

Optimal Investment and Consumption Decision of Family with Life Insurance

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Abstract

We study an optimal portfolio and consumption choice problem of family that combines life insurance of parents who receive deterministic labor income until the fixed time T . We consider utility functions of parents and children separately and assume that parents have uncertain lifetime. If parents die before the time T , children have no income and they choose the optimal consumption and portfolio with remaining wealth combining the insurance benefit. Before the death time of parents, the object of family is to maximize weighted average of utility of parents and children. It is assumed that the utility functions of both parents and children belong to HARA utility class. Using HARA utility we impose the condition that instantaneous consumption rate should be above a given lower bound. We analyze how the changes of weight and other parameters such as lower bound of consumption, hazard rate affect the optimal policies.

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