Optimal hydraulic structures profiles under uncertain seepage head

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Abstract: Most of the hydraulic structures are founded on permeable foundation. There is, however, no procedure to fix the basic barrage parameters, which are depth of sheet piles/cutoffs and the length and thickness of floor, in a cost-effective manner. Changes in hydrological and climatic factors may alter the design seepage head of the hydraulic structures. The variation in seepage head affects the downstream sheet pile depth, overall length of impervious floor, and thickness of impervious floor. The exit gradient, which is considered the most appropriate criterion to ensure safety against piping on permeable foundations, exhibits non linear variation in floor length with variation in depth of downstream sheet pile. These facts complicate the problem and increase the non linearity of the problem. However, an optimization problem may be formulated to obtain the optimum structural dimensions that minimize the cost as well as satisfy the exit gradient criteria. The optimization problem for determining an optimal section for the weirs or barrages normally consists of minimizing the construction cost, earth work, cost of sheet piling, length of impervious floor etc. The subsurface seepage flow is embedded as constraint in the optimization formulation. Uncertainty in design, and hence cost from uncertain seepage head are quantified using fuzzy numbers. Results show that an uncertainty of 15 percent in seepage will result in 22 percent of uncertainty in design represented by overall design cost. The limited evaluation show potential applicability of the proposed method.

Keywords: *Nonlinear Optimization Formulation, Genetic Algorithm, Hydraulic Structures, Barrage Design, Fuzzy Numbers, Uncertainty Characterization.*

1. Introduction

Hydraulic structures such as weirs and barrages are costly water resources projects. A safe and optimal design of hydraulic structures is always being a challenge to water resource researchers. The hydraulic structure such as barrages on alluvial soils is subjected to subsurface seepage. The seepage head causing the seepage vary with variation in flows. Design of hydraulic structures should also insure safety against seepage induced failure of the hydraulic structures.

The variation in seepage head affects the downstream sheet pile depth, overall length of impervious floor, and thickness of impervious floor. The exit gradient, which is considered the most appropriate criterion to ensure safety against seepage induced piping (Khosla, et al., 1936; Asawa, 2005) on permeable foundations, exhibits non linear variation in floor length with variation in depth of down stream sheet pile. These facts complicate the problem and increase the non linearity of the problem. However, an optimization problem may be formulated to obtain the optimum structural dimensions that minimize the cost as well as satisfy the safe exit gradient criteria.

The optimization problem for determining an optimal section for the weirs or barrages consists of minimizing the construction cost, earth work, cost of sheet piling, and length of impervious floor (Garg et al., 2002; Singh, 2007). Earlier work (Garg et al., 2002) discussed the optimal design of barrage profile for single deterministic value of seepage head. This study first solve the of nonlinear optimization formulation problem (NLOP) using genetic algorithm (GA) which gives optimal dimensions of the barrage profile that minimizes unit cost of concrete work, and earthwork and searches the barrage dimension satisfying the exit gradient criteria. The work is then extended to characterize uncertainty in design due to uncertainty in measured value of seepage head, an important hydrogeologic parameter. Uncertainty in design, and hence cost from uncertain head value are quantified using fuzzy numbers

2. Subsurface flow

The general seepage equation under a barrage profile may be written as:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}=0 \qquad (1)$$

This is well known Laplace equation for seepage of water through porous media. This equation implicitly assumes that (i) the soil is homogeneous and isotropic; (ii) the voids are completely filled with water; (iii) no consolidation or expansion of soil takes place; and (iv) flow is steady and obeys Darcy’s law.

For 2-dimensional flow, the seepage equation (1) may be written as:

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}=0 \qquad (2)$$

The need to provide adequate resistance to seepage flow represented by equation (1) both under and around a hydraulic structure may be an important determinant of its geometry (Skutch, 1997).The boundary between hydraulic structural surface and foundation soil represents a potential plane of failure.

Stability under a given hydraulic head could in theory be achieved by an almost limitless combination of vertical and horizontal contact surfaces below the structure provided that the total length of the resultant seepage path were adequately long for that head (Skutch, 1997; Leliavsky, 1979). In practical terms, the designer must decide on an appropriate balance between the length of the horizontal and vertical elements. Present work utilized Khosla's Method of independent variables (Asawa, 2005) to simulate the subsurface behavior in the optimization formulation. Method of independent variables is based on Schwarz-Christoffel transformation to solve the Laplace equation (1) which represents seepage through the subsurface media under a hydraulic structure. A composite structure is split up into a number of simple standard forms each of which has a known solution. The uplift pressures at key points corresponding to each elementary form are calculated on the assumption that each form exists independently. Finally, corrections are to be applied for thickness of floor, and interference effects of piles on each others.

3. Optimal design methodology

Minimize $C\,(L, d_1, d_d) = c_1(f_1) + c_2(f_2) + c_3(f_3) + c_4(f_4) + c_5(f_5)$ (4)

Subject to $$SEG \geq \frac{H}{d_d \pi \sqrt{\lambda}} \qquad (5)$$

$$L^l \leq L \leq {}^u \qquad (6)$$

$$d_1{}^l \leq d_1 \leq d_1{}^u \qquad (7)$$

$$d_d^{\,l} \le d_d \le d_d^{\,u} \quad (8)$$

$$L,\ d_1,\ d_d \ge 0 \quad (9)$$

where C (L, d_1, d_d) is objective function represents total cost of barrage per unit width (Rs/m), and is function of floor length (L), upstream sheet pile depth (d_1) and downstream sheet pile depth (d_d); f_1 is total volume of concrete in the floor per unit width for a given barrage profile and c_1 is cost of concrete floor (Rs/m^3); f_2 is the depth of upstream sheet pile below the concrete floor and c_2 is the cost of upstream sheet pile including driving (Rs/m^2); f_3 is the depth of downstream sheet pile below the concrete floor and c_3 is the cost of downstream sheet pile including driving (Rs/m^2); f_4 is the volume of soil excavated per unit width for laying concrete floor and c_4 is cost of excavation including dewatering (Rs/m^3); f_5 is the volume of soil required in filling per unit width and c_5 is cost of earth filling (Rs/m^3); SEG is safe exit gradient for a given soil formation on which the hydraulic structure is constructed and is function of downstream depth and the length of the floor; $\lambda = \frac{1}{2}[1+\sqrt{1+\alpha^2}]$; $\alpha = \frac{L}{d_d}$; L is total length of the floor; H is the seepage head ; d_1 is the upstream sheet pile depth; d_2 is downstream sheet pile depth; L^l, $d_1^{\,1}$, and $d_d^{\,1}$ is lower bound on L, d_1 and d_d respectively**;** L^u, $d_1^{\,u}$, $d_d^{\,u}$ are upper bound on L, d_1 and d_d respectively. The constraint equation (5) may be written as follows after substituting the value of λ :

$$L - d_d\left(\{2(\frac{H}{d_2\pi(SGE)})^2 - 1\}^2 - 1\right)^{1/2} \ge 0 \quad (10)$$

In the optimization formulation, for a give barrage profile and seepage head H, f_1 is computed by estimating thickness at different key locations of the floor using Khosla's method of independent variables and hence nonlinear function of length of floor (L), upstream sheet pile depth (d_1) and downstream sheet pile depth (d_2). Similarly f_4, and f_5 is nonlinear. The constraint represented by equation (10) is also nonlinear function of length of the floor and downstream sheet pile depth (d_2). Thus both objective function and constraint are nonlinear; make the problem in the category of nonlinear optimization program (NLOP) formulation, which are inherently complex. Characterization of functional parameters is available in literature (Singh, 2007; Garg et al., 2002).

3.1. Characterizing model functional parameters

For a given geometry of a barrage and seepage head H, the optimization model functional parameters f_1, f_2, f_3, f_4 and f_5 are characterized for the barrage profile shown in Fig. 1.

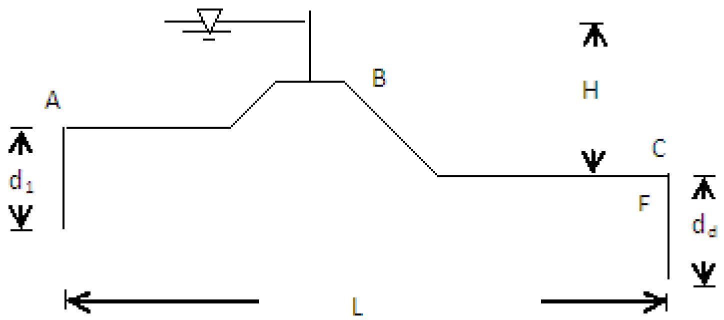

Fig. 1. Schematic of barrage parameters utilized in performance evaluation

Intermediate sheet-piles are not effective in reducing the uplift pressures and only add to the cost of in reducing the uplift pressures and only add to the cost of the barrage (Garg et al., 2002). In present work, no intermediate sheet piles are considered.

3.2. Optimization procedure using genetic algorithm

GA was originally proposed by Holland (Holland, 1975) and further developed by Goldberg (Goldberg, 1989). It is based on the principles of genetics and natural selection.GA's are applicable to a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, non-differentiable, stochastic, or highly nonlinear (Haestad 2003). The GA search starts from a population of many points, rather than starting from just one point. This parallelism means that the search will not become trapped on local optima (Singh and Datta, 2006).

The optimization model represented by equations (4)-(10) and the functional parameters embedded in the optimization model are solved using Genetic Algorithm on M ATLAB platform. The basic steps employed in solution are available in Singh, 2007. Table 1 shows physical parameters obtained by conventional methods for Fig. 2.

Table 1. Physical parameters values of barrage profile

Physical parameters	Values (meters)
*L	105.37
H	7.12
*d_1	5.45
*d_2	5.9

* Decision variables to be optimized

4. Uncertainty characterization in the optimization model

Real-world problems, especially those that involve natural systems, such as soil and water, are complex and composed of many non-deterministic components having non-linear coupling. In dealing with such systems, one has to face a high degree of uncertainty and tolerate imprecision. There is a high degree of local soil variability, and imprecision in the determination of soil parameters and hydrological parameters like seepage head. Statistical techniques have been traditionally used to deal with parametric variation in model inputs, but these require substantial hydrogeologic explorations data for estimates of probability distributions. In the presence of limited, inaccurate or imprecise information, simulation with fuzzy numbers represents an alternative tool to handle parametric uncertainty. Fuzzy sets offer an alternate and simple way to address uncertainties even for limited exploration data sets. In the present work, the optimal design is first obtained assuming a deterministic value of hydrogelogic parameter, safe exit gradient, in optimization model. Uncertainty in safe exit gradient is then characterized using fuzzy numbers. The fuzzified NLOF is then solved using GA.

Uncertainty in general comes in two forms: aleatory (stochastic, random natural variability or noncognitive) and epistemic (cognitive or subjective) (Hofer et al., 2002). Recently, Srinivasan et al. (2007) identified these uncertainties in hydrogeological applications. Aleatory uncertainty refers to uncertainty that cannot be reduced by more exhaustive measurements or by a better model. Epistemic uncertainty, on t he other hand, refers to uncertainty that can be reduced (Ross et al., 2009).

One of the milestones in the evolution of these new uncertainty theories is the seminal paper by Lofti A. Zadeh (1965). He proposed a new mathematical tool in his paper and called this new mathematical tool "fuzzy sets." He proposed the concept of fuzzy algorithms in 1968 (Zadeh, 1968), and together with Bellman, proposed a new approach for decision-making in fuzzy environments in 1970 (Bellman & Zadeh, 1970). Fuzzy set theory has been recently applied in various fields for uncertainty quantification (Cho et al., 2002; Hanss, 2002; Kentel & Aral, 2004; Mauris et al., 2001).

The transformation method presented by Hanss, (2002) uses a fuzzy alpha-cut (FAC) approach based on i nterval arithmetic. The uncertain response reconstructed from a set of deterministic responses, combining the extrema of each interval in every possible way unlike the FAC technique where only a particular level of membership (α -level) values (Hanss & Willner, 1999) for uncertain parameters are used for simulation.

Fuzzy modeling of uncertainty for hydrogeologic parameters such as exit gradient and seepage head is based on Z adeh's extension principle (Zadeh, 1968) and transformation method (TM) (Hanss, 2002). In present study only seepage head is considered to be imprecise. Input seepage head as imprecise parameter, is represented by fuzzy numbers. The resulting output i.e. minimum cost obtained by the optimization model is also fuzzy numbers characterized by their membership functions. The reduced TM (Hanss, 2002) is used in the present study. The measure of uncertainty used is the ratio of the 0.1-level support to the value of which the membership function is equal to 1 (Abebe et al., 2000).

5. Results and discussion

Earlier (mid 19th century), weirs and barrages have been designed and constructed in India on the basis of experience using the technology available at that period of time. Some of them were based on Bligh's creep theory, which proved to be unsafe and uneconomical. Comparison of the parameters of these structures with the proposed approach is, thus, not justified. Therefore, a typical barrage profile, a spillway portion of a barrage, is chosen for illustrating the proposed approach as shown in Fig. 2. The barrage profile shown in Fig. 2 and parameters values given Table 1 is solved employing the methodology presented in this work. The optimized values of parameters for a deterministic seepage head value of 7.12m are shown in Table 2. During the process of optimization, the process of going into new generation continues until the fitness of the population converged i.e. average fitness of population almost matches with the best fitness. This criterion proves the solution to be optimized. The optimized values of parameters for a deterministic seepage head value of 7.12m are shown in Table 2.

Table 2. Optimized parameters for safe exit gradient equal to 1/8 and minimum thickness of floor as 1m

Physical parameters	Values
L	61
d_1	3.1
d_2	9.2

It also resulted in a smaller floor length and overall lower cost. It has shown a savings in the barrage cost ranging from 16.73 percent.

For characterization of uncertainty, seepage head is assumed to vary from 6.0m to 8.19m with central value of 7.12m i.e. almost 15 percent in triangular fuzzy numbers representation. The result of variation in cost is corresponding different degree of membership for seepage head shown in Fig.2. The measure of uncertainty is found to be 22 percent. Since, left and right spread from central value of exit gradient is almost 15 percent, it can be concluded that uncertainty in seepage head reflects comparatively more uncertainty) more than 15 percent) in cost.

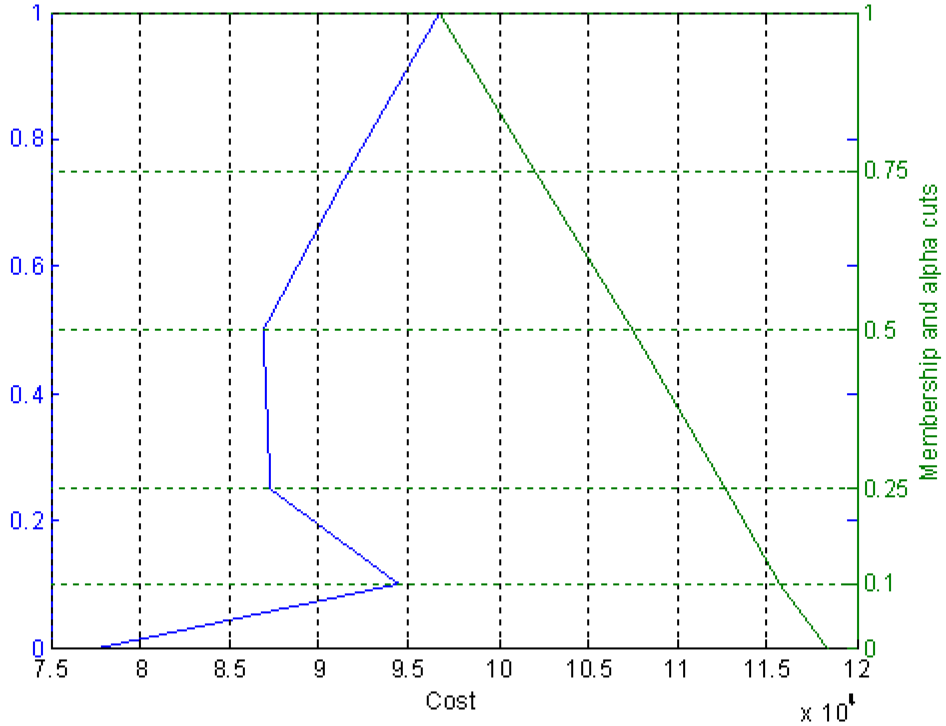

Fig.2. Costs variations corresponding to different α-cuts of seepage head

6. Conclusions

The present work also demonstrates the fuzzy based framework for uncertainty characterization in optimal cost for imprecise hydrologic parameter such as seepage head. The uncertainty in cost is found not to be directly proportional to uncertainty in seepage head. The GA based optimization approach is equally valid for optimal design of other major hydraulic structures.

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