohmen (1969), and Boyer (1975).

<sup>14</sup> A typical example of such a shock is a shift of asset holdings from assets denominated in domestic currency to ones denominated in foreign currency, perhaps in anticipation of a devaluation. Expectations throughout this analysis are taken to be static. See Laffer (1973) for an analysis of this shock in a rigid-price model.

<sup>15</sup> This is a standard result in both the earlier specification of the capital account (Mundell (1963), Sohmen (1969), and Swoboda (1973)) and in portfolio balance approaches (McKinnon and Oates (1966), Boyer (1975), and Dornbusch (1975)).

<sup>16</sup> When shifts of the IS and LM loci are interpreted in terms of policy changes on the part of the authorities, the efficacy of these policies in a closed economy analysis depends upon the slopes of these loci, as any undergraduate student knows.

<sup>17</sup> This conclusion depends upon the assumption that an equilibrium for the economy exists. This would not be the case if both the asset and the goods markets were independent of the exchange rate. In order for the equilibrium to exist one of these markets must depend on that variable, a situation which is not hard to envisage.

<sup>18</sup>In the analysis here it is assumed that there is a single market of each category. If there are more markets in the economy, then the conclusion of this section can be applied to the aggregate of markets in general.

<sup>19</sup>H. Theil (1964) provides an explanation for the use of this form of loss function.

<sup>20</sup>The results derived here can be applied to an aggregate of variables if they are numerous.

<sup>21</sup>These are measures of the horizontal movements of each of these loci. That is, the change in the value of the variable on the horizontal axis due to the shift of one locus assuming the other is perfectly horizontal.

<sup>22</sup>This assumption is made in order to derive specific results. Furthermore, it is the typical way in which the authorities intervene to stabilize the exchange rate. However, as the earlier discussion shows, this analysis can be applied to case in which the goods market is used to intervene. See Parkin (1976) for an interesting application of these principles to the problem of designing an optimal fiscal policy.

<sup>23</sup>The discussion in this paper assumes that the parameters of the private sector are exogenous, and the desired slopes of the equilibrium loci are attained through alteration in the authorities' reaction function. However, the identical problem could be solved assuming that the authorities' behavior is given (for example, they follow a constant money supply policy) and the private sector's behavior can be altered (for example, by changing the interest elasticity of demand for money). This latter solution is carried out by Poole (1970, pp. 205-6).

<sup>24</sup>The optimal slope for the LM curve in these circumstances is vertical even when the demand for money depends upon the exchange rate. The reaction of the authorities should be designed in order to offset any dependence of this sort.

<sup>25</sup>Poole (1970) notes that part of the problem with using an optimal combination policy for closed economy monetary management is that such a policy depends in a complex fashion on the stochastic parameters.

Figure III helps to establish the circumstances under which various policies are optimal. The "perverse" cases Poole cites where the money supply and the indicator move in the opposite direction, are more likely to be optimal in the closed economy analysis, where a constant money stock policy yields a positively sloped LM curve, than in the exchange rate problem where such a policy yields a vertical LM curve.

<sup>26</sup>Poole (1970) points out that an increase in the interest elasticity of demand for money can reduce losses under a money stock rule if  $\rho > \frac{\sigma_p}{\sigma_a}$  (in our notation). The intuitive explanation for this, in our analysis, is as follows: Figure III shows that economies satisfying that inequality have an optimal LM curve which is negatively sloped and flatter than IS. Failing the ability to make the interest elasticity of money demand positive so as to make the actual LM coincide with its optimal position, we should make the elasticity as large as possible negatively so as to get as close as possible to that position.



<sup>27</sup>The values of the stochastic parameters where the optimal LM curve differs substantially from either a horizontal or a vertical position are congregated around point U in Figure III. The rectangular hyperbola FF gets "thick" as the ratio of the variances get large so that all other geodesics are forced out of the (-1,+1) interval as we move to the right in Figure III.

<sup>28</sup>The reader can demonstrate that at no point in either Area A or Area D does the expected loss from a constant exchange rate policy exceed  $\sigma_p^2$  or a constant money stock policy exceed  $\sigma_a^2$ .

<sup>29</sup>This assumption is sufficient but not necessary for the conclusions which follow. The conclusions continue to be valid so long as shocks in the goods market for the private sector are not much smaller than those in the money markets.

<sup>30</sup>This is especially true since, as footnote 11 points out, inflation in the rest of the world appears to be a real shock to a small open economy.

<sup>31</sup>If the authorities do not act independently under fixed exchange rates, the money stock will automatically increase through a rise in reserves. If they act by increasing domestic credit, this increase in reserves can be avoided.

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Appendix

The problem in the text can be generalized in the following manner:

There are three important endogenous magnitudes in the economy: a variable which is not observable contemporaneously that the authorities wish to stabilize (the target variable), denoted by  $e_1$ ; and two variables,  $e_2$  and  $e_3$ , which are contemporaneously observable with one of them,  $e_3$ , entering only the excess demand function of the market in which the authorities intervene. For the problems in the text these variables are:  $e_1$ , the level of nominal income (with either output or prices adjusting);  $e_2$ , the interest rate for the closed economy monetary problem and the exchange rate for the foreign exchange market problem; and  $e_3$ , the money supply.

The desirability of stabilizing the target variable is formalized in a quadratic loss function:

$$L(e_1) = (e_1 - e_1^*)^2$$

where  $e_1^*$  is the desired value of the target variable. The expected value of this function can be written in the form:

$$E[L(e_1)] = E[(e_1 - e_1^*)^2] = E[(e_1 - E[e_1])^2] + (E[e_1] - e_1^*)^2$$

where  $E$  is the expected value operator. This expression shows that the authorities wish to minimize the loss function in two steps. First, they wish to set the values of their policy instruments so that the expected value of the target variable is  $e_1^*$ . When this is done the expected loss function reduces to the simpler form:

$$E[L(e_1)] = E[(e_1 - E[e_1])^2] = \sigma_{e_1}^2$$

It is the further task of the authorities, to minimize the variance of  $e_1$ , upon which this analysis concentrates.

The economy attains equilibrium during every time period even though the value of  $e_1$  is not known until subsequent periods. The equilibrium condition for the aggregate of markets free of government intervention is

$$X(e_1, e_2) = \tilde{v}_p \quad (A1)$$

where  $X( )$  is the excess demand created in these markets by the endogenous variables and  $\tilde{v}_p$  is the random excess supply with mean zero and variance  $\tilde{\sigma}_p$ .

The equilibrium condition of the private sector for the aggregate of markets in which the authorities operate is

$$L(e_1, e_2, e_3) = \tilde{v}_a \quad (A2)$$

where  $L( )$  is the private sector's excess demand in this market and  $\tilde{v}_a$  is the exogenous random element of excess supply arising from the private market, with mean zero and variance  $\tilde{\sigma}_a$ . Uncertainty arises only in the private sector so that the condition of equilibrium of the authorities is that their excess demand,  $G( )$ , be equal to zero:

$$G(e_2, e_3) = 0. \quad (A3)$$

An alternative way of describing this condition is to say that the reaction function is always satisfied.

For small changes in the exogenous and endogenous variables equations (A1) - (A3) can be written in the form

$$\begin{aligned} de_1 + \eta_p de_2 &= \frac{d \tilde{v}_p}{X_1} \\ de_2 + \eta_{p2} de_2 + \eta_{p3} de_3 &= \frac{d \tilde{v}_p}{L_1} \\ + \eta_{a3} de_2 + de_3 &= 0 \end{aligned} \quad (A4)$$

where  $X_1$  is  $\frac{\partial X}{\partial e_1}$  and  $L_1$  is  $\frac{\partial L}{\partial e_1}$  and the  $\eta_i$ 's are the ratios of partial derivatives.

Combining the last two equations of (A4) and redefining the stochastic terms yield the system

$$de_1 + \eta_p de_2 = dv_p$$

$$de_1 + \eta_a de_2 = dv_a$$

where  $dv_p$  has variance  $\sigma_p$  and  $dv_a$  has variance  $\sigma_a$ . The elasticity  $\eta_a$  is related to the elasticities above by the equation

$$\eta_a = \eta_{p2} - \eta_{a3} \eta_{p3} .$$

The elasticity,  $\eta_p$ , of the private sector's equilibrium locus is exogenous. In contrast, the elasticity of the equilibrium locus in the authorities' sector markets is a quantity chosen so as to minimize the variance of  $de_1$ . The optimal value of this can be derived in terms of the parameters of the model.

The change in  $e_1$  can be written as a function of the random variables:

$$de_1 = \frac{\eta_a \cdot dv_p - \eta_p \cdot dv_a}{\eta_a - \eta_p} .$$

The variance of  $e_1$  is then

$$\sigma_{e_1}^2 = \frac{\eta_a^2 \sigma_p^2 + \eta_p^2 \sigma_a^2 - 2\eta_a \eta_p \rho \sigma_a \sigma_p}{(\eta_a - \eta_p)^2} \quad (A5)$$

where  $\rho$  is defined

$$\rho = \frac{\sigma_{ap}}{\sigma_a \sigma_p} .$$

The value of  $\eta_a$  which minimizes the variance of  $e_1$ , denoted by  $\eta_a^*$ , is found by taking the derivative of expression (A5) with respect to  $\eta_a$  and setting it equal to zero. The optimal value of  $\eta_a$  is

$$\eta_a^* = -\eta_p \left( \frac{\rho - \frac{\sigma_a}{\sigma_p}}{\rho - \frac{\sigma_p}{\sigma_a}} \right) . \quad (A6)$$

This formula is more amenable to analysis if the elasticities are replaced by slopes using the following substitutions:

$$s_a^* = \frac{1}{\eta_a^*} ; \quad s_p = \frac{1}{\eta_p} .$$

Then equation (A6) becomes

$$s_a^* = -s_p \left( \frac{\rho - \frac{\sigma_p}{\sigma_a}}{\rho - \frac{\sigma_a}{\sigma_p}} \right) .$$

This is equation (12) in the text.