Optimal Maintenance of Multi-Component Systems: a Review

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Abstract

In this article we give an overview of the literature on multi-component maintenance optimization. We focus on work appearing since the 1991 survey *A survey of maintenance models for multi-unit systems* by Cho and Parlar (1991). This paper builds forth on the review article by Dekker *et al.* (1996), which focusses on economic dependence, and the survey of maintenance policies by Wang (2002), in which some group maintenance and some opportunistic maintenance policies are considered. Our classification scheme is primarily based on the dependence between components (stochastic, structural or economic). Next, we also classify the papers on the basis of the planning aspect (short-term vs long-term), the grouping of maintenance activities (either grouping preventive or corrective maintenance, or opportunistic grouping) and the optimization approach used (heuristic, policy classes or exact algorithms). Finally, we also pay attention to the applications of the models.

Keywords: Maintenance optimization; multi-component systems; literature review; economic dependence; failure interaction; structural dependence; grouping maintenance; opportunistic maintenance; maintenance policies.

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1 Introduction

The last few decades the maintenance of systems has become more and more complex. One reason for this is that systems consist of many components which depend on each other. On the one hand, interactions between components complicate the modelling and optimization of maintenance. On the other hand, interactions also offer the opportunity to group maintenance which may save costs. It follows that planning maintenance actions is a big challenge and it is not surprising that many scholars have studied maintenance optimization problems for multi-component systems. In some articles new solution methods for existing problems are proposed, in other articles a new maintenance policy for multi-component systems is studied. Moreover, the number of papers with practical applications of optimal maintenance of multi-component systems is still growing.

Cho and Parlar (1991) give the following definition of multi-component maintenance models: *"Multi-component maintenance models are concerned with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other (economically / stochastically / structurally)."* So, in these models it is all about making an optimal maintenance planning for systems consisting of components which interact with each other. We will come back later to the concepts of optimality and interaction. For now it is important to remember that the condition of the systems depends on (the state of) the components, which will only function if adequate maintenance actions are performed.

In this article we will give an up-to-date review of the literature on multi-component maintenance optimization. As there have been written several overview articles in the past, we will first give a brief summary of these articles. The article of Cho and Parlar (1991) reviews articles from 1976 up to 1991. The authors divide the literature into five topical categories: machine-interference / repair models, group / block / cannibalization / opportunistic models, inventory / maintenance models, other maintenance / replacement models and inspection / maintenance models. The article of Dekker *et al.* (1996) exclusively deals with multi-component maintenance models that are based on economic dependence. Emphasis is put on articles that have been published after 1991, but there is an overlap with the review of Cho and Parlar (1991). The classification scheme of Dekker *et al.* (1996) differs from that of Cho and Parlar (1991). Firstly, models are classified based on the planning aspect of the model: stationary (long-term) and dynamic (short-term). Secondly, the stationary-grouping models are divided in the categories grouping corrective maintenance, grouping preventive maintenance and opportunistic grouping maintenance. Here, opportunistic grouping is grouping both preventive and corrective maintenance. The dynamic models are divided into two categories: those with a finite horizon and those with a rolling horizon. In a recent article Wang (2002) gives an overview of maintenance policies of deteriorating systems. The emphasis is on policies for single component systems. One section is devoted to opportunistic maintenance policies for multi-component systems. The author primarily considers models with economic dependence.

The existing overview articles prove that there are several ways to categorize articles and models. In section 2 of this review we structure the field and present our comprehensive classification scheme. It differs from the schemes used in the overview articles discussed above. First of all, we distinguish between models with economic, structural and stochastic dependence. Economic dependence implies that grouping maintenance actions either saves costs (economies of scale) or results in higher costs (because of e.g. high downtime costs), as compared to individual maintenance. Stochastic dependence occurs if the condition of components influences the lifetime distribution of other components. Structural dependence applies if components structurally form a part, so that maintenance of a failed component implies maintenance of working components. In sections 3, 4 and 5, we will discuss papers concerning economic, stochastic and structural dependence between components.

In section 6 we will classify articles according to the planning aspect of the maintenance model and the method used to optimize this model. Following the review of Dekker *et al.* (1996) we distinguish between models with a finite and an infinite planning horizon. Models with an infinite planning horizon are called stationary, since they usually provide static rules for maintenance which do not change over the planning horizon. Finite horizon models are called dynamic, since these models can generate dynamic decisions that may change over the planning horizon. In these models short-term information can be taken into account. With respect to the optimization methods, we divide the papers into three categories: exact, heuristic and policy optimization.

Section 7 covers trends and open research areas in multi-component maintenance. Conclusions are drawn in section 8.

2 Structuring the field

In section 2.1 we give a short review of the terminology used in multi-component maintenance optimization models and explain how we searched the literature. In section 2.2 we will present our comprehensive classification scheme.

2.1 Search strategy and terminology

Presenting a scientific review on a certain topic implies that one tries to discuss all relevant articles. Finding these articles, however, is very difficult. It depends on the search engines and databases used, electronic availability of articles and the search strategy. As search engines we used Google Scholar, Scirus and Scopus, and as (online) databases we used ScienceDirect, JStor and MathSciNet. We primarily searched on key words, abstracts and titles, but we also searched within the papers for relevant references. Note that papers published in books or proceedings that are not electronically available, are likely to have escaped.

Terminology is another important issue, as the use of other terms can hide a very interesting paper. The field has been delineated by maintenance, replacement or inspection on one hand and optimization on the other hand. This combination however, provides almost 5000 hits in Google Scholar.

Next, the term multi-component has been used in junction with related terms as opportunistic maintenance (policies), piggyback(ing), joint replacement, joint overhaul, combining maintenance, grouping maintenance, economies of scale and economic dependence. With respect to the term stochastic dependence, we have also searched for synonyms and related terms such as failure interaction, probabilistic dependence and shock damage interaction. This yields in total approximately 500 hits. Relevant articles have been selected from this set by scanning the articles.

The vast literature on maintenance of multi-component systems has been reviewed earlier by others. Therefore, we have also consulted existing reviews and overview articles in this field. Moreover, we have applied a citation search (looking both backwards in time and forwards in time for citations) to all articles found. This citation search is an indirect search method, whereas the above methods are direct methods. The advantage of this method is that one can easily distinguish clusters of related articles.

2.2 Classification scheme

First of all, we classify the multi-component maintenance models on the basis of the dependence / interaction between components in the system considered. Thomas (1986) defines three different types of interactions: economic, structural and stochastic dependence.

Simply said, economic dependence implies that the cost of joint maintenance of a group of components does not equal the total cost of individual maintenance of these components. The effect of this dependence comes to the fore in the execution of maintenance activities. On the one hand, the joint execution of maintenance activities can save costs in some cases (e.g. due to economies of scale). On the other hand, grouping maintenance may also lead to higher costs (e.g. due to manpower restrictions) or may not be allowed. For this reason, we will subdivide the models with economic dependence into two categories: positive and negative economic dependence. That is, we refine the definition of economic dependence as compared to the definition used in the review article of

Dekker *et al.* (1996). Note that in many systems both positive and negative economic dependence between components is present. We will give special attention to the modelling of maintenance optimization of these systems, in particular the k-out-of-n system.

Stochastic dependence occurs if the condition of components influences the lifetime distribution of other components. Synonyms of stochastic dependence are failure interaction or probabilistic dependence. This kind of dependence defines a relationship between components upon failure of a component. For example, it may be the case that the failure of one component induces the failure of other components or causes a shock to other components.

Structural dependence applies if components structurally form a part, so that maintenance of a failed component implies maintenance of working components, or at least dismantling them. So, structural dependence restricts the maintenance manager in his decision on the grouping of maintenance activities.

A second classification of the models is based on the planning aspect: stationary or dynamic. That is, do we make a short-term / operational or a long-term / strategic planning for the maintenance activities? Is the planning horizon finite or infinite? In stationary models, a long-term stable situation is assumed and mostly these models assume an infinite planning horizon. Models of this kind provide static rules for maintenance which do not change over the planning horizon. They generate for example long-term maintenance frequencies for groups of related activities or control limits for carrying out maintenance depending on the state of components. In dynamic models, short-term information such as a varying deterioration of components or unexpected opportunities can be taken into account. These models generate dynamic decisions that may change over the planning horizon.

The last classification we consider is based on the type of optimization method used. This can be either an exact method, a heuristic or a search within classes of policies. Exact optimization methods are designed to find the real optimal solution of a problem. However, if the computing time of the optimization method increases exponentially with the number of components, then exact methods are only desirable to a certain extent. In that case solving problems with many components is impossible and heuristics should be used. Heuristics are local optimization methods that do not pretend to find the global optimum, but can be applied to find a solution to the problem in reasonable time. The quality of such a solution depends on the problem instance. In some cases it is possible to give an upper bound on the gap between the optimal solution and the solution found by the heuristic.

In many papers a maintenance planning is made by optimizing a certain type of policy. Well known maintenance policies are the age and block replacement policies and their extensions. The advantage of policy optimization over other optimization methods is that it gives more insight into the solution of the problem. Note that policy optimization will not always result in the global optimal solution, since there may be another policy which results in a better solution. It may even be the case that the optimal solution cannot be found by a policy. In some cases however, it can be proved that applying a certain maintenance policy results in the exact (global) optimal solution.

3 Economic dependence

In this section we review articles on multi-component systems with economic dependence. We focus on articles appearing since the review of Dekker *et al.* (1996). In sections 3.1 and 3.2 we discuss models with positive and negative dependence, respectively. In section 3.3 we discuss articles on k-out-of-n systems, in which both positive and negative dependence between components is present.

3.1 Positive economic dependence

Positive economic dependence implies that costs can be saved when several components are jointly maintained instead of separately. Compared with the review of Dekker *et al.* (1996) we refine the concept of (positive) economic dependence and distinguish the following forms.

- Economies of scale
    - General
    - Single set-up
    - Multiple set-ups
        - ◇ Hierarchy of set-ups
- Downtime opportunity

The term economies of scale is often used to indicate that combining maintenance activities is cheaper than performing maintenance on components separately. The term economies of scale is very general and it seems to be equal to positive economic dependence. In this paper we will speak of economies of scale when the maintenance cost per component decreases with the number of maintained components. Economies of scale can result from preparatory or set-up activities that can be shared when several components are maintained simultaneously. The cost of this set-up work is often called the *set-up* *cost*. Set-up costs can be saved when maintenance activities on different components are executed simultaneously, since execution of a group of activities requires only one set-up.

In this overview we distinguish between single set-ups and multiple set-ups. In the latter case there usually is a hierarchy of set-ups. For instance, consider a system consisting of two components, which both consist of two subcomponents. Maintenance of the subcomponents of the components may require a set-up at system level and component level. Firstly, this means that the set-up cost at component level is paid only once when the maintenance of two subcomponents of a component is combined. Secondly, the set-up cost at system level is paid only once when all subcomponents are maintained at the same time. Set-up costs usually come back in the objective function of the maintenance problem. If economies of scale are not explicitly modelled by including set-up costs in the objective function, then we classify the model in the category 'general'.

Another form of positive dependence is the downtime opportunity. Component failures can often be regarded as opportunities for preventive maintenance of non-failed components. In a series system a component failure results in a non-operating system. In that case it may be worthwhile to replace other components preventively at the same time. This way the system downtime results in cost savings since more components can be replaced at the same time. Moreover, by grouping corrective and preventive maintenance the downtime can be regulated and in some cases it can even be reduced. Note that if the downtime cost is included in the set-up cost in a certain paper, then we will not classify the paper in the category 'downtime opportunity', but in the category 'set-up cost'. In general however, it is difficult to asses the cost associated with the downtime (see e.g. Smith and Dekker (1997) who approximate the availability and the cost of downtime for a 1-out-of-n system). Therefore, the downtime cost is usually not included in the set-up cost.

In the paragraphs below we discuss articles dealing with positive economic dependence. Our main focus is on the modelling of this dependence.

Economies of scale

General

In comparison with the article of Dekker *et al.* (1996) the category 'general economies of scale' is new. The papers in this category deal with multi-component systems for which joint maintenance of components is cheaper than individual maintenance of components. This form of economies of scale cannot be modelled by introducing a single set-up cost. The cost associated with the maintenance of components is often concave in the number of components that are maintained simultaneously.

Dekker *et al.* (1998a) evaluate a new maintenance concept for the preservation of highways. In road maintenance cost savings can be realized by maintaining larger sections instead of small patches. The road is divided in sectors of 100 metres length. Set-up costs are present in the form of the direct costs associated with the maintenance of different parts of the road. The set-up cost is a function of the number of these parts in a maintenance group. A heuristic search procedure is proposed to find the optimal maintenance planning.

Papadakis and Kleindorfer (2005) introduce the concept of network topology dependencies (NTD) for infrastructure networks. In these networks two types of NTD can be distinguished: contiguity and set-up discounts. Both types define positive economic dependence between components. In the former case savings are realized when costs are paid once when contiguous sections are maintained at the same time. In the latter case savings are realized when costs may be paid once for a neighbourhood of the infrastructure network, independently of how much work is carried out on it. For both types of dependencies a non-linear discount function is defined. The authors consider the problem of maintaining an infrastructure network. It is modelled as an undirected network. Risk measures or failure probabilities for the segments of this network are assumed to be known. A maximum flow minimum cut formulation of the problem is developed. This formulation makes it easier to solve the problem exact and efficiently.

Single set-up

Nearly all articles reviewed by Dekker *et al.* (1996) can be classified in this category. The objective function of the maintenance optimization model usually consists of a fixed cost (the set-up cost) and variable costs. In the articles discussed below, this will not be different.

Castanier *et al.* (2005) consider a two-component series system. Economic dependence between the two components is present in the following way. The set-up cost for inspecting or replacing a component is charged only once if the actions on both components are combined. That is, joint maintenance of components saves costs. In this article the condition of the components is modelled by a stochastic process and it is monitored by non-periodic inspections. In the (opportunistic) maintenance policy several thresholds are defined for doing inspections, corrective and preventive replacements, and opportunistic maintenance. These thresholds are decision variables. Many articles on this type of models have appeared, but most of these articles only consider single component models.

The articles of Scarf and Deara (1998, 2003) consider both economic and stochastic dependence between components in a series system. This combination is scarce in the literature. Positive economic dependence is modelled on the basis that the cost of replacement of one or more components includes a one-off set-up cost whose magnitude does not depend on the number of components replaced. We will discuss these articles in more detail in section 4.

In one of the few case studies found in the literature, Van der Duyn Schouten *et al.* (1998) investigate the problem of replacing lightbulbs in traffic control signals. Each installation consists of three compartments in which lightbulbs serve the green, red, and yellow lights. Maintenance of lightbulbs means replacement, either correctively or preventively. Firstly, positive economic dependence is present in the form of set-up cost, because each replacement action requires a fixed cost in the form of transportation of manpower and equipment. Secondly, the failure of individual bulbs are opportunities for preventive maintenance on other bulbs. The authors propose two types of maintenance policies. In the first policy, also known as the standard indirect-grouping strategy (introduced in maintenance by Goyal and Kusy (1985), for a review of this strategy we refer to Dekker *et al.* (1996)), corrective and preventive replacements are strictly separated. Economies of scale can thus only be achieved by combining preventive replacements of the bulbs. The authors also propose the following opportunistic age-based grouping policy. Upon failure of a lightbulb, this bulb and all other bulbs older than a certain threshold are replaced.

Budai *et al.* (2006) consider a preventive maintenance scheduling problem (PMSP) for a railway system. In this problem (short) routine activities and (long) unique projects for one track have to be scheduled in a certain period. To reduce costs and inconvenience for the travellers and operators, these activities should be scheduled together as much as possible. With respect to the latter, maintenance of different components of one track simultaneously requires only one track possession. In this article time is discretized and the PMSP is written as a mixed-integer linear programming model. Positive dependence is taken into account by the objective function, which is the sum of the total track possession cost and the maintenance cost over a finite horizon. To reduce possible end-of-horizon effects an end-of-horizon valuation is also incorporated in the objective function. Note that the possession cost can be seen as a downtime cost. In this article it is modelled as a fixed / set-up cost. This is the reason that it is classified in this category. Besides this positive dependence there also exists negative dependence between components, since some activities exclude each other. The advantage of a discrete time model is that negative dependence can be incorporated in the model by adding additional restrictions. It appears that the PMSP is a NP hard problem. Heuristics are proposed to find near-optimal solutions in reasonable time.

Multiple set-ups

This is also a new category. The maintenance of different components may require different set-up activities. These set-up activities may be combined when several components are maintained at the same time. We have found one article in this category; it assumes a complex hierarchical set-up structure.

Hierarchical structure of set-ups Van Dijkhuizen (2000) studies the problem of clustering preventive maintenance jobs in a multiple set-up multi-component production system. As far as the authors know, this is the first attempt to model a maintenance problem with a hierarchical (tree-like) set-up structure. Different set-up activities have to be done at different levels in the production system before maintenance can be done. Each component is maintained preventively at an integer multiple of a certain basis interval, which is the same for all components, and corrective maintenance is carried out in between whenever necessary. So, every component has its own maintenance frequency - the frequencies are based on the optimal maintenance planning for single components. Obviously, set-up activities may be combined when several components are maintained at the same time. The problem is to find the maintenance frequencies that minimize the average cost per unit of time. This problem is an extension of the standard-indirect grouping problem (for an overview of this problem see Dekker *et al.* (1996)).

Downtime opportunity

As we stated earlier, the downtime of a system is often an opportunity to combine preventive and corrective maintenance. This is especially true for series systems, where a single failure results in a system downtime. Of course, non-failed components should not be replaced when they are in a good condition, because then useful lifetime is wasted. The maintenance policies proposed in the articles discussed below share this idea.

Gürler and Kaya (2002) propose a new opportunistic maintenance policy for a series system with identical items. The article is an extension of the work by Van der Duyn Schouten and Vanneste (1993), who also propose an opportunistic policy for such a system. In their model, the lifetime of the components is described by several stages, which are classified as good, doubtful, preventive maintenance due and failed. Gürler and Kaya (2002) classify the stages in the same way, but the stages good and doubtful are subdivided into a number of states. The proposed policy is of the control-limit type. Components which are PM due (failed) are preventively (correctively) replaced immediately. The entire system is replaced when a component is PM due or down and the number of components in doubtful states is at least N. Here, N is a decision variable. It appears that this policy achieves significant savings over a policy where the components are maintained individually without any system replacement.

Popova and Wilson (1999) consider m-failure, T-age and (m,T) failure group policies for a system of identical components operating in parallel. According to these policies the system is replaced at the time of the m-th failure, every T time units, or at the minimum time of these events, respectively. These policies were first introduced by Assaf and Shanthikumar (1987), Okumoto and Elsayed (1983) and Ritchken and Wilson (1990), respectively. Popova and Wilson (1999) assume that downtime costs are incurred when failed components are not repaired or replaced. So, when the system operates there is also negative dependence between the components. After all, when the components are left in a failed condition, with the intention to group corrective maintenance, then the downtime costs are incurred. In the maintenance policies a trade-off between these downtime costs and the advantages of grouping (corrective) maintenance is made.

Sheu and Jhang (1996) propose a new two-phase opportunistic maintenance policy for a group of independent identical repairable units. Their model incorporates minimal repair, overhaul, replacement and downtime costs. In the first interval, $(0, T]$, minor failures are removed by minimal repairs and 'catastrophic' failures by replacements. In the second phase, $(T, T + W]$, minor failures are also removed by minimal repairs, but 'catastrophic' failures are left idle. A group maintenance is conducted at time $T + W$ or upon the kth idle, whichever comes first. The generalized group maintenance policy requires inspection at either the fixed time $T + W$ or the time when exactly k units are left idle, whichever comes first. At an inspection, all idle components are replaced with new ones and all operating components are overhauled so that they become as good as new.

Higgins (1998) studies the problem of scheduling railway track maintenance activities and crews. In this problem positive economic dependence is present in the following way. The occupancy of track segments due to maintenance prevents all train movements on those segments. The costs associated with this can be regarded as downtime costs. The maintenance scheduling problem is modelled as a large scale $0 - 1$ programming problem with many (non-linear) restrictions. The objective is to minimize expected interference delay with the train schedule and prioritized finishing time. The downtime costs are modelled by including downtime probabilities in the objective function. The author proposes tabu search to solve the problem. The neighbourhood, which plays a prominent role in local search techniques, is easily defined by swapping the order of activities or maintenance crews.

The article of Sriskandarajah *et al.* (1998) discusses the maintenance scheduling of rolling stock. Multiple train units have to be overhauled before a certain due date. The aim is to find a suitable common due date for each train so that the due dates of individual units do no deviate too much from the common due date. Maintenance done too early or too late is costly since this may cause the 'downtime' of a train. A genetic algorithm is proposed to solve this scheduling problem.

3.2 Negative economic dependence

Negative economic dependence between components occurs when maintaining components simultaneously is more expensive than maintaining components individually. There can be several reasons for this.

- Manpower restrictions
- Safety requirements

- Redundancy / production-loss

Firstly, grouping maintenance results in a peak in manpower needs. Manpower restrictions may even be violated and additional labour needs to be hired, which is costly. The problem here is to find the balance between workload fluctuation and grouping maintenance.

Secondly, there are often restrictions on the use of equipment, when executing maintenance activities simultaneously. For instance, use of equipment may hamper use of other equipment and cause unsafe operations. (Legal) safety requirements often prohibit joint operation.

Thirdly, joint (corrective) maintenance of components in systems in which some kind of redundancy is available may not be beneficial. Although there may exist economies of scale through simultaneous repair of a number of (identical) components, leaving components in a failed condition for some time increases the risk of costly production losses. We will come back to this in section 3.3. Production loss may increase more than linearly with the number of components out of operation. For an example of this type of economic dependence we refer to Stengos and Thomas (1980). The authors give an example of the maintenance of blast furnaces. The disturbance due to maintenance is substantially more, the more furnaces that are out of operation. That is, the cost of overhauling the furnaces increases more than linearly with the number of furnaces out of action.

It appears that maintenance of systems with negative dependence is often modelled in discrete time. The models can be regarded as scheduling problems with many restrictions. These restrictions can easily be incorporated in discrete time models such as (mixed) integer programming models. With respect to these models, there is always the question whether the exact solution can be found efficiently. In other words, the question arises whether the problem is NP-hard. An example of discrete time modelling is given by the article of Grigoriev *et al.* (2006). In this article the so-called periodic maintenance problem (PMP) is studied. In this problem machines have to be serviced regularly to prevent costly production losses. The failures causing these production losses are not modelled. Time is discretized into unit-length periods. In each period at most one machine can be serviced. Apparently, negative economic dependence in the form of manpower restrictions or safety measures play a role in the maintenance of the machines. The problem is to find a cyclic maintenance schedule of a given length T that minimizes total service and operating costs. The operating costs of a machine increase linearly with the number of periods elapsed since last servicing that machine. PMP appears to be a NP-hard problem and the authors propose a number of solution methods. This leads to the first exact solutions for larger sized problems.

In Stengos and Thomas (1980) time is also discretized but the maintenance problem, scheduling the overhaul of two pieces of equipment, is set up as a Markov decision process. The pieces can be in different states and the probability of failure increases with the time since the last overhaul. So in comparison with the problem of Grigoriev *et al.* (2006), pieces can fail during operation. Negative economic dependence is modelled as follows. The cost of overhauling the pieces increases more than linearly with the number of pieces out of action. The objective is to minimize the 'loss of production' cost, which is incurred when a piece is overhauled. The optimal policy is found by a relative value successive approximation algorithm.

In Langdon and Treleaven (1997) the problem of scheduling maintenance for electrical power transmission networks is studied. There is negative economic dependence in the network due to redundancy / production-loss. Grouping certain maintenance activities in the network may prevent a cheap electricity generator from running, so requiring a more expensive generator to be run in its place. That is, some parts of the network should not be maintained simultaneously. These exclusions are modelled by adding restrictions to the MIP formulation of the problem. The authors propose several genetic algorithms and other heuristics to solve the problem.

3.3 k-out-of-n systems

In this section we discuss the different dependencies in the k-out-of-n system in more detail. This system is a typical example of a system with both positive and negative economic dependence between components. A k-out-of-n system functions if at least k components function. If $k = 1$, then it is a parallel system; if $k = n$, then it is a series system. Let us for the moment distinguish between the cases $k = n$ and $k < n$.

In the series system ($k = n$), there is positive economic dependence due to downtime opportunities. The failure of one component results in an expensive downtime of the system and this time can be used to group preventive and corrective maintenance. Negative economic dependence is not explicitly present in the series system.

If $k < n$, then there is redundancy in the system and it fails less often then its individual components. This way a certain reliability can be guaranteed. Typically, the components of this system are identical which allows for economies of scale in the execution of maintenance activities. It is not only possible to obtain savings by grouping preventive maintenance, but also by grouping corrective maintenance. Note that the latter form of grouping is not advantageous in series systems. In other words, the redundant components introduce additional positive dependence in the system. Whereas positive economic dependence is present upon failure of a component, negative economic dependence plays a role as long as the system operates. A single failure of a component may not always be an opportunity to combine maintenance activities. Firstly, grouping corrective and preventive maintenance upon the failure of the component, increases the probability of system failure and costly production losses. Secondly, leaving components in a failed condition for some time, with the intention to group corrective maintenance at a later stage, has the same effect. So, there is a trade-off between the potential loss resulting from a system failure and the benefit of joint maintenance.

One problem of optimizing (age-based) maintenance in k-out-of-n systems is the determination of downtime costs, as a failure does not directly result in system failure. Smith and Dekker (1997) derive the uptime, downtime and costs of maintenance in a 1-out-of-n system (with cold standby), but in general it has appeared to be very difficult to assess the availability and the downtime costs of a k-out-of-n system. In their article, Smith and Dekker (1997) optimize the following age-replacement policy. A component is taken out for preventive maintenance and replaced by a stand-by one, if its age has reached a certain value T_{pm}. Moreover, they determine the number of redundant components needed in the system.

In the maintenance policies considered in the articles below, an attempt is made to balance the negative aspects of downtime costs and the positive aspects of grouping (corrective) maintenance. The opportunistic maintenance policies proposed in these articles are age-based and also contain a threshold for the number of failures (except for the policy introduced by Sheu and Kuo (1994).

In Dekker *et al.* (1998b) the maintenance of light-standards is studied. A light-standard consists of n independent and identical lamps screwed on a lamp assembly. To guarantee a minimum luminance, the lamps are replaced if the number of failed lamps reaches a prespecified number m. In order to replace the lamps the assembly has to be lowered. This set-up activity is an opportunity to combine corrective and preventive maintenance. Several opportunistic age-based variants of the m-failure group replacement policy (in its original form only corrective maintenance is grouped) are considered in this paper. Simulation optimization is used to determine the optimal opportunistic age threshold.

Pham and Wang (2000) introduce imperfect PM and partial failure in a k-out-of-n system. They propose a two-stage opportunistic maintenance policy for the system. In the first stage failures are removed by minimal repair; in the second stage failed components are jointly replaced with operating components when m components have failed, or the entire system is replaced at time T, whichever occurs first. Positive economic dependence is of an opportunistic nature. Joint maintenance requires less time than individual maintenance.

Sheu and Kuo (1994) introduce a general age replacement policy for a k-out-of-n system. Replace the system completely whenever it reaches age T. Their model includes minimal repair, planned and unplanned replacements, and general random repair costs. The long-run expected cost rate is obtained. The aim of the paper is to find the optimal age replacement time T which minimizes the long-run expected cost per unit time of the policy.

The article of Sheu and Liou (1992) will be discussed in section 4, because they assume stochastic dependence between the components of a k-out-of-n system.

4 Stochastic dependence

In the survey of Thomas (1986) multi-component maintenance models with stochastic dependence are considered as a separate class of models. In the more recent overview articles this is not the case. In Cho and Parlar (1991) some articles dealing with failure interaction are discussed, but the modelling of failure interaction between components itself is not. In Wang (2002) nothing is said about systems with failure interaction; articles on this kind of systems only appear in the references. Actually, we are the first ones, since the survey of Thomas (1986), to give a comprehensive review of multi-component maintenance models with stochastic dependence. We do not aim to give solely a list of papers that have appeared. Instead, we want to give insight in the different ways of modelling failure interaction between components and explain the implications of certain approaches and assumptions with respect to practical applicability.

Stochastic dependence, also referred to as failure interaction or probabilistic dependence, implies that the state of components can influence the state of the other components. Here, the state can be given by the age, the failure rate, state of failure or another condition measure. In their seminal work on stochastic dependence, Murthy and Nguyen (1985b) introduce three different types of failure interaction in a two-component system.

Type I failure interaction implies that the failure of component 1 (2) can induce a failure of the other component with probability p (q), and has no effect on the other component with probability $1-p$ $(1-q)$. It follows that there are two types of failures: natural and induced. The natural failures are modelled by random variables, the induced failures are characterized by the probabilities p and q. In Murthy and Nguyen (1985a) the same authors extend type I failure interaction to systems with multiple components. It is assumed that whenever a component fails it induces a total failure of the system with probability p and has no effect on the other components with probability $(1-p)$. In this overview we will consider this to be the definition of type I failure interaction.

Type II failure interaction in a two-component system is defined as follows. The failure of component 2 can induce a failure of component 1 with probability q, whereas every failure of component 1 acts as a shock to component 2, without inducing an instantaneous failure, but affecting its failure rate.

Type III failure interaction implies that the failure of each component affects the failure rate of the other component. That is, every failure of one of the components acts as a shock to the other component.

A potential problem of the failure rate interaction defined by the last two types, is determining the size of the shock. In practice it is very difficult to assess the effect of a failure of one component on the failure rate of another component. Usually there is not much data on the course of the failure rate of a component after the occurrence of a shock. Shocks can also be modelled by adding a (random) amount of damage to the state of another component. Natural failures then occur if the state of a component (measured by the cumulative damage) exceeds a certain level. In this paper we will bring this modelling of type II and III failure interaction together in one definition. That is, we renew the definition of type II failure interaction for multi-component systems. It reads as follows: *The system consists of at least one component which failure either affects the failure rate of at least one of the other components, or causes a (random) amount of damage to the state of at least one of the other components.* It follows that we regard a mixture of induced failures and shock damage as type II failure interaction. Models with type II failure interaction will also be called shock damage models.

In general, the maintenance policies considered in the literature on stochastic dependence, are mainly of an opportunistic nature, since the failure of one component is potential harmful for the other component(s). Modelling failure interaction appears to be quite elaborate. Therefore, most articles only consider two-component systems. Below we review the articles on failure interaction in the following order. Firstly, we will discuss the type I interaction models. For this type of interaction different opportunistic versions of the well-known age and block replacement policies have been proposed. Secondly, the articles on type II interaction will be reviewed. We will see that in most of these articles the occurrence of shocks is modelled as a non-homogeneous Poisson process (NHPP) or that the failure rate of components is adjusted upon failure of other components. Thirdly, we pay attention to articles that consider both types of failure interaction. Finally, we discuss other forms of modelling failure interaction.

Type I

Murthy and Nguyen (1985a) consider two maintenance policies in a multi-component system with type I failure interaction. Under the first policy all failed components are replaced by new ones. When there is no total system failure, then only the single failed component is replaced by a new one. Under the second policy all components, also the functioning component(s), are replaced by new ones. When there is no total system failure, then the single failed component is subjected to minimal repair and made operational. The failure rate of the failed component after repair is the same as that just before failure.

The authors deduce both the expected cost of keeping the system operational for a finite time period as well as the expected cost per unit time, of keeping the system operational for an infinite time period.

Sheu and Liou (1992) consider an optimal replacement policy for a k-out-of-n system subject to shocks. Shocks arrive according to a NHPP. The system is replaced preventively whenever it reaches age $T > 0$ at a fixed cost c_0. If the m-th shock arrives at age $S_m < T$, it causes the simultaneous failure of j components at the same time with probability $p_j(S_m)$ for $j = 0, 1, 2, .., n$, where $\sum_{j=0}^{n} p_j(S_m) = 1$. If $j \geq k$, then the k-out-of-n system is replaced by a new one at a cost c_∞ (unplanned failure replacement). So, the downtime is used to replace all components. If $0 \leq j < k$, then the system is minimally repaired with cost $c_j(S_m)$. After a complete replacement (either a planned or a failure replacement), the shock process is set to zero. All failures subject to shocks are assumed to be instantly detected and repaired. The aim of the paper is to find the optimal T which minimizes the long run expected cost per unit time of the maintenance policy.

The articles of Scarf and Deara (1998, 2003) consider failure-based, (opportunistic) age, and (opportunistic) block replacement policies for a labelled two-component *series* system with type I failure interaction. The articles can be seen as an extension of the article of Murthy and Nguyen (1985b) on failure-based replacement for such systems. Note that since we deal with a series system, the failure of either component causes a system downtime. So, if the system is down, this does not necessarily mean that both components have failed. Economic dependence is modelled on the basis that the cost of replacement of one or more components includes a one-off set-up cost whose magnitude does not depend on the number of components replaced.

The maintenance policies considered in Scarf and Deara (1998) are of the age-based replacement type: replace a component on failure or at age T, whichever is sooner. Failure-based maintenance is viewed as the limiting case ($T \to \infty$) of age-based replacement. As there is also economic dependence between components, the authors consider opportunistic age-based replacement policies: replace a component on failure or at age T or at age $T' < T$ if an opportunity exists.

The policies considered in Scarf and Deara (2003) are of the block replacement type and are extended for two-component systems. The independent block replacement policy is a single component policy and it is of the following form: replace failed components and replace component 1 at times $k\Delta_1, k = 1, 2, ...$ and replace component 2 at times $k\Delta_2, k = 1, 2, ...$ Block replacement can be grouped: replace failed components and replace the system at times $k\Delta, k = 1, 2, ...$ It can also be combined: replace both components (whether failed or not) on failure of the system and replace the system at times $k\Delta, k = 1, 2, ...$ In modified block replacement policies for a two-component system, a component is only replaced at the block replacement times if its age is greater than some critical value. The block replacement times may be independent or grouped, or the components may be combined. Opportunistic modified block replacement policies are of the form: on failure of component 1, if the age of component 2, τ_2, is greater than $b_2^{'}$, then replace both components; otherwise just replace component 1. On failure of component 2, if the age of component 1, τ_1, is greater than $b_1^{'}$, then replace both components; otherwise just replace component 2. At block replacement times for component 1, $k\Delta_1, k = 1, 2, ...$, replace component 1 if $\tau_1 > b_1$ and replace component 2 if $\tau_2 > b_2^{'}$; at block replacement times for component 2, $k\Delta_2, k = 1, 2, ...$, replace component 2 if $\tau_2 > b_2$ and replace component 1 if $\tau_1 > b_1^{'}$ (for suitable chosen thresholds, b_1, $b_1^{'}$, b_2, $b_2^{'}$).

In both articles the maintenance policies are considered in the context of the clutch system used in a bus fleet. This system consists of the clutch assembly (component 2) and the clutch controller (component 1). Actually, the failure of the controller causes a failure of the assembly with probability 1 and the failure of the assembly causes a failure of the controller with probability 0. It is important to mention that the maintenance policies are not only compared on the basis of cost, but also on ease of implementation and system reliability. It is found that an age-based policy is best, but since this implies that components ages have to be monitored, the authors propose to implement a block or modified block policy. Combined modified block replacements seems to be the best alternative for the clutch system under consideration. Combining maintenance of components has the advantage that the system is in general more reliable, although the long run costs per unit time are higher. The economic gains from using a complex policy have to be weighed up against the addition of investment required to implement such policies.

Jhang and Sheu (2000) address the problem of analyzing preventive maintenance policies in a multi-component system with type I failure interaction. The i-th component $1 \leq i \leq N$ has two types of failures. Type 1 failures are minor failures and are removed by a minimal repair. Type 2 failures are catastrophic failures and induce a total failure of the system (i.e. failure of all other components in the system). Type 2 failures are removed by an unplanned / unscheduled replacement of the system. The model takes into account costs for minimal repairs, replacements and preventive maintenance. Generalized age and block replacement policies are proposed. The age replacement policy implies preventive replacement of all components whenever an operating system reaches age T. In the case of a block replacement policy the system is preventively replaced every T years. The expected long-run cost per unit time for each policy is derived and it is discussed how the optimal T can be determined. Various special cases are discussed into detail. Finally, the authors mention the application of their model to the maintenance of mining cables used in hoisting load.

Type II

Satow and Osaki (2003) consider a two-component parallel system. Component 1 is repairable and at failure minimal repair is done. Failures of component 1 occur according to a NHPP. Whenever the component fails it induces a random amount of damage to component 2. The damage is additive and component 2 fails whenever the total damage exceeds a certain failure level. A system failure always occurs whenever component 2 fails, because both components fail simultaneously. By assumption component 2 is not repairable. This means that a failed system needs to be replaced by a new one. Since preventive replacement is cheaper than failure replacement, a two-parameter *preventive* replacement policy is analyzed. The policy takes into account both system age and the total damage of component 2. The system is replaced preventively whenever the total damage of component 2 exceeds k or at time T and it is replaced correctively at system failure. An expression for the expected cost per unit time for long run operation is derived and the policy is optimized analytically for two special cases (the one-parameter policies). Numerical examples show that the policy imposing a limit on the total damage (k) of component 2 outperforms the age T policy. It appears that the two-parameter preventive maintenance policy does not necessarily lead to lower expected costs. This is because in this model the state of component 2 is best indicated by the total damage and its age does not provide any additional information.

Zequeira and Bérenguer (2005) study inspection policies for a two-component parallel standby system. The system operates successfully if at least 1 component functions. Failures can be detected only by periodic inspections. The failure times are modelled as independent random variables. Type II failure interaction is modelled as follows. The failure of one component modifies the (conditional) failure probability of the other component with probability p and does not influence the failure time with probability $1 - p$. Within this respect, the model extends the failure rate interaction models proposed by Murthy and Nguyen (1985b). Inspections are either staggered, i.e. the components are inspected one at a time, or non-staggered, i.e. the components are inspected simultaneously at the same time. It is assumed that there are no economies of scale by doing non-staggered inspections. Numerical experiments prove that for the case of constant hazard rates, staggered inspections outperform non-staggered inspections on the expected average cost per unit time criterion. The authors explain this counter-intuitive result as follows. When inspections are staggered, at least one component is in an operating condition more frequently than when inspections are not staggered.

Lai and Chen (2006) consider a two-component system with failure rate interaction. The lifetimes of the components are modelled by random variables with increasing failure rates. Component 1 is repairable and it undergoes minimal repair at failures. That is, component 1 failures occur according to a NHPP. Upon failure of component 1 the failure rate of component 2 is modified (increased). Failures of component 2 induce the failure of component 1 and consequently the failure of the system. The authors propose the following maintenance policy. The system is completely replaced upon failure, or preventively replaced at age T, whichever occurs first. The expected average cost per unit time is derived and the policy is optimized with respect to parameter T. The optimum turns out to be unique.

Barros *et al.* (2006) introduce imperfect monitoring in a two-component parallel system. It is assumed that the failure of component i is detected with probability $1 - p_i$ and is not detected with probability p_i. The components have exponential lifetimes and when a component fails the extra stress is placed on the surviving one for which the failure rate is increased. Moreover, independent shocks occur according to a Poisson process. These shocks correspond to common cause failures and induce a system failure. The following maintenance policy is proposed. Replace the system upon failure (either due to a shock or failure of the components separately), or preventively at time T, whichever occurs first. Assuming that preventive replacement is cheaper, the total expected discounted cost over an unbounded horizon is minimized. Numerical examples show the relevance of taking into account monitoring problems in the maintenance model. The model is applied to a parallel system of electronic components. When one fails the overwork is placed on the surviving one and the delivery rate is kept to the same level.

Type I and II

Murthy and Nguyen (1985b) derive the expected cost of operating a two-component system with type I or type II failure interaction for both a finite and an infinite time period. They consider a simple, non-opportunistic, maintenance policy. Always replace failed components immediately. This means that the system is only renewed if a natural failure induces a failure of the other component.

Nakagawa and Murthy (1993) elaborate on the ideas of Murthy and Nguyen (1985b). They consider two types of failure interaction between two components. In the first case the failure of component 1 induces a failure of component 2 with a certain probability. In the second case the failure of component 1 causes a random amount of damage to the other component. In the latter case the damage accumulates and the system fails when the total damage exceeds a specified level. Failures of component 1 are modelled as a NHPP with increasing intensity function. The following maintenance policy is examined. The system is replaced at failure of component 2 or at the *N-th* failure of component of component 1, whichever occurs first. For both models the optimal number of failures before replacing the system as to minimize the expected cost per unit time over an infinite horizon is derived. The maintenance policy for the shock damage model is extended as follows: the system is also replaced at time T. This results in a two-parameter maintenance policy which is also optimized. The authors give an application of their models to the chemical industry; component 1 is a pneumatic pump and component 2 is a metal container. The failure of the pneumatic pump may either lead to an explosion, causing system failure (model 1), or lead to a reduction in the wall thickness of the container (model 2). The extension of model 2 captures the introduction of preventive maintenance of the container at time T.

Other types

Özekici (1988) considers a reliability system of n components. The state of the system is given by the random vector $\mathbf{X}_t$ of the ages of the components at time t, that is $\mathbf{X}_t = (\mathbf{X}_t^1, ..., \mathbf{X}_t^n)$. It is assumed that $\mathbf{X}_t^i \geq 0$ for all $t > 0$ and $i = 1, ..., N$, where $\mathbf{X}_t^i = \infty$ implies that component i is in a failed state at time t. The stochastic structure of the system is that the stochastic process $\mathbf{X} = \{\mathbf{X}_t : t > 0\}$ with state-space $[0, \infty)$ is a positive, increasing, right-continuous, and quasi-left continuous, strong Markov process. Stochastic dependence between the components is modelled by making the age (state) of a component at time t dependent on the age of the system up to time t. The failure interaction considered here differs from type I and II failure interaction defined above. It is worth to mention that this paper is written independently of the work of Murthy and Nguyen (1985b; 1985a).

Maintenance is modelled as follows. There are periodic overhauls at which the state of the system is inspected and a replacement decision is made on the components based on the observation of the system. In this paper the cost structure of the maintenance decision is very general and consists of two types: costs which only depend on the number of replaced components and costs which depend on the state of the system at the time of inspection. Economic dependence between components is 'hidden' in the former costs. Replacing a group of components together is cheaper than replacing the components separately or in smaller subgroup. The optimal replacement problem is formulated as a Markov decision process. The author proposes a very general class of replacement policies, for which the decision to replace a component depends on the age of all components. It appears to be possible to characterize the optimal solution to the replacement problem. Unfortunately, it cannot be proved that there exists a single critical age for the system, which describes the optimal replacement problem. The author provides some intuitive results, e.g. it is not optimal to replace new components and if the age of components that have to be replaced is increased, then the optimal policy does not change. He also gives an important counter-intuitive result: it is not true that more components are replaced as the system gets older.

5 Structural dependence

Structural dependence means that some operating components have to be replaced, or at least dismantled, before failed components can be replaced or repaired. In other words, structural dependence between components indicates that they cannot be maintained independently, but together. It is not about failure dependence, but about maintenance dependence. Since the failure of a component offers an opportunity to replace other components, opportunistic policies are expected to perform well on systems with structural dependence between components. Obviously, preventive maintenance may also be advantageous, since maintenance of structural dependent components can be grouped.

There may be several reasons for structural dependence. For example a bicycle chain and a cassette form a union which should always be replaced together, rather than individually. Another example is from Dekker *et al.* (1998a), which considers road maintenance. Several deterioration mechanisms affect roads, e.g. longitudinal and transversal unevenness, cracking and ravelling. For each mechanism one may define a virtual component, but if one applies a maintenance action to such a component it also affects the state with respect to the other failure mechanisms.

The seminal paper in this category is from Sasieni (1956). He considers the production of rubber tyres. The machine which produces the tyres consists of two “bladders”; one tyre is produced on each bladder simultaneously. Upon failure of a bladder, the machine must be stripped down before replacement can be done. This means that the other bladder can be replaced at the same time. Note that immediate replacement is not mandatory, but a failed bladder will produce faulty tyres. Two maintenance policies are analyzed and optimized. The first one is a preventive maintenance policy. Bladders which have made a predetermined number of tyres (m) without failure are replaced. The second one is an opportunistic version of the first policy. When a machine is stripped to replace one bladder, replace the other bladder if it has produced than $n \leq m$ tyres.

6 Planning horizon & optimization methods

In this section we will classify articles on the basis of the planning horizon of the maintenance model and the optimization methods used to solve this model. Actually, these two concepts are related. The majority of the articles reviewed here, assumes an infinite horizon. This assumption facilitates the mathematical analysis; it is often possible to derive analytical expressions for optimal control parameters and the corresponding optimal costs. So, in the category infinite horizon (stationary grouping) models policy optimization is the most popular optimization method. For convenience we will not give a list of the articles in this category.

Finite-horizon models consider the system in this horizon only, and hence assume implicitly that the system is not used afterwards, unless a so-called *residual value* is incorporated to estimate the industrial value of the system at the end of the horizon. In the article of Budai *et al.* (2006) the so-called end-of-horizon effect is eliminated by adding an additional term to the objective function. This term values the last interval.

The optimization methods applied to finite horizon models are either exact methods or heuristics.[1] Exact methods always find the global optimum solution of a problem. If the complexity of a optimization problem is high and the computing time of the exact method increases exponentially with the size of the problem, then heuristics can be used to find a near-optimal solution in reasonable time.

The scheduling problem studied by Grigoriev Grigoriev *et al.* (2006) appears to be NP-hard. Instead of defining heuristics, the authors choose to work on a relatively fast exact method. Column-generation and a branch-and-price technique are utilized to find the exact solution of larger-sized problems. The problem considered by Papadakis and Kleindorfer (2005) is first modelled as a mixed integer linear programming problem, but it appears that it can also be formulated as a max-flow min-cut problem in an undirected network. For this problem efficient algorithms exist and thus, an exact method is applied.

Langdon and Treleaven (1997), Sriskandarajah *et al.* (1998), Higgins (1998) and Budai *et al.* (2006) propose heuristics to solve complex scheduling problems. The first two articles utilize genetic algorithms. Higgins (1998) applies tabu search and Budai *et al.* (2006) define different heuristics that are based on intuitive arguments. In all four articles the heuristics perform well; a good solution is found within reasonable time.

7 Trends and open areas

In this section we give our opinion on the future of optimal maintenance of multi-component systems. We first analyze the trends in modelling multi-component maintenance and we will then mention the open research areas in this field.

Trends

The last few years several articles have appeared on optimal maintenance of systems with stochastic dependence. In particular, the shock-damage models have got many attention. One explanation for this is that type II failure interaction can be modelled in several ways, whereas there is not much room for extensions in the type I failure model. Another reason

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[1] Actually, if the maintenance policy is relatively easy, it is sometimes possible to determine the expected maintenance costs over a finite period of time. For instance, Murthy and Nguyen (1985b; 1985a) consider failure-based policies in a system with stochastic dependence and derive an expression for the expected cost of operating the system for a finite time.

is that since the field of stochastic dependence is not very broad yet, it is easy to add a new feature such as minimal repair or imperfect monitoring to an existing model. Thirdly, many existing opportunistic maintenance policies for systems with economic dependence, have not yet been applied to systems with (type II) failure interaction.

Another upcoming field in multi-component maintenance modelling is the class of finite horizon maintenance scheduling problems. Finite horizon models can be regarded as dynamic models, because short-term information can be taken into account. Maintenance scheduling problems are often modelled in discrete time as mixed integer linear programming problems. These problems can be NP-hard and in that case heuristics or local search methods have to be developed in order to solve the problems to near-optimality efficiently. In the last decade tabu search, genetic algorithms and problem specific heuristics have already been applied to maintenance scheduling problems (see Langdon and Treleaven (1997), Sriskandarajah *et al.* (1998), Higgins (1998) and Budai *et al.* (2006)). However, there is still need for better local search algorithms.

Open areas

To our opinion more work can be done in the following areas:

Finite horizon models On the one hand, the class of infinite horizon models has been studied extensively in literature. Based on renewal-reward theory many maintenance policies for stationary grouping models have been analyzed. On the other hand, the class of finite horizon models, which includes many maintenance scheduling problems, has never had that much attention. However, maintenance of multi-component systems has to be made operational. Therefore, finite horizon and especially rolling horizon models, which also take short-term into account, have to be developed. In order to solve these models heuristics / local search methods should be further developed. Exact algorithms also need more attention. The article of Grigoriev *et al.* (2006) shows that some scheduling problems of reasonable size can be optimized exactly in reasonable time.

Case-studies This review shows that case-studies are not represented very well in the field. This is surprising, since maintenance is something which should be done in practice and not in theory. To our opinion many models are just (mathematical) extensions of existing models and most of the times models are not validated empirically. Case-studies can lead to new models, both in the context of cost structures and dependencies between components.

Modelling multiple set-up activities In this article we have subdivided the category "economic dependence" into a number of subcategories. It appears that examples of modelling maintenance of systems with multiple set-up activities are scarce. Therefore, this seems to be a promising field for further research. After all, in many production systems complex set-up structures exist.

Structural dependence The field of structural dependence is wide open. To our opinion there have only been published a few articles on this topic.

Stochastic dependence Two decades ago Murthy and Nguyen published two articles on the maintenance of systems with stochastic dependence. Although this topic has had much attention since then, most articles still deal with two-component systems. So, there is still a lot of work to do on modelling maintenance of systems with failure interaction consisting of more than two components.

Combination of dependencies In this article we have seen one example of the combination of structural and economic dependence (Scarf and Deara (1998; 2003)). We have also review some papers with both positive and negative economic dependence. Obviously, the combination of different types of interaction results in difficult optimization models. So, this is also an opportunity for researchers to come up with some new models.

Simulation optimization We have already said that much work has been done on maintenance policies for the class of infinite horizon models. Many maintenance policies are not analytically tractable and simulation is needed to analyze these policies. We observe that the optimization of policies via simulation is often done by using algorithms for deterministic optimization problems. Methods such as simulated annealing and response surface methodology may be more efficient. This should be investigated further.

8 Conclusions

In this article we have reviewed the literature on optimal maintenance of multi-component maintenance. We have first classified articles on the type of dependence between components: economic, stochastic and structural dependence. Subsequently, we have also subdivided these classes into new categories. For example, we have introduced the categories positive and negative economic dependence. We have paid attention to articles with both forms of interaction. Moreover, we have defined several subcategories in the class of models with positive economic dependence. With respect to articles in the class of stochastic dependence, we are the first to review these articles systematically.

Another classification has been made on the basis of the planning horizon models and optimization methods. We have focussed our attention on the use of heuristics and exact methods in finite horizon models. We have concluded that this is a promising open research area, especially because the infinite horizon models have always got more attention.

We have given our opinion on the trends and the open areas in the literature on multi-component maintenance. We have observed a shift from infinite horizon models to finite horizon models and from economic to stochastic dependence. This immediately defines the open research areas, which also include topics such as case-studies, modelling combinations of dependencies between components and modelling multiple set-up activities.

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