

# Optimal Operation of Industrial CHP-based Power Systems in Liberalized Energy Markets

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**Abstract** – This paper presents the implementation of a new method in calculating the optimal unit commitment and economic dispatch of industrial enterprises with its own energy supply system, mostly with CHP-plants. To yield the most synergetic effects, both the marketplace and the demand of disposable industrial processes are taken into account. Because of the complexity of the formulated mathematical model and, thus, of the optimization problem, a new Dynamic Search Strategy (DSS) based on Mixed Integer Linear Programming (MILP) is presented. With DSS, a considerable acceleration of calculation time could be achieved.

**Keywords:** CHP-based power systems, short-term operation planning, independent power producers, cost analysis, industrial load management, liberalized energy markets, mixed integer linear programming.

## I. INTRODUCTION

Whereas the European Union gives deadlines for the stepwise opening of the national electricity markets, the German Government has decided to open the German market for all customers in one single step. Since the new economic law coming into effect on April 29th, 1998, even the tariff customers may change their supplying utility. In practice, particularly the industrial customers with its high energy consumption are the focus of interest of the public utilities.

Industrial branches with a high need of energy often run its own power systems. The main difference between industrial and public utility power systems is the variety of different types of energy needed, e.g. steam, hot water, air pressure, electric power etc. (see Fig. 1). Therefore, combined heat and power (CHP-)plants can frequently be found in industrial power systems, which are often fired by natural gas. Furthermore, waste heat and residual materials from the production processes have to be taken into account. Third, some industrial processes allow to apply methods of load management. Particularly this point has often been ignored since today, but will be of growing importance in a competitive environment where flexibility of energy consumption and generation respectively will surely have its own economical benefit.

The short-term optimal operation of their power system with a planning period from half the day to one week is an important task for industrial enterprises with the objective to minimize the operational costs. Because of the liberalization of the electricity and gas market, a variety of new opportunities arises for industrial power systems to

play an active part in the energy markets.

Besides the possibility to simply change the supplying utility to get lower electricity prices, industrial enterprises will soon be able to buy and sell spot energy at a power exchange. This makes it necessary to economically assess the effects of certain offers on the whole industrial power system. Furthermore, the surplus of electric energy may be sold as an Independent Power Producer (IPP) at a power exchange or bilaterally to another (industrial) customer taking the grid tariffs into account. Therefore, the objective changes from minimizing the cost to maximizing the economical benefit.

Because of the complexity of this optimization problem, only computer based algorithms are able to calculate exact solutions in a reasonable time. This paper presents a new approach to maximize the economical benefit of industrial power systems under the conditions of liberalized energy markets.

This paper is structured as follows: After defining all relevant variables and symbols needed to formulate the optimization problem in section 2, the necessary mathematical equations are developed and described in section 3. Section 4 explains the methodology and the expected advantages of the DSS, followed by a brief numerical example to demonstrate its effectiveness on real optimization problem. The conclusions drawn in section 5 round off this paper.

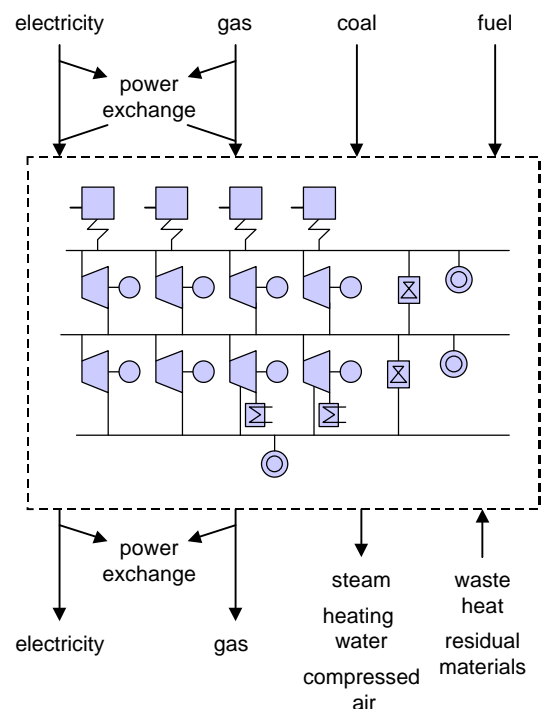


Fig. 1. General structure of industrial power systems.

## II. NOTATION

### A. Variables

$Obj$	objective function value
$i, I$	index/number of time steps
$k, K$	index/number of supplying contracts
$m, M$	index/number of purchasing contracts
$D$	duration
$c, C$	variable/given costs per time
$t, T$	variable/given takings per time
$p, P$	variable/given power
$r, R$	variable/given reserve power
$a, A$	index/number of positive contributions to a node
$b, B$	index/number of negative contributions to a node
$\gamma, \Gamma$	continuous/binary variable (contracts)
$s, S$	index/number of piecewise linear segments
$u, U$	index/number of converting units
$\lambda, \Lambda$	continuous/binary variable (units)
$\sigma, \Sigma$	continuous/binary variable (start-up)
$\pi$	continuous variable (power gradient)
$\Omega$	binary variable (interruption)
$F$	frequency
$\mu$	continuous variable (load postponement)
$n, N$	index/number of balancing nodes
$\rho, \vartheta$	continuous variables (reserve)
$\psi$	binary variable (shut-down)
$j, J$	index/number of postponable time steps
$z, Z$	index/number of down-time dependent start-up costs
$e, E$	variable/given energy
$G$	minimum improvement of $Obj$
$Q$	accuracy of integer solution's $Obj$
$f$	factor
$x$	number of subsequent unsuccessful trials to find an integer solution

### B. Descriptions:

$T$	transaction
$beg, end$	begin, end
$in, out$	in, out
$S$	start-up
$+, -$	positive/negative change between two intervals
$D$	duration
$F$	frequency
$PB$	payback
$max$	maximum

## III. PROBLEM FORMULATION

### A. Objective

The main task of industrial power systems is to supply their customers with all kinds of energy needed to guarantee a reliable production. In the past, a rather close relationship existed between the industrial power system and the supplied industrial processes. Generally, both could organizationally be found in one and the same enterprise. The liberalization of the energy sector and, thus, the competition even in the industrial energy supply shows new opportunities for industrial power systems to buy and,

particularly, to sell energy. Thereby, a restructuring can already be noticed in the industry towards a profit center based industrial energy supply with the goal to maximize the profit. This means at the same time that the industrial processes can no longer only be seen as simply a load, but as paying customers. Keeping these reflections in mind, the objective function of the short-term optimal operation problem can be written as

$$Obj = \sum_{i=1}^I \sum_{k=1}^K t_{i,k} \cdot D_i - \sum_{i=1}^I \left( \sum_{m=1}^M c_{i,m} + \sum_{u=1}^U c_{i,u} \right) \cdot D_i \rightarrow \max! \quad (1)$$

### B. General modeling approach

The general concept of modeling power systems consists of separated models for purchasing and supplying contracts respectively, power converting units and storage devices. The latter had already been reported on by [5] and is therefore not considered in this paper. The models are described as abstract black boxes formulating the mathematical correlation between the input and output variables (e.g. gas consumption rate vs. generated electric power and steam mass flow). All items are connected by balancing nodes. Therefore, different types of converting units, e.g. boilers, steam or gas turbines, steam pipes etc. can be simply modeled in the same way.

### C. Balancing nodes

At each node, two different types of balancing equations have to be fulfilled, the power and the reserve balancing equation.

$$\sum_{a=1}^{A_n} P_{i,a} - \sum_{b=1}^B P_{i,b} = 0 \quad (2)$$

Whereas the power balancing equation (2) is immediately comprehensible and has also been used in former papers, the formulation of the reserve requirements should be subject to closer examination. In contrast to other considered power systems in the literature, we find a multistage power system with several elements connected in series or in parallel (see Fig. 1). Therefore, it is no longer sufficient to use the formulation

$$\sum_{a=1}^{A_n} r_{i,a} - R_{i,n} \geq 0 \quad (3)$$

for every node related to a load. Fig. 2 shows a simplified example. Let us suppose that the reserve needed for load L would be additional 20 units (e.g. MW). From (3) we find that the reserve requirement could be matched because  $T_1$  and  $T_2$  could produce additional 30 units each. In fact,  $B_1$  runs at maximum power and, thus, the apparent reserve of  $T_1$  and  $T_2$  doesn't really exist. As a result, the following requirement is introduced using reserve variables for every node, forcing for the discussed example to commit also  $B_2$ .

$$\sum_{a=1}^{A_n} r_{i,a} - \sum_{b=1}^{B_n} r_{i,b} \geq 0 \quad \forall n \in \{1, \dots, N\} \quad (4)$$

### D. Purchasing contracts

Two types of purchasing contracts can generally be distinguished. On the one hand, there are certain amounts of fuel like heating oil and/or coal available which had already been bought and which costs per unit must be

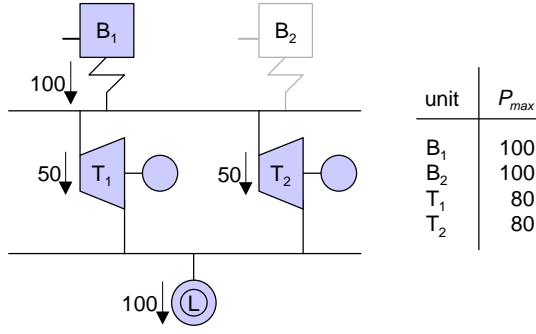


Fig. 2. Example of unbalanced reserve requirements.

considered.

$$c_{i,m} = \gamma_{i,m} \cdot C_m \quad (5)$$

$$p_{i,m} = \gamma_{i,m} \cdot P_m \quad (6)$$

$$r_{i,m} = (\rho_{i,m} - \gamma_{i,m}) \cdot P_m \quad (7)$$

On the other hand, electricity, heat and gas purchasing contracts usually contain prices related to the purchased power, so we have to formulate

$$c_{i,m} = \sum_{s=1}^{S_m} \gamma_{i,m,s} \cdot (C_{i,m,s+1} - C_{i,m,s}) \quad (8)$$

$$p_{i,m} = \sum_{s=1}^{S_m} \gamma_{i,m,s} \cdot (P_{i,m,s+1} - P_{i,m,s}) \quad (9)$$

$$r_{i,m} = \sum_{s=1}^{S_m} (\rho_{i,m,s} - \gamma_{i,m,s}) \cdot (P_{i,m,s+1} - P_{i,m,s}). \quad (10)$$

When participating in a power exchange, offers are to be selected from the variety of existing offers which could be used to maximize the objective function value. Therefore, all reasonable offers have to be introduced into the optimization problem, and the optimal solution will show whether an offer should be realized or rejected.

For modeling an offer consisting of a fix power over a certain period of time, the following equations are needed.

$$c_{i^{beg},m} = \Gamma_{i^{beg},m} \cdot (C_{i^{beg},m} + C_{i^{beg},m}^T) \quad (11)$$

$$p_{i,m} = \Gamma_{i^{beg},m} \cdot P_{i,m} \quad \forall i \in [i^{beg}; i^{end}] \quad (12)$$

$$r_{i,m} = p_{i,m} \quad (13)$$

If the offered power is not fix but has to be between a given upper and lower limit, we have to introduce additional continuous variables

$$c_{i,m} = \Gamma_{i^{beg},m} \cdot (C_{i,m,1} + C_m^T) + \sum_{s=1}^{S_m} \gamma_{i,m,s} \cdot (C_{i,m,s+1} - C_{i,m,s}) \quad (14)$$

$$p_{i,m} = \Gamma_{i^{beg},m} \cdot P_{i,m,1} + \sum_{s=1}^{S_m} \gamma_{i,m,s} \cdot (P_{i,m,s+1} - P_{i,m,s}) \quad (15)$$

$$r_{i,m} = \Gamma_{i^{beg},m} \cdot P_{i,m,1} + \sum_{s=1}^{S_m} (\rho_{i,m,s} - \gamma_{i,m,s}) \cdot (P_{i,m,s+1} - P_{i,m,s}) \quad (16)$$

$$\sum_{s=1}^{S_m} \gamma_{i,m,s} \leq \Gamma_{i^{beg},m} \cdot S_m \quad (17)$$

$$\sum_{s=1}^{S_m} \rho_{i,m,s} \leq \Gamma_{i^{beg},m} \cdot S_m; \quad \forall i \in [i^{beg}; i^{end}] \quad (18)$$

### E. Converting units

Generating or, more generally, converting units are used to convert e.g. electrical power to pressured air. These units are mathematically described as black boxes,

containing a piecewise linear relation between the input and the output variables [6]. Additionally, the input and output variables are introduced into their corresponding balancing nodes. In practice, converting units may have more than one input and one output variable like, for example, boilers needing gas and, at the same time at a lower extend, electricity to produce steam. In these cases, additional variables are introduced to the problem using the same set of variables. Because of their importance to industrial power systems, CHP-based power plants with extraction-condensing turbines and, thus, two degrees of freedom to produce heat and electricity have to be taken into account. The used mathematical model uses two sets of continuous variables considering the operating range between both heat and electricity generation. It is described more detailed in [6].

Although minimum up- and down-times are realized in the presented mathematical model, more realistic schedules could be found in formulating start-up costs depending upon down-time. The basic idea is illustrated in Fig. 3.

The start-up costs are

$$c_{i,u}^S = \sum_{i,u} \sigma_{i,u} \cdot C_{\infty,u}^S - \sum_{z=1}^{Z_u} \sigma_{i,z,u} \cdot C_{z,u}^S \quad (19)$$

where  $\sum_{i,u}$  is the start-up variable defined by

$$\Lambda_{i,u} - \Lambda_{i-1,u} - \sum_{i,u} \leq 0. \quad (20)$$

Because  $\sigma_{i,z,u}$  reduces the start-up costs, restrictions have to be found ensuring that only  $\sigma_{i,z,u}$  is greater zero where  $z$  corresponds to the actual down-time. To reduce the start-up costs as much as possible,  $\sigma_{i,z,u}$  will then always be set to one by the optimization algorithm.

On condition that

$$C_{1,u}^S > C_{2,u}^S > \dots > C_{Z_u,u}^S > 0 \quad (21)$$

which always can be met (see Fig. 3), (22) and (23) formulate the necessary conditions.

$$\Lambda_{i-z-1,u} - \sigma_{i,z,u} \geq 0 \quad \forall z \in \{1, \dots, Z_u\} \quad (22)$$

$$\sum_{z=1}^{Z_u} \sigma_{i,z,u} \leq \sum_{i,u} \quad (23)$$

Furthermore, cost based limited power changes are introduced to avoid oscillating dispatches of the committed units. In the start-up and the shut-down interval, the unit's minimum power should be used. Thus, a shut-down variable  $\Psi_{i,u}$  is defined by

$$\Lambda_{i,u} - \Lambda_{i+1,u} - \Psi_{i,u} \leq 0 \quad (24)$$

$$\sum_{i,u} \leq \lambda_{i,u,1} \quad (25)$$

$$\Psi_{i,u} \leq \lambda_{i,u,1} \cdot \quad (26)$$

The cost based limited power changes only should be

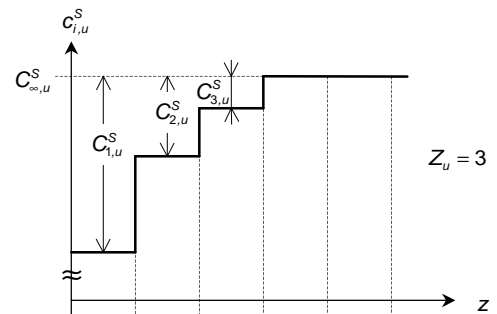


Fig. 3. Illustration of the time dependent start-up costs.

active if the actual interval is nor the start-up nor the shut-down interval.

$$p_{i,u}^+ \geq p_{i,u} - p_{i-1,u} - \Sigma_{i,u} \cdot P_{S_u} \quad (27)$$

$$p_{i,u}^- \geq p_{i-1,u} - p_{i,u} - \Psi_{i,u} \cdot P_{S_u} \quad (28)$$

Whereas  $\Sigma_{i,u}$  only flips to one if the unit is started because of the start-up costs,  $\Psi_{i,u}$  will be set to one whenever possible to deactivate (28) and to avoid the cost of a negative power gradient. The following equations ensure that only in the case of a shut-down  $\Psi_{i,u}$  will be one.

$$\Lambda_{i,u} + \Lambda_{i+1,u} \geq \Psi_{i,u} \quad (29)$$

$$1 - \Lambda_{i+1,u} \geq \Psi_{i,u} \quad (30)$$

The resulting costs of positive and negative power gradients can be formulated as

$$p_{i,u}^+ = \sum_{s=1}^{S_u^+} \pi_{i,u,s}^+ \cdot (P_{u,s+1}^+ - P_{u,s}^+) \quad (31)$$

$$c_{i,u}^+ = \sum_{s=1}^{S_u^+} \pi_{i,u,s}^+ \cdot (C_{u,s+1}^+ - C_{u,s}^+) \quad (32)$$

$$p_{i,u}^- = \sum_{s=1}^{S_u^-} \pi_{i,u,s}^- \cdot (P_{u,s+1}^- - P_{u,s}^-) \quad (33)$$

$$c_{i,u}^- = \sum_{s=1}^{S_u^-} \pi_{i,u,s}^- \cdot (C_{u,s+1}^- - C_{u,s}^-). \quad (34)$$

#### F. Supplying contracts

In the past, the coverage of industrial customer's load was treated as obligatory and, therefore, had been mathematically considered as an equality constraint setting all produced power at a time step equal to the demand. Through the liberalizing of the energy markets, the objective of operational planning changes from satisfying the customer's demand while minimizing the operational cost towards maximizing the profit. Apart from the costs for energy supply, this means consequently to consider the price the customer pays for being supplied. Three short examples should illustrate the problem.

First, innovative and more flexible supplying contracts should make economically use of the idle potential of industrial load management [3]. If, for example, some interruptible industrial processes can be scheduled and only costs are considered, the optimal schedule will surely contain all loads interrupted because this leads to minimum costs. Thus, the price the customer would have paid for the non-delivered energy has also to be taken into account. Second, time-dependent tariffs are in Germany commonly used for electricity to give financial incentives to shift peak load to off-peak hours. Industrial power systems are affected by this circumstance in two ways. Both purchasing and supplying contracts may contain time-dependent tariffs and, therefore, shifting the customer's load to off-peak hours has not to be necessarily economically better. Third, the possibility to realize spot energy transactions at a power exchange also shows the need to consider the energy prices.

A simple supplying contract without any possibilities of load management can be formulated as

$$p_{i,k} = P_{i,k} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot (P_{i,k,s+1} - P_{i,k,s}) \quad (35)$$

$$r_{i,k} = R_{i,k} \quad (36)$$

$$t_{i,k} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot (T_{i,k,s+1} - T_{i,k,s}). \quad (37)$$

To modify this contract towards an interruptible contract, (35) and (36) have to be modified to

$$p_{i,k} = P_{i,k} \cdot \Gamma_{i,k} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot (P_{i,k,s+1} - P_{i,k,s}) \quad (38)$$

$$r_{i,k} = R_{i,k} \cdot \Gamma_{i,k}. \quad (39)$$

In practice, interruptible contracts contain certain conditions to restrict the duration (40) and the frequency (41 – 42) of interruptions and the minimum supplying time without a interruption (43).

$$\sum_{j=i}^{i+I^D} \Gamma_{j,q} \geq 1 \quad (40)$$

$$\Gamma_{i-1,k} - \Gamma_{i,k} - \Omega_{i,k} \leq 0 \quad (41)$$

$$\sum_{i=1}^{I^F} \Omega_{i,k} \leq F_k \quad (42)$$

$$\sum_{j=i-I^D}^{i-1} \Gamma_{j,k} \geq \Omega_{i,k} \cdot D_k \quad (43)$$

In some cases, the underlying loads of a supplying contract show a so-called payback attitude which is illustrated in Fig. 4 [1]. The additional payback power can be described by a relation to the interrupted energy (Fig. 5).

$$\tilde{p}_{i,k}^{PB} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot \tilde{P}_{k,s}^{PB} \quad (44)$$

$$\tilde{e}_{i,k}^{PB} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot \tilde{E}_{k,s}^{PB} \quad (45)$$

$$\sum_{s=1}^{S_k} \gamma_{i,k,s} - \Gamma_{i,k} \cdot S_k \leq 0; \quad \{\lambda_{i,k,s}, \dots, \lambda_{i,k,S_k}\} \in SOS2 \quad (46)$$

For the mathematical model, the payback power is added to the right hand side of (38).

$$p_{i,k} = P_{i,k} \cdot \Gamma_{i,k} + p_{i,k}^{PB} \quad (47)$$

Even with payback, the total load must not be greater than a given limit, representing the technical background of the controllable load.

$$p_{i,k} \leq P_k^{max} \quad (48)$$

Now the interrupted energy can be calculated for each time interval by

$$e_{i,k}^{PB} = f_1 \cdot e_{i-1,k}^{PB} + f_2 \cdot P_i \cdot D_i \cdot (1 - \Gamma_{i,k}) - f_3 \cdot D_i \cdot \tilde{p}_{i,k}^{PB} \quad (49)$$

$$e_{0,k}^{PB} = e_{1,k}^{PB} = 0. \quad (50)$$

The factors  $f_1$ ,  $f_2$  and  $f_3$  could be used to adapt the model to the real technical conditions. If, for example,

$$f_1 = f_2 = f_3 = 1, \quad (51)$$

the payback energy is equal to the interrupted energy. The variables  $\tilde{e}_{i,k}^{PB}$  and  $e_{i,k}^{PB}$  must be equal only if the load is not interrupted. Otherwise it could arrive that too much payback energy is supplied, depending on the energy prices. If the load is interrupted,  $\tilde{e}_{i,k}^{PB}$  and  $\tilde{p}_{i,k}^{PB}$  are equal to zero anyway because of (46).

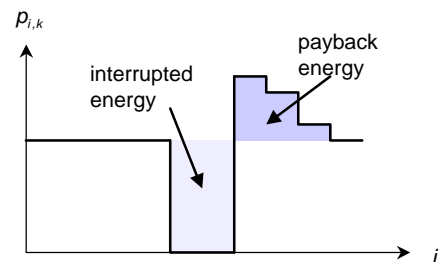


Fig. 4. Example of a load interruption with payback.

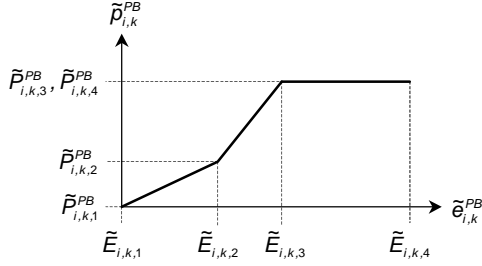


Fig. 5. Relation between payback power and payback energy.

$$\tilde{e}_{i,k}^{PB} \geq e_{i,k}^{PB} - (1 - \Gamma_{i,k}) \quad (52)$$

$$\tilde{e}_{i,k}^{PB} \leq e_{i,k}^{PB} \quad (53)$$

Equation (53) only is restrictive if the load is not interrupted and results together with (52) to the wanted relation between  $\tilde{e}_{i,k}^{PB}$  and  $e_{i,k}^{PB}$ .

A last but frequently found method of industrial load management concerns postponable processes. In these cases, the load pattern remains unchanged but the moment of starting the process can be controlled. Therefore, the following equations have to be formulated.

$$P_{i,k} = \sum_{j=0}^{J_k} \mu_{j,k} \cdot P_{i+j,k} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot (P_{i,k,s+1} - P_{i,k,s}) \quad (54)$$

$$r_{i,k} = \sum_{j=0}^{J_k} \mu_{j,k} \cdot R_{i+j,k} \quad (55)$$

$$\sum_{j=0}^J \mu_{i,k} = 1; \quad \{\mu_{0,k}, \dots, \mu_{J,k}\} \in SOS1 \quad (56)$$

$$t_{i,k} = \sum_{s=1}^{S_k} \gamma_{i,k,s} \cdot (T_{i,k,s+1} - T_{i,k,s}) \quad (57)$$

#### IV. SOLUTION ALGORITHM

In preceding works, it was shown that MILP is a well suited algorithm to solve particularly CHP-based power systems [4]. Moreover, the MILP fulfills exactly every restriction introduced and is able to find the global solution. Nevertheless, compared to other algorithms the MILP may be rather slow. With regard to the complexity of the problem presented in this paper, big effort has to be taken to assure reasonable computational times.

The standard MILP algorithm first computes the primal solution that serves as the absolute lower limit of the objective function value and is used in the subsequent branch-and-bound algorithm (B&B) to find a feasible integer solution [2]. For the practical problems considered in this paper, the computational time for computing the primal solution is rather short and can be neglected compared to the time the standard B&B-algorithm (sB&B) needs to calculate the global solution. By investigating a variety of optimizations with sB&B, the main drawbacks of sB&B were identified as follows. First, several objective function values differ only slightly from each other, so valuable computation time is wasted with respect to the obtained results. Second, the last integer solution found cannot be treated as the global solution unless the whole branching tree contains no subproblems. Particularly the time between finding the last solution and the proof that there are no better solutions can be unacceptable long. Meanwhile, the user simply has to wait and gets no information about the progress of the optimization.

Therefore, the new Dynamic Search Strategy (DSS) concentrates on acceleration of the sB&B.

The steps of DSS are illustrated in Fig. 6. In the first step, the algorithm tries to find a first integer solution with the primal objective function value as a natural lower limit (Fig. 6, a). Then, the upper limit given by the objective function value of the integer solution is decreased to obtain a given minimum improvement to the next integer solution (Fig. 6, b).

$$Obj(UL_1) = (1 - G) \cdot Obj(IS_1) \quad (58)$$

$$Q_1 = (IS_1 - PS) / IS_1 \quad (59)$$

The basic idea of the DSS in contrast to sB&B is not only to search integer solutions, i.e. to decrease the upper limit like the sB&B (search from above, SFA), but also to increase the lower limit (search from below, SFB). This is used to decrease the accuracy of integer solutions which becomes then

$$Q_1 = (IS_1 - LL_1) / IS_1 \quad (60)$$

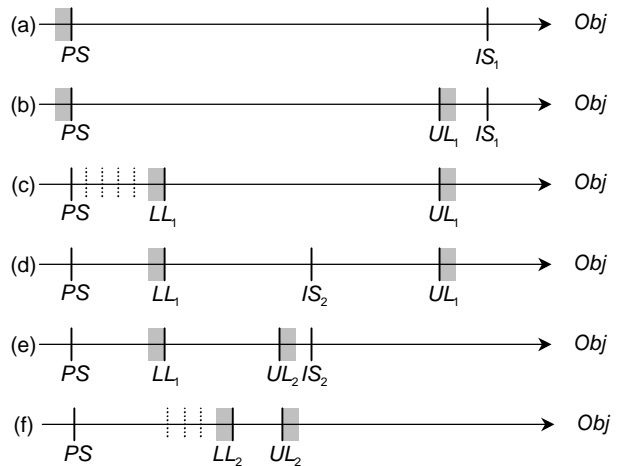
Increasing the lower limit is achieved by successively defining an actual limit only a little greater than the primal objective function value. Then, all subproblems are branched until no subproblem's objective function value is lower or equal than the actual limit. Generally, it is expected that no integer solution can be found between the existing and the actual lower limit, otherwise the DSS stops and the global solution is found.

The DSS changes alternately between SFA (to find better integer solutions) and SFB (to decrease the accuracy) until the accuracy remains under a given value. The changes are controlled by setting the computation time  $D^{SFA}$  for SFA and  $D^{SFA}$  for SFB, respectively:

$$D^{SFA} = f_4 \cdot D^{DSS} \cdot \cos(\pi \cdot f_5 \cdot x) \quad (61)$$

$$D^{SFB} = f_4 \cdot D^{DSS} - D^{SFA} \quad (62)$$

The time  $D^{DSS}$  is set by the time needed calculating the first integer solution, whereas empirical values of  $f_4 = 2$  and  $f_5 = 0,05$  show best results. After the first integer solution found, (61) and (62) force the algorithm to use twice the time needed for the first solution to find a second one. If



PS: primal solution  $UL_q$ : upper limit q  $Obj$ : objective value  
 $IS_q$ : integer solution q  $LL_q$ : lower limit q

Fig. 6. Exemplary sequence of the DSS.

this is not successful (Fig. 6, b), then  $x$  is decreased by one and the algorithm changes to SFB for a short time (Fig. 6, c). Then SFA finds a second integer solution (Fig. 6, d), but no third (Fig. 6, e), so SFB increases again the lower limit. Supposing that there should be no further integer solution,  $x$  will increase steadily. Thus, the algorithm will decrease the time available for SFA while increasing the time for SFB and vice versa (if there should be a further solution, anyway,  $x$  will be reset to zero).

As an additional feature, DSS does not only provide better information about the progress of the optimization but also calculates the marginal prices and costs per time from the shadow prices of the simplex algorithm for each contract.

## V. NUMERICAL EXAMPLE

To demonstrate the efficiency of the proposed DSS algorithm solving the mathematical model presented in this paper, a numerical example will be discussed below. The model of the considered power system consists of 81 converting units, 12 purchasing contracts, 62 balancing nodes, and 20 supplying contracts. The resulting mathematical optimization problems contains 4338 variables with 1440 of them binary and 144 SOS2, 5227 restrictions and a coefficient matrix containing 21538 non-zero elements. For the calculation of the optimization problem, a DEC Alpha AXP 500/500 was used. Two calculations were carried out to compare DSS to sB&B.

The results are presented in Fig. 7 where the output data of the two algorithms over time is shown. While sB&B only issues the objective value when a new integer solution could be found, DSS provides additionally the actual upper and lower limits and the resulting accuracy.

Fig. 7 shows that DSS is able to calculate better integer solutions in less time than sB&B. Moreover, the computed accuracy keeps the power system operator waiting for the optimal schedule up to date about the progress of the optimization which could be valuable for the operator's acceptance of the obtained results.

## VI. CONCLUSIONS

This paper reports on a new mathematical model calculating the optimal unit commitment and economic dispatch of industrial enterprises with its own energy supply system, mostly with CHP-plants. To yield the most synergetic effects, both the marketplace and the demand of disposable industrial processes are taken into account. Therefore, equations are developed to mathematically describe the different types of conversion units, purchasing and supplying contracts. Because of the complexity of the formulated mathematical model and, thus, of the optimization problem, the new Dynamic Search Strategy (DSS) based on Mixed Integer Linear Programming (MILP) is presented. The shown results comparing standard branch-and-bound algorithm (sB&B) to DSS demonstrate that with DSS, a considerable acceleration of calculation time can be achieved. Furthermore, the marginal prices and costs per time can be calculated and analyzed to support economic decisions.

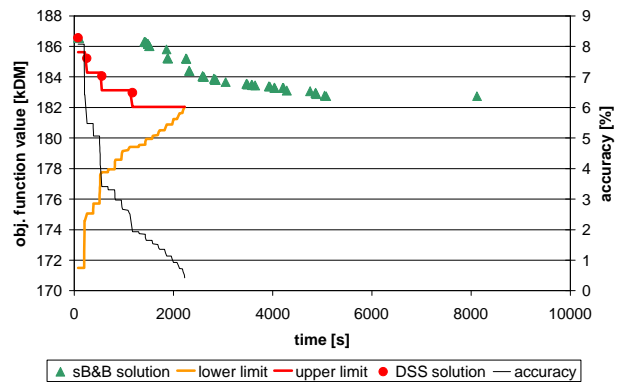


Fig. 7. Numerical example of the DSS.

## VII. REFERENCES

- [1] R. A. Abdo, G. Lokken and R. F. Bischke, "Load Management Implementations: Decisions, Opportunities and Operation", *IEEE Trans. Power Apparatus and Systems*, vol. 101, no. 10, October 1982, pp. 3902-3906.
- [2] E. M. L. Beale, *Introduction to Optimization*, Wiley, Chichester, 1988.
- [3] C. O. Björk, *Industrial Load Management - Theory, Practice and Simulations*, Elsevier Science Publishers, Amsterdam, 1989.
- [4] H. Braun, K. W. Edwin and R. Schroeder, "Rechnerunterstützte Optimierung der Lastaufteilung für Chemieunternehmen", *Energiewirtschaftliche Tagesfragen*, vol. 40, no. 10, October 1990, pp. 693-697.
- [5] R. Pfeiffer and J. Verstege, "Committing and Dispatching Power Units and Storage Devices in Cogeneration Systems with Renewable Energy Sources", *Proc. of the 4th International Conference on Power System Control and Management*, pp. 21-25, April 16-18 1996, London.
- [6] T. Seeger and J. Verstege, "Short Term Scheduling in Cogeneration Systems", *Proc. of the 17th Power Industry Computer Application Conference*, pp. 106-112, May 7-10 1991, Baltimore.

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