Optimal maintenance decision for line reparable units (LRU) for an aircraft system

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Abstract

Maintenance decisions concerning repair of the Line Replaceable Unit (LRU) of an aircraft fleet need to be considered carefully while deciding the phasing out of the fleet. This is important for achieving higher degree of cost effectiveness and fleet availability at desired level. Discard rate and phasing out period for an aircraft fleet are the critical parameters for determining optimum time to stop the maintenance of LRU. The economic value of remaining useful life of an aircraft fleet should be taken into consideration by salvaging the LRU at the end of the phasing out. The paper suggests a methodology to arrive the time that will minimize the total life cycle cost and provide us economic basis to withdraw the maintenance resources. A mathematical model has been developed for the discard rate of aircrafts based on failure rate, mission life and remaining useful life of the aircrafts in the fleet. This will assist in fulfilling the managing demand of LRU while phasing out of the aircraft fleet.

Keywords: Maintenance decision, Line replaceable units (LRU), Discard rate, LCC, Phasing out time of aircraft fleet, Availability, Linear programming

1. Introduction and background

Military equipments such as aircrafts are expensive complex structures that break down because components are either worn out or damaged during operations. To support operational readiness (or availability), spare components and maintenance resources are required. However, since spares and resources are costly, consume space and become obsolete over time, there is a trade-off between life cycle cost and availability. The purpose of the planner is to sustain the demands for equipment to support operations with respect to cost and availability. The line replaceable units (LRU) are a complex component of an airplane, ship or spacecraft that is designed to be replaced quickly at the flight line or airport ramp area. A LRU is a black box (sealed unit), such as a radio or other auxiliary equipment

Many LRUs for commercial aircraft are designed according to (Aeronautical Radio Incorporated) ARINC specifications. ARINC is a company owned by a number of airlines that sells specifications and designates standards. LRUs are also defined by manufacturers like Airbus and Boeing and by various military organizations. Military LRUs are typically designed to interface according to data bus standards such as MIL-STD-1553

When an aircraft develops a malfunction, the failed LRU are removed from it and replaced when spare LRU are available; otherwise, backorders are established (Sherbrooke, 1971).

2. Problem description

The background of this paper is to build a model for optimizing maintenance when phasing-out an aircraft fleet i.e. to optimize maintenance on repairable components and find optimal point to stop maintenance action and discard maintenance equipment for LRU. The remaining useful life of an aircraft is estimated based on mission hours and the operational hours. Determination of optimal time to stop maintenance depends on various factors such as discard rate of aircrafts in the fleet, failure and repair rate of LRU.

For obtaining more accuracy in the result, the realistic complexity has been integrated in the model by including different types of aircraft based on different mission hours and different age group categories based on operated hours in past.

The cost parameters and availability parameter are jointly considered in the model to make the decision concerning maintenance schedule. The main concept of the present model is to minimize the total cost while keeping desired availability of LRU in the maintenance shop.

3. Model formulation

The present paper comprises the methodology in the form a mathematical model which deals with determination of optimal maintenance time for LRU. In the model the discard rate, failure and repair rate of LRU and phase-out period are the key variables. Mathematical formulation is done for discard rate, failure rate and instantaneous number of aircrafts and number of LRU at any given instant of time. Figure 3 shows the model architecture that describes the flow process of the model development.

Definitions

1) ***Mission hours*****:** The expected operational hours for an aircraft before its discard.
2) ***Phase out period*****:** This is the time horizon when all the aircrafts in the fleet will be discarded.
3) ***Calendar time*****:** The name itself describes that time follows calendar. In the paper optimum time to stop the maintenance and phasing out period are follows calendar time.
4) ***Operational time*****:** This correspondence the time during when aircraft in operation. Mission hours and operational time belongs to this category.
5) ***Discard rate for fleet*****:** It is defined as the number of aircrafts discarded from aircraft fleet per time unit.

Notations

T	Phasing out period for the fleet
$N_a(0)$	Number of in service aircrafts in the fleet at time $t = 0$ (Initial stock level of LRU)
$N_a(t)$	Number of in service aircrafts in the fleet at any given time t
$N_l(0)$	Number of LRU at time $t = 0$
$N_l(t)$	Number of LRU at any given time t
t_s	Optimum time to stop the maintenance of LRU
λ_a^d	Discard rate of aircrafts in the fleet
λ	Failure rate of LRU (Failure rate of aircraft, see section 3.7)
μ_a	LRU replacement rate in the aircraft
μ_l	Repair rate for the LRU
k_s	Fraction of in service LRU from discarded aircraft in the fleet
k_r	Fraction of LRU for repair from discarded aircraft in the fleet
k_d	Fraction of LRU for discard from discarded aircraft in the fleet
S_l	Salvage value of in service LRU
m	Total number of types of aircrafts in the fleet based on mission hours
n	Number of age categories for aircrafts in the fleet
ϕ	Fraction to determine required LRU to attain desired availability
i	Types of aircrafts
j	Types of age groups
T_i	Mission hours of aircraft of type i

N_{ij}	Number of aircrafts of type i belongs to j age group
C_l^H	Holding cost per LRU
C_l^R	Repair cost per LRU
C_l^d	Discard cost per LRU
K	Conversion factor between calendar time and operational time

3.1 Assumption and limitations

1. Only corrective maintenance is considered in the optimization model.
2. LRU are assumed to be identical from all the aircrafts in the fleet.
3. Availability of aircraft assumed to be dependent only on the availability of in service LRU.
4. Fraction for obtaining LRU in service k_w , repairable LRU k_r and discardable LRU k_d are assumed to be same from each discarded aircraft.
5. Failure rate of LRU is identical as failure rate of aircraft as model is mainly focused on the only on repair of LRU.
6. A constant conversion factor (K) is considered between calendar and operational time and it is same for all the aircrafts in the fleet.
7. After the failure, the failed components are repaired immediately. (No queuing)

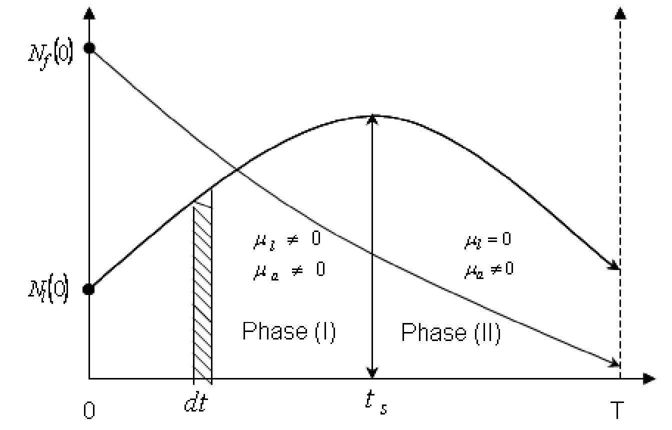

Fig. 1 Phase out graph

The number of LRU in the maintenance shop at any given instant of time described by Figure2

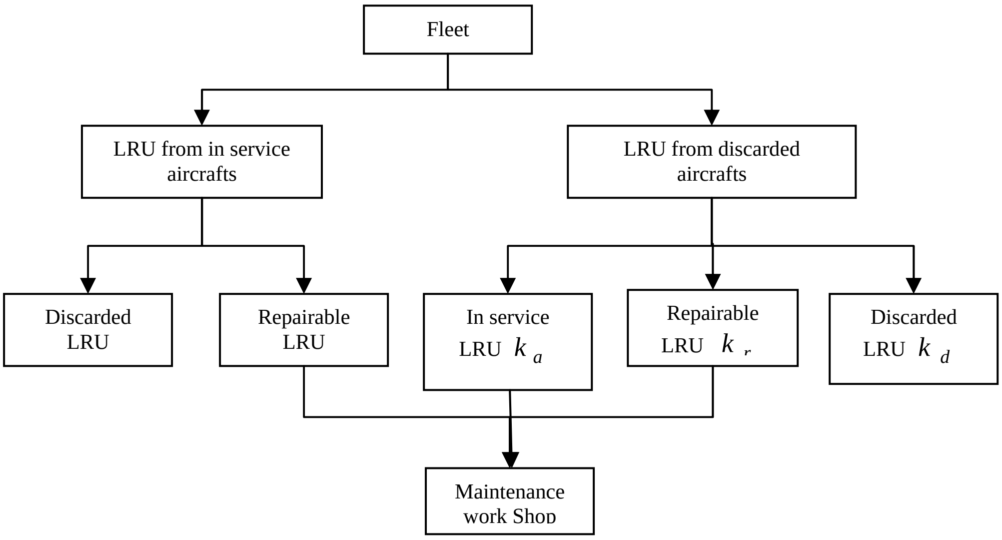

Fig. 2 Resources for obtaining LRU at maintenance shop

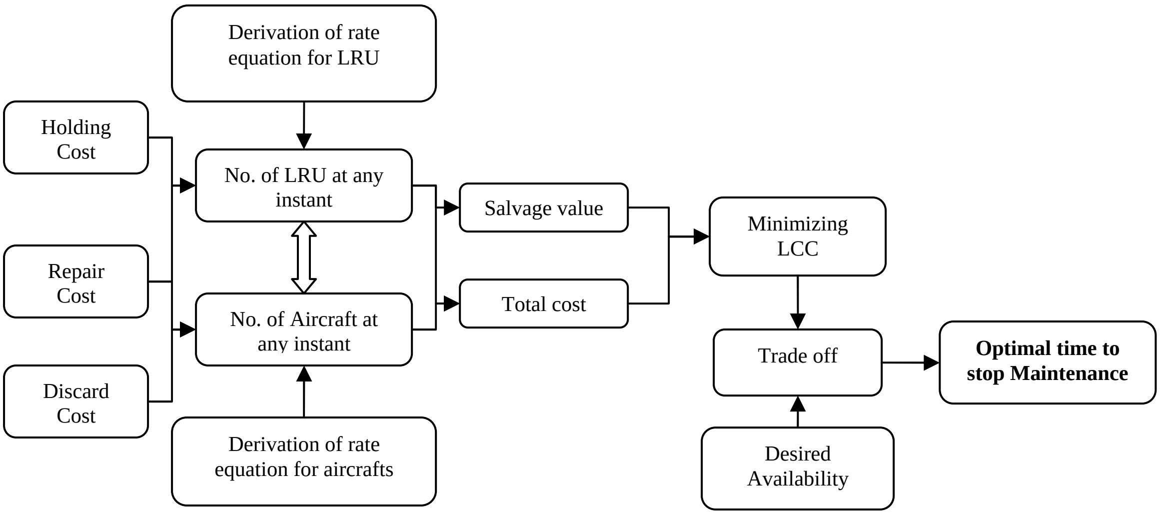

Fig.3. Model process architecture

3.2 Derivation of rate equation for the LRU at given instant of time:

The rate of increasing in service LRU $\frac{dN_l}{dt}$ at any given instant of time is expressed in terms of failure and repair rate of LRU and discard rate of aircraft in the fleet.

Rate of increasing number of LRU in service = (LRU in service receiving from aircraft + repair rate of LRU – failure rate of LRU – LRU for repair from discarded aircraft)

$$\frac{dN_l}{dt} = \lambda_a^d \cdot k_s + \mu_l - \lambda - \lambda_a^d \cdot k_r \qquad (1)$$

3.3 Derivation of rate equation for aircraft at given instant of time:

The rate of changing of number of available aircrafts $\frac{dN_a}{dt}$ at any given instant of time is expressed in terms of failure rate, repair rate, and discard rate of aircraft in the fleet.

Rate of increasing number of available aircrafts in the fleet = (Repair rate of aircraft – failure rate of aircraft– discard rate aircraft)

$$\frac{dN_a}{dt} = \mu_a - \lambda - \lambda_a^d \tag{2}$$

3.4 Desired fleet availability consideration

Desired availability of a fleet depends on availability of group of aircrafts and mission profile. Hence, the individual availability an aircraft will affect the availability of group of aircraft. Therefore model formulation should be done in such a way so that desired availability should be taken care off. For that a constant parameter "ϕ" is introduced, which defines a relation between the number of aircraft and corresponding number of in service LRU. It tells the desired number of available LRU at maintenance shop. In the paper the failure rate of LRU is taken constant hence the value of "ϕ" should also be constant. For increasing failure rate the value of "ϕ" should be change accordingly.

$$N_l(t) = \phi \cdot N_a(t) \tag{3}$$

3.5 Instantaneous or Point Availability of LRU

In the paper it is assumed that the availability of an aircrafts in the fleet is strictly depends on availability of LRU. Due to the rate of change in the fleet size corresponds to variability in desired availability of LRU at any given instant of time.

3.6 Relation between calendar time and operational time

In the model both calendar and operation times have been used and it determines the optimum time to stop the maintenance that is based on calendar time. Model also incorporates the operational hours hence both these time categories should be normalized to one type of category (calendar time). In order to reduce the complexity the conversion factor K is considered to convert operational hours in to calendar time.

*Calendar time = K * (Operational time) (K>1)*

3.7 Formulation for the average failure rate λ of LRU in the fleet

This paper comprises modeling for the fleet of aircrafts, which assumes that discard is carried out based on specified age or mission hours of aircraft i.e. that failure rate of the aircraft and LRU is constant and uniformly distributed. The policy of discarding the aircrafts from the fleet follows age replacement policy (Barlow, 1965) that is irrespective of remaining life. If T is the life time for aircraft then the mean failure rate will be $\lambda = \frac{1}{T}$, if the fleet consists "m" types of aircrafts then average failure rate for the fleet will be expressed by equation (4)

$$\lambda = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{T_i}$$

To convert operation time into calendar time parameter T replaced by KT

$$\lambda = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{K \cdot T_i} \tag{4}$$

3.8 Formulation for the discard rate λ_a^d of aircrafts in the fleet

The discard rate is the most important parameter in the determination of optimal time to stop the maintenance. Therefore, the mathematical formulation must consider these relevant parameters that include age of aircraft based on operated hours and mission hours. In table1, where aircrafts are categorized based on these two categories as mentioned in rows and columns. In the mathematical formulation there are "m" type of aircraft with mission flight hours T_i (*i=1 to m*) as mentioned in the row. Aircrafts of different age group are categorized based on the different operated hours (j = 1 to n) as mentioned in the column.

		Types of Aircraft with different Mission Profile				
Different Age group		T_1	T_2	T_3	-	T_m
	$(0-t_1)$	N_{11}	N_{12}	N_{13}	-	N_{1m}
	(t_1-t_2)	N_{21}	N_{22}	N_{23}	-	N_{2m}
	(t_2-t_3)	N_{31}	N_{32}	N_{33}	-	N_{3m}
	$(t_{n-1}-t_n)$	N_{n1}	N_{n2}	N_{n3}	-	N_{nm}

Table1. Number of aircrafts of different age group and different mission profile

In table 1, N_{nm} represents the number of aircrafts, which falls into the category of m^{th} type of aircraft with n^{th} type of age group. The discard rate of an aircraft with mission profile T at any given instant of time t can be expressed as follows (see derivation).

$$Discard\ Rate = \frac{1}{Time\ left\ to\ discard} \quad = \quad \frac{1}{\left(T - t\right)}$$

Derivation: The discard rate at any given instant of time can be written as conditional probability of its present life, which can be expressed by given expression below. Uniform distribution is considered for failure rate function, as the discard of an aircraft is based on its mission hours.

$$\lambda_a^d = \frac{f(t)}{R(t)} = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{1 - \int_{-\infty}^{t} f(u) \cdot du} = \frac{1/T}{1 - t/T} = \frac{1}{T - t}$$

Average discard rate for N_{ij} number of aircrafts with mission profile T_i hours and t_j hours old will be expressed as $\frac{N_{ij}}{T_i - \left(t + t_j\right)}$, In this paper the total number of categories are $n \times m$ (see table(1)), therefore the generalized mathematical formula for average discard rate λ_a^d for the fleet will be.

$$\lambda_a^d = \frac{1}{n \times m} \sum_{i=}^{m} \sum_{j=1}^{n} \frac{N_{ij}}{T_i - \left(t + t_j\right)}$$

For converting operation time into calendar time by replacing T by KT

$$\lambda_a^d = \frac{1}{n \times m} \sum_{i=}^{m} \sum_{j=1}^{n} \frac{N_{ij}}{KT_i - \left(t + K t_j\right)} \tag{5}$$

From equation (2), (4) and (5), the rate of changing the fleet size will re written as following

$$\frac{dN_a}{dt} = \mu_a - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{K \cdot T_i} - \frac{1}{n \times m} \sum_{i=}^{m} \sum_{j=1}^{n} \frac{N_{ij}}{T_i - \left(t + K \cdot t_j\right)}$$

$$\Rightarrow dN_a = \left(\mu_a - \frac{1}{m} \sum_{i=1}^{m} \frac{1}{KT_i} - \frac{1}{n \times m} \sum_{i=}^{m} \sum_{j=1}^{n} \frac{N_{ij}}{KT_i - \left(t + Kt_j\right)} \right) \cdot dt \tag{6}$$

Phasing out period (T): Phasing-out time for the aircrafts fleet duration mainly depends on rate of change number of aircrafts present in the fleet. Phasing out period will be determined by the discard rate equation (6) for the aircrafts in the fleet by putting boundary conditions on fleet size. Where, at $t = 0,\ N_a(t) = N_a(0) \quad ;; \quad t = T,\ N_a(t) = 0$

$$\int_{N_a(0)}^{0} dN = -\int_{0}^{T} \lambda_a^d \cdot dt$$

After solving,

$$e^{n \times m \times N_a(0)} = \prod_{i=1}^{m} \prod_{j=1}^{n} \frac{K\left(T_i - t_j\right)}{\left(KT_i - \left(T + Kt_j\right)\right)} \tag{7}$$

It is difficult to express the value of phasing out period (T) in terms of other variables mentioned in above equation. The value of T can be determined by using numerical method.

4. Cost modeling

The optimization is based on trade off analysis between Life Cycle Cost and desired availability. For determining the optimum time to stop the maintenance of LRU the phasing out period is divided in two phases (see figure 1) (0 to t_s and t_s to T). In the first phase of phasing out period both LRU and aircraft is getting repaired, while in the second phase LRU repair is stopped. Cost calculation is done individually for both phases.

4.1 Cost formulation during time interval (0 to t_s) C_{0-t_s}

In the first phase of cost formulation repair cost and holding cost are considered for LRU while repair cost and discard cost are considered for aircrafts.

Total cost = Repair cost for LRU + Holding cost (*Safety stock for availability* + *LRU for repair in the maintenance workshop*) + Repair cost for aircrafts + Discard cost aircrafts.

$$\int_{0}^{C_{0-t_s}} dc = \int_{0}^{t_s} \left(\lambda + \lambda_a^d \cdot k_r\right) \cdot C_l^R \cdot dt + \int_{0}^{t_s} dN_l \cdot C_l^H + \int_{0}^{t_s} dN_a \cdot C_a^R + \int_{0}^{t_s} \lambda_a^d \cdot k_d \cdot C_l^d \cdot dt$$

Repair cost for LRU =Number of LRU's for repair during time *dt* ***** Repair cost per LRU

Holding stock = Safety stock + LRU for repair at workshop

Replacing dN_a and λ_a^d from expression (6) and (5) respectively and integrating formula (8), we gets total cost for the first phase.

$$C_{0-t_s} = \left(\lambda \cdot C_l^R + \left(\phi \cdot C_l^H + C_a^R\right) \cdot \mu_a - \left(\phi \cdot C_l^H + C_a^R\right) \cdot \frac{1}{m} \sum_{i=1}^{m} \frac{1}{KT_i} \right) \cdot t_s$$
$$+ \left[k_r \cdot C_l^R + k_d \cdot C_a^d - \phi \cdot C_l^H - C_a^R\right] \cdot \frac{1}{n.m} \cdot \sum_{i=1}^{m} \sum_{j=1}^{n} N_{i\,j} \cdot \ln\left(\frac{KT_i - Kt_j}{KT_i - t_s - Kt_j} \right) \tag{9}$$

4.2 Cost formulation during time interval (t_s to T) C_{t_s-T} :

In the second phase all repair activities are stopped for LRU since there are enough spare LRU in stock. During time interval of this phase, following cost is considered.

Total cost = Holding cost (*Safety stock for availability*) + Repair cost for aircrafts + Discard cost for LRU.

$$\int_{0}^{C_{t_s-T}} dc = \int_{t_s}^{T} \phi \cdot dN_a . C_l^H + \int_{t_s}^{T} dN_a \cdot C_a^R + \int_{t_s}^{T} \lambda_a^d \cdot k_d \cdot C_l^d \cdot dt \tag{10}$$

Similarly as first phase, Replacing dN_a and λ_a^d by formula (6) and formula (5) respectively and integrating formula (14), we gets total cost in the second phase.

$$\begin{aligned} C_{t_s-T} = & \left(C_a^R \cdot \mu_a - \left(\phi \cdot C_l^H + C_a^R\right) \cdot \frac{1}{m} \sum_{i=1}^{m} \frac{1}{KT_i} \right) \cdot \left(T - t_s \right) \\ & + \left[k_d \cdot C_l^d - \phi \cdot C_l^H - C_a^R \right] \cdot \frac{1}{n.m} \cdot \sum_{i=1}^{m} \sum_{j=1}^{n} N_{ij} \cdot \ln \left(\frac{KT_i - t_s - Kt_j}{KT_i - T - Kt_j} \right) \end{aligned} \tag{11}$$

4.3 Salvage value of LRU at the end of phase-out period:

The salvage value will consist only LRU left at the end of phasing out period as number of aircrafts at the end of phasing out period are considered as zero.

$$S = N_l^S(T) \cdot S_l$$

The value of $N_l^S(T)$ can be determined using rate equation (1), and replacing discard rate the salvage value comes out to be

$$\int_{N_l(0)}^{N_l^S(T)} dN_l = \int_0^T \lambda_a^d \cdot \left(k_a - k_r\right) \cdot dt + \int_0^T \left(\mu_l - \lambda\right) \cdot dt$$

$$S = \left[N_l(0) + \frac{\left(k_a - k_r\right)}{n \cdot m} \cdot \sum_{i=1}^{m} \sum_{j=1}^{n} N_{ij} \cdot \ln \left(\frac{KT_i - T}{KT_i - T - Kt_j} \right) + \left(\mu_l - \lambda\right) \cdot T \right] \cdot S_l^S \tag{12}$$

5. Linear programming formulation for Life Cycle Cost

5.1 Objective function

The objective function minimizes life cycle cost comprising total repair cost and holding cost and salvage value. The value of C_{0-t_s} , C_{t_s-T} and S from expressions (9), (11) and (12), we gets the expression for Life Cycle Cost

Minimize (LCC): $Min\ C_{LCC} = C_{0-t_s} + C_{t_s-T} - S$ (13)

Subjected to: $N_l^S(t) \geq \phi \cdot N_a(t)$ (For maintaining desired availability) (14)

Number of LRU's in service at time $\geq \phi \times$ (Number of aircrafts at the same time)

5.2 Solution approach

The model contains a large number of input variables due to it depends on various cost aspects. In order to consider the desired availability relation (3) was introduced that directly proportional to aircraft availability. A trade-off will be generated between LCC and fleet availability. For each value of ϕ correspondence given availability with corresponding optimum maintenance time at lowest LCC. The value of phase out period (T) will be determined from equation (7) and substituted in the objective function. For a given data set for the various variables of model optimization tool box in MATLAB software should be used for solving linear programming to determine optimal maintenance time for various values of availability i.e. different set of values of ϕ.

6. Conclusion

This paper introduced a concept and methodology to assist maintenance policy maker for taking optimum maintenance decision for line replaceable units in the phasing out of aircraft fleet. Mathematical formulation has been done to determine the optimal maintenance schedule and phasing out period. An approach has been suggested for developing mathematical relationship among discard rate of aircrafts, phasing out period and number of LRU in the maintenance work shop. The derived relation leads to a decision support tool for taking optimum maintenance decision.

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