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Optimal Multibit Digital to Analog Conversion

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Abstract— **When employing multibit data converters the necessity arises to compensate for Digital to Analog Converter (DAC) element mismatch. The most widespread compensation techniques are based on Digital Element Matching (DEM) and, if properly designed, these can achieve almost arbitrary DAC mismatch noise shaping. This paper gives a closed form expression for the optimal DEM noise shaping profile. It depends upon the spectrum of the analog signal to be quantized and may also include a frequency weighting filter which reflects perceptual criteria.**

I. Introduction

Analog to digital conversion is essential for the efficient storage and transmission of signals [1], [2]. Of particular interest are *feedback based* quantization schemes (or simply, feedback quantizers). Examples of feedback quantizers are Sigma-Delta converters [2]–[4], DPCM converters [5] and their many variants. The main advantage of feedback quantizers, when compared to pulse coded modulation based schemes, is that they can attain high performance at moderate oversampling ratios and with low bit scalar quantizers [2], [3], [6]. This is a consequence of the fact that a well designed feedback quantizer pushes the effects of quantization to frequencies that are outside the band of interest [3], [4], [6]. The resulting signal can then be filtered and decimated to the Nyquist frequency to eliminate much of the quantization effect, and to recover a low processing bit rate [3], [4].

When designing a feedback quantizer there exist, at least, three tuning parameters: the number of bits in the quantizer, the oversampling ratio (OSR) and the noise shaping filter [3], [4], [6]. Increasing either the number of bits, the OSR or the filter complexity may lead to improved quantizer performance. However, there are several limitations. Firstly, for high speed applications an increase in the OSR can lead to processing rates far beyond those sensible given standard circuit technology [7], [8]. Secondly, it is well known that the use of low bit quantizers and high complexity noise shaping filters may lead to stability problems in feedback quantizers [3], [9]. To obtain less stringent stability conditions, as well as to reduce quantization noise without having to use excesive OSRs, it is useful to consider multibit quantizers [2], [4], [6].

Multibit feedback quantizers may give good performance with moderate OSR and with less stability problems. There is, however, a significant disadvantage. The feedback path in a feedback quantizer comprises a Digital to Analog Converter (DAC). Whilst it can be regarded as ideal in the case of single-bit quantization (see, e.g., [3], [4], [6]), this is not so in the case of multibit quantization. The quality of the DAC depends strongly on the matching of discrete elements such as current sources or capacitors [3], [10]. With element mismatch, which is unavoidable, the resulting performance of the data converter may be seriously affected[1]. This is a consequence of the fact that the component mismatch appears as a noise source added to the signal to be quantized. This noise source is usually referred to as DAC mismatch noise.

There exist two main groups of techniques aimed at dealing with DAC component mismatch [2]. These have been developed not only for feedback quantization schemes, but also for more general situations. The first one relies on special fabrication processes or on individual trimming of the DAC components (see [3] and the references therein). These techniques are, usually, too expensive to become standard solutions to the mismatch problem. The second group of techniques relies on signal processing strategies. These, in turn, can be classified as calibration techniques (using analog or digital signal processing) [11]–[14] and as Dynamic Element Matching (DEM) based techniques [8], [10], [15]–[20]. Perhaps, the most simple and effective techniques are DEM based.

The basic principle behind DEM schemes is to use the DAC elements in such a way that the mismatch is averaged out and, therefore, has only a small effect on the converter output. The most basic DEM technique uses random access to the DAC elements [17], yielding a mismatch noise that can be regarded as white [16]. Another popular DEM technique is data weighted averaging (DWA), which uses the DAC elements in a cyclic fashion [10]. This leads to *first order* mismatch noise shaping [16], thus improving the signal to noise ratio at low frequencies. The main drawback of DWA is that it may originate tones [10], [16]. To reduce these tones several modifications to the basic DWA algorithm have been proposed, including bi-directional DWA [21], pseudo DWA [8], randomized DWA [15], [22], partitioned DWA [23], rotated DWA [24], [25], incremental DWA [26], etc. All these techniques try to maintain the noise shaping principle of DWA, but lower the likelihood of tones through appropriate modifications. More general schemes that have the potential to achieve almost arbitrary noise shaping profiles have also been proposed in, e.g., [19], [27].

An interesting alternative DAC mismatch compensation technique has been recently proposed in [7]. In that work,

[1]Note that in single bit feedback quantizers the DAC needs to distinguish between two levels only. This makes the matching of the discrete components irrelevant.

DEM is embellished through the use of pre and post filtering. In principle, the technique in [7] can be applied in conjunction with any DEM strategy and is aimed at improving its mismatch noise shaping characteristics.

The above works motivate us to investigate the question of how to design noise shaping profiles associated with DEM schemes. In the present work, we will provide an explicit expression for the DEM noise shaping profile which minimizes a weighted measure of the data converter reconstruction error. Furthermore, we will illustrate the performance gains that can be obtained with optimal DEM noise shaping when compared to previous schemes documented in the literature.

The remainder of this paper is organized as follows: Section II describes the general DAC mismatch mitigation problem. In Section III, we characterize optimal DEM noise shaping profiles in an explicit fashion. Section IV describes the application of our results to the case of feedback quantizer DACs. An example is provided in Section V. Section VI draws conclusions.

Notation: We focus on discrete time signals and, without loss of generality, normalize the sampling interval to unity. We use standard vector space notation for signals. For example, x denotes $\{x(k)\}_{k\in\mathbb{N}_0}$. We also use z as the argument of the z-transform and also as the forward shift operator, where the meaning is clear from the context.

All signals in this paper are assumed to be wide sense stationary stochastic processes with zero mean, finite variance, and rational power spectral density (PSD). Given a process x, we denote its variance by σ_x^2. We also recall the well known fact that

$$\sigma_x^2 \triangleq \mathcal{E}\left\{x(k)^2\right\} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\Omega_x(e^{j\omega})\right|^2 d\omega,$$

where $\Omega_x(z)$ is a rational transfer function such that $\left|\Omega_x(e^{j\omega})\right|^2$ equals the PSD of x.

II. DAC-DEM Architecture

We will first focus on the data conversion scheme shown in Figure 1. In that figure, r is the (discrete time) analog input signal, ADC is an analog to digital converter, and DAC-DEM is a digital to analog converter, realized using unit elements and equipped with an appropriate DEM strategy (see, e.g., [2]–[4]). The signal y corresponds to a quantized (i.e., digital) version of r and $\hat{r}$ is a (discrete-time) analog signal which approximates r.[2]

The output of the ADC in Figure 1 is modelled via

$$y = T(z)r + N(z)q, \tag{1}$$

where $T(z)$ and $N(z)$ are appropriate rational transfer functions and q corresponds to quantization noise[3]. Usually, $T(z)$

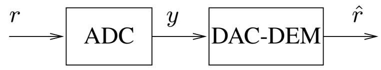

Fig. 1. Standard data conversion scheme.

is termed *signal transfer function* and $N(z)$, *noise transfer function* (see, e.g., [4]). Note that (1) describes every ADC architecture in which an additive noise model is employed to model quantization (see, e.g., [3]–[6] and also [28], [29]). In particular, if $N(z) = T(z) = 1$, then (1) reduces to the output of a pulse coded modulation scheme (see, e.g., [6]). On the other hand, if $N(z) = 1 - z^{-1}$ and $T(z) = z^{-1}$, then (1) models the output of a first order Sigma-Delta converter, as described in, e.g., [2], [3], [6].

We will assume that the DAC consists of M nominally identical elements whose values are added together in accordance with the input digital code and the employed DEM scheme (see, e.g., [2]). Each DAC element has a value e_i ($i \in \{1, 2, \cdots, M\}$), nominally equal to $\bar{e}$. Due to fabrication imperfections, e_i can be modelled as a realization of an independent Gaussian random variable with mean $\bar{e}$ and variance σ_e^2. It is convenient to define M control signals, c_i ($i \in \{1, 2, \cdots, M\}$), as follows:

$$c_i = \begin{cases} 1 & \text{if the } i^{th} \text{ element is selected,} \\ 0 & \text{if the } i^{th} \text{ element is not selected.} \end{cases}$$

Therefore, the output of the DAC-DEM block in Figure 1 can be written as

$$\hat{r} = \sum_{i=1}^{M} c_i e_i.$$

If the DAC was ideal, i.e., if $\sigma_e^2 = 0$, then $\hat{r} = y$ (which, in turn, is designed to resemble r). However, in practice there always exists DAC element mismatch. Therefore, $\hat{r}$ differs from the analog correspondence of y. This difference, say m, is called *DAC mismatch noise* and is given by

$$m \triangleq \hat{r} - y = \sum_{i=1}^{M} c_i(e_i - \bar{e}). \tag{2}$$

An important property of DAC mismatch noise is that many DEM techniques are such that m can be accurately described via

$$m = D(z)\eta, \tag{3}$$

where $D(z)$ is a stable rational transfer function and η is a zero mean white noise sequence, uncorrelated to the input r and to the quantization noise q. We will refer to $D(z)$ as the *DEM noise shaping profile*. For example, if Butterfly randomization is employed, then $D(z) = 1$ [16], [17]. In the case of DWA, we have $D(z) = 1 - z^{-1}$ [10], [16]. More general DEM schemes, such as those proposed in [7], [19], [27], allow one to implement arbitrary DEM noise shaping profiles.

In the remainder of this work we will show how to design $D(z)$ such as to optimize overall converter performance.

[2]We note that our subsequent analysis holds, *mutatis mutandis*, if one also considers a reconstruction filter in the scheme of Figure 1 (see also Section IV).

[3]It is implicit in (1) that the main source of noise in the ADC output is quantization. This means that if the ADC is a feedback quantizer, then its feedback DAC is assumed ideal (as in the case of software based quantizers). We will study the design of the feedback DAC in Section IV.

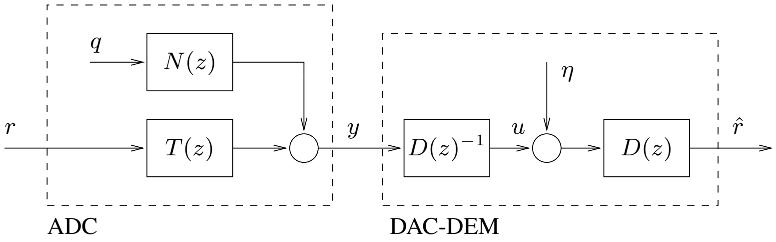

Fig. 2. Linear model for the considered data converter architecture.

III. Optimal DEM Noise Shaping Design

According to Section II, the setup in Figure 1 can be modelled as in Figure 2. Consequently, the output of the considered data conversion architecture obeys

$$\hat{r} = T(z)r + N(z)q + D(z)\eta. \tag{4}$$

The signal transfer function $T(z)$ and the noise transfer function $N(z)$ stem from the ADC design, which is a very developed research area (see, e.g., [3], [4], [6]). The choice of $D(z)$ is, however, less well understood.

We will next show how to design the DEM noise shaping profile having overall data converter performance in mind. For that purpose we will assume that the ADC is given (i.e., that $T(z)$ and $N(z)$ are specified) and will derive an expression for the optimal profile.

We will adopt a model where the variance of η, namely σ_η^2, is proportional to the variance of u (see Figure 2), i.e.,

$$\lambda \triangleq \frac{\sigma_u^2}{\sigma_\eta^2} \tag{5}$$

is a fixed constant. The constant λ depends on the statistics of u and on the DAC-DEM parameters. For example, if Butterfly randomization is employed and u is "busy signal", then

$$\lambda \approx k\frac{M}{\sigma_e} \tag{6}$$

for some constant k [2], [16], [17]. In the case of DWA, a similar expression for λ holds [10], [16].

Remark 1: We note that the scheme in Figure 2 corresponds to the scheme considered in [7], although in that work $D(z)$ (and also $D(z)^{-1}$) are actual filters and not only models of the noise shaping provided by the DEM strategy. Our setting encompasses both the architecture in [7] and the schemes considered in [10], [16], [16], [17], [19], [27], as discussed previously. ■

A key element of our approach is that we will focus on the overall converter performance, i.e., we will concentrate on the difference between the analog signals r and $\hat{r}$. To that end, we define the weighted reconstruction error as

$$e_W \triangleq W(z)(r - \hat{r}), \tag{7}$$

where the weighting filter $W(z)$ is stable and proper. This filter is application dependant and models the importance that reconstruction errors have at different frequencies. For example, in audio applications, $W(z)$ may model the ear sensitivity to low level noise power (see, e.g., [30])[4]. We will measure the data converter performance by means of the variance of e_W, namely

$$\sigma_{e_W}^2 \triangleq \mathcal{E}\left\{e_W(k)^2\right\}. \tag{8}$$

To simplify matters we will assume that the ADC and the number of DAC elements M are such that

$$\frac{\sigma_q^2}{\lambda} \approx 0, \tag{9}$$

where σ_q^2 refers to the variance of the quantization noise q. For a $b-$bit ADC, σ_q^2 is (roughly) of the order of 2^{-2b} (see, e.g., [4], [5]). Therefore, to satisfy (9), it is sufficient to have a large number of bits in the ADC and/or a large number of elements in the DAC and/or small enough DAC element mismatch variance (recall (6)). This is resonable in the context of multibit data converters.

Our result is stated in the following theorem:

Theorem 1: Consider (4) and (5), assume that r, q and η are uncorrelated, and suppose that (9) holds with equality. If $T(z)$ and $N(z)$ are given, then the DEM noise shaping profile that minimizes $\sigma_{e_W}^2$, say $D_{opt}(z)$, satisfies

$$\left|D_{opt}(e^{j\omega})\right|^2 = \beta\left|W(e^{j\omega})^{-1}T(e^{j\omega})\Omega_r(e^{j\omega})\right|, \tag{10}$$

$\forall\omega \in [-\pi, \pi]$. In (10), β is an arbitrary positive constant.

Proof: Using (4) and (7) it is immediate to see that

$$e_W = W(z)(1 - T(z))r - W(z)N(z)q - W(z)D(z)\eta.$$

This leads to[5]

$$\sigma_{e_W}^2 = ||W(z)(1 - T(z))\Omega_r(z)||_2^2 + \sigma_q^2\,||W(z)N(z)||_2^2 + \sigma_\eta^2\,||W(z)D(z)||_2^2, \tag{11}$$

where $\left|\Omega_r(e^{j\omega})\right|^2$ is the PSD of r. Since (5) holds, σ_η^2 is not pre-specified. It depends on the variance of u.

From Figure 2 it follows that

$$u = D(z)^{-1}T(z)r + D(z)^{-1}N(z)q. \tag{12}$$

Using (5) in (12) leads to

$$\sigma_\eta^2 = \frac{\left|\left|D(z)^{-1}T(z)\Omega_r(z)\right|\right|_2^2}{\lambda} + \sigma_q^2\frac{\left|\left|D(z)^{-1}N(z)\right|\right|_2^2}{\lambda}. \tag{13}$$

[4]Note that $W(z)$ may also be implicit in the choice of $T(z)$ and $N(z)$ (see, e.g., [30], [31]).

[5]For every rational $H(z)$, $||H(z)||_2^2 \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega})\right|^2 d\omega$, provided the integral exists.

Since (9) holds with equality, (11) and (13) lead to

$$\sigma_{e_W}^2 = ||W(z)(1-T(z))\Omega_r(z)||_2^2 + \sigma_q^2\, ||W(z)N(z)||_2^2 + \frac{\left|\left|D(z)^{-1}T(z)\Omega_r(z)\right|\right|_2^2 ||W(z)D(z)||_2^2}{\lambda}. \quad (14)$$

We recall that both $N(z)$ and $T(z)$ are given. Thus, the first two terms in the right hand side of (14) are fixed. Therefore, the optimal $D(z)$ minimizes the third term in (14). Use of the Cauchy-Schwartz inequality (see also [5], [32]) leads immediately to (10). ■

The above result provides optimal DEM noise shaping profiles in terms of the input PSD, the weighting filter frequency response and the signal transfer function of the ADC. It is worth noting that the proposed choice for $D(z)$ is independent of the DAC and quantizer parameters. This allows us to conjecture that our designs will have good robustness properties.

It is interesting to note that Theorem 1 implies that choices for $D(z)$ considered in the literature are optimal for specific situations. For example,

$$D(z) = D_n(z) \triangleq \left(\frac{z-1}{z}\right)^n, n \in \{0,1,2\},$$

encompasses Butterfly randomization for $n=0$, and so-called *first order* and *second order* DAC mismatch noise shaping for $n=1$ and $n=2$, respectively (see, e.g., [4]). Theorem 1 allows us to conclude that $D_n(z)$ is *optimal* if and only if

$$\left|W(e^{j\omega})\right| = \left|\frac{T(e^{j\omega})\Omega_r(e^{j\omega})}{(e^{j\omega}-1)^{2n}}\right|, \forall \omega \in [-\pi, \pi]. \quad (15)$$

In contrast to (15), our result provides additional flexibility, tailoring the inclusion of DEM noise shaping to any application involving data converters.

To implement an optimal DEM scheme, i.e., one that uses $D_{opt}(z)$, one can rely upon the schemes proposed in [19], [27], where the DAC elements are accessed in such a way that arbitrary noise shaping can be achieved. Alternatively, and if available technology allows one to do so, one could also use the architecture proposed in [7] (see also Remark 1).

IV. Application to the Design of Feedback DACs in Feedback Quantizers

In this section we specialize our DEM noise shaping results to feedback quantizers with DAC in the feedback path, such as Sigma-Delta converters. We will focus on the architecture depicted in Figure 3. In that figure, r is the analog input, $\mathcal{Q}$ is a b-bit scalar quantizer, DAC-DEM is a DAC equipped with DEM, as described in Section II, $L(z)$ is the so-called loop filter, and $P(z)$ is a reconstruction (or interpolation) filter.

In relation to Figure 3, we define the quantization noise sequence by

$$q \triangleq y - v.$$

As usual in the feedback quantization literature (see, e.g., [3]–[6] and also [28], [29]), we adopt a white noise model for q.

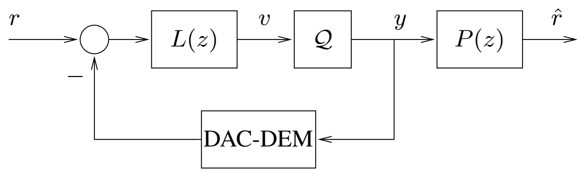

Fig. 3. Feedback quantizer with non ideal DAC in the feedback path.

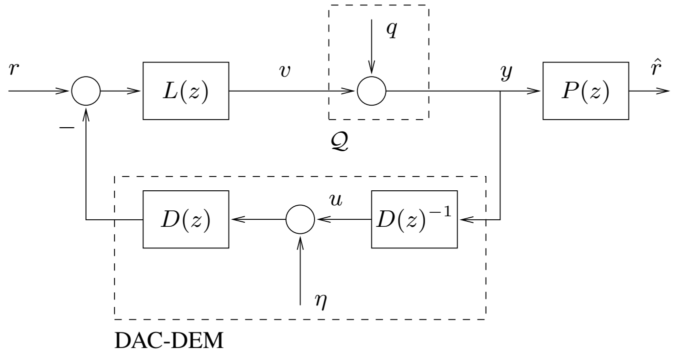

Fig. 4. Linear model for feedback quantizer with non ideal DAC in the feedback path.

More precisely, we assume that q is a zero mean white noise sequence, uncorrelated to the input r, and whose variance, σ_q^2, is proportional to the variance of the input to the quantizer v, i.e.,

$$\gamma \triangleq \frac{\sigma_v^2}{\sigma_q^2} \quad (16)$$

is a known (and fixed) constant.

The constant γ depends on the statistics of v and on (half) the scalar quantizer dynamic range V [5], [33]. For example, if v is a sequence of identical Gaussian random variables with variance σ_v^2, and $V = \alpha\sigma_v$,[6] then

$$\gamma = \frac{3}{\alpha^2}(2^b-1)^2.$$

The above model, when combined with the linear model for the DAC presented in Section II, yields the feedback quantizer model in Figure 4. In this scheme, it holds that

$$\hat{r} = P(z)T(z)r + P(z)N(z)q + P(z)T(z)D(z)\eta, \quad (17)$$

where $T(z) = L(z)(1+L(z))^{-1}$ and $N(z) = 1 - T(z)$ (compare to (4)).

To simplify subsequent derivations, we will assume that γ and λ, see (5) and (16), are such that

$$\frac{1}{(\gamma - C)(\lambda - D)} \approx 0 \quad (18)$$

for every real C and D such that $\gamma - C > 0$ and $\lambda - D > 0$. This assumption holds under the same conditions as those underlying (9).

As in the data conversion architecture examined before, we will measure the performance of the feedback quantizer by

[6] α is usually referred to as the *quantizer overload factor*; $\alpha = 4$ is a common choice (see, e.g., [5]).

means of the weighted variance of the reconstruction error e_W defined in (7). In this case, we have the following result:

***Theorem** 2:* Consider (17), (5) and (16), assume that r, q and η are uncorrelated, and suppose that (18) holds with equality. If $L(z)$ is given, then the DEM noise shaping profile that minimizes $\sigma^2_{e_W}$, say $D_{opt}(z)$, satisfies

$$\left|D_{opt}(e^{j\omega})\right|^2 = \beta\left|W(e^{j\omega})^{-1}P(e^{j\omega})^{-1}\Omega_r(e^{j\omega})\right|, \quad (19)$$

$\forall\omega \in [-\pi, \pi]$. As before, β is an arbitrary positive constant.

Proof: From (17), and assuming that r, q and η are uncorrelated, it follows that

$$\sigma^2_{e_W} = ||W(z)(1 - P(z)T(z))\Omega_r(z)||_2^2 + \sigma_q^2 ||W(z)P(z)N(z)||_2^2 + \sigma_\eta^2 ||W(z)P(z)T(z)D(z)||_2^2. \quad (20)$$

Since (16) and (5) hold, σ^2_q and σ^2_η depend on the variances of v and u, respectively. A little algebra shows that

$$\sigma_q^2 = \frac{||T(z)\Omega_r(z)||_2^2}{\gamma - ||T(z)||_2^2} + \frac{\sigma_u^2 ||T(z)D(z)||_2^2}{\lambda(\gamma - ||T(z)||_2^2)}, \quad (21)$$

$$\sigma_\eta^2 = \frac{\left|\left|D(z)^{-1}T(z)\Omega_r(z)\right|\right|_2^2}{\lambda - ||T(z)||_2^2} + \frac{\sigma_v^2 \left|\left|S(z)D(z)^{-1}\right|\right|_2^2}{\gamma(\lambda - ||T(z)||_2^2)}. \quad (22)$$

Since (18) holds with equality, (20), (21) and (22) lead to

$$\sigma^2_{e_W} = ||W(z)(1 - P(z)T(z))\Omega_r(z)||_2^2 + \frac{||T(z)\Omega_r(z)||_2^2 \, ||W(z)P(z)N(z)||_2^2}{\gamma - ||T(z)||_2^2} + \frac{\left|\left|D(z)^{-1}T(z)\Omega_r(z)\right|\right|_2^2 ||W(z)P(z)T(z)D(z)||_2^2}{\lambda - ||T(z)||_2^2}.$$

Since $L(z)$ is given, the result follows by proceeding as in the proof of Theorem 1. ■

Theorem 2 gives a closed form expression for the optimal DEM filter to be employed in the feedback DAC of feedback quantizers. As before, the usual DEM filter choices made in the literature, see, e.g., [4], arise as special cases of our result.

V. Example

In this section we illustrate the ideas developed in this paper via an example. To that end, we use the DEM scheme proposed in [27] to achieve the noise shaping profile suggested by our results. In particular, we employ the routines in the Matlab© toolbox described in [4]. This simulation tool provides fairly accurate behavioural simulations of Sigma-Delta converters and DEM techniques.

We will consider the setup in Figure 1. The ADC is taken as a 3-bit first order Sigma-Delta converter, which gives $T(z) = z^{-1}$ and $N(z) = (z - 1)z^{-1}$ (see (1)). The DAC is assumed to consist of $M = 8$ elements. To emphasize the effect of DAC element mismatch, we assume that the variance of each element is $\sigma_e^2 = 0.25$. The input r is chosen to resemble audio and with PSD given by

$$\left|\Omega_r(e^{j\omega})\right|^2 = \left|\frac{0.26(e^{j\omega} + 0.51)(e^{j\omega} - 1)^2}{(e^{j\omega} - 0.78)(e^{j\omega} - 0.82)(e^{j\omega} - 0.53)(e^{j\omega} - 0.39)}\right|^2,$$

while the weighting filter is chosen to be the low pass filter

$$W(z) = \frac{0.22595}{(z - 0.778)(z - 0.8282)}.$$

To study the benefits of DEM mismatch compensation, we apply Theorem 1. This gives the optimal filter[7]

$$D_{opt}(z) = \frac{1.0644(z + 0.4284)(z - 0.9998)}{(z + 0.1896)(z - 0.4756)}.$$

We next fix the realization of DAC elements and consider 100 realizations of the input r. Figure 5 shows the (sample) variance of the weighted error, see (8), for every input realization. In that figure, "Ideal DAC" refers to a case where $\sigma_e^2 = 0$, "No DEM" refers to a case where there is DEM mismatch and no compensation, "Optimal DEM" refers to our proposal; the other two curves refer to first and second order DAC mismatch noise shaping. It is appreciated that DEM mismatch compensation is, as expected, beneficial for reducing $\sigma^2_{e_W}$. Furthermore, the use of first and second order mismatch noise shaping provides worse performance when compared with the optimal filter suggested by Theorem 1. Indeed, the average weighted error variances for each design are as follows:

DAC conversion scheme	$\sigma^2_{e_W}$
Ideal DAC	0.0037
No DEM	0.0085
Optimal DEM	0.0042
1st order DEM	0.0053
2nd order DEM	0.0051

This means that, for the situation studied, optimal DEM filtering provides a 24.3% reduction in the weighted reconstruction error variance (relative to the ideal case), when compared to second order noise shaping. Despite the fact that in the simulations σ_e^2 is fairly large, our design achieves a performance which is only 13.5% far from that of an ideal DAC with no element mismatch.

VI. Conclusions

This work has proposed a systematic way to tackle the problem of DEM mismatch noise shaping profile design. By adopting a linear DAC mismatch noise model, we have provided a closed form expression for the optimal noise shaping profile. This expression is stated in terms of the input signal spectrum, a weighting filter and the signal transfer function associated to the ADC generating the signal to be converted back to analog form. Further work may include the joint optimization of DEM noise shaping profile and ADC transfer functions, as well as experimental validations.

[7]Note that $D_{opt}(z)$ in (10) is not necessarily rational. Since the method in [27] requires a rational DEM noise shaping profile specification, we have used a simple curve fitting routine to adjust a rational filter to the square root of the right hand side in (10).

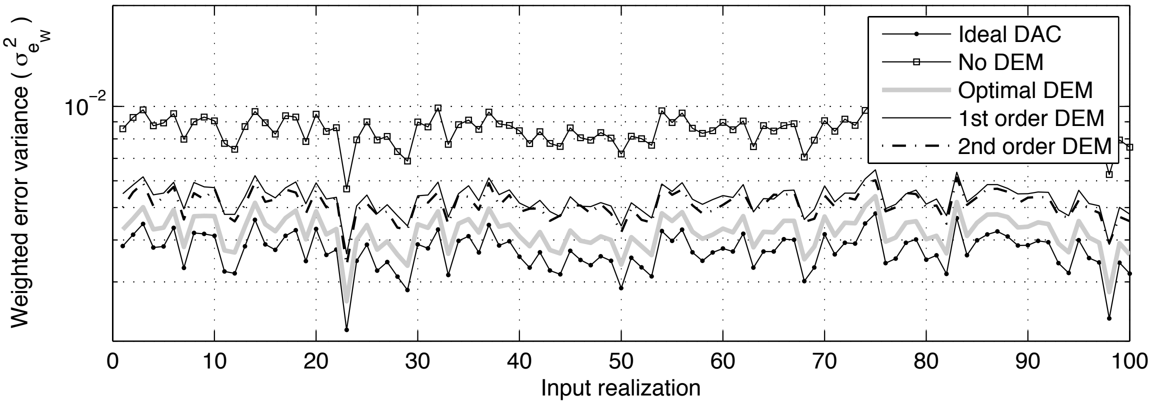

Fig. 5. Weighted error variance as a function of the input realization number.

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