Optimal Information Extraction in Energy-Limited Wireless Sensor Networks

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Abstract

The current practice in wireless sensor networks is to develop functional system designs and protocols for information extraction using intuition and heuristics, and validate them through simulations and implementations. We address the need for a complementary formal methodology by developing non-linear optimization models of static WSN that yield fundamental performance bounds and optimal designs. We present models both for maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception), and for minimizing the energy usage subject to information constraints. Other constraints in these models correspond to fairness and channel capacity (assuming noise but no interference). We also discuss extensions of these models that can handle data aggregation, interference and even node mobility. We present results and illustrations from computational experiments using these models that show how the optimal solution varies as a function of the energy/information constraints, network size, fairness constraints, and reception power. We also compare the performance of some simple heuristics with respect to the optimal solutions.

1 Introduction

Wireless sensor networks (WSN) are an emerging technology which seem ready to revolutionize the availability and quality of information in a wide array of application areas. This new technology has come about due to the rapid advances in embedded microprocessors, wireless communications, and MEMS sensors over the past decade.

As we set out to design and implement these kinds of systems, however, one fact becomes clear. *In the area of WSN there is a significant gap between practice and formal understanding: proposed system designs and protocols are rapidly out-pacing analysis.* There are very few formal models for analyzing the fundamental performance of information routing in wireless sensor networks.

Such models are necessary to understand the theoretical bounds on performance and how they are affected by different design parameters such as topology, number of nodes, energy levels, and fairness. We take an optimization approach in this paper. Due to the underlying equations that describe the capacity of physical channels, we will rely on convex non-linear programming techniques. We present models both for maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception) and for minimizing the energy usage subject to information constraints. Fairness constraints are also modeled. We will also discuss extensions of these models that can handle data aggregation, interference and even node mobility.

Optimization models can aid us in two complementary ways. The first involves designing a WSN for a given application. The best network configuration for an application is often difficult to determine due to the variability in problem parameters that characterize the diverse applications to which this technology can be applied. These parameters include the quality of information requested, the energy cost of sensing and receiving information, and node positions. The most appropriate network parameters for the application in question can be determined by comparing the optimal performance for different parameter settings.

The second way in which optimization models can improve our understanding of WSN concerns the operation of a sensor network. Here an optimization model provides the means to evaluate proposed protocols for information routing. Much of the current literature in sensor and ad-hoc wireless networks consists of practical proposals for new protocols for information routing. Typically, simulation results are used to examine the impact of various parameters on the effectiveness of the protocol. Comparisons are usually performed with respect to some baseline heuristic strategies or with alternative protocols. Iterated over time, this procedure yields practical, implementable protocols with successively better performance characteristics. However, if we do not know the fundamental bounds imposed by the underlying problem structure, then it will not be clear how the implemented protocol differs from optimal performance. It is important to know the fundamental performance bounds and optimal solutions to determine if there is room for additional improvement of a given protocol.

The rest of the paper is organized as follows. In section 2, we discuss some recent papers on information routing in wireless sensor networks and optimization models of wireless networks to place our work in context. In section 3, we define our notation and present a basic operational optimization model. We expand this model in section 4 to a more general pair of optimization problems that can be used for WSN design problems and discuss extensions. We present computational results based on these models in section 5, and conclude with a discussion in section 6.

2 Related Work

Wireless sensor networks, consisting of large numbers of unattended devices capable of communication, computation and sensing are fast becoming a hot research area [2], [3]. Some of the early work in this area has developed the hardware and software needed for such sensor systems - e.g. Smart Dust Motes [11], TinyOS [13], and PicoRadio [15] projects from UC Berkeley; the Wireless Integrated Network Sensors (WINS) project [1] at UCLA; and the μAMPS project at MIT [14].

Several papers describe novel querying and routing mechanisms suitable for sensor networks. These include LEACH [18], SPIN protocols [17], Directed Diffusion [19], Rumor Routing [26] , ACQUIRE [23], GHT [28], and DIMENSIONS [29]. Protocols have also been developed for querying sensor networks as distributed databases [20], [21], [22], [27]. In most of this work, the performance of the proposed querying and routing mechanisms is validated through simulations or implementation, without reference to an optimum benchmark solution.

There are also some papers presenting first-order performance analysis of such protocols. For example, in [23], the performance of ACQUIRE is analyzed using mathematical modeling. Different variants of Directed Diffusion are modeled and analyzed in [31]. However, the body of literature on the optimal behavior and fundamental limitations of these kinds of wireless networks is still considerably small.

In [6], the authors show that for an arbitrary communication pattern, the per-node throughput in a multi-hop wireless network goes to zero asymptotically as the size of the network increases. A similar negative result is shown for the many-to-one capacity of wireless sensor networks in [30]. The impact of spatial correlation on capacity for joint routing-compression is described in [9], [10]. The complexity of optimal aggregation and its impact on data gathering in sensor networks is discussed in [16]. An aggregation tree construction that yields an $O(\log k)$ to the optimum for all concave aggregation functions is described in [32].

Most closely related to our work presented here are papers relating to optimization models of general multi-hop wireless networks as well as wireless sensor networks (which can be considered a special case of the former with a many-to-one data flow instead of arbitrary communication between pairs of nodes). The most important work in this area in recent years has been the work by Toumpis and Goldsmith on capacity regions for wireless networks [4], [5]. Using a linear-programming optimization based formulation (similar in spirit to our work), the authors study the characteristics of the maximum information throughput that can be obtained in a network with arbitrary topology. One key difference from our work is that Toumpis and Goldsmith focus on general-purpose wireless networks and do not incorporate energy or fairness constraints in their modeling. They also do not use constraints corresponding to the non-linear channel capacity.

The non-linear physical channel constraints are considered in the optimization models discussed in [33], [34]. In these works the authors consider a similar model to ours (jointly optimizing the routing as well as power control and bandwidth allocation). They also treat the constraints imposed by interference in their models. Again a significant difference between our work and these models is that they do not focus on sensor networks where energy and fairness constraints are important.

Optimization models have also been used to study maximum lifetime conditions for sensor networks. Bhardwaj and Chandrakasan [8] develop upper bounds on the lifetime of networks based on optimum role assignments to sensors (e.g. whether they should act as routers or aggregators). Kalpakis *et al.* [24] formulate a linear programming problem to schedule flows within the network in such a way as to maximize the network lifetime. Our work incorporates a number of different constraints from these, such as the non-linear physical channel constraint (which allows for joint optimization of power control and routing) and fairness constraints.

3 Notation and Preliminary Model

Our first optimization model considers the problem of operating an existing WSN in the most efficient manner. Assume we have placed n sensor nodes in fixed locations, each with a limited energy supply E_i, and let d_{ij} denote the physical distance between nodes i and j. The purpose of this network is to extract as much information as possible to a given sink node (node $n+1$ with unlimited energy resources – a reasonable assumption if the sink is "plugged in"). Each node consumes C units of energy per-bit received and β units of energy per-bit sensed.

We assume that the sensor nodes can adjust both the information flow rate and the transmission power, which are denoted f_{ij} and P_{ij} for the link between nodes i and j, respectively. The relation between the flow rate and transmission power on a link is given by Shannon's capacity equation for an AWGN channel, assuming a square-law signal decay:

$$f_{ij} \leq \log\left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta}\right) . \tag{1}$$

This expression assumes that the decay factor of the medium is d_{ij}^{-2}, the communication channel has a noise of η, and that all transmissions are scheduled (either through time or frequency division multiplexing) such that they are non-interfering.

The objective therefore is to find the coordinated operation of all nodes by setting transmission powers and flow rates in order to maximize the amount of information that reaches the sink:

$$\sum_{j=1}^{n} f_{jn+1} . \tag{2}$$

We assume that there is no data aggregation in this model, and additionally we guarantee end-to-end fairness of our solution by explicitly enforcing that each node sends at most a fraction α_i of the total information that reaches the sink.

$$\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 \qquad i = 1:n \tag{3}$$

$$\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} \qquad i = 1:n \tag{4}$$

The fractions must satisfy $\sum_{i=1}^{n} \alpha_i \geq 1$ and $\alpha_i \geq 0$ for all $i = 1:n$ for there to be a feasible solution. The total energy consumed at node i, which we denote ε_i, is the sum of the energy consumed sensing, transmitting and receiving. The constraint that this energy does not exceed the available energy E_i for all sensor nodes is given by

$$\varepsilon_i := \beta \left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \right) + \sum_{j=1}^{n+1} P_{ij} + \sum_{j=1}^{n} C f_{ji} \leq E_i \quad i = 1:n \tag{5}$$

Combining the objective in Expression (2) with the constraints in Expressions (1), (3), (4), (5), and the fact that the flow rates and transmission powers are non-negative we obtain the following non-linear optimization model

$$\begin{aligned}
\max \quad & \sum_{j=1}^{n} f_{jn+1} & \\
\text{s.t.} \quad & \sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 & i = 1:n \\
& \sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} & i = 1:n \\
& \beta \left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \right) + \sum_{j=1}^{n+1} P_{ij} + \sum_{j=1}^{n} C f_{ji} \leq E_i & i = 1:n \\
& f_{ij} \leq \log \left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta} \right) & i = 1:n,\ j = 1:n+1 \\
& f_{ij} \geq 0, P_{ij} \geq 0 & i = 1:n,\ j = 1:n+1
\end{aligned} \tag{6}$$

4 Design Models

Consider now the problem of designing a WSN for a given application. In this context we now have the liberty to determine the position of the nodes and the amount of energy to place at each node. A node that should have more energy for the overall performance of the entire network can be given a bigger battery. The problem of actively optimizing the location of the nodes poses serious difficulties, as for example the non-linear inequality (1) becomes non-convex. We avoid this problem in the current work.

Below we present a pair of optimization problems that investigate the trade-off between minimum energy requirements and maximum information possible for a given network topology. We end the section discussing generalizations from these models and their different domains and applications.

The problem of deciding how to distribute a given overall amount of energy is easier to implement. Adding all the consumption of energy for every node i given in Expression (5) we obtain the following expression for the overall energy consumed by the sensor nodes of the WSN:

$$\begin{aligned}\sum_{i=1}^{n} \varepsilon_i &= \sum_{i=1}^{n}\left(\beta\left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji}\right) + \sum_{j=1}^{n+1} P_{ij} + \sum_{j=1}^{n} C f_{ji}\right) \\ &= \beta \sum_{i=1}^{n} f_{in+1} + \sum_{i=1}^{n}\sum_{j=1}^{n+1} P_{ij} + C \sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij} \\ &= \sum_{i=1}^{n} (\beta f_{in+1} + P_{in+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n} (C f_{ij} + P_{ij}) \ .\end{aligned}$$

Note that this expression for the total energy consumed has a nice interpretation: the first term represents the cost of sensing the information that is sent to the sink and the cost of transmitting it to the sink, while the second term represents simply the cost of transmitting information among all pairs of nodes $i - j$, it is composed of P_{ij} the energy node i spends transmitting, and $C f_{ij}$ the cost node j spends receiving.

Assuming we have an overall energy budget of $E_{\max}$ to distribute among the sensor nodes, the previous expression for the total energy consumed gives the following overall energy bound:

$$\sum_{i=1}^{n} (\beta f_{in+1} + P_{in+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n} (C f_{ij} + P_{ij}) \leq E_{\max} \ . \tag{7}$$

Replacing the energy bound constraint (5) in Problem (6) by this later global energy constraint we obtain the following non-linear optimization problem, which allows the optimization problem to determine the optimal energy distribution.

$$\begin{array}{lll} \max & \displaystyle\sum_{j=1}^{n} f_{jn+1} & \\ \text{s.t.} & \displaystyle\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 & i = 1:n \\ & \displaystyle\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} & i = 1:n \\ & \displaystyle\sum_{i=1}^{n} (\beta f_{in+1} + P_{in+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n} (C f_{ij} + P_{ij}) \leq E_{\max} & \\ & \displaystyle f_{ij} \leq \log\left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta}\right) & i = 1:n,\ j = 1:n+1 \\ & f_{ij} \geq 0, P_{ij} \geq 0 & i = 1:n,\ j = 1:n+1 \end{array} \tag{8}$$

A "dual" problem to Problem (8) above, is the problem of minimizing the overall energy usage, while guaranteeing that we extract at least $f_{\min}$ information to the sink. This problem can be stated as

$$
\begin{array}{lll}
\min & \sum_{i=1}^{n} (\beta f_{in+1} + P_{in+1}) + \sum_{i=1}^{n}\sum_{j=1}^{n} (C f_{ij} + P_{ij}) & \\
\text{s.t.} & \sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq 0 & i = 1:n \\
& \sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \leq \alpha_i \sum_{j=1}^{n} f_{jn+1} & i = 1:n \\
& \sum_{j=1}^{n} f_{jn+1} \geq f_{\min} & \\
& f_{ij} \leq \log\left(1 + \frac{P_{ij} d_{ij}^{-2}}{\eta}\right) & i = 1:n,\ j = 1:n+1 \\
& f_{ij} \geq 0, P_{ij} \geq 0 & i = 1:n,\ j = 1:n+1
\end{array}
\tag{9}
$$

4.1 Relation between design problems

The relationship between Problems (8) and (9) above is the subject of the following proposition. In it we denote by $(f, P) = (f_{12}, \ldots, f_{nn+1}, P_{12}, \ldots, P_{nn+1})$ the vector of flow rate and transmission power variables.

Proposition 1 *If (f^*, P^*) is the optimal solution to Problem (9) with $f_{\min}$, then (f^*, P^*) is the optimal solution to Problem (8) with*

$$E_{\max} = \sum_{i=1}^{n} \left(\beta f^*_{in+1} + P^*_{in+1}\right) + \sum_{i=1}^{n}\sum_{j=1}^{n} \left(C f^*_{ij} + P^*_{ij}\right) \ .$$

Conversely, if (f^*, P^*) is the optimal solution to Problem (8) with $E_{\max}$, then (f^*, P^*) is the optimal solution to Problem (9) with $f_{\min} = \sum_{j=1}^{n} f^*_{jn+1}$.

Proof: First note that any (f^*, P^*) optimal for Problem (9) must satisfy $\sum_{j=1}^{n} f^*_{jn+1} = f_{\min}$. If this were not the case, we can define $a = \frac{f_{\min}}{\sum_{j=1}^{n} f^*_{jn+1}} < 1$ and $\widetilde{P}_{ij} = \eta d_{ij}^2 \left(e^{a f^*_{ij}} - 1\right)$, then the solution $(af^*, \widetilde{P})$ would be feasible for Problem (9) with a strictly better objective function, a contradiction. Let E^* denote the optimal objective function value of Problem (9). Consider now Problem (8) with $E_{\max} = E^*$, then (f^*, P^*) is feasible for (8) with an objective function value of $\sum_{j=1}^{n} f^*_{jn+1} = f_{\min}$. Let $(\widehat{f}, \widehat{P})$ denote the optimal solution for (8), then $\sum_{j=1}^{n} \widehat{f}_{jn+1} \geq f_{\min}$. Therefore $(\widehat{f}, \widehat{P})$ is feasible for (9) with a total energy cost less than E^*, and thus an optimal solution for (9), which in turn implies that $\sum_{j=1}^{n} \widehat{f}_{jn+1} = f_{\min}$. We conclude that (f^*, P^*) is optimal for Problem (8).

The proof of the converse assertion is analogous, where we use the continuity of the constraint functions to show that any optimal solution of Problem (8) must satisfy that

$$\sum_{i=1}^{n} \left(\beta f^*_{in+1} + P^*_{in+1}\right) + \sum_{i=1}^{n}\sum_{j=1}^{n} \left(C f^*_{ij} + P^*_{ij}\right) = E_{\max} \ . \qquad \blacksquare$$

The relationship between Problems (8) and (9) can also be observed computationally. In Figure 1 we plot both the maximal information extracted as a function of the energy bound and minimum energy needed as a function of the information bound. The experiments that originated these results considered the same WSN with all nodes placed in a straight line, the sink node at one end, 10 sensor nodes uniformly distributed from a distance 1 to 10 of the sink, and the following values for other problem parameters: $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $\alpha_i = 0.20$ for all i. The minimum information bound was varied from $f_{\min} = 1$, to $f_{\min} = 20$ when solving Problem (9), and the maximum energy bound was varied from $E_{\max} = 0.01$ to $E_{\max} = 0.2$ when solving Problem (8).

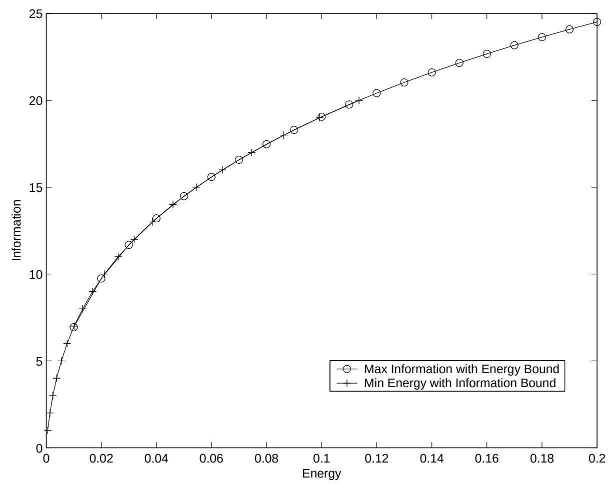

Figure 1: Optimal energy versus information

However there is one striking difference between these two optimization problems, Problem (9) exhibits consistently a faster convergence than Problem (8). This is illustrated by the following table which summarizes the number of interior point iterations that an algorithm takes to solve each problem, as we increase the number of nodes. The experiments considered all nodes placed in a straight line, with the sink node at one end, all sensor nodes uniformly distributed from 1 to 10 of the sink, and the following values for other problem parameters: $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $\alpha_i = 0.20$ for all i. The minimum information bound was set at 10 for Problem (9) and the maximum energy bound was set at 0.01 for Problem (8).

Table 1: Convergence comparison for Problems (1) and (2). Line topology with $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, $\alpha_i = 0.25$, and either information bound of 10 or energy bound of 0.01.

No. nodes	Problem (1) IPM iterations	Problem (2) IPM iterations
4	19	19
7	19	21
10	20	19
15	21	22
20	27	22
25	22	24
30	27	116
40	39	447
50	31	272
60	39	89
70	49	371
80	34	347

4.2 Problem properties

The discussion in this subsection highlights some important properties of Problem (9), which can be reduced to a much simpler form. This simplification, which does not have a direct translation to Problem (8), provides insight into the differences in the observed computational convergence.

To simplify our presentation we will use the arc-incidence matrix, as it is used in the Network Flows literature; see for example [25]. For a network with $n+1$ nodes and m arcs, the arc-incidence matrix, usually denoted by N, is a $n+1$ by m matrix with coefficients equal to 0, 1 or -1. The matrix is defined by

$$N_{i(k,l)} = \begin{cases} 1 & \text{if } i = k \\ -1 & \text{if } i = l \\ 0 & \text{otherwise .} \end{cases}$$

We can write the flow constraints, Expressions (3) and (4), in Problems (8) and (9) (the first two constraints), using matrix N as

$$0 \leq Nf \leq \alpha \sum_{j=1}^{n} f_{jn+1} \ .$$

Proposition 2 *Define $\kappa_j = C$ if $j = 1 : n$ and $\kappa_{n+1} = \beta$. Problem (9) obtains the same optimal solution as Problem (10) below:*

$$\begin{array}{lll} \min & \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij} + \eta d_{ij}^2 \left(e^{f_{ij}} - 1 \right) & \\ \text{s.t.} & 0 \leq Nf \leq \alpha f_{\min} & \\ & \sum_{j=1}^{n} f_{jn+1} = f_{\min} & \\ & f_{ij} \geq 0 & i = 1:n, j = 1:n+1 \ . \end{array} \tag{10}$$

Proof: Consider (f^*, P^*) any optimal solution for Problem (9). Then we have that f^* is feasible for (10), with a possibly better objective value since $\eta d_{ij}^2 \left(e^{f_{ij}^*} - 1 \right) \leq P_{ij}^*$. Conversely, given any f feasible for (10), define $P_{ij} = \eta d_{ij}^2 \left(e^{f_{ij}} - 1 \right)$, and (f, P) is feasible for (9) with the same objective value. This completes the proof. ■

For f a vector of flow rates, let us denote by $z(f)$ the objective function of Problem (10),

$$z(f) = \sum_{i=1}^{n} \sum_{j=1}^{n+1} \kappa_j f_{ij} + d_{ij}^2 \left(e^{f_{ij}} - 1 \right) \ ,$$

note that $z(f)$ is a convex function. We also denote by $S(y)$ the feasible region of Problem (10) and $\phi(y)$ the optimal objective function value of Problem (10), both as a function of the information bound $f_{\min} = y$. Therefore Problem (10) can be rewritten as

$$\begin{array}{rll} \phi(f_{\min}) = & \min & z(f) \\ & \text{s.t.} & f \in S(f_{\min}) \end{array} \tag{11}$$

Proposition 3 *The function $\phi(y)$ is convex for all $y \geq 0$*

Proof: First note that Problem (10) is feasible for any information bound $f_{\min} = y \geq 0$ and the objective function $z(f) \geq 0$, therefore $\phi(y)$ is well defined for all $y \geq 0$. Consider two information bound values

$y_1, y_2 \geq 0$ and let f_1^* and f_2^* be the respective optimal solutions for Problem (10). From the convexity of the constraints we can show that for any $\lambda \in [0,1]$, $\lambda f_1^* + (1-\lambda) f_2^* \in S(\lambda y_1 + (1-\lambda) y_2)$. Then the convexity follows from

$$\phi(\lambda y_1 + (1-\lambda)y_2) \leq z(\lambda f_1^* + (1-\lambda) f_2^*) \leq \lambda z(f_1^*) + (1-\lambda) z(f_2^*) = \lambda \phi(y_1) + (1-\lambda)\phi(y_2) \ . \qquad \blacksquare$$

4.3 Model variations

The following are some variations on these models that incorporate additional constraints or different variables:

- We can formulate all problems above only in terms of variables f_{ij} representing the number of bits transmitted from i to j (rather than the bit rate). In this setting each sensor node transmits with a fixed power, and therefore at a fixed rate. More bits are transmitted by taking a longer period of time. The resulting problem is a simpler linear programming model. A bit version of Problem (6), for example, is obtained by replacing Expression (1) by $f_{ij} \leq B$ and Expression (5) by

$$\beta \left(\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \right) + \sum_{j=1}^{n+1} T_{ij} f_{ij} + \sum_{j=1}^{n} C f_{ji} \leq E_i \ ,$$

  where T_{ij} is the per-bit transmission energy cost. The upper bound on the total bits effectively limits the length of a round. An interesting open question is to quantify the loss of efficiency on the network by considering this more constrained model.

- An alternate objective can be to maximize a weighted fairness. This could be achieved by replacing the objective in Problem (6) by $\max \sum_{i=1}^{n} w_i \alpha_i$ and by modifying the constraint (4) to

$$\sum_{j=1}^{n+1} f_{ij} - \sum_{j=1}^{n} f_{ji} \geq \alpha_i \ .$$

- Additional requirements that can be modeled by incorporating a simple linear constraint are: a per node limit on the total energy available to each node and a limit on the amount of information that can be sensed by each node during a round.

- Multiple time periods can be easily represented in these models. The only constraint that links different time periods is the energy constraint (Expression (5) in Problem (6) for example). The total energy available has to be distributed across all time periods.

- The problems above do not allow for data aggregation, i.e. all data that is sensed must leave the network through the sink. To expand the model to consider data aggregation we need to relax the lower bound in the flow constraints, Expression (3), to some negative value. To model data aggregation reasonably we should consider multi-commodity flows, in order to identify what is the data that can be aggregated.

The variations on the problem above are fairly straightforward and the only potential complications in solving the new models are that considering multiple time periods or multi-commodity flows makes the problem larger. Below are a pair of variations on the model that create non-convex optimization problems, and are therefore much harder to solve. We mention them here to show possible future research directions.

- The models above assume that the network has scheduled communications on all links (using TDMA or FDMA). For a CDMA like environment, interference poses a non-convex constraint. There are some techniques that can be used to handle such constraints approximately [34].

- Models can also consider the possibility of mobile nodes, in which locations and therefore inter-node distances can be varied as a design parameter at the expense of some energy for motion. However, this also introduces a non-convex constraint.

5 Computational Experiments

We performed our computational experiments with the non-linear solver LOQO 6.02 called from AMPL scripts. We used the NEOS server for optimization to perform our computations; see [35].

Throughout our computational experiments we have considered two different types of network topologies that are easily scalable and that we denote the *line topology* and the *square topology*. In the line topology we considered WSN where all sensor nodes lie uniformly distributed in a line of length L, with the sink node placed at one end. The square topology considers $n = k^2$ nodes uniformly distributed on a square grid with sides of length L ($[0, L] \times [0, L]$) and the sink is located outside that square.

Our computational experiments illustrate different possible uses for optimization models for WSN. We first show how to use Problem (10) to obtain the optimal amount of information that should be extracted from a given WSN. We then use the optimization model to benchmark the performance of two very simple heuristics that set energy levels and route information for different WSN. Our last three studies investigate the effect of different problem parameters on the performance of the sensor network: we show the effect of changing the fairness pattern on the minimum energy required to extract certain information, the effect of the fairness patterns on optimal energy distribution and routing patterns, and the effect of the reception cost on the routing behavior of the WSN.

5.1 Optimal level of information extraction

In our previous discussion of the equivalence between Problems (8) and (9) we presented computational results that showed the trade-off between the information to be extracted from the WSN and the energy needed to do so. In Figure 1 we note that each extra unit of information demands an increasing amount of energy. Granted this observation is for that particular example, we will therefore investigate this further.

We now address the question of whether for any WSN each extra unit of information demands an increasing amount of energy. The answer is yes and we show this by proving that for any WSN the subgradient of $\phi(y)$, $\partial\phi(y)$, is an increasing positive multifunction. The fact that $\partial\phi(y)$ is monotonic increasing is due to the convexity of $\phi(y)$ (Proposition 3), so we simply have to show that it is positive. We work with subgradients because $\phi(y)$ could be non-differentiable.

Proposition 4 *Assume that either $\beta > 0$ or $d_{in+1} > 0$ for all $i = 1 : n$. Then for any $y > 0$, all subgradients of $\phi(y)$ are positive.*

Proof: The assumptions $\beta > 0$ or $d_{in+1} > 0$ for all $i = 1 : n$ are to remove a pathological case where a sensor node located at the sink can supply the sink with y information at zero cost, in this case $\phi(y) = 0$ and $\phi'(y) = 0$.

Consider any $y > 0$, then from the fact that y flow has to reach the sink and the assumptions of this proposition, we have that $\phi(y) > 0 = \phi(0)$. The subgradient set of $\phi(y)$ at y is defined by

$$\partial\phi(y) = \{\xi \mid \phi(y+z) \geq \phi(y) + \xi z, \text{ for all } z \text{ s.t. } y+z \geq 0\} \ .$$

Consider any $\xi \in \partial\phi(y)$. The subgradient satisfies $\phi(0) \geq \phi(y) + \xi(-y)$, which implies that $\xi \geq \frac{\phi(y)}{y} > 0$. ■

In the context of a commercial application of a WSN and given the increasing energy cost of additional information for all WSN, it is reasonable to try to determine the optimal amount of information to extract from a given WSN. In a commercial setting it is reasonable to assume that there is some monetary value for information from a WSN, for example a dollars per unit of information, likewise it is reasonable to assume that the cost of energy is b dollars per unit of energy. In this setting we can explicitly compare the trade-off between information extraction and energy consumption by maximizing the net return function $V(y) = ay - b\phi(y)$, where y is the amount of information extracted. The maximum level of net return is obtained for the solution y^* that satisfies $0 \in \partial V(y^*)$, since $\phi(y)$ is convex, which implies that the optimal information level satisfies $a/b \in \partial\phi(y^*)$. Optimization solvers also provide subgradient values of $\partial\phi(y)$ as the dual variable on the information constraint, for illustrative purposes we plot the dual variable for an example in Figure 2. The values in $\partial\phi(y)$ quantify the change in the objective function with respect to changes in y. Because of the monotonicity of the subgradient, we can perform a binary search to obtain the information level y^* which has $a/b \in \partial\phi(y^*)$ and thus gives the optimal level of return. This leads to an approach where to obtain a solution $\hat{y}$ such that $|y^* - \hat{y}| \leq \varepsilon$, we solve a polynomial number ($O(\log(\frac{1}{\varepsilon}))$ of Problems (10).

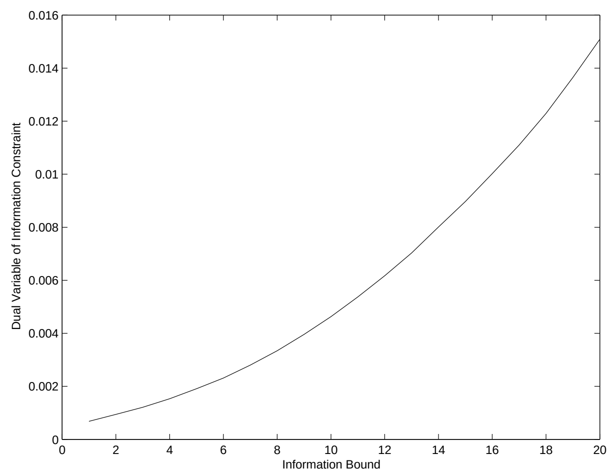

Figure 2: Dual variable of information bound for different information constraints

5.2 Comparison of Optimal Performance v.s. Heuristics

In this subsection we explore how the optimal performance given by Problem (10) compares to two very simple heuristics for assigning energy to nodes and distributing the information.

In our first heuristic we will only allow transmissions from nodes directly to the sink. The simplified version of Problem (10) under this assumption is

$$\begin{aligned} \min \quad & \sum_{i=1}^{n} \left(\beta f_i + \eta d_i^2 e^{f_i} - \eta d_i^2\right) \\ \text{s.t.} \quad & \sum_{i=1}^{n} f_i = f_{\min} \\ & f_i \leq \alpha_i f_{\min} \\ & f_i \geq 0 \ , \end{aligned}$$

where f_i represents the information flow from i to the sink. A seemingly efficient solution to this problem is to assign as much information as possible to the nodes with smallest objective function contribution. (This does not lead to the optimal as the optimal sets the flow at each node with a positive flow such that their objective contribution is the same.) We achieve this solution by the following heuristic, which we denote the

Direct Heuristic: (1) sort the nodes according to their distance to the sink, that is $d_1 \le d_2 \le \ldots \le d_n$, (2) set the flow from $i = 1$ to n to $f_i = \alpha_i f_{\min}$ until $\sum_{i=1}^n f_i = f_{\min}$, and (3) set all remaining flows to zero.

Our second heuristic, which we denote the Hop Heuristic, routes all information from a node to the closest node in the direction of the sink. This heuristic can be generally described as follows, Hop Heuristic: (1) sort the nodes according to their distance to the sink, (2) set the amount of flow generated at i from $i = 1$ to n to $\alpha_i f_{\min}$ until $\sum_{i=1}^n f_i = f_{\min}$, (3) determine the shortest path from every node providing information to the sink, (4) send all the information from i to the next node on the shortest path from i to the sink.

In Figures 3, 4, 5, 6, and 7 below we present how the optimal energy levels compare with the energy levels obtained from the heuristic procedures. The experiments considered linear and square topologies, and present the different energy levels as we increase the number of nodes in the network. For the both types of problems we considered the following problem parameters $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $f_{\min} = 10$. The linear topologies examples considered from 4 to 80 sensor nodes placed in a line uniformly distributed a distance 1 to 10 from the sink. The square topologies considered from 4 to 81 nodes uniformly distributed on a grid in the square $[0, 1] \times [0, 1]$ with the sink located at $(-0.3, 0.5)$. For both types of problems we considered two different uniform fairness patterns. One constant in n, with $\alpha_i = 0.25$ for all i, and the other with $\alpha_i = 2/n$ for all i.

Due to the fact that for the constant fairness pattern ($\alpha_i = 0.25$), both heuristic procedures will generate all information from the 4 closest nodes, the performance of both heuristics is very similar. We therefore opted to ignore the Hop Heuristic in this case and only plot the Direct Heuristic to compare with the optimal performance.

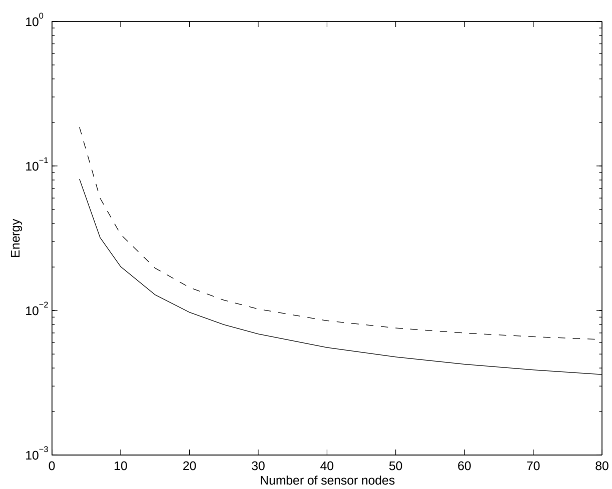

Figure 3: Minimal Energy and Direct Heuristic, for fixed fairness 0.25 as a function of the number of nodes, linear topology

A striking observation from Figure 4 is the poor performance of the Hop Heuristic. The reason for this is that in the line topology all the information is routed through the node that is closest to the sink, this node then is forced to spend a significant amount of energy to transmit it to the sink. In the Direct Heuristic and in the optimal solution no node transmits all the information, so the transmission powers can be much smaller.

The performance of the Hop Heuristic for the square topology is better, as can be noted in Figure 6. In this example, all the information is first routed to the nodes that are on the face of the square next to the sink, from where they can be transmitted directly. Therefore there is no “bottleneck” node that has to send all the information and incur in the exponential transmission cost.

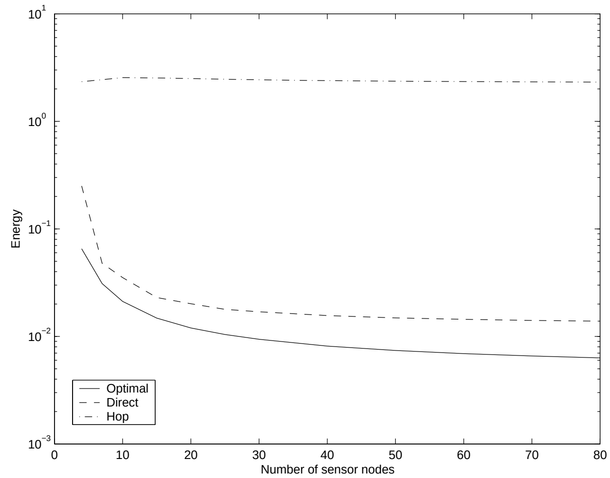

Figure 4: Minimal Energy and Heuristics, for variable fairness $2/n$ as a function of the number of nodes, linear topology

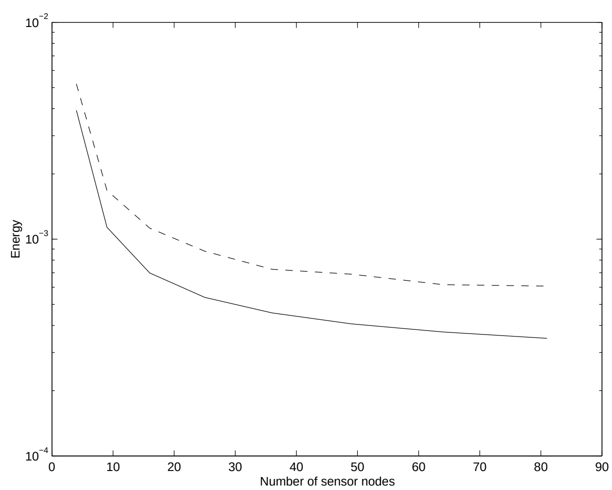

Figure 5: Minimal Energy and Direct Heuristic, for fixed fairness 0.25 as a function of the number of nodes, square topology

Note that the energy is plotted in a log axis, therefore although that in general the differences persist the Direct Heuristic provides a reasonable approximation to the optimal solution for large n. This statement however should be taken with a grain of salt, as the topology of the problem and problem parameters do influence the proximity of the heuristic to the optimal solution. For instance, in the example with square topology and fairness $\alpha_i = 2/n$ the heuristic obtains the same energy level as the optimal solution for n larger than 40. In fact the optimal solution routes the information directly to the sink just as the heuristic solution, see Figure 6. This observation breaks down if we simply scale the example by 100 (the sensor nodes are now uniformly distributed on the square $[0, 100] \times [0, 100]$ with the sink node at $(-30, 50)$), see Figure 7.

5.3 Effect of fairness on optimal energy

From Problem (10) above we note that the energy needed to extract a certain amount of information, depends in an exponential fashion on the amount of information to extract. Consider a given WSN with n sensor nodes, with a fixed topology, fixed energy cost coefficients β and C, and on a communication channel with a fixed noise η. We now obtain an upper bound on the amount of energy to extract any information level $f_{\min}$ with any fairness pattern α, such that $\sum_{i=1}^{n} \alpha_i \geq 1$. The optimal energy level E^* for the WSN will be less than or equal to $\widehat{E}$, the energy needed to route the information for a completely fair WSN, that is when $\alpha_i = \frac{1}{n}$ for all $i = 1, \ldots, n$. Analogously $\widehat{E}$ will be less than or equal to the energy needed to have each node route $\frac{1}{n} f_{\min}$ directly to the sink, since this is one of many feasible ways of routing that information. This means that

$$\begin{aligned} \widehat{E} &\leq \beta \sum_{i=1}^{n} \frac{1}{n} f_{\min} + \eta \sum_{i=1}^{n} d_{in+1}^2 \left(e^{\frac{1}{n} f_{min}} - 1 \right) \\ &= \beta f_{\min} + \sum_{i=1}^{n} d_{in+1}^2 \eta \left(e^{\frac{1}{n} f_{min}} - 1 \right) . \end{aligned}$$

This expression provides an upper bound on the energy needed to extract $f_{\min}$ information from a given WSN with any fairness pattern α. We plot this upper bound, as well as the energy needed for a WSN with uniform alpha patterns (we considered $\alpha_i = 0.1, 0.2$ and 1 on every sensor node) in Figure 8. The computational experiment considered a network with 11 sensor nodes, equally spaced on a line at a distance .1 to 1.1 from the sink, with other problem parameters set at $C = 0.00005$, $\beta = 0.00001$, and $\eta = 0.001$. We computed the optimal energy needed to extract for different information bound levels.

The process of obtaining a lower bound is trickier, as the optimal solution for the completely unfair WSN (i.e. $\alpha_i = 1$ for all $i = 1, \ldots, n$) still looks for the shortest route and origin to obtain its information. Simple bounds, like having all the information originate at the node closest to the sink, which has a cost of

$$\beta f_{\min} + \eta (\min\{d_{1n+1}, \ldots, d_{nn+1}\})^2 \left(e^{f_{min}} - 1 \right) ,$$

provide functions that are not bounds and increase very rapidly. We plot this function, and the energy cost of sending all the information from the node furthest from the sink, in Figure 9. Here we also plot the upper bound on the minimum energy obtained previously for comparison purposes. Valid lower bounds on the optimal energy for any fairness pattern are provided by the linear functions constructed from the subgradients of $\phi(y)$ for the completely unfair WSN.

5.4 Effect of fairness patterns on performance

In this subsection we study how different fairness patterns affect the optimal energy distribution in the WSN and also what is the form of the optimal routing of the information.

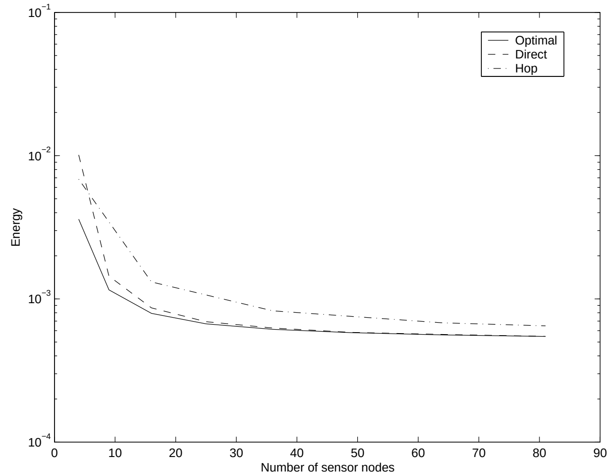

Figure 6: Minimal Energy and Heuristics, for variable fairness $2/n$ as a function of the number of nodes, square topology on $[0,1]^2$

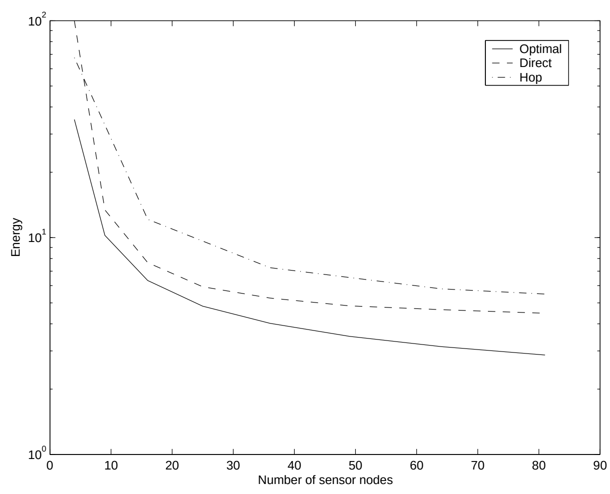

Figure 7: Minimal Energy and Heuristics, for variable fairness $2/n$ as a function of the number of nodes, square topology on $[0,100]^2$

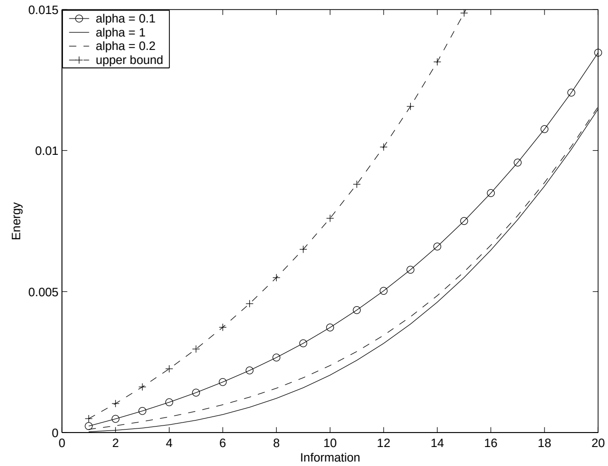

Figure 8: Information vs energy for different α

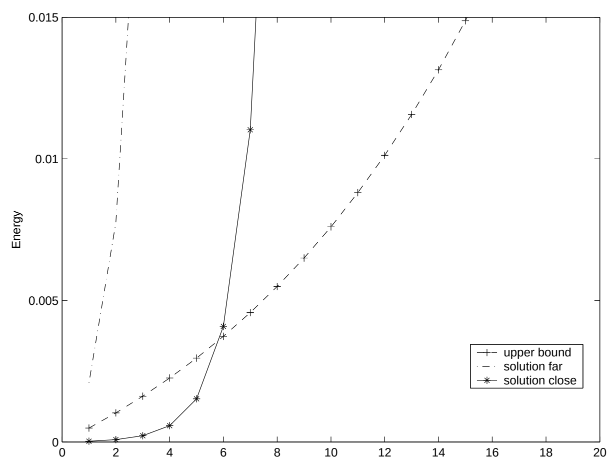

Figure 9: Information vs energy for different α

We consider an example with 25 sensor nodes uniformly distributed on a grid $[0, 10] \times [0, 10]$ and a sink node located at $(-3, 5)$. Other problem parameters were set at $\beta = 0.00001$, $C = 0.00005$, $\eta = 0.0001$, and $f_{\min} = 10$. Below we present four examples, each considering a different fairness pattern on the network. The first pattern considered a totally unfair network, where every node could potentially send all the information, $\alpha_i = 1$ for all i, The optimal energy distribution and flow rates are presented in Figure 10. We note that although all the information could potentially originate from a single node, the optimal is to use several nodes, the ones that are within a certain radius from the sink to obtain all the information. Also note that in this solution the information is routed directly to the sink.

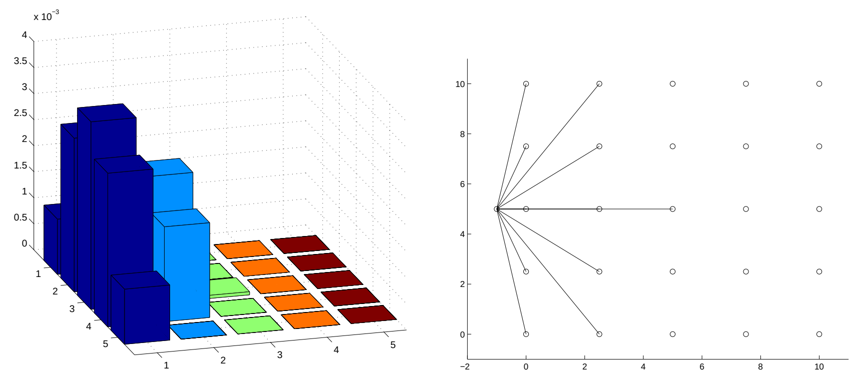

Figure 10: Optimal energy distribution and flow rates, $\alpha = 1$ for all nodes

Our second experiment considers a more restrictive uniform fairness pattern. We set $\alpha_i = 0.05$ for all i. Although this is not a totally fair system, now every node can send at most 5% of the total information, so at least 20 of the 25 nodes must be used. As can be seen from the optimal energy distribution and flow rates in Figure 11. Also now some information originates far enough that it is beneficial to route the information through other nodes to be efficient, for example, the information that originates in nodes that are on the 4th column is routed through some other node.

Both the reason for not sending all the information from a single node in the first example, and deciding to route the information that originates far away are decisions that are minimizing the contribution of the cost of the power to transmit, which can be seen from the objective function of (10) is $\eta d_i(e^{f_{ij}} - 1)$. The reason to send information from nodes that are further away that available capacity is not to increase the exponent f_{ij} too much, and the reason to route the information that is far away is to keep at zero the contribution of terms which have a big d_{ij}.

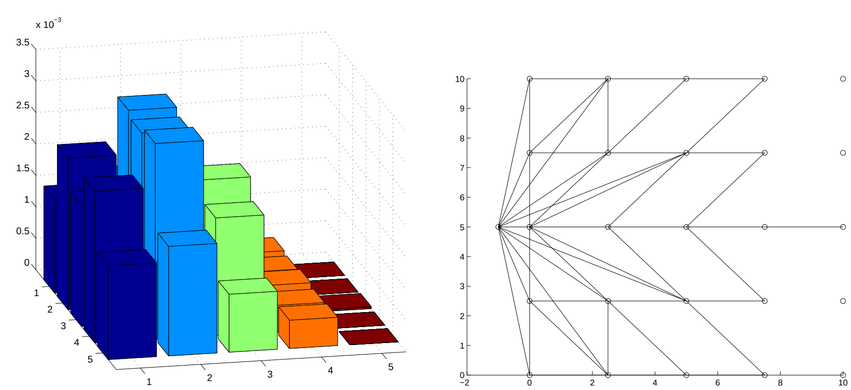

Figure 11: Optimal energy distribution and flow rates, $\alpha = 0.05$ for all nodes

Examples three and four consider skewed fairness patterns, which increase as we approach the upper right corner of the grid. Both patterns have the same form, in which the nodes in the diagonal from the lower left to the upper right, and all nodes that are directly above and to the right of that diagonal node take the same value. The third example considers α gradually taking the values 0, 0.125, 0.25, 0.375, and 0.5; while example four considers α varying though 0, 0.035, 0.07, 0.105, and 0.14.

The optimal energy distribution and flow rates for example three can be seen in Figure 12. The optimal energy distribution and flow rates for example four are in Figure 13.

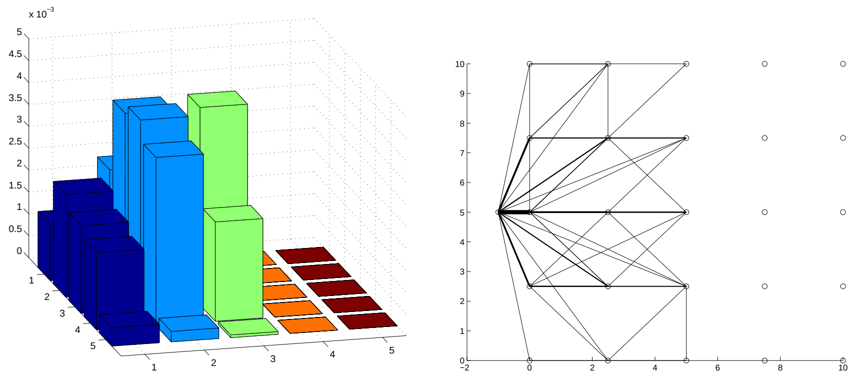

Figure 12: Optimal energy distribution and flow rates, α concentrated on upper right corner, maximum value $\alpha = 0.5$

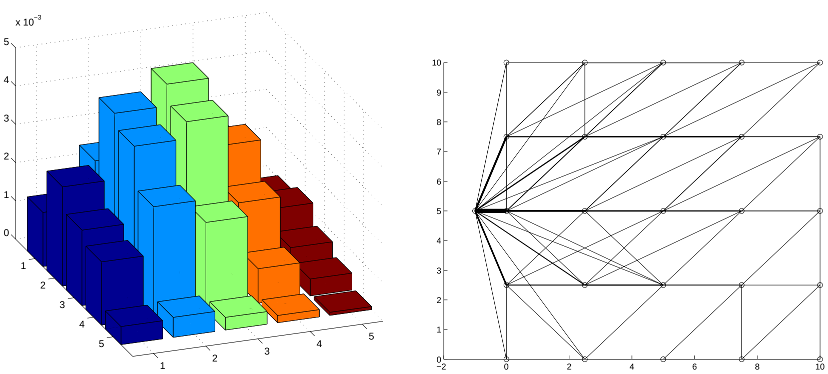

Figure 13: Optimal energy distribution and flow rates, α concentrated on upper right corner, maximum value $\alpha = 0.2$

In the last two examples we note that the optimal energy distribution is no longer symmetric, as it should be due to the fact that the fairness pattern is skewed. We also observe a peculiar phenomenon, information is routed through a *longer* path to get to the sink. The reason for this is again the exponential cost of the transmission power, if all the nodes in the shortest paths are already sending a certain amount of information at a certain power level, it might be more economical to route an additional unit through a longer path of nodes that are not transmitting. Granted, in this example the actual amount of information that was routed on longer paths is minimal, and probably would not occur if we consider discrete phenomena, such as that if a transmission power is below a certain threshold, no transmission actually occurs.

5.5 Types of solution

Predicting properties of the optimal solution can help us in obtaining efficient distributed protocols to route the information in the WSN. A very useful insight would be to determine when the optimal solution prefers to route the information directly and when it is more efficient to hop through a different node. A complete answer involving all the problem's parameters is too convoluted at this moment. An optimization model can provide the optimal routing solution, and we can observe how the hopping behavior is affected for different values of the reception cost C. For this experiment we preferred to take a different approach, we consider a very simple example that is also amenable to an analytical solution.

To provide an initial solution to this question we consider a very simplified problem consisting of only two sensor nodes, one of which provides all the information (that is $\alpha_1 = 0$ and $\alpha_2 = 1$). The question is to try to predict when node 2 will prefer to send the information directly to the sink and when it will prefer to route it through node 1. In order to avoid a trivial solution we place node 1 closer to the sink than node two, in fact for simplicity we place it exactly mid-way between node 2 and the sink. Assume also that we have $f_{\min} = 1$, that we have a sensing cost of β, and noise parameter of η. Clearly the decision of whether to route information or not will be affected for different reception costs C. We note that for small values of C node 2 will find more attractive to route its information through node 1. In this case node two has the alternative to route part of the information and send the rest directly to the sink. For high values of C the network will decide that it's too expensive to route information through node 1 and node 2 will send everything directly. Here we investigate for what values of C will the network decide to hop and for which to send the information directly.

With the use of optimization models we solve for the optimal routing behavior given different values of the reception cost C. In Figure 14 we plot the total value of flow that is sent directly to the sink from node 2 for different reception cost values. This computational example additionally has the following parameters values $\beta = 0.00001$ and $\eta = 0.1$.

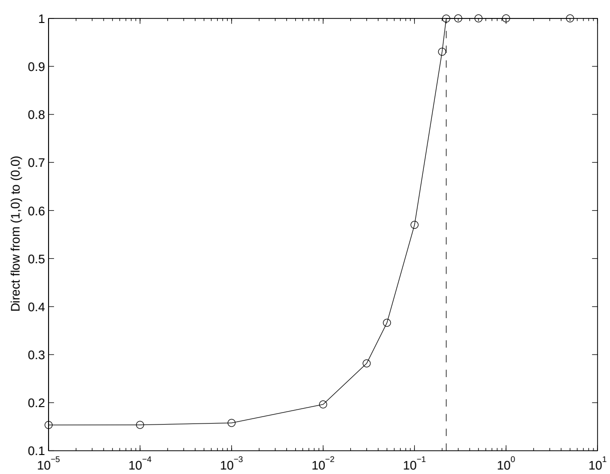

Figure 14: Flow to the sink without hopping as a function of the reception cost C

We note that for reception costs higher than a critical value (plotted as a dashed vertical line) node 2 sends all the information directly to the sink. We also note that it is never optimal to hop all the information, as for any reception cost there is some fraction of the information being sent directly. We finally note that there is a dramatic change in the type of the routing solution as C varies from 10^{-2} to 0.3.

Due to the simplicity of this example, we can analyze the results further. We are simply comparing the solution in which we route all the information directly at a cost

$$h_s = \beta + \eta(e-1)$$

with the case in which we send f_1 from node 2 to node 1 and then to the sink, and $f_2 = 1 - f_1$ directly form node 2 to the sink, at a cost

$$\begin{aligned} h_c(f_1) &= \beta + \frac{1}{4}\eta(e^{f_1} - 1) + Cf_1 + \frac{1}{4}\eta(e^{f_1} - 1) + \eta(e^{1-f_1} - 1) \\ &= \beta + Cf_1 + \frac{1}{2}\eta(e^{f_1} - 1) + \eta(e^{1-f_1} - 1) \ . \end{aligned}$$

The amount of information that will be routed will be the minimizer of function $h_c(f_1)$ on the domain $[0, 1]$.

We need to determine for what values of C will $h_c(f_1) < h_s$ for some $f_1 \in (0, 1]$, which means that it is more convenient to route f_1 than to send everything directly. Equivalently we will determine for what value of C, the function $H_C(f_1) = h_c(f_1) - h_s \geq 0$ for all $f_1 \in [0, 1]$, these are the values of C that will make it more convenient to send directly rather than route the information. It is easy to show that $H_C(f_1)$ is a convex function and $H_C(0) = 0$, therefore to guarantee that $H_C(f_1) \geq 0$ for all $f_1 \geq 0$ it is sufficient to show that $H'_C(f_1) \geq 0$. This last condition reduces to $C \geq \eta(e - \frac{1}{2})$. This critical value for the reception cost is $C = 0.221828$ for the problem parameters of this example. We plot this value a vertical line in Figure 14. Note that the WSN prefers to route all the information precisely at that critical value.

6 Conclusions

In this paper we addressed the need for a systematic methodology by developing formal non-linear optimization models of static WSN that yield fundamental performance bounds and optimal designs. We presented models for two problems: 1. maximizing the total information gathered subject to energy constraints (on sensing, transmission and reception) and 2. minimizing the energy usage subject to information constraints. Other constraints in these models correspond to fairness and channel capacity (assuming noise without interference).

We showed that the two problems are in fact equivalent to each other in terms of a correspondence between optimal solutions and constraints. However, it turns out that the second model is computationally more efficient. We provided some analytical insight into this property of the second model.

We then presented a number of results from computational experiments showing how the optimum information extraction varies with energy, for different fairness constraints. We discussed how the dual variable can be used by a designer to determine a desired trade-off between information extraction and energy expenditure.

We also presented results showing how some simple heuristics (sending directly to sink and shortest multi-hop paths) compare to the optimal solution as network size increases. One interesting result is that when there are no fairness constraints the optimal way for all nodes to send directly to the sink is for them to send information in such a way that their contributions to the objective function are all equal. We also illustrated optimal energy distribution and flow patterns for scenarios with different fairness constraints.

Another novel result presented here pertains to the condition when the optimal solution should involve multi-hop routing and when it shouldn't. We identified a threshold for the reception cost beyond which the optimal solution sends the information directly to the sink rather than doing multi-hop routing, for a simple example.

There are a number of natural extensions of this work we would like to undertake in the future. Many of these involve the model variations we mentioned in section 4.3 — in particular enriching our models to incorporate data aggregation, mobility and interference (which would be meaningful in a CDMA environment as opposed to the interference-free TDMA/FDMA scheduled access assumed in this paper).

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