
Optimal Planning and Management of Hybrid Vehicles in Smart Grid

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Abstract

Smart grid can be expressed as a combination of power network substructures with an extensive telecommunication network which is able to provide a two-way communication and use of advanced sensors in order improve efficiency, system reliability, transport security, and power consumption. Loads in this network are divided into two groups, linear and non-linear. The majority of these loads on the network, such as rectifiers, electric vehicles are non-linear. The non-linear loads can cause odd harmonics in the network and can damage transformers. In this article, management and planning of hybrid vehicles for total harmonic index reduction and also annual cost reduction has been considered.

Keywords: *Smart grid, Electric vehicles, Harmonic distortion.*

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1 Introduction

In electric engineering industry, voltage regulation and power loss reduction is accessible by a combination of stable and unstable capacitors rather than voltage regulators [1-4]. Now days, smart grid technology is exposed to rapid changes for modernizing smart grids to deal with increasing energy demand and also for increasing electric vehicles in the future [5]. This article addresses the major problem of management of power quality for smart distribution grids and a load management strategy based on installing capacitors is proposed for total harmonic distortion reduction in power distribution and transformers and also annual cost reduction. Smart grids provide an excellent occasion for better power quality management and keep acceptable total harmonic distortion in power networks [6].

The integration of computer automation in all levels enables smart grid to improve and adapt itself to new situation fast also it can improve system reliability and safety and efficiency manage the continuation of services to customers in an optimal and economic way [7].

So far, many researches about installing capacitors and electric vehicles in smart grids have been done to improve system load profile. In installing capacitors, numerical algorithm is presented for optimization of parallel capacitor size in distribution system with non-sinusoidal voltage in which values such as effective voltage and total harmonic distortion is used for calculation of capacitors capacity [8]. In next researches, this investigation has been done by using particle swarm optimization algorithm and found similar results [9]. One important problem, in both smart grids and traditional grids is the availability of power quality issue and particularly the availability of harmonics. Different researches have been done on houses loads harmonic and play in electric vehicles [10-16].

2 Harmonic load flow

In real distribution systems, loss reduction and voltage regulation is doable by using stable capacitors or having keying ability and also voltage regulator but the combination of this situation with non-sinusoidal voltage of substation, makes difficult the determination of the capacity of capacitors. Therefore, for simplification this problem, can consider stable capacitors in system main frequency and harmonic frequencies.

2.1 Modeling system in harmonic and basic frequencies

Figure 1 shows radial distribution n bus system which is a usual bus i includes one load and a group of electric vehicles and a parallel capacitor.

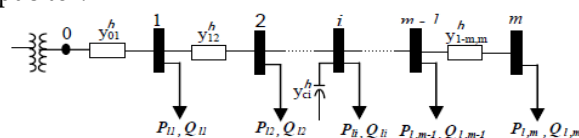


Figure1: the diagram of linear feeder of radial smart system in basic frequency

Every variable used in this figure shows a specific quantity in the following the meaning of which is presented:

$y_{i,i+1}^1 = 1/(R_{i,i+1} + X_{i,i+1})$: Admittance line between bus i and $i+1$

$R_{i,i+1}$: Feeder resistance between bus i and $i+1$

$X_{i,i+1}$: Self-reactance between bus i and $i+1$

P_{li} : Active load bus i

Q_{li} : Reactive load bus i

y_{ci}^1 : Admittance of parallel capacitor bus i

If i th bus is only a capacitor, P_{li} and Q_{li} are set to zero. Since reactive power produced depends on voltage used for that by parallel capacitors, parallel capacitors were presented as constant admittance instead of constant reactive power generators. Quantity and bus voltage phase angle obtained by solving mismatch equations equation (1).

$$\begin{aligned}
 Pli - \sum_{j=i-1}^{i+1} |Y_{ji}^1| |V_j^1| |V_i^1| \cos(\delta_i^1 - \delta_j^1 - \theta_{ji}^1) &= 0 \\
 Qli - \sum_{j=i-1}^{i+1} |Y_{ji}^1| |V_j^1| |V_i^1| \sin(\delta_i^1 - \delta_j^1 - \theta_{ji}^1) &= 0 \\
 i = 1, 2, \dots, m
 \end{aligned}
 \tag{1}$$

Which in equation (1) quantity of Y_{ji}^1 calculated by using equation (2).

$$Y_{ji}^1 = |Y_{ji}^1| \angle \theta_{ji}^1 = \begin{cases} -y_{ji}^1, & \text{if } j \neq i, \\ y_{i-1,i}^1 + y_{i+1,i}^1 + y_{ci}^1, & \text{if } j = i, \end{cases}
 \tag{2}$$

$V_i^1 (= |V_i^1| \angle \delta_i^1)$ Is basic voltage in bus i . If no capacitor is in bus i , y_{ci}^1 isn't appeared in equations above. For solving equations of mismatch, it may use standard way of Newton Raphson load flow. In power frequency, power loss between buses i and $i+1$ is calculated by equation (3).

$$P_{loss(i,i+1)}^1 = R_{i,i+1} \left[|V_{i+1}^1 - V_i^1| |y_{i,i+1}^1| \right]^2
 \tag{3}$$

2.2 The model of system distribution in harmonic frequencies

In higher frequencies of distribution lines models of parallel capacitors, are available but mixed features of consumer loads are not known. Therefore it is recommended that are model of generalized linear load is used which includes are parallel resistant with an inductance which the direction of reactive power and reactive are selected in basic frequency. If skin-effect is ignored in higher frequencies, the n th resultant of load admittance of harmonic frequencies parallel capacitor admittance and feeder admittance are calculated by equations (4), (5), (6) respectively:

$$y_{ii}^n = \frac{P_{li}^n}{|V_i^1|^2} - j \frac{Q_{li}^n}{n |V_i^1|^2}
 \tag{4}$$

$$y_{ci}^n = n y_{ci}^1
 \tag{5}$$

$$y_{i,i+1}^n = \frac{1}{R_{i,i+1} + jnX_{i,i+1}}
 \tag{6}$$

The equivalent torque distribution system in n th harmonic is shown in figure 2.

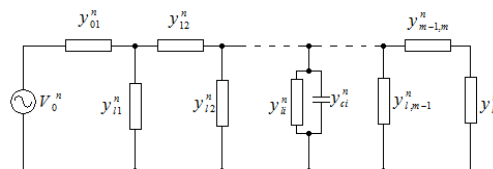


Figure2: The equivalent torque distribution system in n th harmonic frequency

Harmonic voltages are calculated by linear equations system which is shown in equation (7).

$$\begin{bmatrix} Y_{11}^n & Y_{12}^n & 0 & \dots & 0 \\ Y_{21}^n & Y_{22}^n & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & Y_{n-1,n-1}^n & Y_{n-1,n}^n & V_{n-1}^n \\ 0 & \dots & 0 & Y_{n-1,n}^n & Y_{nn}^n \end{bmatrix} \begin{bmatrix} V_1^n \\ V_2^n \\ \dots \\ V_{n-1}^n \\ V_n^n \end{bmatrix} = \begin{bmatrix} y_{11}^n V_0^n \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix}
 \tag{7}$$

Traditionally, power system operators provide quality and storage reliability for consumers by contact voltage bus in permitted areas. Harmonic, rapid increase in using switching tools solid-state resulted in remarkable wave in recent decades.

So, the quality of power is measured by the level of distortion and storage wave.

In each bus i , voltage rms is calculated by equation (8).

$$|V_i| = \sqrt{\sum_{n=1}^N |V_i^n|^2},
 \tag{8}$$

In equation (8), N is the maximum levels of harmonic which is considered and should be in acceptable are the total harmonic distortion of voltage wave which is presented by equation (9); generally it is necessary not to be higher than accepted maximum.

$$THD_i (\%) = \frac{100}{|V_i^1|} \sqrt{\sum_{n=1}^N |V_i^n|^2},
 \tag{9}$$

Equations (8), (9), are used in limitation of optimization problems. Next step is the optimization of capacitor's capacity in determined bus of radial distribution system for less reduction total loss reduction in all system frequencies are calculated from equation (10).

$$P_{loss} = \sum_{n=1}^N \left(\sum_{i=0}^{m-1} P_{loss(i,i+1)}^n \right)
 \tag{10}$$

In real stat, the capacitor's capacity has standard values and cannot be included every values. The acceptable maximum of capacitors is obtained by relation (11).

$$Q_{max}^c = LQ_0^c \tag{11}$$

When L is integer, the location of parallel capacitor, there are L selections for capacitor capacity. The goal function in optimization algorithm is calculated as equation (12) regarding power loss and capacitor capacity.

$$F = K^p P_{loss} + \sum_{j=1}^k K_j^c Q_j^c \tag{12}$$

Where K^p is annual cost for every unit of loss and its value is considered 168 \$/KV according to reference [8], subtitle j in equation (12) implying buses include capacitors in addition borders condition should be considered.

$$THD_i \leq THD_{max} \quad i = 1, 2, \dots, m \tag{13}$$

Where V_{min} , V_{max} are acceptable maximum and minimum value for voltage for every bus. Also, high level condition for THD value is considered as follow in optimization algorithm.

3 Vehicle management and planning

3.1 Charging vehicle management and planning

The network in this study is 9-bus IEEE network. This network is radial which resistance values and inductance are presented in appendix (a). Also 24 hours active and reactive power 9-bus is gathered in appendix (b).

In this study MATLAB calculating software is used in which equations related load flow is studied first that mentioned in equation (1) and (2). In this software, for obtaining voltage and angle, every bus is presented.

In next step network harmonic voltage are calculated by using equations (4) and (7). In this study first, fifth and seventh harmonics are studies which fifth and seventh harmonic voltage are 3% and 4% of first harmonic. After obtaining harmonic voltages of every bus, for

obtaining total voltage for each bus, we use equation (8). Next step is the calculation of maximum harmonic distortion (THD) which equation (9) is used.

For obtaining total system loss, equation (10) is used. Optimization algorithm is as two-goal algorithm which the first goal is to minimize maximum THD, equation (9) and to minimize annual cost according to equation (12). Equation (12) consists of two parts. The first part is related to annual loss cost which system total loss multiply by annual cost equals to 168 \$/Kwh. The second part is related to installing capacitors cost, which is obtained by the total size of capacitors which are installed in buses multiply by cost of each capacitor which is presented in appendix (c). From these two goal function, NSGA-II algorithm is applied for the calculation of the best capacitor size and also the best time for charging vehicles and finding optimal parking.

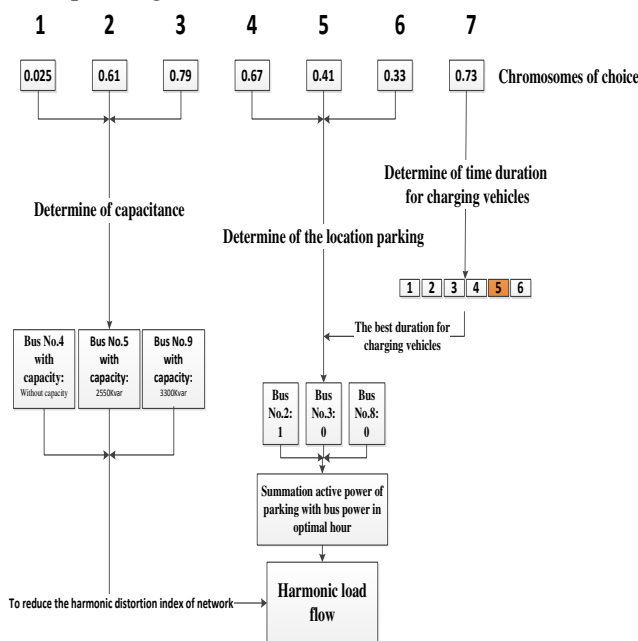


Figure3: Optimization algorithm for capacitor determination location of parking and time duration.

According to algorithm in figure (3), 7 chromosomes are used. The minimum level of each chromosome is zero and maximum level is one. The three first chromosomes are considered for determining capacitor size and the next chromosome are considered for determining the location of parking and the last chromosome is related to duration optimal time for charging vehicles. Capacitors should be installed in buses

4, 5 and 9. Also parking's are installed 2, 3, and 8. Every parking has the capacity of 10 electric vehicles which these vehicles come into the parking for charging as a group and after 4 hours come out from parking. The capacity of charge for each vehicles is 4.3 kW which considering the battery efficiency equals to 90%, the capacity of vehicles is considered 4.8 KW in this simulation. As a whole, every parking has the capacity of 48 KW. After obtaining chromosome, we should follow figure (3) for charging them into numbers. As we see in figure (3), the first three boxes are related to capacitors which for changing into capacitor size, at first it multiplied by 27.9 and then changed them into integer numbers, by using *floor* order in MATLAB and multiplied by 150 in order to obtain capacitor size. (appendix c) $Capacity(1:3)=floor(x(1:3)*27.9)$

As mentioned before, boxes four to six is related to available parking's in boxes. These numbers should be binary, means if the number is 1, it means, parking can change vehicle, and if the number is zero, it means it is not allowed to charge. For this goal, we should use *round* order to change the number chain into binary.

$$parking(4:6)=round(x(4:6))$$

And the seventh box is related to duration of time for changing, which for changing it we use following order.

$$floor(x(7)*5.9)+1$$

Obtained numbers are from 1 to 6 which everyone shows the duration of time as table (1).

Table 1. Four hours time duration for charging vehicles.

Duration	Hours
1	1-2-3-4
2	5-6-7-8
3	9-10-11-12
4	13-14-15-16
5	17-18-19-20
6	21-22-23-24

For optimization algorithm NSGA-II, the number of frequency is considered 50 and the number of population is considered 50. During 6 time performance, the number of right answers obtained in every phase is 50 which as a whole, 300 answers are obtained from NSGA-II. Obtained answers have similarities which minimizes to 50 after clearing answers (figure 4). Surly, we cannot say from these obtained

answers through this group we can use fuzzy inference system. According to figure (4), 50 options are for cost and THD maximum. From which minimum and maximum values were calculated.

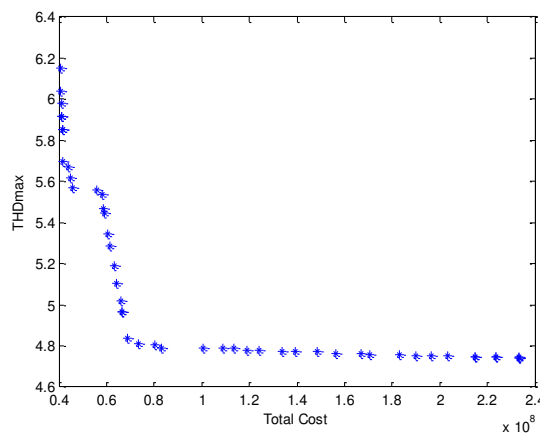


Figure4: Pareto Front annual cost and maximum harmonic distortion

So according to fuzzy inference above, the best option is option 43 which can be investigated carefully in table 2.

Table 2. The most optimal answer from optimization algorithm NSGA-II

pop	C4	C5	C9	P2	P3	P8	time	cost	THDmax
43	0	2250	3300	1	1	0	2	1.2E+8	4.784

Maximum harmonic distortion in 43 people is 4.7449 and annual cost is 12E+7 and time duration is 2 for charging vehicles, that is, at 5, 6, 7, and 8 o'clock which network load is not peak, vehicles can be charged for four hours in installed parking's in buses 2 and 3 and also bus 9 isn't allowed to charge a vehicle for loss reduction 8 distortions caused by vehicles, we should install capacitors in size 2250 and 3300 KW in buses 5 and 9 and installing capacitor at bus 4 is not economic.

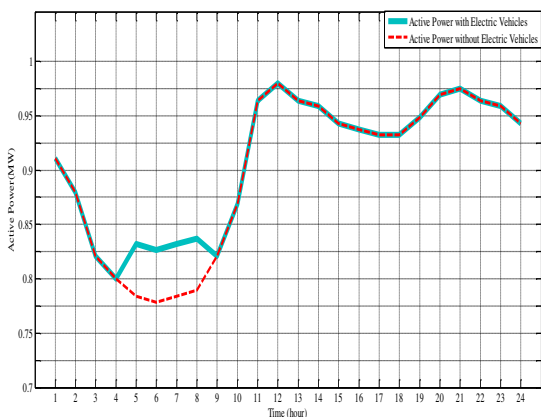


Figure5: 24-hour active power bus 2 with and without electric vehicle.

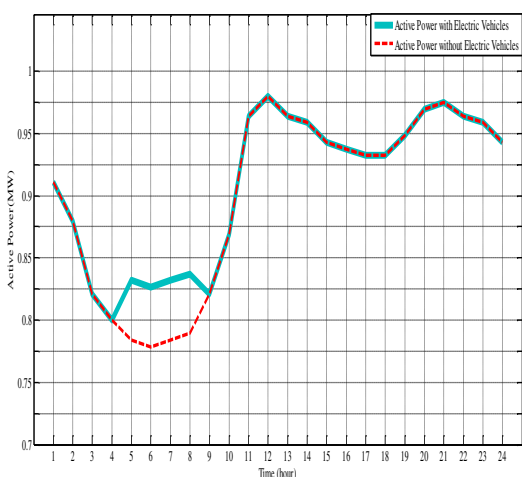


Figure6: 24-hour active power bus 3 with and without electric vehicle

Regarding above information, figure 5 and 6 which is related to bus 2 and 3, it is clear that at low load hours, (5 to 8 in the morning) vehicles are charged and active power diagram for buses increased and doesn't hurt the system.

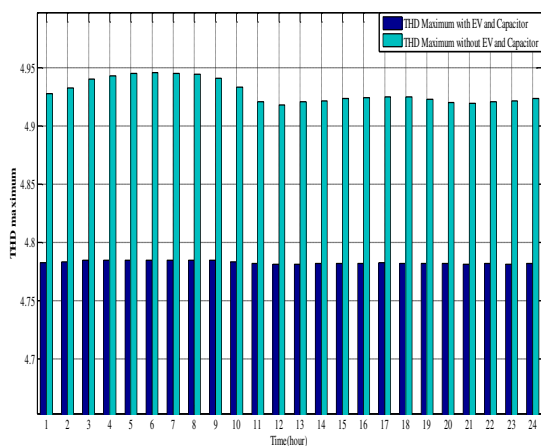


Figure7: 24-hour maximum harmonic distortion index

In normal mode, system has high THD which is unevenly. For example, maximum harmonic distortion occurred at 5 o'clock. When vehicles are charging and capacitors are installed in network, maximum THD has reached its value to 4/77% and as we can see in figure 7, it has equal THD value during a day.

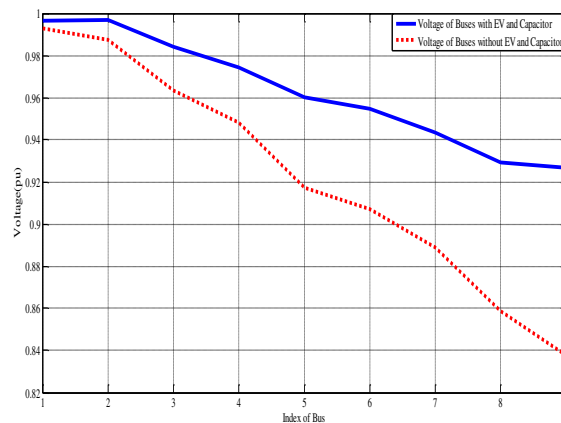


Figure8: Profile voltage 9-bus at peak hour

Peak hour system is 12 pm at this system has voltage drop and high losses. As we can see in figure8 voltage profile improved after installing capacitors and charging vehicles and improved minimum voltage from 0.84 per unit to 0.924 per unit and voltage profile has the smother mode to state before installing capacitors and charging vehicles.

3.2 Management and planning of charging electric vehicles and power injection (9 buses network)

In this section, connecting the cars to network for injecting and also for load for charging is considered. Its simulation is like previous part, but for obtaining annual cost we should use equation (15).

$$F = K^p P_{loss} + K^{p.injection} P_{injection} + \sum_{j=1}^k K_j^c Q_j^c \quad (15)$$

$K^{p.injection}$ Is the injection annual cost which is considered 200 \$/KW and $P_{injection}$ is electric vehicles injective power to network and its algorithm is like figure 9.

In this algorithm, we should consider this condition that vehicles cannot be charged and inject power at the same time.

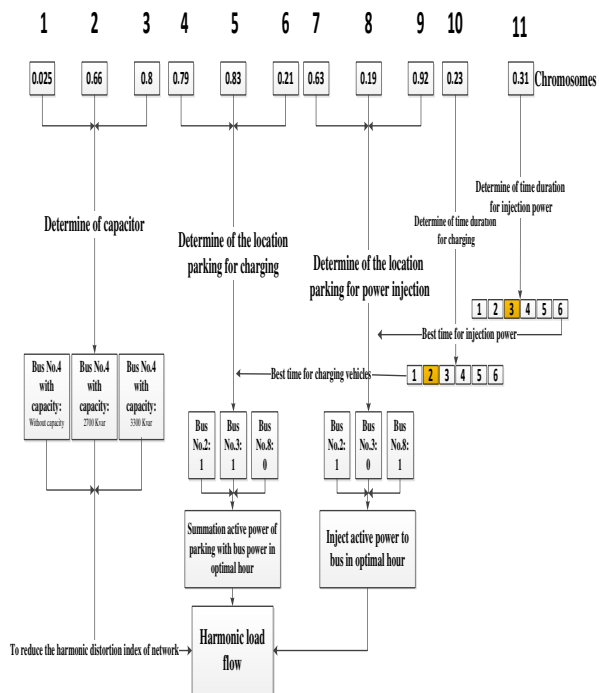


Figure9: Optimization algorithm for determining capacitor, the location of parking for charging and power injection and their time duration

In this simulation, the locations of capacitors are located at buses 4, 5 and 9. Also parking are at buses 2, 3 and 8. Electric vehicles in addition to charging, can inject power to network.

This algorithm consists of 11 chromosome, which the first 3 chromosome are related to determining capacitor, and chromosome 4 to 6 are related to determining parking for charging vehicles and chromosomes 7 to 9 are related to determining parking for injecting power to network. Also for determining time duration for charging and discharging vehicles chromosomes 10 and 11 are respectively.

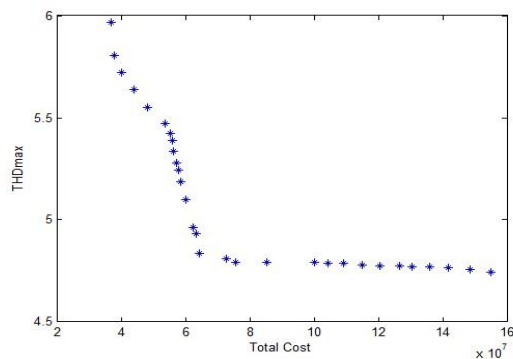


Figure10:Pareto Front annual cost and harmonic distortion maximum (charge and power injection)

For changing code chain we use knowledge in previous part. For optimization algorithm NSGA-II the number of frequencies is considered 50 and the number of population is considered 50. During three times performance, the number of obtained answers in each phase is 50 which as a whole 150 answers were obtained. Obtained answers have similarities which minimizes to 50 after deleting some answers (figure 10).

For finding best answer we use fuzzy inference. In this phase, maximum harmonic distortion and annual cost are considered 4/8 and 12E+7 respectively. So, according to fuzzy inference the best option is option 19 which is investigated in table 3 carefully.

Table 3: the most optimal answer from algorithm NSGA-II

(charge state and power injection)

pop	C4	C5	C9	P2
19	0	2700	3300	1
P3	P8	D2	D3	D8
1	0	1	0	1
time	Time-d	cost	THD	
2	3	1E+8	4.79	

Maximum harmonic distortion is 4.79 and annual cost is 100124011. In time duration 2, from 5 to 8 o'clock, electric vehicles can refer to parking's located in buses 2 and 3 for charging and in time duration 3, from 9 to 12, can refer to parking's located in buses 2 and 8 for injecting their extra power. Also, for improving voltage profile, we should install 2700 and 3300 KW capacitor in buses 5 and 9.

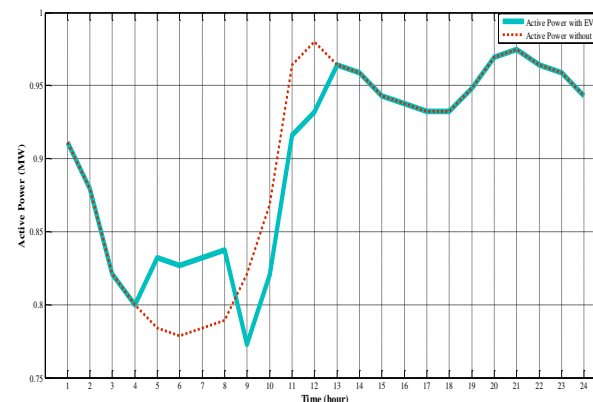


Figure11: 24-hours bus 2 active power before and after electric vehicles in network (injection and charge)

In hours when the load is low, vehicles went to parking for charging and when the load is at its peak (12 o'clock) which its duration is at 9 to 12, vehicles can't be charged, but vehicles should inject to the network which have extra active power.

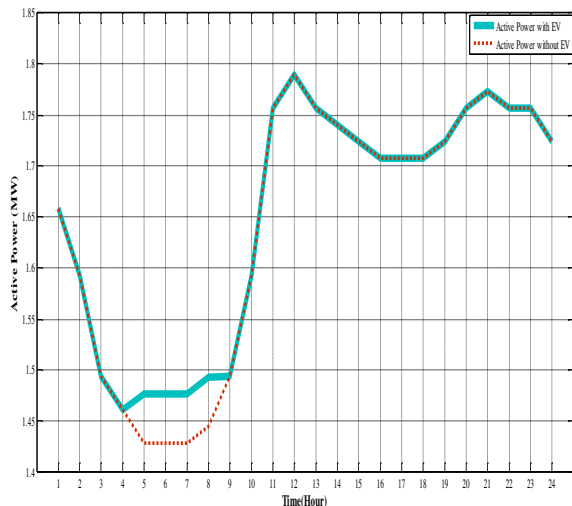


Figure12:24-hours power active bus 3 before and after electric vehicles in network.

In bus 3, electric vehicles can go to parking for charging but there is no need to inject power for loss reduction and harmonic distortion when the load is at its peak. As you can see in figure 12, vehicles were charged at the time when load is low.

3.3 Charging electric vehicles management and planning and inject power into 85 buses distribution network.

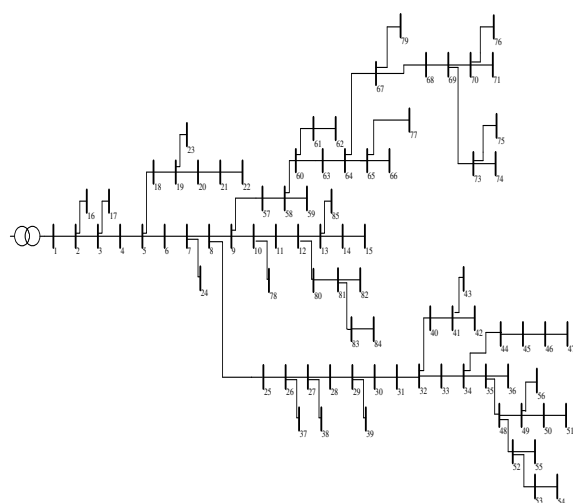


Figure13:the linear plan of 85 buses distribution network.

Figure 13 is a standard 85 buses network (modified [17]) which in this study is simulated. This network consists of several linear and nonlinear loads. This network has five capacitors which should be installed at buses 8, 11, 31, 36, 63. Parking's are located at buses 6, 14, 28, 33, 60, 69 for charging and power injection.

In optimization NSGA-II, 19 chromosome are used which the five first chromosome relate to capacitors and 6 to 11 to parking for charging the vehicles chromosome 12 to time duration for charging vehicles and chromosomes 13 to 18 are used for power injection parking and chromosome 19 is used for time duration for injecting power for charging string code, we use previous sections. The best choice can be seen in table 4.

Table 4: The most optimal answer from algorithm NSGA-II (Charging state and 85 buses power injection system)

C8	C11	C31	C36	C63
2400	900	1650	1200	2700
P6	P14	P28	P33	P60
0	1	1	0	0
P69	time	P6	P14	P28
0	2	1	0	0
P33	P60	P69	Time-d	
0	1	0	3	

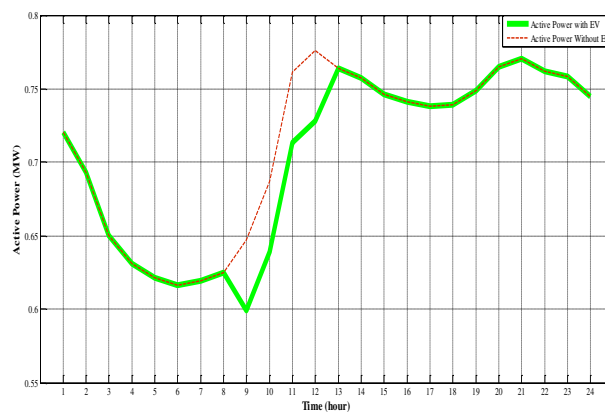


Figure14: 24- hours active power for bus 6 after and before simulation (85 buses).

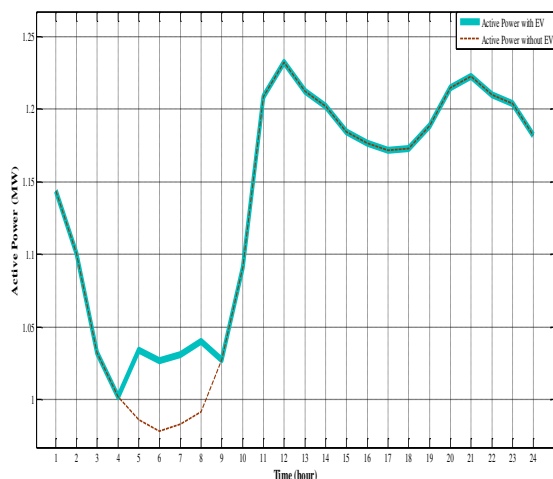


Figure15: 24- hours active power for bus 28 after and before simulation (85 buses).

4 Conclusions

Result obtained from this study can be presented as follow:

In load flow, harmonic voltage can cause power loss increase.

By increase limitation on total harmonic distortion, economic profit decreases in short term, but due to the network isn't under high harmonic pressure, and less damage hurts network devices, we can gain more economical profit in long term.

Regarding voltage control feature by capacitors network voltage profile can be improved after locating capacitors.

Capacitors can cause power loss decrease at peak load hours. When the load is load, it can, how reverse effect.

By connecting vehicle to network and injecting harmonic resulted from charging this vehicles profile voltage drop can be seen which can be improved after locating capacitors.

By connecting vehicles, power loss increases, which this increase is remarkable in peak load hours, and by using algorithm NSGA-II, the best time for charging these vehicles is considered. It's clear the best time for charging this vehicles non-peak time.

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Table (Appendix B): 24-hour values of active power of 9 buses

H	B1	B2	B3	B4	B5	B6	B7	B8	B9
1	1766	942.9	1724	1535	1549	749	1104	938	1570
2	1708	911.1	1658	1484	1496	725	1067	910	1523
3	1642	879.3	1593	1429	1444	697	1027	875	1464
4	1539	821	1494	1337	1348	654	964	819	1371
5	1495	799.8	1461	1301	1314	635	935	791	1324
6	1473	784	1428	1279	1288	625	921	784	1312
7	1458	778.7	1428	1268	1279	620	913	777	1300
8	1466	784	1428	1275	1288	623	918	777	1300
9	1480	789.2	1445	1286	1296	629	926	784	1312
10	1532	821	1494	1334	1348	651	959	812	1359
11	1627	868.7	1593	1414	1427	691	1019	868	1453
12	1803	964.1	1757	1568	1583	766	1129	812	1605
13	1840	980	1790	1598	1610	781	1150	868	1640
14	1810	964.1	1757	1572	1583	768	1131	959	1605
15	1796	958.8	1740	1561	1575	762	1122	952	1593
16	1766	942.9	1724	1535	1549	751	1106	938	1570
17	1752	937.6	1707	1524	1540	746	1099	931	1558
18	1752	932.3	1707	1521	1531	742	1094	931	1558
19	1752	932.3	1707	1521	1531	743	1095	931	1558
20	1774	948.2	1724	1543	1557	753	1110	945	1581
21	1810	969.4	1757	1576	1592	770	1134	966	1617
22	1825	974.7	1773	1587	1601	775	1141	973	1628
23	1803	964.1	1757	1568	1583	766	1129	959	1605
24	1796	958.8	1757	1561	1575	763	1124	952	1593

APPENDIX

Table (Appendix A): ohmic values and reactance on buses.

From i	To i+1	$R_{i,i+1} (\Omega)$	$X_{i,i+1} (\Omega)$
0	1	0.1233	0.4127
1	2	0.0140	0.6051
2	3	0.7463	1.2050
3	4	0.6984	0.6084
4	5	1.9831	1.7276
5	6	0.9053	0.7886
6	7	2.0552	1.1640
7	8	4.7953	2.7160
8	9	5.3434	3.0264

Table (Appendix B): 24-hour values of reactive power of 9 buses

H	B1	B2	B3	B4	B5	B6	B7	B8	B9
1	441	328	432	1764	579	106	58	124	191
2	428	318	418	1712	559	102	56	120	185
3	412	305	396	1642	538	98	54	116	179
4	387	285	375	1543	503	92	50	109	168
5	374	278	361	1496	490	89	49	105	162
6	367	273	361	1473	483	88	48	104	160
7	364	270	354	1462	476	87	48	102	157
8	367	273	354	1467	483	88	48	104	160
9	371	275	361	1485	483	89	48	105	162
10	383	285	375	1537	503	92	50	108	166
11	406	303	396	1630	531	97	53	115	177
12	450	335	439	1805	593	108	59	127	196
13	460	340	446	1840	600	110	60	130	200
14	454	335	439	1811	593	108	59	127	196
15	447	333	439	1793	586	107	58	126	194
16	441	328	432	1770	579	106	58	124	191
17	438	325	425	1758	572	105	57	124	191
18	438	325	425	1753	572	105	57	123	189
19	438	325	425	1753	572	105	57	123	189
20	444	328	432	1776	579	106	58	124	191
21	454	335	439	1817	593	108	59	127	196
22	457	338	446	1828	600	109	60	129	198
23	450	335	439	1805	593	108	59	127	196
24	450	333	439	1799	586	107	59	127	196

Table (Appendix C): the size and cost of capacitors.

j	1	2	3	4	5	6	7
Q _j (kvar)	150	300	450	600	750	900	1050
K _j (\$/kvar)	0.500	0.350	0.253	0.220	0.276	0.183	0.228
j	8	9	10	11	12	13	14
Q _j (kvar)	1200	1350	1500	1650	1800	1950	2100
K _j (\$/kvar)	0.170	0.207	0.201	0.193	0.187	0.211	0.176
j	15	16	17	18	19	20	21
Q _j (kvar)	2250	2400	2550	2700	2850	3000	3150
K _j (\$/kvar)	0.197	0.170	0.189	0.187	0.183	0.180	0.195
j	22	23	24	25	26	27	
Q _j (kvar)	3300	3450	3600	3750	3900	4050	
K _j (\$/kvar)	0.174	0.188	0.170	0.183	0.182	0.179	