Open access • Journal Article • DOI:10.2139/SSRN.871037

Optimal Monetary and Fiscal Policy in a Currency Union — Source link

Tommaso Monacelli, Jordi Galí

Institutions: Bocconi University

Published on: 01 Dec 2005 - Social Science Research Network

Topics: Fiscal union, Currency union, Monetary base, Fiscal policy and Devaluation

NBER WORKING PAPER SERIES

OPTIMAL MONETARY AND FISCAL POLICY
IN A CURRENCY UNION

Jordi Gali
Tommaso Monacelli

Working Paper 11815
http://www.nber.org/papers/w11815

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2005

Galí thanks the Fundación Ramón Areces and the European Commission (MAPMU RTN). For comments, we thank participants in seminars at Boston College, GIIS Geneve, Lausanne, Paris I, Humboldt, Oslo and in the EUI conference on "Open Macro Models and Policy in the Development of the European Economy", and the CREI-CEPR conference on "Designing a Macroeconomic Policy Framework for Europe". Anton Nakov and Lutz Weinke provided excellent research assistance. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

Optimal Monetary and Fiscal Policy in a Currency Union
Jordi Gali and Tommaso Monacelli
NBER Working Paper No. 11815
December 2005
JEL No. E52, F41, E62

ABSTRACT

We lay out a tractable model for fiscal and monetary policy analysis in a currency union, and analyze its implications for the optimal design of such policies. Monetary policy is conducted by a common central bank, which sets the interest rate for the union as a whole. Fiscal policy is implemented at the country level, through the choice of government spending level. The model incorporates country-specific shocks and nominal rigidities. Under our assumptions, the optimal monetary policy requires that inflation be stabilized at the union level. On the other hand, the relinquishment of an independent monetary policy, coupled with nominal price rigidities, generates a stabilization role for fiscal policy, one beyond the efficient provision of public goods. Interestingly, the stabilizing role for fiscal policy is shown to be desirable not only from the viewpoint of each individual country, but also from that of the union as a whole. In addition, our paper offers some insights on two aspects of policy design in currency unions: (i) the conditions for equilibrium determinacy and (ii) the effects of exogenous government spending variations.

Jordi Gali
Department of Economics (E52-359)
MIT
50 Memorial Drive
Cambridge, MA 02142
and NBER
jgali@mit.edu

Tommaso Monacelli
IGIER-Universita Bocconi
tommaso.monacelli@uni-bocconi.it

1 Introduction

The creation of the European Monetary Union (EMU) has led to an array of new challenges for policymakers. Those challenges have been reflected most visibly in the controversies surrounding the implementation and proposed reforms of the Stability and Growth Pact, as well as in the frequent criticisms of the interest rate policy of the European Central Bank. From the perspective of macroeconomic theory, the issues raised by EMU have created an urgent need for an analytical framework that would allow us to evaluate alternative monetary and fiscal policy arrangements for EMU, or other monetary unions that may emerge in the future. In the present paper we propose a tractable framework suitable for the analysis of fiscal and monetary policy in a currency union, and study its implications for the optimal design of such policies from the viewpoint of the union as a whole.

In our opinion that analytical framework has to meet several desiderata. First, it has to incorporate some of the main features characterizing the optimizing models with nominal rigidities that have been developed and used for monetary policy analysis in recent years. Secondly, it should contain a fiscal policy sector, with a purposeful fiscal authority. Thirdly, the framework should comprise many open economies, linked by trade and financial flows.

It is worth noticing that while several examples of optimizing sticky price models of the world economy can be found in the literature, tractability often requires that they be restricted to two-country world economies.[1] Yet, while such a framework may be useful to discuss issues pertaining to the links between two large economies (say, the U.S. and the euro area), it can hardly be viewed as a realistic description of the incentives and constraints facing policymakers in a monetary union like EMU, currently made up of twelve countries (each with an independent fiscal authority), but expected to accommodate as many as thirteen additional members over the next few years. Clearly, and in contrast with models featuring two large economies, the majority of the countries in EMU are small relative to the union as a whole. As a result, their policy decisions, taken in isolation, are likely to have very little impact

[1] See, among others, Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Benigno and Benigno (2003), Bacchetta and van Wincoop (2000), Devereux and Engle (2003), Pappa (2003), Kollmann (2001), Chari , Kehoe and McGrattan (2003). Only a subset of these contributions feature a role for a fiscal sector. For a recent analysis of monetary-fiscal policy interaction in a two-country setting and flexible exchange rates see Lombardo and Sutherland (2004). For a two-country analysis more specifically tailored to a monetary union, see Ferrero (2005).

on other countries. While it should certainly be possible, as a matter of principle, to modify some of the existing two-country models to incorporate an arbitrarily large number of countries (i.e. an N-country model, for large N), it is clear that such undertaking would render the resulting model virtually intractable.

In the present paper we propose a tractable framework for policy analysis in a monetary union that meets the three desiderata listed above. First, we introduce nominal rigidities by assuming a staggered price setting structure, analogous to the one embedded in the workhorse model used for monetary policy analysis in closed economies, which we treat as a useful benchmark. Secondly, we model the currency union as being made up by a continuum of small open economies, subject to imperfectly correlated productivity shocks. This approach allows one to overcome the tractability problems associated with "large N," by making each economy of negligible size relative to the rest of the world . Finally, we incorporate a fiscal policy sector, by allowing for country-specific levels of public consumption, and by having the latter yield utility to domestic households.

Our analysis focuses on the optimal fiscal and monetary policies from the viewpoint of the currency union as a whole. In particular we determine the monetary and fiscal policy rules that maximize a second-order approximation to the integral of utilities of the representative households inhabiting the different countries in the union.

Two main results emerge from that analysis. First, we show that it is optimal for the (common) monetary authority to stabilize inflation in the union as a whole. Attaining that goal generally requires offsetting the threats to price stability that may arise from the joint impact of the fiscal policies implemented at the country level. Our finding would thus seem to provides a rationale for a monetary policy strategy like the one adopted by the European Central Bank, i.e. one that focuses on attaining price stability for the union as a whole.[2] It is important to stress, however, that the optimality of that policy is conditional on the national fiscal authorities simultaneously implementing their part of the optimal policy package. The latter implies a neutral fiscal stance in the aggregate–in a sense to be made precise below–, which poses no inflationary pressures on the union. As discussed below, in the

[2]Benigno (2004) obtains a similar result in the context of a currency union model without a fiscal sector. His analysis focuses on the implications of asymmetries across countries on the definition of the relevant price index to be stabilized. Our focus is instead on the interaction between monetary and fiscal policies.

absence of such coordinated response by the national fiscal authorities, the union's central bank may find it optimal to deviate from a strict inflation targeting policy.

Second, under the optimal policy arrangement, each country's fiscal authority plays a dual role, trading-off between the provision of an efficient level of public goods and the stabilization of domestic inflation and output gap. Interestingly, we find that the existence of such a stabilizing role for fiscal policy is desirable not only from the viewpoint of each individual country, but also from that of the union as a whole. Our simulations under the optimal policy mix of a representative economy's response to an idiosyncratic productivity shock show that the strength of the countercyclical fiscal response increases with the importance of nominal rigidities. Our findings on this front call into question the desirability of imposing external constraints on a currency union's members ability to conduct countercyclical fiscal policies that seek to limit the size of the domestic output gap and inflation differentials resulting from idiosyncratic shocks.

In addition to the main results just described, our paper sheds new light on two additional aspects of policy design in currency unions, in the presence of nominal rigidities. The first issue pertains to the conditions for equilibrium determinacy. As is well known from the closed economy literature, in order to guarantee the uniqueness of equilibrium the central bank must eventually adjust the nominal interest rates more than one-for-one with changes in inflation, a property generally referred to as the "Taylor principle."[3] When joining a currency union, a small economy relinquishes its ability to meet the Taylor principle, since variations in its rate of inflation that are the result of purely idiosyncratic shocks will have a small (infinitesimal, in our model) effect on union-wide inflation, and will thus induce little or no response from the union's central bank. This may raise doubts regarding the possibility of guaranteeing a unique equilibrium and avoiding unnecessary sunspot fluctuations in that context. Our analysis demonstrates that the equilibrium path for country-level variables will be uniquely determined so long as the equilibrium is determinate for the union as a whole. This can in turn be guaranteed by having the union's central bank follow an interest rate rule that satisfies the usual Taylor principle.

Secondly, we provide an analysis of the effects of an *exogenous* change in government spending in a small open economy belonging to a monetary union (or, equivalently, under a hard peg). While in the closed economy counterpart the effects of

[3] See, e.g., Woodford (2001).

a change in government spending are ambiguous, –since they always depend on the endogenous response of monetary policy to the fiscal intervention[4]– this is not case for a country in a currency union: in the latter case an increase in government spending always raises output and the price level in the short run, after which a period of sustained deflation is needed to restore the initial terms of trade.

The paper is organized as follows. In Section 2 we develop the basic model. In Section 3 we characterize the equilibrium dynamics in a currency union, from the perspective of both a single member economy and of the union as a whole. In Section 4 we study optimal monetary and fiscal policy in a currency union. We take for granted an institutional arrangement in which monetary policy is conducted by a common central bank, whereas fiscal policy is conducted at the level of each member country. We contrast the case of full price flexibility to the more realistic one involving nominal rigidities. Section 5 concludes and suggests extensions for future work.

2 A Currency Union Model

We model the currency union as a closed system, made up of a *continuum of small open economies* represented by the unit interval. Each economy, indexed by $i \in [0,1]$ is of measure zero; as a result, its domestic policy decisions do not have any impact on the rest of the union. While different economies are subject to imperfectly correlated shocks, we assume that they share identical preferences, technology, and market structure.[5]

Next we describe in detail the problem facing households and firms in our model economy.

2.1 Households

Consider a typical country belonging to the monetary union (say, country i). We assume it is inhabited by a representative infinitely lived household seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t^i, N_t^i, G_t^i) \tag{1}$$

[4]See, for instance, Linnemann and Schabert (2003).

[5]In Galí and Monacelli (2005) we use a similar modelling formalism, though the focus of the paper–the design of monetary policy by a single, small open economy with its own central bank–is very different from the one in the present paper.

where C_t^i, N_t^i denote, respectively, private consumption and hours of work, while G_t^i is an index of public consumption, described in a separate section below.

More precisely, C_t^i is a composite consumption index defined by

$$C_t^i \equiv \frac{(C_{i,t}^i)^{1-\alpha}\ (C_{F,t}^i)^{\alpha}}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} \tag{2}$$

where $C_{i,t}^i$ is an index of country i's consumption of domestic goods (i.e., goods produced in country i itself) given by the CES function

$$C_{i,t}^i \equiv \left(\int_0^1 C_{i,t}^i(j)^{\frac{\epsilon-1}{\epsilon}}\ dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $j \in [0,1]$ denotes the type of good (within the set produced in country i).[6]

Variable $C_{F,t}^i$ is an index of country i's consumption of imported goods, given by:

$$C_{F,t}^i \equiv \exp \int_0^1 c_{f,t}^i\ df$$

where $c_{f,t}^i \equiv \log C_{f,t}^i$ is, in turn, the log of an index of the quantity of goods consumed by country i's households that are produced in (and, hence, imported from) country f. That index is defined in a way symmetric to (3), that is:

$$C_{f,t}^i \equiv \left(\int_0^1 C_{f,t}^i(j)^{\frac{\epsilon-1}{\epsilon}}\ dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

Notice that in the specification of preferences described above $\alpha \in [0,1]$ is the weight on imported goods in the utility of private consumption. Given that the weight of the home economy in the union is infinitesimal, a value for α strictly less than one reflects the presence of home bias in private consumption, implying that households in different countries will have different consumption baskets.[7] Equivalently, we can think of α as an index of openness.

Finally, notice that parameter $\epsilon > 1$ denotes the elasticity of substitution between varieties produced within any given country, independently of the producing country.

Maximization of (1) is subject to a sequence of budget constraints of the form:

$$\int_0^1 P_t^i(j) C_{i,t}^i(j)\ dj + \int_0^1 \int_0^1 P_t^f(j) C_{f,t}^i(j)\ dj\ df\ + E_t\{Q_{t,t+1} D_{t+1}^i\} \leq D_t^i + W_t^i N_t^i - T_t^i \tag{5}$$

[6] As discussed below, each country produces a continuum of differentiated goods, represented by the unit interval. Each good is produced by a separate firm. No good is produced in more than one country.

[7] As a result, CPI inflation differentials across countries may emerge, even if the law of one price holds for each individual good.

for $t = 0, 1, 2, ...$, where $P_t^f(j)$ is the price of good j produced in country f (expressed in units of the single currency). D_{t+1}^i is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (and which includes shares in firms, local and foreign), W_t^i is the nominal wage, and T_t^i denotes lump-sum taxes.

We assume that households have access to a complete set of contingent claims, traded across the union. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs, common across countries. Also, implicit in the notation in (5)–which features a single country index for each price–is the assumption that the *law of one price* holds across the union.

The optimal allocation of any given expenditure on the goods produced in a given country yields the demand functions:

$$C_{i,t}^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} C_{i,t}^i \qquad ; \qquad C_{f,t}^i(j) = \left(\frac{P_t^f(j)}{P_t^f}\right)^{-\epsilon} C_{f,t}^i \tag{6}$$

for all $i, f, j \in [0,1]$. $P_t^i \equiv \left(\int_0^1 P_t^i(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}}$ represents country i's *domestic* price index (i.e., an index of prices of domestically produced goods), for all $i \in [0,1]$. Notice that, as a consequence of the law of one price, $P_t^f \equiv \left(\int_0^1 P_t^f(j)^{1-\epsilon}\, dj\right)^{\frac{1}{1-\epsilon}}$ is the price index for the bundle of goods imported from country f, as well as the latter's domestic price index. It follows from (6) that $\int_0^1 P_t^i(j) C_{i,t}^i(j)\, dj = P_t^i C_{i,t}^i$ and $\int_0^1 P_t^f(j) C_{f,t}^i(j)\, dj = P_t^f C_{f,t}^i$.

Furthermore, the optimal allocation of expenditures on imported goods by country of origin implies:

$$P_t^f C_{f,t}^i = P_t^* C_{F,t}^i \tag{7}$$

for all $f \in [0,1]$, where $P_t^* \equiv \exp \int_0^1 p_t^f\, df$ is the union-wide price index. From the viewpoint of any individual country, P_t^* is also a price index for imported goods. Notice that (7) implies that we can write total expenditures on imported goods as $\int_0^1 P_t^f C_{f,t}^i\, df = P_t^* C_{F,t}^i$

Finally, and letting $P_{c,t}^i \equiv (P_t^i)^{1-\alpha}(P_t^*)^{\alpha}$ denote the *consumer price index* (CPI) in country i, the optimal allocation of expenditures between domestic and imported goods in that country is given by:

$$P_t^i C_{i,t}^i = (1-\alpha)\ P_{c,t}^i C_t^i \qquad ; \qquad P_t^* C_{F,t}^i = \alpha\ P_{c,t}^i C_t^i \tag{8}$$

Combining all previous results, we can write total consumption expenditures by country i's households $P_t^i C_{i,t}^i + P_t^* C_{F,t}^i = P_{c,t}^i C_t^i$. Thus, and conditional on an optimal allocation of expenditures, the period budget constraint can be rewritten as:

$$P_{c,t}^{i} C_{t}^{i} + E_{t}\{Q_{t,t+1} \; D_{t+1}^{i}\} \leq D_{t}^{i} + W_{t}^{i} N_{t}^{i} + T_{t}^{i} \tag{9}$$

In what follows we assume that the period utility takes the simple form

$$U(C, N, G) \equiv (1-\chi) \; \log C + \chi \; \log G - \frac{N^{1+\varphi}}{1+\varphi} \tag{10}$$

where parameter $\chi \in [0,1)$ measures the weight attached to public consumption (relative to private consumption).

The remaining optimality conditions for country i's households are thus given by:

$$C_{t}^{i} \; (N_{t}^{i})^{\varphi} = (1-\chi) \; \frac{W_{t}^{i}}{P_{c,t}^{i}} \tag{11}$$

$$\beta \; \left(\frac{C_{t}^{i}}{C_{t+1}^{i}}\right) \left(\frac{P_{c,t}^{i}}{P_{c,t+1}^{i}}\right) = Q_{t,t+1} \tag{12}$$

which are assumed to hold for all period and states of nature (at t and $t+1$, in the case of (12)). Taking conditional expectations on both sides of (12) and rearranging terms we obtain a conventional Euler equation:

$$\beta R_{t}^{*} \; E_{t}\left\{\left(\frac{C_{t}^{i}}{C_{t+1}^{i}}\right) \left(\frac{P_{c,t}^{i}}{P_{c,t+1}^{i}}\right)\right\} = 1 \tag{13}$$

where $R_{t}^{*} = \frac{1}{E_{t}\{Q_{t,t+1}\}}$ is the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency in $t+1$ or, for short, the (gross) nominal interest rate. Below we assume that the union's central bank uses that interest rate as its main instrument of monetary policy.

For future reference it is useful to note that (11) and (13) can be respectively written in log-linearized form as:

$$w_{t}^{i} - p_{c,t}^{i} = c_{t}^{i} + \varphi \; n_{t}^{i} - \log(1-\chi)$$

$$c_{t}^{i} = E_{t}\{c_{t+1}^{i}\} - (r_{t}^{*} - E_{t}\{\pi_{c,t+1}^{i}\} - \rho) \tag{14}$$

where, as before, lower case letters denote the logs of the respective variables, $\rho \equiv -\log\beta$ is the time discount rate, and $\pi_{c,t}^{i} \equiv p_{c,t}^{i} - p_{c,t-1}^{i}$ is CPI inflation. The above optimality conditions hold for all $i \in [0,1]$

2.1.1 Some Definitions and Identities

Before proceeding with our analysis, we introduce several assumptions and definitions, and derive a number of identities that are extensively used below.

We start by defining the *bilateral terms of trade* between countries i and f as $\mathcal{S}^i_{f,t} \equiv \frac{P^f_t}{P^i_t}$, i.e., the price of country f's domestically produced goods in terms of country i's. The *effective terms of trade* for country i are thus given by

$$\begin{aligned} \mathcal{S}^i_t &\equiv \frac{P^*_t}{P^i_t} \\ &= \exp \int_0^1 (p^f_t - p^i_t)\ df \\ &= \exp \int_0^1 s^i_{f,t}\ df \end{aligned}$$

where $s^i_{f,t} \equiv \log \mathcal{S}^i_{f,t}$. Equivalently, in logs, we have $s^i_t = \int_0^1 s^i_{f,t}\ df$.

Notice also that the CPI and the domestic price levels are related according to:

$$P^i_{c,t} = P^i_t\ (\mathcal{S}^i_t)^\alpha$$

or, in logs:

$$p^i_{c,t} = p^i_t + \alpha\ s^i_t \tag{15}$$

Hence, it follows that *domestic inflation* – defined as the rate of change in the price index for domestically produced goods, i.e., $\pi^i_t \equiv p^i_t - p^i_{t-1}$ – and *CPI inflation* are linked according to the equation:

$$\pi^i_{c,t} = \pi^i_t + \alpha\ \Delta s^i_t \tag{16}$$

which makes the gap between our two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the index of openness α.

Notice that the distinction between CPI inflation and domestic inflation, while meaningful at the level of each country, vanishes for the currency union as a whole. Formally, integrating (15) over $i \in [0, 1]$ and using the fact that $\int_0^1 s^i_t\ di = 0$, yields the basic equality:

$$p^*_{c,t} = p^*_t$$

and, hence, $\pi^*_{c,t} = \pi^*_t$.

2.1.2 International Risk Sharing

Under the assumption of complete markets for state-contingent securities across the union, a first order condition analogous to (12) will hold for the representative household in any other country, say country f:

$$\beta \left(\frac{C_t^f}{C_{t+1}^f}\right)\left(\frac{P_{c,t}^f}{P_{c,t+1}^f}\right) = Q_{t,t+1} \tag{17}$$

Combining (12) and (17), we obtain:

$$C_t^i = \vartheta_i \; C_t^f \; (\mathcal{S}_{f,t}^i)^{1-\alpha} \tag{18}$$

for all $i, f \in [0,1]$ and all t, and where ϑ_i is a constant which will generally depend on initial conditions regarding relative net asset positions. Henceforth, and without loss of generality, we assume symmetric initial conditions (i.e., zero net foreign asset holdings for all countries, combined with an ex-ante identical environment), in which case we have $\vartheta_i = \vartheta = 1$ for all $i \in [0,1]$.

Taking logs on both sides of (18) and integrating over f we obtain

$$c_t^i = c_t^* + (1-\alpha)\; s_t^i \tag{19}$$

where $c_t^* \equiv \int_0^1 c_t^f \; df$ is the (log) aggregate consumption index for the union as a whole.

2.2 Optimal Allocation of Government Purchases

Country i's public consumption index is given by

$$G_t^i \equiv \left(\int_0^1 G_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$$

where $G_t^i(j)$ is the quantity of domestic good j purchased by the government. For simplicity, we assume that government purchases are fully allocated to domestically produced goods.[8]

For any given level of public consumption G_t^i (whose determination is a central focus of the analysis below), the government allocates expenditures across goods in

[8] For OECD countries, there is evidence of strong home bias in government procurement, over and above that observed in private consumption . See for instance Trionfetti (2000) and Brulhart and Trionfetti (2004).

order to minimize total cost. This yields the following set of government demand schedules, analogous to those associated with private consumption:

$$G_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i} \right)^{-\epsilon} \; G_t^i$$

In order to focus our attention on the determination of its aggregate level and its effects (rather than the distortions induced by its financing), we assume that government spending is entirely financed by means of lump sum taxes (accruing to domestic residents).

2.3 Firms

2.3.1 Technology

Each country has a continuum of firms represented by the interval $[0, 1]$. Each firm produces a differentiated good with a linear technology:

$$Y_t^i(j) = A_t^i \; N_t^i(j) \tag{20}$$

for all $i, j \in [0, 1]$, where A_t^i is a country-specific productivity shifter. The latter is assumed to follow an AR(1) process (in logs):

$$a_t^i = \rho_a \; a_{t-1}^i + \varepsilon_t^i$$

where $a_t^i \equiv \log A_t^i$, $\rho_a \in [0, 1]$, and $\{\varepsilon_t^i\}$ is white noise.

The assumption of a linear technology implies that the real marginal cost (expressed in terms of domestic goods) is common across firms in any given country, and given (in logs) by

$$mc_t^i = -\log(1 - \tau^i) + w_t^i - p_t^i - a_t^i$$

where τ^i is a (constant) employment subsidy whose role is discussed below.

Let $Y_t^i \equiv \left[\int_0^1 Y_t^i(j)^{\frac{\epsilon-1}{\epsilon}} \; dj \right]^{\frac{\epsilon}{\epsilon-1}}$ denote the aggregate output index for country i. The amount of labor hired is thus given by

$$N_t^i = \int_0^1 N_t^i(j) \; dj = \frac{Y_t^i \; Z_t^i}{A_t^i} \tag{21}$$

where $Z_t^i \equiv \int_0^1 \frac{Y_t^i(j)}{Y_t^i} \; dj$. In the Appendix we show that equilibrium variations in $z_t^i \equiv \log Z_t^i$ around the perfect foresight steady state are of second order. Thus, and up to a first order approximation, the following relationship between aggregate employment and output holds for all $i \in [0, 1]$:

$$y_t^i = a_t^i + n_t^i \tag{22}$$

2.3.2 Price setting

Firms are assumed to set prices in a staggered fashion, as in Calvo (1983). Hence, a measure $1-\theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. As is well known, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule:[9]

$$\overline{p}_t^i = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \; E_t\{mc_{t+k}^i + p_{t+k}^i\} \tag{23}$$

where $\overline{p}_t^i$ denotes the (log) of newly set prices in country i (same for all firms reoptimizing), and $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$ is the (log) of the optimal markup in the corresponding flexible price economy (or, equivalently, the markup prevailing in a zero inflation steady state).

3 Equilibrium Dynamics

3.1 Aggregate Demand and Output Determination

The clearing of market for good j produced in country i requires

$$\begin{aligned}
Y_t^i(j) &= C_{i,t}^i(j) + \int_0^1 C_{i,t}^f(j)\; df + G_t^i(j) \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} \left[(1-\alpha)\left(\frac{P_{c,t}^i}{P_t^i}\right) C_t^i + \alpha\int_0^1 \left(\frac{P_{c,t}^f}{P_t^i}\right) C_t^f \; df + G_t^i\right] \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} \left[(1-\alpha)(\mathcal{S}_t^i)^{\alpha}\; C_t^i + \alpha(\mathcal{S}_t^i)^{\alpha}\int_0^1 (\mathcal{S}_{f,t}^i)^{1-\alpha}\; C_t^f \; df + G_t^i\right] \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} \left[C_t^i\; (\mathcal{S}_t^i)^{\alpha} + G_t^i\right]
\end{aligned} \tag{24}$$

and where the last equality makes use of (18). An analogous condition must hold for all $i, j \in [0,1]$ and all t.

Plugging the previous condition into the definition of country i's aggregate output $Y_t^i \equiv \left(\int_0^1 Y_t^i(j)^{1-\frac{1}{\epsilon}}\; dj\right)^{\frac{\epsilon}{\epsilon-1}}$ we obtain the following aggregate goods market clearing

[9] The approximation is carried out around a zero inflation steady state. See the appendix in Galí and Monacelli (2005) for a derivation in the context of a model with an identical price-setting block.

condition for country i:

$$Y_t^i = C_t^i \ (\mathcal{S}_t^i)^\alpha + G_t^i \tag{25}$$

A log-linear first order approximation of that market clearing condition around a (symmetric) steady state is given by:[10]

$$\widehat{y}_t^i = (1-\gamma) \ (\widehat{c}_t^i + \alpha \ s_t^i) + \gamma \ \widehat{g}_t^i \tag{26}$$

where a "$\frown$" symbol is used to denote log deviations of a variable from its steady state value, e.g. $\widehat{x}_t \equiv x_t - x$, and where $\gamma \equiv \frac{G}{Y}$ denotes the steady state government spending share.

Using (19) and the terms of trade definition, we can rewrite (26) as follows:

$$\widehat{y}_t^i = \gamma \ \widehat{g}_t^i + (1-\gamma) \ \widehat{c}_t^* - (1-\gamma) \ (p_t^i - p_t^*) \tag{27}$$

The previous equation establishes that domestic output is positively related to government spending, union-wide consumption (which is an index for the strength of foreign demand), and inversely related to domestic prices (relative to average prices in the union).

Notice that we can integrate (27) over $i \in [0,1]$ in order to obtain the union-wide goods market clearing condition:

$$\widehat{y}_t^* = \gamma \ \widehat{g}_t^* + (1-\gamma) \ \widehat{c}_t^* \tag{28}$$

where $\widehat{y}_t^* \equiv \int_0^1 \widehat{y}_t^i \ di$, and $\widehat{g}_t^* \equiv \int_0^1 \widehat{g}_t^i \ di$.

Similarly, integrating (14) over $i \equiv [0,1]$ and combining the resulting difference equation with (28), yields the following union-wide dynamic IS equation:

$$\widehat{y}_t^* = E_t\{\widehat{y}_{t+1}^*\} - (1-\gamma)(r_t^* - E_t\{\pi_{t+1}^*\} - \rho) - \gamma E_t\{\Delta \widehat{g}_{t+1}^*\} \tag{29}$$

where $\pi_t^* \equiv \int_0^1 \pi_{i,t} \ di$. We can solve the previous equation forward and, under the assumption that $\lim_{T\to\infty} E_t\{\widehat{g}_{t+T}^*\} = \lim_{T\to\infty} E_t\{\widehat{y}_{t+T}^*\} = 0$, write it in level form as:

$$\widehat{y}_t^* = \gamma \ \widehat{g}_t^* - (1-\gamma) \sum_{k=0}^{\infty} E_t\{r_{t+k}^* - \pi_{t+k+1}^* - \rho\}$$

[10] The derivation makes use of a first order Taylor expansion of $\log(Y_t^i - G_t^i)$, as shown in the Appendix. We also use the fact that in a symmetric steady state $\mathcal{S}^i = 1$ (and hence $s^i = 0$) for all $i \in [0,1]$.

Hence, we see that fluctuations in union-wide output will result from variations in union-wide government spending and expected long-term rates, with the weight attached to both factors being positively and negatively related, respectively, to the steady state share of government spending in output.

3.2 The Supply Side: Marginal Cost and Inflation Dynamics

Given our assumption of price setting à la Calvo, the dynamics of *domestic* inflation in terms of real marginal cost in each individual country are described by the difference equation

$$\pi_t^i = \beta \; E_t\{\pi_{t+1}^i\} + \lambda \; \widehat{mc}_t^i \tag{30}$$

where $\widehat{mc}_t^i = mc_t^i + \mu$ denotes the (log) deviation of real marginal cost from its steady state, and $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$.[11]

Using some of the previous results, we can further derive the following expression for marginal cost:

$$\begin{aligned} mc_t^i &= w_t^i - p_t^i - a_t^i - \log(1-\tau^i) \\ &= (w_t^i - p_{c,t}^i) + (p_{c,t}^i - p_t^i) - a_t^i - \log(1-\tau^i) \\ &= c_t^i + \varphi \; n_t^i + \alpha \; s_t^i - a_t^i - \log(1-\tau^i) - \log(1-\chi) \end{aligned} \tag{31}$$

We can now combine (31) with (22) and (26) to obtain an expression for marginal cost as a function of output and government spending, all expressed in deviations from steady state (and up to a first order approximation):

$$\widehat{mc}_t^i = \left(\frac{1}{1-\gamma} + \varphi\right) \; \widehat{y}_t^i - \frac{\gamma}{1-\gamma} \; \widehat{g}_t^i - (1+\varphi) \; a_t^i \tag{32}$$

The intuition for the negative relationship between marginal cost and government spending is easy to grasp: *given output*, an increase in government spending crowds out domestic consumption and/or generates a real appreciation, both of which tend to reduce real marginal cost through their negative effect on the product wage.[12] In addition, we see that the elasticity of real marginal cost with respect to output is increasing in the government share γ. The reason is simple: in response to a

[11] Notice that under our assumptions the fact that each individual economy is open does not affect the form of the equation relating domestic inflation to real marginal cost. See Galí and Monacelli (2005) for further discussion and a formal derivation.

[12] Notice that the corresponding elasticity is increasing in γ, since the greater the weight of government spending in aggregate demand the larger will be the percent decline in consumption needed to keep output constant.

given percent increase in output, and given an unchanged current level of current government spending $\widehat{g}_t^i$ and technology a_t^i, a larger γ is associated with a larger percent increase in consumption and/or the terms of trade. As a result, a larger increase in the product wage and, hence, marginal cost will obtain.

Combining (30) and (32) we can derive a version of the new Keynesian Phillips curve (NKPC), applying to each economy in the union:

$$\pi_t^i = \beta \ E_t\{\pi_{t+1}^i\} + \lambda \left(\frac{1}{1-\gamma} + \varphi \right) \ \widehat{y}_t^i - \frac{\lambda\gamma}{1-\gamma} \ \widehat{g}_t^i - \lambda(1+\varphi) \ a_t^i \tag{33}$$

Notice also that by integrating the previous equation over $i \equiv [0,1]$ we can obtain the corresponding new Keynesian Phillips curve for the union as a whole:

$$\pi_t^* = \beta \ E_t\{\pi_{t+1}^*\} + \lambda \left(\frac{1}{1-\gamma} + \varphi \right) \ \widehat{y}_t^* - \frac{\lambda\gamma}{1-\gamma} \ \widehat{g}_t^* - \lambda(1+\varphi) \ a_t^* \tag{34}$$

where $a_t^* \equiv \int_0^1 a_t^i \ di$.

We have now derived the set of log-linear equilibrium conditions for inflation and output in each individual country (summarized by (27), and (33)), as well as for the union as a whole (given by (29) and (34)), as a function of government spending (local and union-wide) and the common interest rate. Given the equilibrium path of those variable, one can use (14) (or, equivalently, (12)) to back out equilibrium consumption in each country.

Next we turn to the analysis of some properties of that equilibrium, before we plunge into the central question of optimal policy design. We start with a brief (but important) digression on the conditions for equilibrium determinacy in a currency union.

3.3 Equilibrium Determinacy in the Currency Union: A Digression

We start our digression by noticing that the linearized equilibrium dynamics for the currency union as a whole are analogous to those in the baseline, closed economy new Keynesian model. In particular, and given an exogenous stationary process for $\{g_t^*, a_t^*\}$, the equilibrium dynamics for union-wide inflation π_t^* and output $\widehat{y}_t^*$, are described by equations (29) and (34). In order "to close the model" those two equations should be supplemented with an additional equation describing how the central bank sets the interest rate r_t^*.

As is well known from the closed-economy literature, to the extent that the interest rate rule adopted by the central bank satisfies the so-called "Taylor principle," the equilibrium path for output and inflation is uniquely pinned down, as a function of the exogenous shocks (see Bullard and Mitra (2001), Woodford (2001)).

Consider, for the sake of concreteness, the following interest rate rule for the union's central bank:

$$r_t^* = \rho + \phi_\pi \ \pi_t^* + \phi_a \ a_t^* + \phi_g \ \widehat{g}_t^* \tag{35}$$

where ϕ_π is assumed to be non-negative. Under that specification of monetary policy, a straightforward application of the findings of Bullard and Mitra (2001) to our model, implies that the equilibrium for the union will be uniquely determined if and only if $\phi_\pi > 1$, i.e., if the central bank adjusts the short-term nominal rate more than one-for-one in response to variations in union-wide inflation π_t^*.[13] Given the equilibrium values for $\widehat{y}_t^*$, one can easily back out aggregate consumption $\widehat{c}_t^*$ (using (28)), as well as other variables of interest.

Suppose that the interest rate rule followed by the union's central bank guarantees a unique equilibrium for union-wide variables. What can we say about the uniqueness of equilibrium in each of the member countries?

Notice that for each individual country's economy one can also derive conditions analogous to the closed economy. The corresponding inflation equation is already given by (33). The corresponding dynamic IS equation can be easily derived by combining (26) with (14) and (16), which yields

$$\widehat{y}_t^i = E_t\{\widehat{y}_{t+1}^i\} - (1-\gamma)(r_t^* - E_t\{\pi_{t+1}^i\} - \rho) - \gamma E_t\{\Delta \widehat{g}_{t+1}^i\} \tag{36}$$

While the previous equations take the form of the analogous conditions for the standard closed economy model, there exists an important difference: even if the union's central bank follows an interest rule satisfying the Taylor principle, its setting of the interest rate r_t^* no longer responds systematically to domestic inflation π_t^i (or domestic output $\widehat{y}_t^i$, for that matter), since the latter has only an infinitesimal weight in aggregate inflation π_t^* (or in $\widehat{y}_t^*$, in the case of output).[14] Hence, a straightforward application of the Taylor principle logic seems to imply that equilibrium should indeed

[13] In Section 4 we show that the optimal monetary policy in the currency union can be implemented by a policy of this form, with a specific choice of coefficients ϕ_a and ϕ_g, but an arbitrary $\phi_\pi > 1$.

[14] Even if fundamental shocks are highly correlated across countries (thus allowing for potentially high correlation between π_t^i and π_t^*), it is still the case that r_t^* will not respond to an eventual change in π_t^i that is driven by revisions in expectations unrelated to economic fundamentals, the source of potential indeterminacy.

be indeterminate in each individual economy, since from the latter's point of view r_t^* can be viewed as exogenous. That logic, nevertheless, is incorrect, for it fails to take into account an additional condition -given by equation (27)- that must be satisfied in equilibrium for each individual economy, and which we repeat here for convenience:

$$\widehat{y}_t^i = \gamma \ \widehat{g}_t^i + (1-\gamma)\widehat{c}_t^* - (1-\gamma)(p_t^i - p_t^*)$$

That condition establishes a link between the *levels* of domestic output and domestic prices (with the remaining variables being exogenous) which is absent in the closed economy benchmark model. That link is a consequence of the effects of the terms of trade on demand, combined with the one-to-one mapping between domestic prices and the terms of trade (given p_t^*) which results from the assumption of a common currency (and, hence, a constant exchange rate). Roughly speaking, the adoption of a hard peg against a currency of a country (or a currency union) whose price level is uniquely pinned down by its own monetary regime, acts as a substitute for the adoption of an autonomous monetary policy satisfying the Taylor principle.

The previous point can be demonstrated quite easily. First note that we can combine (27) and (28) to yield:

$$\widehat{y}_t^i - \widehat{y}_t^* = \gamma \ (\widehat{g}_t^i - \widehat{g}_t^*) - (1-\gamma)(p_t^i - p_t^*) \tag{37}$$

Second, subtracting (34) from (33) and combining the resulting expression with (37) allows us to derive, after some straightforward algebra, the following difference equation for country i's terms of trade, $s_t^i \equiv p_t^* - p_t^i$,

$$s_t^i = \omega \ s_{t-1}^i + \omega\beta \ E_t\{s_{t+1}^i\} + \omega \ u_t^i \tag{38}$$

where $\omega \equiv \frac{1}{1+\beta+\lambda[1+\varphi(1-\gamma)]} \in [0, \frac{1}{1+\beta})$ and $u_t^i = -\lambda\varphi\gamma(\widehat{g}_t^i - \widehat{g}_t^*) + \lambda(1+\varphi)(a_t^i - a_t^*)$

The above difference equation (38) has a unique stationary solution, of the form:

$$s_t^i = \delta \ s_{t-1}^i + \delta \sum_{k=0}^{\infty} (\beta\delta)^k E_t\{u_{t+k}^i\} \tag{39}$$

where $\delta \equiv \frac{1-\sqrt{1-4\beta\omega^2}}{2\omega\beta} \in (0,1)$.

Given the equilibrium path for the terms of trade $\{s_t^i\}$, determined by (39), we can back out the equilibrium levels of domestic prices and output using the definition of the terms of trade and (37).

3.4 The Effects of Domestic Government Spending Shocks

While not the focus of the present paper it is useful to consider the effects of an exogenous change in government spending, in order to understand the mechanisms that may make it a useful policy tool in the absence of an autonomous monetary policy.

For concreteness, let us assume that government spending follows a exogenous AR(1) process

$$\widehat{g}_t^i = \rho_g \ \widehat{g}_{t-1}^i + \varepsilon_{g,t}^i$$

Without loss of generality, we assume that the union's economy is in a perfect foresight, zero inflation steady state and we set $p_t^* = \widehat{g}_t^* = a_t^* = a_t^i = 0$ for all t. Under the previous assumptions, we have $u_t = -\lambda\varphi\gamma \ \widehat{g}_t^i$. It then follows from (39) that

$$p_t^i = \delta \ p_{t-1}^i + \psi_g \ \widehat{g}_t^i \tag{40}$$

where $\psi_g \equiv \frac{\delta\lambda\varphi\gamma}{1-\beta\delta\rho_g} > 0$.

Hence, a positive shock to domestic government spending leads to a persistent *rise* in the domestic price level, though the latter eventually returns to its original level (given stationarity of $\widehat{g}_t^i$). Equivalently, domestic inflation initially increases, but eventually turns negative. How long inflation remains positive after the shock will depend on both δ and ρ_g.

Given the response of domestic prices $\{dp_{+k}^i\}_{k=0}^{\infty}$, the effect of on domestic output on impact and over time can be derived from equation (27):

$$d\widehat{y}_{+k}^i = \gamma \ \rho_g^k - (1-\gamma) \ dp_{+k}^i$$

Notice that the effect on impact is given by

$$d\widehat{y}_{+0}^i = \gamma - (1-\gamma) \ \psi_g$$

As prices approach the "full stickiness" limit ($\lambda \rightarrow 0$, $\delta \rightarrow 1$, $\psi_g \rightarrow 0$) we have $d\widehat{y}_{+0}^i = \gamma$, in other words, output increases one for one with government spending, since there is no crowding out effect resulting from higher domestic prices (we have a unit "level" multiplier). When prices are not completely sticky ($\lambda > 0$) the price level rises, thus dampening the direct effect of government spending on output. Hence, $d\widehat{y}_{+0}^i < \gamma$, with the "level" multiplier being less than one. That crowding out effect will be larger the more persistent is the shock (the higher ρ_g), since $\frac{\partial \psi_g}{\partial \rho_g} > 0$. [15]

Notice that the sign and qualitative pattern of the economy's response to a change in government spending are unambiguous when the country belongs to a currency union, as in the analysis above. This is in contrast to a closed economy or an open economy with autonomous monetary policy, since in those cases the effects of a fiscal shock depend on the endogenous response of monetary policy to the fiscal intervention.[16]

Figure 1 displays the effects on output, the output gap, the domestic price level and inflation of a one percent rise in government spending for alternative values of the price stickiness parameter ϑ. The output gap is defined here as the deviation of output from its level under fully flexible prices (and given by equation (32) for $\widehat{mc}_t^i = 0$). Hence we see that a rise in government spending leads to a terms of trade appreciation (rise in the price level) and a rise in output. The latter effect is stronger when prices are more rigid. When prices are flexible (and the output gap is by definition zero), the effect on output is dampened but never to such an extent that the output multiplier turns negative.

4 Optimal Fiscal and Monetary Policy Design

Next we derive and characterize the optimal fiscal-monetary regime in the currency union. The institutional constraints are as follows. Monetary policy is conducted in a centralized fashion by a common central bank, which sets the short-term nominal rate r_t^*. Fiscal policy is conducted by each country's fiscal authority, which determines the steady state level of government spending G^i, a constant employment subsidy τ^i and–most importantly, given our focus–follows a rule describing short term variations in government spending $\{\widehat{g}_t^i\}$ in response to shocks of different nature.

We seek to derive the monetary and fiscal policy rules that maximize the welfare of the union as a whole, given those assumed institutional constraints. We start by analyzing the social planner's problem. Then we show under what conditions the efficient allocation can be supported as an equilibrium, under the assumption of

[15] That crowding effect is never strong enough to generate a negative response of output. In fact, under flexible prices (constant markup), equation (32) reduces to

$$y_t^i = \Omega \; g_t^i$$

where $\Omega \equiv \frac{\gamma}{1+\varphi(1-\gamma)} > 0$ and $\frac{\partial \Omega}{\partial \gamma} > 0$.

[16] See, for instance, Linnemann and Schabert (2003).

flexible prices. Finally, we derive the optimal (second-best) policies in the presence of nominal rigidities.

4.1 The Social Planner's Problem

The union's optimal allocation in any given period can be described as the solution to the following social planner's problem:

$$\max \int_0^1 U(C_t^i, N_t^i, G_t^i)\ di$$

subject to the technological and resource constraints

$$Y_t^i = A_t^i\ N_t^i$$

$$Y_t^i = C_{i,t}^i + \int_0^1 C_{i,t}^f\ df + G_t^i \tag{41}$$

for all $i \in [0,1]$. Notice that the previous constraints already embed the optimal condition whereby the different good types in any given country should be produced and consumed in identical quantities.[17]

Under our specification of preferences, the optimality conditions for the social planner's problem are:

$$\frac{(N_t^i)^\varphi}{A_t^i} = \frac{(1-\chi)(1-\alpha)}{C_{i,t}^i} = \int_0^1 \frac{(1-\chi)\alpha}{C_{i,t}^f}\ df = \frac{\chi}{G_t^i}$$

for all $i \in [0,1]$. In words, the marginal loss of utility for a household in country i of producing an additional unit of the composite good, given by $(N_t^i)^\varphi / A_t^i$, must be equal, at the margin, to the utility gain resulting from any of the three possible uses of that additional output: consumption by domestic households, consumption by all households in the union, and domestic government spending.

Using the resource constraint (41), and the fact that $Y_t^i = A_t^i N_t^i$, we can guess and verify that the solution to the social planner's problem is given by:

$$N_t^i = 1 \tag{42}$$

$$Y_t^i = A_t^i \tag{43}$$

$$C_{i,t}^i = (1-\chi)(1-\alpha)\ A_t^i \tag{44}$$

[17] That condition in turn implies that $Z_t^i = 1$ in (21), for all $i \in [0,1]$

$$C_{i,t}^{f} = (1-\chi)\alpha \ A_t^i \tag{45}$$

$$G_t^i = \chi \ A_t^i \tag{46}$$

for all $i, f \in [0,1]$, and all t.

Combining (44) and (45), together with definition of country i's total consumption index (2), we can derive an expression for the latter under the optimal allocation (in logs):

$$c_t^i = (1-\alpha) \ a_t^i + \alpha \int_0^1 a_t^f \ df + \log(1-\chi)$$

or, in levels,

$$C_t^i = (1-\chi) \ (A_t^i)^{1-\alpha} (A_t^*)^{\alpha}$$

where $A_t^* \equiv \exp \int_0^1 a_t^f \ df$ is an index of union-wide productivity.

Aggregating over countries, we obtain the corresponding optimal allocation for the union as a whole:

$$Y_t^* = A_t^*$$

$$C_t^* = (1-\chi) \ A_t^*$$

$$G_t^* = \chi \ A_t^*$$

4.2 Decentralization of the Efficient Allocation under Flexible Prices

Before we turn to the interesting case of optimal policy in the presence of nominal rigidities, it is useful to examine the case of flexible prices, since it constitutes a useful benchmark as shown below.

We start by showing how, under certain conditions, the union-wide optimal allocation derived above can be supported as an equilibrium in the presence of *flexible prices*. Letting variables with an upper bar denote their values in a flexible price equilibrium we have

$$\begin{aligned} 1-\frac{1}{\epsilon} &= \overline{MC}_t^i \\ &= \frac{(1-\tau^i)}{A_t^i(1-\chi)} \ \overline{C}_t^i \ (\overline{N}_t^i)^{\varphi} \ (\overline{S}_t^i)^{\alpha} \\ &= \frac{(1-\tau^i)}{A_t^i(1-\chi)} \ \overline{C}_t^i \ (\overline{N}_t^i)^{\varphi} \ \frac{\overline{Y}_t^i - \overline{G}_t^i}{\overline{C}_t^i} \\ &= \frac{1-\tau^i}{1-\chi} \ (1-(\overline{G}_t^i/\overline{Y}_t^i)) \ (\overline{N}_t^i)^{1+\varphi} \end{aligned}$$

In order for the equilibrium allocation under flexible prices to correspond to the union's socially optimal allocation the following conditions must be satisfied for all $i \in [0,1]$ and t. First, the subsidy τ^i must be set at a level

$$\overline{\tau}^i = \frac{1}{\epsilon} \tag{47}$$

Secondly, government spending must be set according to the rule[18]

$$\overline{G}_t^i = \chi \; A_t^i \tag{48}$$

If both conditions are satisfied for all $i \in [0,1]$, the flexible price equilibrium will yield the level of employment and output in each country that is optimal from the union's perspective, i.e., $\overline{Y}_t^i = A_t^i$ and $\overline{N}_t^i = 1$, for all $i \in [0,1]$, and all t.[19] It is easy to check that the remaining optimality conditions will also be satisfied as a result of households' optimization.

Notice that in the economy with flexible prices, the lack of an autonomous monetary policy is of no consequence for the attainment of the optimal allocation, for monetary policy is neutral in that environment (it can only influence the path of prices). As a result, local fiscal authorities can focus exclusively on the efficient provision of public consumption goods, according to rule (48) (shadowing the central planner's decisions on that front). In our example economy that rule implies a constant government spending share $\overline{G}_t^i / \overline{Y}_t^i = \gamma = \chi$ for all t.

While the level of prices in the union and in each individual country is determined by the monetary policy regime, each country's terms of trade as well as the inflation differentials vis a vis the union are fully determined by real factors in the present scenario. More specifically, note that the path for the terms of trade that will support the efficient allocation is given by:

$$\overline{S}_t^i = (\overline{C}_t^i / \overline{C}_t^*)^{\frac{1}{1-\alpha}} = A_t^i / A_t^* \tag{49}$$

for all $i \in [0,1]$, and all t. Given the definition of the terms of trade it follows that the inflation differential will be inversely proportional to the productivity growth

[18] Or, equivalently, $\overline{G}_t^i = \chi \; \overline{Y}_t^i$

[19] In contrast with Galí and Monacelli (2005), where the optimal allocation problem is analyzed from the viewpoint of a small open economy, here the choice of the subsidy is not affected by any desire to influence the terms of trade in a country's favor. The reason is simple: that goal cannot be attained by all countries simultaneously, and hence it serves no purpose when trying to decentralize the solution to the union's social planner problem. As a result the only role played by the subsidy is to offset firms' market power.

differential:

$$\pi_t^i - \pi_t^* = -(\Delta a_t^i - \Delta a_t^*)$$

In the following section we assume that (47) is satisfied, so that the only remaining non-offset distortion is the presence of nominal rigidities. Our aim is to determine the optimal design of policy in such an environment, when there is a single monetary policy but decentralized fiscal policies.

4.3 Optimal Policy Design in the Presence of Nominal Rigidities

In the presence of nominal rigidities it will generally be impossible for a monetary union to attain the optimal allocation. The reasons are well understood. First, staggered price setting implies that the level of employment and output within each country may differ from the efficient one at any point in time, both in the aggregate and across sectors (i.e., good types). This will be true even if the distortion associated with market power is offset by means of a subsidy, as discussed above. Secondly, the sluggish adjustment of prices, combined with the impossibility of nominal exchange rate adjustments (inherent to a currency union), implies that the changes in terms of trade that would be required to support the optimal allocation cannot occur instantaneously.

As shown in Galí and Monacelli (2005) in the context of a related model, when each individual country has its own currency and an autonomous monetary policy (as opposed to the monetary union case considered here), a monetary policy that succeeds in stabilizing the domestic price level in each country would replicate the flexible price equilibrium and, hence, the optimal allocation. Under a currency union, however, and to the extent that different countries experience asymmetric shocks leading to discrepancies among their natural interest rates, the lack of a country-specific monetary policy (i.e., an independent interest rate setting) makes it impossible to attain that outcome. As a result, the union as a whole will experience some deviations from the optimal allocation and, accordingly, some welfare losses. What is the monetary/fiscal policy mix that will minimize those losses? That is the question we address next.

4.4 Union Members' Tradeoffs

Let $\overline{y}_t^i = a_t^i$ and $\overline{g}_t^i = \log \chi + a_t^i$ denote the (logs) of output and government spending in country i associated with the union-wide efficient allocation (or equivalently, with the flexible price equilibrium under an optimal policy). We use the notation $\widetilde{y}_t^i$ and $\widetilde{g}_t^i$ to denote the log deviations of country i's output and government spending from those benchmark levels, i.e., $\widetilde{y}_t^i \equiv y_t^i - \overline{y}_t^i$ and $\widetilde{g}_t^i \equiv g_t^i - \overline{g}_t^i$, which we henceforth refer to as country i's *output gap* and *government spending gap*, respectively.

It will prove convenient to define the following measure of the fiscal stance:

$$\begin{aligned} \widetilde{f}_t^i &\equiv \widetilde{g}_t^i - \widetilde{y}_t^i \\ &= (g_t^i - y_t^i) - \log \chi \end{aligned}$$

which we henceforth refer as the *fiscal gap*.[20]

Using (32), together with the fact that $\overline{y}_t^i - y^i = \overline{g}_t^i - g^i = a_t^i$ (where variables without time subscripts denote steady state values), we can derive the following relationship between the real marginal cost, and the output and fiscal gaps:

$$\begin{aligned} \widehat{mc}_t^i &= \left(\frac{1}{1-\chi} + \varphi \right) \widetilde{y}_t^i - \frac{\chi}{1-\chi} \widetilde{g}_t^i \\ &= (1+\varphi)\ \widetilde{y}_t^i - \frac{\chi}{1-\chi} \widetilde{f}_t^i \end{aligned}$$

where we have imposed an optimal steady state government spending share ($\gamma = \chi$).

We can combine the previous expression with (30) to obtain a version of the new Keynesian Phillips curve for each union member, expressing domestic inflation in terms of the corresponding output and fiscal gaps:

$$\pi_t^i = \beta\ E_t\{\pi_{t+1}^i\} + \lambda\,(1+\varphi)\ \widetilde{y}_t^i - \frac{\lambda\chi}{1-\chi} \widetilde{f}_t^i \tag{50}$$

In addition we can combine (27), (28) and (49), to obtain an equation determining the change in the output gap differential as a function of the differentials in fiscal gap changes, inflation and productivity growth:

$$\Delta\widetilde{y}_t^i - \Delta\widetilde{y}_t^* = \frac{\chi}{1-\chi}(\Delta\widetilde{f}_t^i - \Delta\widetilde{f}_t^*) - [(\pi_t^i - \pi_t^*) + (\Delta a_t^i - \Delta a_t^*)] \tag{51}$$

The previous two equations describe the evolution of country i's output gap and price level as a function of the domestic fiscal gap, given the productivity differential

[20] Strictly speaking, $\overline{g}_t^i$ and, hence, $\widetilde{g}_t^i$ are only well defined if $\chi > 0$, which we assume for the remainder of this section.

and the union wide fiscal and output gaps. They also make clear the nature of the tradeoffs facing policymakers in the union. To illustrate those tradeoffs, assume that $\widetilde{y}_t^* = \widetilde{f}_t^* = p_t^* = 0$. Consider equation (50), describing the evolution of the price level in country i. As in the familiar closed economy benchmark, that equation implies that prices could be fully stabilized by closing the output and fiscal gaps at all times, thus trying to replicate the flexible price equilibrium allocation. Yet, (51) makes clear that this will not be feasible in the presence of asymmetric productivity shocks since, in that case, closing the output gap (without creating a fiscal gap) requires that the terms of trade and, hence, domestic prices, adjust.

4.5 Union-Wide Tradeoffs

The evolution of inflation, the output gap, and the fiscal gap for the currency union is described by two aggregate equilibrium relationships familiar from the closed economy case. Thus, by integrating (50), we can derive an equation describing union-wide inflation in terms of the corresponding gaps:

$$\pi_t^* = \beta \; E_t\{\pi_{t+1}^*\} + \lambda\,(1+\varphi)\;\widetilde{y}_t^* - \frac{\lambda\chi}{1-\chi}\;\widetilde{f}_t^* \tag{52}$$

The union's output gap is determined by a dynamic IS-type equation, which we can derive using (29):

$$\begin{aligned} \widetilde{y}_t^* &= E_t\{\widetilde{y}_{t+1}^*\} - (1-\chi)(r_t^* - E_t\{\pi_{t+1}^*\} - \overline{rr}_t^*) - \chi E_t\{\Delta\widetilde{g}_{t+1}^*\} \\ &= \frac{\chi}{1-\chi}\widetilde{f}_t^* - (r_t^* - E_t\{\pi_{t+1}^*\} - \overline{rr}_t^*) + E_t\{\widetilde{y}_{t+1}^*\} - \frac{\chi}{1-\chi}E_t\{\widetilde{f}_{t+1}^*\} \end{aligned} \tag{53}$$

where $\overline{rr}_t^*$ is the union's natural rate of interest, given by

$$\begin{aligned} \overline{rr}_t^* &= \rho + (1-\chi)^{-1}(E_t\{\Delta\overline{y}_{t+1}^*\} - \chi E_t\{\Delta\overline{g}_{t+1}^*\}) \\ &= \rho + E_t\{\Delta\overline{y}_{t+1}^*\} \\ &= \rho + E_t\{\Delta\overline{a}_{t+1}^*\} \end{aligned}$$

Notice that, to the extent that the union's aggregate fiscal gap $\widetilde{f}_t^*$ remains stable at zero, there is no tradeoff between stabilization of the output gap and inflation for the union as a whole. In that case the outcome $\widetilde{y}_t^* = \widetilde{\pi}_t^* = 0$ could be easily attained by having the central bank follow a rule of the sort

$$r_t = \overline{rr}_t^* + \phi_\pi\;\pi_t^*$$

On the other hand, if the *aggregated* decisions of the local fiscal authorities lead to fluctuations in the union-wide fiscal gap, the job of the single central bank is made considerably more difficult. To illustrate this formally, notice that we can integrate (53) and combine it with (52) to yield:

$$\begin{aligned}
\pi_t^* &= \beta \; E_t\{\pi_{t+1}^*\} + \frac{\lambda\varphi\chi}{1-\chi} \; \widetilde{f}_t^* - \sum_{k=0}^{\infty} E_t(r_{t+k}^* - \pi_{t+k+1}^* - \overline{rr}_{t+k}^*) \\
&= \frac{\lambda\varphi\chi}{1-\chi} \; \sum_{k=0}^{\infty} \beta^k \; E_t\{\widetilde{f}_{t+k}^*\} - (1-\beta)\sum_{k=0}^{\infty} \beta^k \; E_t\{(r_{t+k}^* - \pi_{t+k+1}^* - \overline{rr}_{t+k}^*)\}
\end{aligned}$$

Notice that a positive union-wide fiscal gap, current and/or anticipated, will generate upward pressure on current inflation. That pressure can only be partly offset by having the central bank run a tighter monetary policy, which would require raising current and/or future interest rates above their natural level, thus dampening the expansionary impact of members' fiscal policies on the union's output gap and inflation. Below we show that this is indeed the sort of rule that the union's central bank should adopt, as part of the optimal monetary-fiscal policy mix for the union.

4.6 The Optimal Policy Problem

What is the path for the fiscal gap, for each country and in the aggregate, that is consistent with maximization of the union's welfare? What are the resulting optimal output gap and price level paths for the union consistent with that optimal choice?

In the Appendix we show that a second order approximation to the *sum* of utilities of union households about an efficient steady state takes the form:

$$\mathbb{W} \simeq -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \; \int_0^1 \left(\frac{\epsilon}{\lambda} \; (\pi_t^i)^2 + (1+\varphi) \; (\widetilde{y}_t^i)^2 + \frac{\chi}{1-\chi} (\widetilde{f}_t^i)^2 \right) \; di + tips \tag{54}$$

where *tips* denotes terms that are independent of policy.

We define the optimal policy mix for the currency union as the set of rules for the fiscal gaps $\{\widetilde{f}_t^i\}$ for all $i \in [0,1]$ and the common interest rate $\{r_t^*\}$, along with with the associated second best outcomes π_t^i, $\widetilde{y}_t^i$ for all t, that maximize (54), subject to (50), (51), and the "aggregation" constraints

$$\pi_t^* = \int_0^1 \pi_t^i \; di \quad ; \quad \widetilde{y}_t^* = \int_0^1 \widetilde{y}_t^i \; di \quad ; \quad \widetilde{f}_t^* = \int_0^1 \widetilde{f}_t^i \; di \tag{55}$$

The optimal policy problem can be solved in two stages. First, we determine the processes $\{\pi_t^i, \widetilde{y}_t^i, \widetilde{f}_t^i\}$, for all $i \in [0,1]$, that maximize (54) subject to (50), (51) and (55). Secondly, given the solution to that first-stage problem, we determine the interest rate rule that will support the implied paths for the union-wide inflation, output gap and fiscal gap, using (53).

The optimality conditions associated with the first-stage problem are given by:

$$\frac{\epsilon}{\lambda}\pi_t^i + \Delta\psi_{\pi,t}^i + \psi_{y,t}^i - \psi_{\pi,t}^* = 0 \tag{56}$$

$$(1+\varphi)\,\widetilde{y}_t^i - \lambda(1+\varphi)\psi_{\pi,t}^i + (1-\beta L^{-1})\psi_{y,t}^i - \psi_{y,t}^* = 0 \tag{57}$$

$$\frac{\chi}{1-\chi}\widetilde{f}_t^i + \frac{\lambda\chi}{1-\chi}\psi_{\pi,t}^i - \frac{\chi}{1-\chi}(1-\beta L^{-1})\psi_{y,t}^i - \psi_{f,t}^* = 0 \tag{58}$$

$$-\int_0^1 \psi_{y,t}^i di + \psi_{\pi,t}^* = 0 \tag{59}$$

$$-(1-\beta L^{-1})\int_0^1 \psi_{y,t}^i di + \psi_{y,t}^* = 0 \tag{60}$$

$$\frac{\chi}{1-\chi}(1-\beta L^{-1})\int_0^1 \psi_{y,t}^i di + \psi_{f,t}^* = 0 \tag{61}$$

for all $i \in [0,1]$ and $t = 0,1,2,...$, where $\{\psi_{\pi,t}^i, \psi_{y,t}^i\}$, $\psi_{\pi,t}^*$, $\psi_{y,t}^*$, and $\psi_{f,t}^*$ are the (discounted) Lagrange multipliers associated with constraints in (50), (51) and (55), and $\psi_{\pi,-1}^i = 0$.

Integrating (56) over $i \in [0,1]$, combining the resulting equation with (59), we obtain:

$$\frac{\epsilon}{\lambda}\pi_t^* + \int_0^1 \Delta\psi_{\pi,t}^i di = 0$$

Similarly, integrating (57) over $i \in [0,1]$, combining the resulting equation with (60), we obtain:

$$\widetilde{y}_t^* - \lambda\int_0^1 \psi_{\pi,t}^i di = 0$$

Both can be combined to yield

$$\epsilon\,\pi_t^* + \Delta\widetilde{y}_t^* = 0 \tag{62}$$

for $t = 0,1,2,...$

Integrating (58) over $i \in [0,1]$, combining the resulting equation with (61) and the result above, we obtain:

$$\widetilde{f}_t^* = -\widetilde{y}_t^* \tag{63}$$

Notice that (62) and (63), together with the union-wide equilibrium conditions (52) and (53), imply that the equilibrium under the optimal policy will satisfy

$$\pi_t^* = \widetilde{y}_t^* = \widetilde{f}_t^* = 0 \tag{64}$$

for all t. This is one of the central results emerging from our analysis. In words, we can state it as follows: the combined monetary-fiscal policy mix must be such that, at the union level, inflation, the output gap and the fiscal gap remain at a constant (zero) value, at all times. That condition requires, in turn, that the equilibrium interest rate r_t^* equals the union-wide natural rate $\overline{rr}_t^*$ at all times. As argued above, and *conditional* on $\widetilde{f}_t^* = 0$ for all t, the union's central bank can implement the desired outcome by adopting a policy rule of the form:

$$r_t = \overline{rr}_t^* + \phi_\pi \ \pi_t^*$$

What are the paths of inflation and the output gap for each union member associated with the optimal policy? What fiscal policy will support those paths?

Combining (57) and (58), and noticing that (60) and (61) imply $\frac{\chi}{1-\chi}\psi_{y,t}^* + \psi_{f,t}^* = 0$, we obtain:

$$(1+\varphi)\ \widetilde{y}_t^i + \widetilde{f}_t^i = \lambda\varphi\ \psi_{\pi,t}^i \tag{65}$$

In this second best environment, as long as prices are less than fully flexible, we have $\psi_{\pi,t}^i > 0$. Hence (65) immediately implies that, unlike the union-wide policy prescription (64), setting $\widetilde{f}_t^i = \widetilde{y}_t^i = 0$ for each member country i cannot be an equilibrium under the optimal policy.

To fully characterize the equilibrium dynamics, we notice that the aggregate multiplier $\psi_{\pi,t}^* = \int_0^1 \psi_{y,t}^i di$ (from (59)) must evolve exogenously from the viewpoint of the single member country. By substituting (59), (60) and (61) into (56), (57) and (58), we define a rational expectations equilibrium under commitment in country i as an allocation for $\left\{\pi_t^i,\, \widetilde{y}_t^i,\, \widetilde{f}_t^i,\, \psi_{\pi,t}^i,\, \psi_{y,t}^i\right\}$ that satisfies (50), (51), (56), (57), (58), for any given $\left\{\psi_{\pi,t}^*\right\}$ and stochastic processes $\{a_t^i,\, a_t^*\}$, along with the initial condition $\psi_{\pi,-1}^i = 0$. Next we illustrate the implied equilibrium dynamics and the optimal policy responses by means of some simulations.

4.6.1 Dynamic Simulations

In this section we illustrate the equilibrium behavior of the prototype member economy under the commitment policy described above. We resort to a series of dynamic simulations, and adopt the following benchmark parameterization. We assume $\varphi = 3$, which implies a labor supply elasticity of $\frac{1}{3}$. We assume a steady-state markup $\mu = 1.2$, which implies that ϵ, the elasticity of substitution between differentiated goods (of the same origin), is 6. Parameter θ is set to a benchmark value of 0.75 (a value consistent with an average period of one year between price adjustments), although below we report results of sensitivity analysis on this parameter. We assume $\beta = 0.99$, which implies a riskless annual return of about 4 percent in the steady state. As for the fiscal sector, we parameterize the steady state share of government spending in output as $\gamma = \chi = 0.25$, roughly the average of government final consumption for the euro zone.

We follow the real business cycle literature (King and Rebelo (1999)) and assume the following autoregressive process for labor productivity in country i:

$$a_t^i = \ 0.95 \ a_{t-1}^i \ + \ \varepsilon_{i,t}^a$$

Figure 2 displays impulse responses to a one percent (asymmetric) rise in productivity in the domestic economy for alternative values of the price stickiness parameter θ. In particular, $\theta = 0$ represents the limiting case of full (domestic) price flexibility. The figure is representative of the main result of the paper.

Consider first the case of full price flexibility ($\theta = 0$). In that case there is no loss of efficiency associated with inflation, since the latter no longer creates any relative price distortions. Hence, as shown in the figure, it is optimal to fully close the fiscal gap and the output gap, in response to asymmetric movements in productivity.[21]. As a result, it is optimal to have the union member fully absorb the rise in productivity through an adjustment in the terms of trade brought about by a change in the domestic price level, while maintaining output and government spending at their first-best levels.

To the extent that price stickiness is present ($\theta > 0$), there are welfare losses associated with departures from price stability, in addition to those stemming from nonzero output and fiscal gaps. However -as discussed above- the flexible price/efficient allocation is not feasible under the currency union regime. In particular, the rise in productivity must be absorbed only via a gradual and persistent fall in the price level, with the consequent relative price distortions. As a result, the optimal policy mix requires expanding the fiscal gap to bring about the rise in demand necessary to accommodate the desired expansion in output, thus smoothing the adjustment

[21] In fact, under price flexibility, equation (50) does not act as a constraint on the evolution of domestic prices. Hence, optimal policy in this case must satisfy equation (65) with $\psi_{\pi,t}^i = 0$.

of prices over time. To see that formally, notice that in the equilibrium under the optimal policy equation (51) simplifies to:

$$\widetilde{y}_t^i - \frac{\chi}{1-\chi}\widetilde{f}_t^i + p_t^i = -\widetilde{a}_t^i$$

where $\widetilde{a}_t^i \equiv a_t^i - a_t^*$ (and where, without loss of generality, we have normalized $p_t^* = 0$). Hence, to the extent that the price level reacts gradually, the rise in productivity will be absorbed via a combination of a fall in the output gap and a rise in the fiscal gap. In general, the local fiscal authority is required to trade-off movements in inflation on the one hand with movements in the output and fiscal gap on the other. The higher the degree of price rigidity, the larger the implied fluctuations of both gaps under the optimal policy.

Notice that, under our benchmark parameterization, welfare losses from any given output gap variation are of an order of magnitude larger than the ones implied by the same variation in the fiscal gap. This explains why in Figure 2 the implied volatility of the fiscal gap is larger than the one in the output gap. The optimal balance between the two variables will in general depend on the relative weights attached to the quadratic terms in $\widetilde{y}_t^i$ and $\widetilde{f}_t^i$ in the welfare loss function (54). These weights depend in turn on parameters φ and χ. The lower the elasticity of labor supply (i.e., the larger φ) the smaller the adjustment in the output gap (relative to the fiscal gap), whereas the larger χ (the share of government spending in the optimal steady state) the lower the adjustment brought about via the fiscal gap (relative to the output gap).

5 Conclusions

We have developed a tractable multicountry framework suitable for monetary and fiscal policy analysis in a currency union. As an application of the model, we have determined the optimal monetary-fiscal policy mix in the presence of idiosyncratic shocks to productivity. Given our assumed nominal rigidities, the presence of those shocks, combined with the impossibility of resorting to nominal exchange rate adjustments, induces an inefficient response of the terms of trade that generates room for fiscal policy as stabilization tool. In particular, the union-wide optimal policy calls for variations in local government spending that go beyond the mere efficient provision of public goods. On the other hand, our findings suggest that the union's central bank should stick to a policy that aims to stabilize the price level in the union as a whole, thus resisting any temptation to accommodate the inflationary pressures that may arise from the aggregation of local fiscal policies.

Our framework calls for extensions on a series of grounds. In order to meet our self-imposed tractability requirement, we restrict ourselves to less-than-general parametric specifications for utility and technology, and ignore capital accumulation. Furthermore, our model ignores other aspects that are likely to be relevant for the design of optimal policies. Missing elements include, among others, the presence of sticky wages (along with sticky prices), the need to rely on distortionary taxes, the effects of government debt policies, and the likely existence of non-fully Ricardian behavior on the part of households. Finally, our framework assumes the presence of complete international financial markets. By relaxing the assumption of perfect risk-sharing, we could generate a complementary role for fiscal policy as a cross-country insurance tool. The emergence of a potential conflict between the latter and the stabilization role described in the present paper is likely to constitute an interesting line worth exploring in future research. We plan to pursue some of those extensions in future work.

Appendix

For notational simplicity we omit country subscripts, unless needed.

Taylor expansion of $\log(Y_t - G_t)$

Let $\gamma \equiv \frac{G}{Y}$ the steady state government spending share. Define $\widehat{y}_t \equiv \log \frac{Y_t}{Y}$ and $\widehat{g}_t \equiv \log \frac{G_t}{G}$. A second-order Taylor expansion of $\log(Y_t - G_t)$ about the steady state yields:

$$
\begin{aligned}
\log(Y_t - G_t) &= \log((1-\gamma)Y) + \frac{1}{1-\gamma}\left(\frac{Y_t - Y}{Y}\right) - \frac{\gamma}{1-\gamma}\left(\frac{G_t - G}{G}\right) \\
&\quad -\frac{1}{2}\frac{1}{(1-\gamma)^2}\left(\left(\frac{Y_t - Y}{Y}\right)^2 + \gamma\left(\frac{G_t - G}{G}\right)^2 - 2\gamma\left(\frac{Y_t - Y}{Y}\right)\left(\frac{G_t - G}{G}\right)\right) \\
&= \log((1-\gamma)Y) + \frac{1}{1-\gamma}\left(\widehat{y}_t - \gamma\, \widehat{g}_t\right) \\
&\quad +\frac{1}{2}\frac{1}{1-\gamma}\left(\widehat{y}_t^2 - \gamma\, \widehat{g}_t^2\right) - \frac{1}{2}\frac{1}{(1-\gamma)^2}\left(\widehat{y}_t - \gamma \widehat{g}_t\right)^2 \\
&= \log((1-\gamma)Y) + \frac{1}{1-\gamma}(\widehat{y}_t - \gamma\, \widehat{g}_t) - \frac{1}{2}\frac{\gamma}{(1-\gamma)^2}\left(\widehat{g}_t - \widehat{y}_t\right)^2
\end{aligned}
$$

Let $\widetilde{y}_t = y_t - \overline{y}_t$ and $\widetilde{g}_t = g_t - \overline{g}_t$ denote the output and fiscal gaps, respectively, as defined in the text. Note that= $\widehat{y}_t = \widetilde{y}_t + (\overline{y}_t - y)$ and $\widehat{g}_t = \widetilde{g}_t + (\overline{g}_t - g)$. Hence, $\widehat{g}_t - \widehat{y}_t = \widetilde{g}_t - \widetilde{y}_t + (\overline{g}_t - \overline{y}_t) - \log\gamma$.

Quite generally, $\overline{g}_t$ and $\overline{y}_t$ will depend on exogenous shocks only. In the present model, $\overline{g}_t - \overline{y}_t = \log\chi$. Thus, when considering fluctuations about the efficient steady state (with $\gamma = \chi$) we have $\widehat{g}_t - \widehat{y}_t = \widetilde{g}_t - \widetilde{y}_t$, allowing to write:

$$
\log(Y_t - G_t) \simeq \frac{1}{1-\chi}\left(\widetilde{y}_t - \chi\, \widetilde{g}_t\right) - \frac{1}{2}\frac{\chi}{(1-\chi)^2}\left(\widetilde{g}_t - \widetilde{y}_t\right)^2 + tips
$$

Taylor expansion of $\int_0^1 \log C_t^i \, di$

From (25) in the text we have:

$$
\log C_t^i = c_t^i = \log(Y_t^i - G_t^i) - \alpha s_t^i
$$

Using the fact that $\int_0^1 s_t^i \; di = 0$ and assuming a common (optimal) steady state in all countries we have:

$$\begin{aligned}\int_0^1 \log C_t^i \; di &= \int_0^1 \log(Y_t^i - G_t^i) \; di \\ &\simeq \frac{1}{1-\chi} \int_0^1 \left(\widetilde{y}_t^i - \chi \; \widetilde{g}_t^i\right) \; di - \frac{1}{2} \frac{\chi}{(1-\chi)^2} \int_0^1 (\widetilde{y}_t^i - \widetilde{g}_t^i)^2 \; di\end{aligned}$$

Taylor expansion of $\frac{N_t^{1+\varphi}}{1+\varphi}$

A second order Taylor expansion of the disutility of labor about a steady state is given by

$$\begin{aligned}\frac{N_t^{1+\varphi}}{1+\varphi} &\simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\left(\frac{N_t - N}{N}\right) + \frac{\varphi}{2} N^{1+\varphi}\left(\frac{N_t - N}{N}\right)^2 \\ &= \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right) + \frac{\varphi}{2} N^{1+\varphi}\widehat{n}_t^2 \\ &= \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\widehat{n}_t + \frac{1}{2} N^{1+\varphi}(1+\varphi)\; \widehat{n}_t^2\end{aligned}$$

where $\widehat{n}_t \equiv \log \frac{N_t}{N}$. In the model in the text, the steady state about which the economy fluctuates under the optimal policy is given by $\overline{N} = 1$. Hence, we have

$$\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \widehat{n}_t + \frac{1}{2}(1+\varphi)\; \widehat{n}_t^2 + tips$$

The next step consists in rewriting the previous expression in terms of the output gap. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right) \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$, we have

$$\begin{aligned}\widehat{n}_t &= \widehat{y}_t - a_t + z_t \\ &= \widetilde{y}_t + z_t\end{aligned}$$

where $z_t \equiv \log \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$, and where we use the fact that $\overline{y}_t = a_t$.

The following lemma shows that z_t is proportional to the cross-sectional distribution of relative prices (and, hence, of second order).

Lemma 1: $z_t \simeq \frac{\epsilon}{2}\ var_j\{p_t(j)\}$

Proof: see Appendix 2.

Using the previous results we can thus rewrite the second order approximation to the disutility of labor about that steady state in terms of the output gap and the price dispersion terms as:

$$\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \widetilde{y}_t + z_t + \frac{1}{2}(1+\varphi)\ \widetilde{y}_t^2 + tips$$

Collecting results and reintroducing country subscripts, we can write the second order approximation to aggregate welfare in the monetary union as follows:

$$\begin{aligned}
\mathbb{U}_t &\equiv \int_0^1 U(C_t^i, G_t^i, N_t^i) di \\
&= (1-\chi)\int_0^1 \log C_t^i\ di + \chi \int_0^1 \log G_t^i\ di - \int_0^1 \frac{(N_t^i)^{1+\varphi}}{1+\varphi}\ di \\
&\simeq \int_0^1 (\widetilde{y}_t^i - \chi\ \widetilde{g}_t^i)\ di - \frac{1}{2}\frac{\chi}{(1-\chi)}\int_0^1 (\widetilde{g}_t^i - \widetilde{y}_t^i)^2\ di \\
&\quad + \chi \int_0^1 \widetilde{g}_t^i\ di - \int_0^1 (\widetilde{y}_t^i + z_t^i + \frac{1}{2}(1+\varphi)\ (\widetilde{y}_t^i)^2)\ di + tips \\
&= -\int_0^1 \left(z_t^i + \frac{1}{2}(1+\varphi)\ (\widetilde{y}_t^i)^2 + \frac{1}{2}\frac{\chi}{1-\chi}(\widetilde{g}_t^i - \widetilde{y}_t^i)^2 \right)\ di + tips
\end{aligned}$$

In order to express utility in terms of inflation we make use of the following Lemma:

Lemma 2: $\frac{1}{2}\sum_{t=0}^{\infty} \beta^t\ z_t^i = \frac{\epsilon}{\lambda}\sum_{t=0}^{\infty} \beta^t\ (\pi_t^i)^2$

Proof: see appendix 2.

Now we can write the discounted sum of utilities across households as:

$$\begin{aligned}
\mathbb{W}_t &\equiv \int_0^1 \sum_{t=0}^{\infty} \beta^t\ U(C_t^i, G_t^i, N_t^i) di \\
&\simeq -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{\epsilon}{\lambda}\ (\pi_t^i)^2 + (1+\varphi)\ (\widetilde{y}_t^i)^2 + \frac{\chi}{1-\chi}(\widetilde{g}_t^i - \widetilde{y}_t^i)^2 \right)\ di
\end{aligned}$$

Appendix 2: Proofs of Lemmas 1 and 2

Lemma 1: $z_t \simeq \frac{\epsilon}{2}\ var_j\{p_t(j)\}$

Proof: let $\widehat{p}_t(j) \equiv p_t(j) - p_t$. Notice that,

$$\begin{aligned} \left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon} &= \exp\left[(1-\epsilon)\ \widehat{p}_t(j)\right] \\ &\simeq 1 + (1-\epsilon)\ \widehat{p}_t(j) + \frac{(1-\epsilon)^2}{2}\ \widehat{p}_t(j)^2 \end{aligned}$$

Furthermore, from the definition of P_t, we have $1 = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di$. Hence, it follows that

$$E_j\{\widehat{p}_t(j)\} = \frac{(\epsilon-1)}{2}\ E_j\{\widehat{p}_t(j)^2\}$$

In addition, a second order approximation to $\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}$, yields:

$$\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} \simeq 1 - \epsilon\ \widehat{p}_t(j) + \frac{\epsilon^2}{2}\ \widehat{p}_t(j)^2$$

Combining the two previous results, it follows that

$$\begin{aligned} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} di &= 1 + \frac{\epsilon}{2}\ E_j\{\widehat{p}_t(j)^2\} \\ &= 1 + \frac{\epsilon}{2}\ var_j\{p_t(j)\} \end{aligned}$$

from which it follows that $z_t \simeq \frac{\epsilon}{2}\ var_j\{p_t(j)\}$

Lemma 2: $\sum_{t=0}^{\infty} \beta^t\ z_t = \frac{1}{2}\frac{\epsilon}{\lambda} \sum_{t=0}^{\infty} \beta^t\ \pi_t^2$

Proof: we make use of the following property of the Calvo model, as shown in Woodford (2001, NBER WP8071):

$$\sum_{t=0}^{\infty} \beta^t\ var_j\{p_t(j)\} = \frac{1}{\lambda} \sum_{t=0}^{\infty} \beta^t\ \pi_t^2$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$, as in the text. The desired result follows trivially from Lemma 1.

References

Bacchetta, Philippe, and Eric van Wincoop (2000): “Does Exchange Rate Stability Increase Trade and Welfare?”, *American Economic Review*, 90:5, 1093-1109.

Benigno, Gianluca, and Benigno, Pierpaolo (2003): “Price Stability in Open Economies,” *Review of Economic Studies*, vol. 70, no. 4, 743-764.

Benigno, Pierpaolo (2004): “Optimal Monetary Policy in a Currency Area,” *Journal of International Economics*, vol. 63, issue 2, 293-320.

Blanchard O. and C. Khan (1980): "The Solution of Linear Difference Models with Rational Expectations", *Econometrica* 48: 1305-1311.

Brulhart M. and F. Trionfetti (2004), "Public Expenditure, International Specialisation and Agglomeration", *European Economic Review* 48, 851-881.

Bullard, James and K. Mitra (2001): “Learning About Monetary Policy Rules”, *Journal of Monetary Economics,* September 2002. 49(6), 1105-1129.

Calvo, Guillermo, 1983, “Staggered Prices in a Utility Maximizing Framework,” *Journal of Monetary Economics*, 12, 383-398.

Chari, V.V., Patrick Kehoe, and Ellen McGrattan (2002): “Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business Cycles,” *Review of Economic Studies* 69, 533-563

Clarida, Richard, Jordi Galí, and Mark Gertler (2000): “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory,” *Quarterly Journal of Economics*, vol. 105, issue 1, 147-180.

Clarida, Richard, Jordi Galí, and Mark Gertler (2001): “Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach,” *American Economic Review*, vol. 91, no. 2, 248-252.

Clarida, Richard, Jordi Galí, and Mark Gertler (2002): “A Simple Framework for International Monetary Policy Analysis,” *Journal of Monetary Economics*, vol. 49, no. 5, 879-904.

Corsetti, Giancarlo and Paolo Pesenti (2001): “Welfare and Macroeconomic Interdependence,” *Quarterly Journal of Economics* vol. CXVI,issue 2, 421-446.

Devereux, Michael B. and Charles Engel (2003): "Monetary Policy in the Open Economy Revisited: Exchange Rate Flexibility and Price Setting Behavior", *Review of Economic Studies,* December.

Ferrero, A. (2005) “Fiscal and Monetary Rules for a Currency Union”, mimeo New York University.

Galí, Jordi (2003): “New Perspectives on Monetary Policy, Inflation, and the

Business Cycle," in *Advances in Economics and Econometrics*, volume III, edited by M. Dewatripont, L. Hansen, and S. Turnovsky, Cambridge University Press

Galí, Jordi and Mark Gertler (1999): "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, vol. 44, no. 2, 195-222.

Galí, Jordi, and Tommaso Monacelli (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies,* vol. 72, issue 3, 2005, 707-734.

King Robert. and S. Rebelo, (1999), "Resuscitating Real Business Cycles," in J.B. Taylor, and M. Woodford, eds., *Handbook of Macroeconomics*, Amsterdam: North-Holland.

Kollmann, Robert (2001): "The Exchange Rate in a Dynamic Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation," *Journal of International Economics* vol.*55, 243-262.*

Linnemann, Ludger and A. Schabert (2003), "Fiscal Policy in the New Neoclassical Synthesis", *Journal of Money, Credit, and Banking* 35, 2003, 911-929.

Lombardo G. and A. Sutherland (2004), "Monetary and Fiscal Interactions in Open Economies", *Journal of Macroeconomics*, 26, 319-348.

Obstfeld, Maurice and Kenneth Rogoff (1995): "Exchange Rate Dynamics Redux," *Journal of Political Economy* 103, no. 3, 624-660.

Obstfeld, Maurice and Kenneth Rogoff (1999): "New Directions for Stochastic Open Economy Models," *Journal of International Economics*, vol. 50, no. 1, 117-153.

Pappa, Evi (2003): "Should the Fed and the ECB Cooperate ? Optimal Monetary Policy in a Two-Country World," *Journal of Monetary Economics* forthcoming.

Rotemberg, Julio and Michael Woodford (1999): "Interest Rate Rules in an Estimated Sticky Price Model," in J.B. Taylor ed., *Monetary Policy Rules*, University of Chicago Press.

Trionfetti F. (2000), "Discriminatory Public Procurement and International Trade", Blackwell Publishers.

Woodford, Michael (2001) "The Taylor Rule and Optimal Monetary Policy", *American Economic Review,* 91(2): 232-237.

Woodford, Michael (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Yun, Tack (1996), "Monetary Policy, Nominal Price Rigidity, and Business Cycles", *Journal of Monetary Economics*, 37:345-70.

Figure 1. Impulse Responses to a Government Consumption Shock

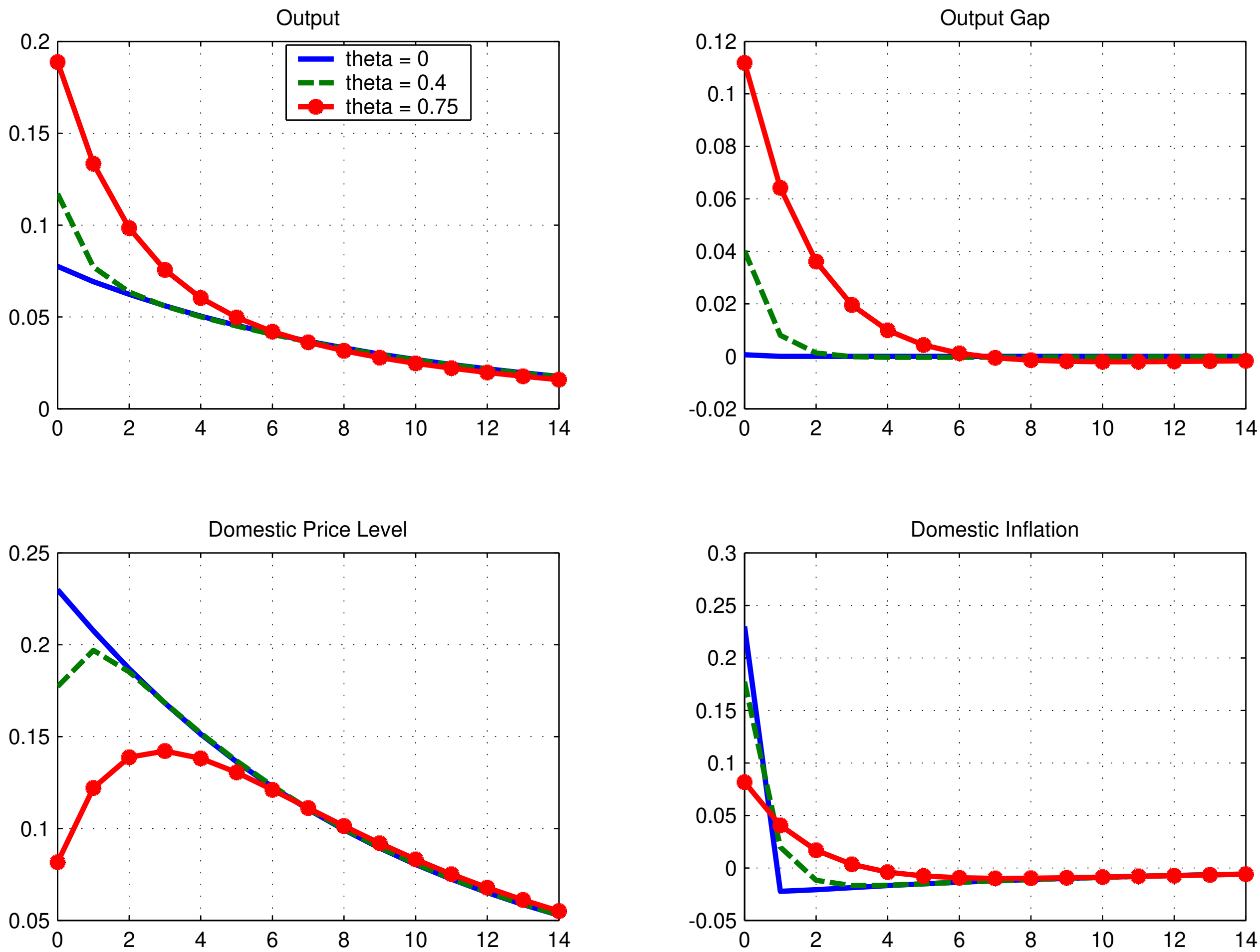

Figure 2. Productivity Shock under the Optimal Policy