



Optimal life insurance purchase and consumption/investment under uncertain lifetime

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Abstract

In this paper, we consider optimal insurance and consumption rules for a wage earner whose lifetime is random. The wage earner is endowed with an initial wealth, and he also receives an income continuously, but this may be terminated by the wage earner's premature death. We use dynamic programming to analyze this problem and derive the optimal insurance and consumption rules. Explicit solutions are found for the family of CRRA utilities, and the demand for life insurance is studied by examining our solutions and doing numerical experiments.

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1. Introduction

In this paper, we consider the optimal life insurance purchase and consumption strategies for a wage earner subject to mortality risk in a continuous-time economy. Decisions are made continuously about these two strategies for all time $t \in [0, T]$, where the fixed

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planning horizon T can be interpreted as the retirement time of the wage earner. The wage earner also receives his/her income at rate $y(t) \geq 0$ continuously, but this is terminated by the wage earner's death or retirement, whichever happens first. We use a random variable to model the wage earner's lifetime. The life insurance offered has an instantaneous term: the bigger the insurance premium rate paid by the wage earner, the bigger the claim paid to his/her family upon premature death. Income not consumed or used to buy insurance is invested at a riskless interest rate. The problem is to find the strategies that are best in terms of both the family's consumption for all $t \leq T$ as well as the terminal time – T wealth.

Beginning in the 1960s, many researchers constructed quantitative models to analyze the demand for life insurance and the rate of investment for an individual under uncertainty. We are going to review some papers which contributed to this effort.

Yaari (1965) is a starting point for modern research on the demand for life insurance. Yaari considered the problem of life insurance under an uncertain lifetime for an individual; this is the sole source of the uncertainty. The individual's objective was to maximize

$$E \left[\int_0^T U(c(t)) dt \right],$$

where T , the individual's lifetime, is the random variable which takes values on $[0, \bar{T}]$, \bar{T} is some given positive number which represents for the maximum possible lifetime for the consumer, and U is a utility function. Note that the horizon is random in the above functional, but the above functional can be rewritten into the following equivalent form:

$$\int_0^{\bar{T}} \bar{F}(t) U(c(t)) dt,$$

where $\bar{F}(t)$ is the probability that the individual will be alive at time t . Note that now the horizon is a fixed time. This simple idea provides a useful method to analyze the optimization problem with a random life time. Since then, numerous literature has been built on Yaari's pioneering work. However, Leung (1994) pointed out that Yaari's model cannot have an interior solution which lasts until the maximum lifetime for the optimal consumption. What's more, we cannot employ dynamic programming to analyze this kind of model since we can not appropriately define the terminal condition for the HJB equation within the frame of Yaari's model (e.g., Ye, 2006).

Campbell (1980), Fischer (1973), Lewis (1989) and Iwaki and Komoribayashi (2004) examined the demand for life insurance from different perspectives. Campbell (1980) considered the insurance problem in a very short time, $[t, t + \Delta t]$, used a local analysis (Taylor expansion) to greatly simplify the problem, and then derived the insurance policy in terms of the present value of the future income, the current wealth, and other parameters. Since the local analysis has no information about the present value of the future income even if one knows the income stream, Campbell had to assume that the present value of the future income is given exogenously. Lewis (1989) examined the demand for life insurance from the perspective of the beneficiaries. Iwaki and Komoribayashi (2004) considered the optimal insurance from the perspective of households using a martingale method. Households are only allowed to buy life insurance at time 0 in their model, and so the households cannot change the amount of life insurance regardless of what happens after time 0, although the market for life insurance exists.

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