Optimal Planning and Scheduling of Offshore Oil Field Infrastructure Investment and Operations

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A multiperiod mixed-integer linear programming (MILP) model formulation is presented for the planning and scheduling of investment and operation in offshore oil field facilities. The formulation employs a general objective function that optimizes a selected economic indicator (e.g., net present value). For a given planning horizon, the decision variables in the model are the choice of reservoirs to develop, selection from among candidate well sites, the well drilling and platform installation schedule, capacities of well and production platforms, and the fluid production rates from wells for each time period. The formulation incorporates the nonlinear reservoir performance, surface pressure constraints, and drilling rig resource constraints. The resulting MILP model contains several thousand binary variables and is intractable using a full space branch and bound technique. A sequential decomposition strategy using aggregation of time periods and wells, followed by successive disaggregation, is proposed. Two examples are presented to illustrate the performance of the algorithm.

1. Introduction

The scheduling of well and facility operations is a very relevant problem in offshore oil field development. The problem is characterized by a long planning horizon (typically 10 years) and a large number of choices of platforms, wells, and fields and their interconnecting pipeline infrastructure. Resource constraints such as availability of the drilling rigs make the requirement for proper scheduling more imperative to utilize resources efficiently. The sequencing of installation of well and production platforms is essential to ensure their availability before drilling wells. The operational design of the well and production platforms, and the time of installation, are critical as they involve significant investment costs, motivating the need to optimize these decisions to maximize the return on investment. Thus, oil field development represents a complex and expensive undertaking in the oil industry.

Since the advent of linear programming, several works on oil field development have been published. The problems studied include facility location and allocation, production planning, and scheduling. Most of the published work address these problems individually (as discussed in a later section) and are greatly limited in their scope of applicability. This is due to the size of the resulting formulations and their combinatorial complexity. As a result, most problems solved to date are either for single or few reservoirs, or for very few time periods. In addition, problems addressed so far have either assumed fixed production profiles (heuristically determined) or are limited to evaluating different predetermined investment and production policies. Most of the formulations presented so far have also been limited by the capability of solving these problems using existing solvers and conventional solution techniques.

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In this paper, the offshore oil field development problem is formulated as a multiperiod planning, problem, integrating the facility allocation, production planning, and scheduling within a single model. The formulation incorporates simultaneously the reservoir performance, surface pressure constraints, and oil rig resource constraints. An effective decomposition algorithm is presented to solve the resulting large-scale and combinatorially complex problems.

The paper is organized as follows. A background description of the problem and previous work in this area are described in following sections. The complete problem formulation is presented in section 2. The proposed decomposition algorithm is presented in section 3, followed by results of test problems in section 4.

1.1. Background. To explain the variables and parameters involved in the optimization problem, a brief description of the field layout is presented below. Figure 1 illustrates a generalized configuration of an offshore complex. The site contains a number of fields (F) with each of them containing several reservoirs (R). A reservoir contains one or more wells (W) through which oil and/or gas is produced. The wells are drilled from a well platform (WP) using drilling rigs, either vertical to or at an angle to the surface. Pipes are laid out in a network from the WP to the W which delivers the oil to the WP. Usually, there is two-phase flow in these lines due to the presence of gas and oil. The choice of W drilled from a WP is made by determining the best allocation of wells and location of the WPs. Well platforms collect oil from associated wells and deliver it to a production platform (PP), where the gas and oil are separated and the oil is then sent to the sales terminal. Clearly, the location of the PP, WP, and allocation of W to them is itself a complex optimization problem, as the costs of drilling are affected by the length of pipes required. In this work, we assume that the potential location of PP and WP is given. In

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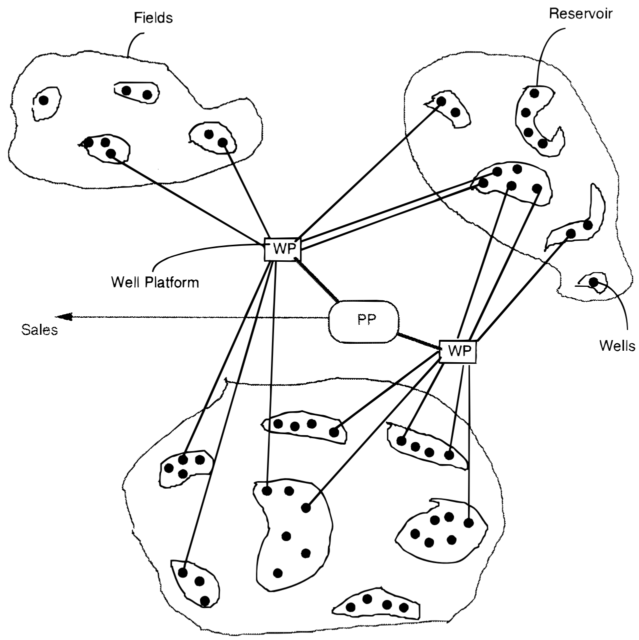

Figure 1. Configuration of fields, well platforms, and production platforms.

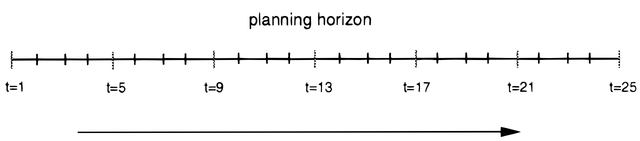

Figure 2. Time periods over planning horizon.

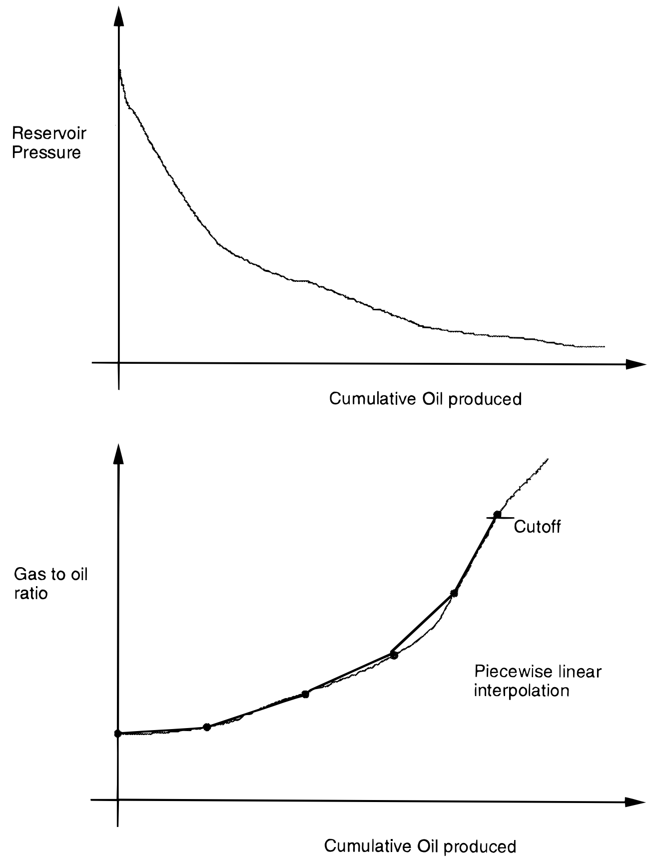

Figure 3. Reservoir performance characteristics.

addition, the potential allocation of wells to WP is assumed to be specified. Note that wells from a reservoir may be potentially allocated to more than one well platform and one WP may produce wells from more than one reservoir. However, once a choice is made, each well is assigned to a single WP. Similarly, a PP may service the production from more than one WP, F, or R. The initial oil and gas reserves in the reservoirs, as well as the distribution of the productivity index (defined as the ratio of oil flow from a well to the difference in reservoir and well bore pressure) of wells, are estimated from geologic studies.

The development of the entire facility over a planning horizon involves deciding where to drill the wells, allocation of wells to well platforms, installation of WPs and PPs, the scheduling of the drilling of oil wells, and operational planning of well production rates. The planning horizon may be represented by multiple periods of operation, where the length of the time periods determines the times when decisions are implemented. Figure 2 shows a typical planning horizon with quarterly time periods. It will be assumed that operating conditions are constant within a time period within the context of the planning horizon.

The primary issues involved in the problem are the reservoir behavior as a function of time and the surface pressure constraints. When oil is extracted from a reservoir (through its constituent wells), then the pressure in the reservoir and gas-to-oil ratio changes nonlinearly as a function of the cumulative oil removed from the reservoir (see Figure 3). The driving force for the production of oil from wells is the pressure difference between the reservoir and the bottom hole of the well. The productivity index of the well at the sand face is the reservoir parameter that determines the oil flow rate for a given pressure difference.

In this work, a maximum pressure drop between the reservoir pressure and the bottom hole pressure is used to limit the oil flow rate. An inflow performance relationship determines the bottom hole pressure, and a linear pressure drop relation determines the well head pressure as a function of the bottom hole pressure. Thus, as the cumulative oil produced from the reservoir increases, the reservoir pressure decreases, leading to a drop in the driving force and therefore the oil production rate. The productivity index (PI) of a well is defined by

$$\text{oil flow} = (\text{productivity index}) \times (\text{pressure differential between reservoir and well bore})$$

The PI depends on the permeability-thickness product (Kh), which is obtained from a geologic map of each reservoir. Thus, the productivity index is proportional to Kh. The Kh map, however, has uncertainty at potential well locations. The PIs are therefore obtained from random sampling from a normal distribution for a given mean and standard deviation of the mapped Kh for each reservoir. The PI is assumed to be constant throughout the planning horizon. The pressure constraints that determine the production of oil are the pressure balance at well and production platforms. Pressure chokes are provided on pipes to control the pressure drops to balance pressures at the junction of manifolds. The pressure drop in pipes is modeled as a linear function of the oil and gas flow rates. The surface pressure constraints are justified from field experience, where higher pressure wells choke lower pressure wells, resulting in potentially lower flows from wells in reservoirs at a lower pressure.

The cost of the well and production platforms are modeled as a sum of a fixed cost and a variable cost that is a function of the capacity of the platform. The platform capacity is determined as the largest flow in all time periods. The cost of drilling a well is a function of the length of the pipe and the completion of drilling. For the model, the cost of each well is assigned by random sampling from a normal distribution for a given mean and standard deviation for the length of pipe for all wells from the associated well platform.

The objective function is based on the total net present value (NPV), which is calculated from the sum of discounted investment costs and the sum of discounted revenues from sale of oil. Depreciation and royalties paid may also be included in the calculation of the NPV.

The various decisions that influence the total profit are as follows.

(1) Design variables: (a) Number and location of production platforms and well platforms and their oil and gas rate capacities. (b) Number and location of wells to be drilled for production.

(2) Scheduling and operation variables: (a) The time periods when WP and PP are installed and when the wells are drilled. (b) The location (scheduling) of drilling rigs on WPs. (c) Production rates of oil for each time period.

The oil production rate from a well is determined by its productivity index and the reservoir pressure. The well is usually capped when the GOR (gas-to-oil ratio) exceeds a certain threshold limit or when the pressure in the reservoir is lower than a minimum pressure.

Additional resource constraints in the scheduling problem involve the availability of drilling rigs. During each time period, a limited number of wells (one per month in the model) can be drilled using the rig. A well is drilled using the drilling rig located on the associated well platform. The movement of drilling rigs between well platforms results in a loss of time (typically 1 month) as well as a fixed cost. As a result, the allocation and scheduling of rigs for drilling is essential in the final operational plan.

1.2. Previous Work. Previous work in the optimization of offshore field development may be classified into two broad categories.

1.2.1. Location−Allocation Problem. One of the earliest works by Devine and Lesso (1972) solves the problem of determining the continuous two-dimensional location of production platforms and the allocation of wells to well platforms. The trade-offs associated with the costs involve the capacity of the platforms and the cost of piping from wells to the platforms. They proposed an iterative two-stage algorithm that first fixes the allocation of wells to the platforms and then determines the location of platforms. In the next stage, the location determined in the first stage is fixed and the allocation problem is solved to get a new set of well allocation variables. The problem, however, does not include the scheduling of well drilling or production planning of wells. Garcia-Diaz et al. (1996) presented a network representation for the same problem and proposed a Lagrangean relaxation solution method.

1.2.2. Production Planning and Scheduling. Lee and Aranofsky (1958) formulated a production planning problem that expressed the performance of reservoirs linearly as a function of time. Sullivan (1982) used a nonlinear reservoir performance equation and approximated it by using piecewise linear interpolation using integer variables. Both these works did not include the scheduling of drilling of wells and surface pressure constraints. Aranofsky and Williams (1962) proposed an LP model for scheduling of well drilling, assuming a preset production profile. In addition, the model had the drawback of fractional solutions for the number of wells drilled in a time period. Bohannon (1970) proposed a mixed-integer linear programming (MILP) model for development of multireservoir systems, assuming a predetermined linear decline of production rate with cumulative oil produced. Frair and Devine (1973) proposed a model that simultaneously included the location−allocation of wells, scheduling of facility operation, and production rates for different time periods. However, the model did not include the reservoir performance equations and assumed a linear production decline curve for each reservoir. Costa (1975) and Dogru (1975) proposed models for optimal platform location and scheduling of well drilling, but they did not include any production planning. Harding et al. (1996) tested sequential quadratic programming, simulated annealing, and a genetic algorithm for planning the multiple field development. They assumed a predetermined target production rate and prescheduled the capital investment within a fixed time window.

This paper presents an actual industrial problem, where an offshore oil field was considered for development. At the time when the problem was presented to us, the potential locations of the platforms and the potential wells to be drilled were predetermined. The problem consisted of determining the actual number and location of platforms, actual wells to be drilled and their interconnections to the platform, and the production planning and scheduling of the oil field. Although numerous models have been proposed in the literature, there are no multiperiod planning models that simultaneously incorporate the scheduling of facility operations, design of platforms, and surface constraints. As shown in this work, this problem can be formulated as a multiperiod mixed-integer programming problem for which a sequential decomposition algorithm is proposed.

It is worthwhile to note that, in this paper, potential locations of well platforms may be considered, and the algorithm chooses the set of platforms and their interconnections to wells as a solution of the optimization problem. Thus, a superstructure containing multiple possible locations of platforms and their potential matchings to wells may be considered within the framework presented in this paper.

2. Problem Formulation

In this section, a mathematical model for the offshore oil field development problem is presented. The major assumptions in this model are the following: (1) Each reservoir contains a single homogeneous mixture at the same pressure. (2) Multiple wells in a reservoir produce independently on the basis of each well's PI and the reservoir's average pressure. (3) The location of potential wells and platforms as well as their potential interconnections to wells are known. (4) Pressure drop in pipes and wells is linear in the oil flow rate and gas flow rate. (5) The well oil flow rate is proportional to the pressure difference between the well head and well bore. (6) The productivity index of each well is constant across the planning horizon. (7) Each well has a single

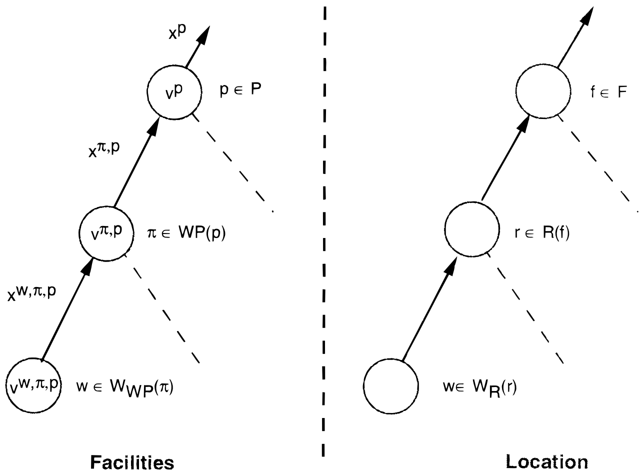

Figure 4. Description of facility and location variables.

completion in a single reservoir. This implies that a drilled well is capped after extracting oil from the well. Therefore, it is not reused to drill from this well to other well locations in the reservoir.

The model is configured hierarchically in terms of sets to represent the physical configuration in a general form. The following sets, indices, variables, and parameters are defined. We omit superscripts in the variable definition for the sake of simplicity.

(1) Sets and Indices

PP = set of production platforms

p = production platform $p \in$ PP

WP(p) = set of well platforms associated with platform p

π = well platform $\pi \in$ WP(p)

F = set of fields

f = field $f \in F$

$R(f)$ = set of reservoirs associated with field f

r = reservoir $r \in R(f)$

$W_{WP}(\pi)$ = set of wells associated with well platform π

$W_R(r)$ = set of wells associated with reservoir r

$W_{WP,R}(r,\pi)$ = set of wells associated with reservoir r and well platform π

w = well $w \in W_{(.)}(.)$

t = time periods

τ = aggregated time periods

D = set of drilling rigs

k = drilling rigs $k \in D$

$J(r)$ = linear interpolation pieces used for reservoir r

j = index of piece used in piecewise linear interpolation $j \in J$

The hierarchy of indices for the definition of the configuration of the entire field is w, π, and p, where p is at the highest level and w is at the lowest level. (w,π,p) is an ordered triplet that defines the associated variables and parameters at the level of wells. We note that since the configuration is hierarchically defined, the pair (π,p) would define the variables at the level of well platforms. Also, since a well is located in a reservoir and is associated with a well platform, we define three sets of wells on the basis of the context of association. Thus, $W_{WP}(\pi)$ denotes wells associated with well platform $\pi \in$ WP, $W_R(r)$ denotes wells associated with reservoir $r \in R$, and $W_{WP,R}(r,\pi)$ denotes the set of wells within a given reservoir $r \in R$ that is associated with a given well platform $\pi \in$ WP. Thus,

$$W_{WP,R}(r,\pi) = \{w | w \in W_{WP}(\pi) \cap W_R(r)\} \quad \forall\ r \in R(f),\ f \in F,\ \pi \in WP(p),\ p \in PP$$

Figure 4 shows the hierarchy of facilities and locations and the associated variables defined in this model.

As indicated in Figure 4, the pipelines connecting different facilities are modeled as part of the associated facilities. For instance, the pipe from WP to PP is considered a part of the associated WP. Similarly, the flow rates in pipes are modeled using flow variables associated with that facility.

(2) Continuous Variables

x_t = oil/gas flow in period t

$\hat{x}_t$ = cumulative oil/gas flow amount up to period t

l_t = oil flow (mass) in period t

g_t = gas flow (volumetric) in period t

ϕ_t = gas-to-oil ratio in period t

v_t = pressure in period t

δ_t = pressure drop at choke in period t

d_t = design variable in period t

e_t = design expansion variable in period t

$Z_{k,t}$ = number of times drilling rig k is moved in period t

$\lambda_{j,t}$ = interpolation variable in period t

The expansion variable e_t is used to model the discounted variable cost of installation of the well and production platform using linear terms.

(3) Binary Variables

z_t = 1 if facility (well or platforms) is drilled/installed in period t

$zd_{k,t}$ = 1 if drilling rig k is located on facility in period t

$ZT_{k,t}$ = 1 if drilling rig k is moved after period t

$y_{j,t}$ = 1 if piece j used for linear interpolation in period t

= 0 otherwise

(4) Parameters

ρ = productivity index of well

P_{max} = maximum pressure drop from well bore to well head

GOR_{max} = maximum GOR (gas-to-oil ratio)

t_a = number of periods in an aggregated time period τ

T_a = number of aggregated time periods, $\tau = 1 ... T_a$

M_w = maximum number of wells drilled by a rig in a time period

Δt = length of time period t

Ω^u = upper bound parameter (defined by the respective equation)

α = pressure drop coefficient for oil flow rate

β = pressure drop coefficient for GOR

c_{1t} = discounted revenue price coefficient for oil sales

c_{2t} = discounted fixed cost coefficient for capital investment

c_{3t} = discounted variable cost coefficient for capital investment

c_{4t} = discounted cost coefficient for moving rigs

(5) Superscripts

(w,π,p) = variables associated with well $w \in W$, with well platform π and production platform p

(π,p) = variables associated with well platform π and production platform p

(p) = variables associated with production platform p

(r) = variables associated with reservoir r

(6) Other Notations

$\overline{(.)}$ = formulation with fixed value of variable

$\langle(.)\rangle$ = aggregation with respect to well variables and parameter

The model equations are as follows.

(1) Mass Balance. The sum of flow of oil/gas from all wells $w \in W_{\mathrm{WP}}(\pi)$ associated with a well platform $\pi \in \mathrm{WP}(p)$ is the total flow of oil/gas at the well platform. Similarly, flows from well platforms are added to determine the flow at the production platforms.

$$\sum_{w \in W_{\mathrm{WP}}(\pi)} x_t^{w,\pi,p} = x_t^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (1)$$

$$\sum_{\pi \in \mathrm{WP}(p)} x_t^{\pi,p} = x_t^p \qquad \forall\ p \in \mathrm{PP} \quad (2)$$

$$\sum_{p \in \mathrm{PP}} x_t^p = x_t^{\mathrm{total}} \quad (3)$$

$$\text{for } t = 1 \ldots T$$

(2) Pressure Balance at Well and Production Platforms. The pressure at the well platform $\pi \in \mathrm{WP}(p)$ is the pressure at the wells $w \in W_{\mathrm{WP}}(\pi)$ associated with well platform π minus the pressure drop in the corresponding pipe. The pressure drop is expressed as a linear function of the oil and gas flow rate. Here, δ is defined as the pressure drop across the pressure chokes on the lines:

$$v_t^{\pi,p} = v_t^{w,\pi,p} - \alpha x_t^{w,\pi,p} - \beta g_t^{w,\pi,p} - \delta_t^{w,\pi,p} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (4)$$

$$v_t^{p} = v_t^{\pi,p} - \alpha x_t^{\pi,p} - \beta g_t^{\pi,p} - \delta_t^{\pi,p} \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (5)$$

$$\text{for } t = 1 \ldots T$$

(3) Flow Constraints in Wells. The maximum flow of oil is related to the productivity index of the well and the allowable pressure drop. The gas flow is limited by the maximum allowable GOR:

$$x_t^{w,\pi,p} = l_t^{w,\pi,p} + g_t^{w,\pi,p} \quad (6)$$

$$l_t^{w,\pi,p} \le \rho^{w,\pi,p} P_{\max} \quad (7)$$

$$g_t^{w,\pi,p} \le l_t^{w,\pi,p} \mathrm{GOR}_{\max} \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (8)$$

$$\text{for } t = 1 \ldots T$$

(4) Cumulative Flow Amount from Wells up to Period θ. The cumulative amount of oil from a well is calculated from the sum of the amount of oil from all periods up to time period θ. Thus,

$$\hat{x}_\theta^{w,\pi,p} = \sum_{t=1}^{\theta-1} x_t^{w,\pi,p} \Delta t \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (9)$$

$$\text{for } \theta = 1 \ldots T$$

The cumulative oil amount from a reservoir is obtained by summing the cumulative amounts from all wells associated with the reservoir. We define a new set as follows:

$$W_{F,R}(f,r) = \{(w,\pi,p) | w \in W_{\mathrm{WP}}(\pi) \cap W_R(r),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP}\} \qquad \forall\ r \in R(f),\ f \in F$$

Thus,

$$\hat{x}_\theta^{r,f} = \sum_{(w,\pi,p) \in W_{F,R}(f,r)} \hat{x}_\theta^{w,\pi,p} \qquad \forall\ r \in R(f),\ f \in F \quad (10)$$

$$\text{for } \theta = 1 \ldots T$$

(5) Piecewise Linear Interpolation at Well and Reservoir Level. The pressure and GOR in a reservoir is a nonlinear function of the cumulative oil produced (see Figure 3). The actual relation is taken to be a function of the initial pressure, the oil's PVT properties, and the initial oil volume. The pressure and GOR are then calculated using piecewise linear interpolation. The equations are explained in detail in Appendix A. The oil and gas flow rates from individual wells in a reservoir are also calculated using piecewise linear interpolation:

$$v_t^{r,f} = h_1(\hat{x}_t^{r,f}, y_{j,t}^{r,f}, \lambda_{j,t}^{r,f}, \tilde{v}_t^{r,f}) \qquad \forall\ r \in R(f),\ f \in F \quad \text{for } t = 1 \ldots T \quad (11)$$

$$x_t^{w,\pi,p} = h_1(\hat{x}_t^{w,\pi,p}, y_{j,t}^{w,\pi,p}, \lambda_{j,t}^{w,\pi,p}, \tilde{x}_t^{w,\pi,p}) \quad (12)$$

$$\phi_t^{w,\pi,p} = h_1(\hat{x}_t^{w,\pi,p}, y_{j,t}^{w,\pi,p}, \lambda_{j,t}^{w,\pi,p}, \tilde{w}_t^{w,\pi,p}) \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (13)$$

$$\text{for } t = 1 \ldots T$$

(6) Logical Constraints for Installation and Flow from Facilities. The wells may be drilled only once in one of the time periods. Also, the facilities (well and production platforms) may be installed only in one of the time periods.

$$\sum_{t=1}^{T} z_t^{w,\pi,p} \le 1 \qquad \forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (14)$$

$$\sum_{t=1}^{T} z_t^{\pi,p} \le 1 \qquad \forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP} \quad (15)$$

$$\sum_{t=1}^{T} z_t^{p} \le 1 \qquad \forall\ p \in \mathrm{PP} \quad (16)$$

$$\text{for } t = 1 \ldots T$$

Note that the variables z_t may be treated as special ordered sets (SOS) (see Nemhauser and Wolsey (1988)), by introducing a dummy period t_d such that

$$\sum_{t=1}^{T} z_t + z_{t_d} = 1 \qquad (17)$$

By treating the investment binaries as SOS variables, one can potentially reduce the number of nodes enumerated in a branch and bound tree.

The flow of oil and gas from a facility can be non-zero only after it is installed:

$$x_\theta^{w,\pi,p} \le \Omega^u \sum_{t=1}^{\theta} z_t^{w,\pi,p} \qquad \forall\ w \in W_{WP}(\pi), \pi \in WP(p), p \in PP \quad (18)$$

$$x_\theta^{\pi,p} \le \Omega^u \sum_{t=1}^{\theta} z_t^{\pi,p} \qquad \forall\ \pi \in WP(p), p \in PP \quad (19)$$

$$x_\theta^{p} \le \Omega^u \sum_{t=1}^{\theta} z_t^{p} \qquad p \in PP \quad (20)$$

$$\text{for } \theta = 1 \ldots T$$

Note that the upper bound Ω^u is defined accordingly based on the variables in the equation. In eq 18, Ω^u is equal to the maximum well oil flow rate $x_\theta^{w,\pi,p}$. The well platform associated with a well must be installed before drilling that well. Similarly, production platforms must be installed before the associated well platforms. This condition may be modeled as follows:

$$z_\theta^{w,\pi,p} \le \sum_{t=1}^{\theta} z_t^{\pi,p} \qquad \forall\ w \in W_{WP}(\pi), \pi \in WP(p), p \in PP \quad (21)$$

$$z_\theta^{\pi,p} \le \sum_{t=1}^{\theta} z_t^{p} \qquad \pi \in WP(p), p \in PP \quad (22)$$

$$\text{for } \theta = 1 \ldots T$$

(7) Design of Facility. The design capacity of a facility is determined by the maximum flow among all periods $t = 1 \ldots T$. To model the variable design cost using linear terms, a design *expansion* variable is used, which is equal to the design capacity for the time period when the facility is installed. Note that the expansion variable e may be non-zero only in one time period (from eqs 15 and 25), which represents the period when the facility is installed. For the case of well platforms, the equations are

$$x_t^{\pi,p} \le d_t^{\pi,p} \qquad (23)$$

$$d_t^{\pi,p} = d_{t-1}^{\pi,p} + e_t^{\pi,p} \qquad (24)$$

$$e_t^{\pi,p} \le \Omega^u z_t^{\pi,p} \qquad \forall\ \pi \in WP(p), p \in PP \quad (25)$$

$$\text{for } t = 1 \ldots T$$

Similar equations may also be written for the production platforms.

(8) Number of Wells Drilled in Time Period t. The maximum number of wells that can be drilled in a time period is dependent on the length of the time period, the time taken by a drilling rig to drill a well, and the number of available drilling rigs. The drilling rig resource constraints are explained in detail in Appendix B.

$$\sum_{w \in W_{WP}(\pi)} z_t^{w,\pi,p} \le h_2(Z_{k,t}^p, ZT_{k,t}^{\pi,p}, M_w) \qquad \forall\ \pi \in WP(p), p \in PP \quad (26)$$

$$\text{for } t = 1 \ldots T$$

(9) Drilling Rig Scheduling Constraints. The number of times a drilling rig is moved depends on the allocation of drilling rigs to well platforms for drilling wells. The equations are explained in detail in Appendix B.

$$Z_{k,t}^p = h_3(z_t^{w,\pi,p}) \qquad \forall\ k \in D, w \in W_{WP}(\pi), \pi \in WP(p), p \in PP \quad (27)$$

$$\text{for } t = 1 \ldots T$$

(10) Flow Profile Constraints. An additional operational constraint is that the flows should be a nonincreasing function of time in order to ensure a smooth flow profile.

$$x_t^{w,\pi,p} \ge x_{t+1}^{w,\pi,p} - \Omega^U\Big(1 - \sum_{t'=1}^{t} z_{t'}^{w,\pi,p}\Big) \qquad \forall\ w \in W_{WP}(\pi), \pi \in WP(p), p \in P \quad (28)$$

$$\text{for } t = 1 \ldots T$$

This constraint may be excluded in the model to accommodate a flexible well performance.

(11) Objective Function. The objective function to be maximized is the discounted profit (Ψ) which includes revenues from sale of oil, investment cost and depreciation of facilities, and cost of moving rigs:

$$\max \Psi = \sum_{t}^{T}\Big\{c_{1t}x_t^{\text{total}} - \sum_{p\in PP}\Big[(c_{2t}^p z_t^p + c_{3t}^p e_t^p) + \sum_{\pi\in WP(p)}\Big\{(c_{2t}^{\pi,p} z_t^{\pi,p} + c_{3t}^{\pi,p} e_t^{\pi,p}) + \sum_{w\in W_{WP}(\pi)} c_{2t}^{w,\pi,p} z_t^{w,\pi,p}\Big\}\Big] - c_{4t}\sum_{k\in D}\Big(\sum_{p\in PP}\Big[Z_{k,t}^p + \sum_{\pi\in WP(p)} ZT_{k,t}^{\pi,p}\Big]\Big)\Big\} \quad (29)$$

In the equation above, the first sum corresponds to the income from sales, the second and third term are the investment costs of the production and well platforms, respectively, the fourth term represents the drilling costs of wells, and the last term is the sum of costs for moving the drilling rigs.

2.1. Multiperiod Planning Model Formulation. The model consisting of eqs 1−29 is the complete formulation of the multiperiod field development problem (MP) as presented below:

$$\text{MP:} \ \max \Psi \text{ in eq 29}$$

subject to

$$\text{eqs } 1-28$$

The number of equations in the model can be very large when the number of time periods increases. The combinatorial complexity of the model, which is due to the extremely large number of binary variables, makes it computationally intractable for solving real world problems. We note that the binary variable decisions involved are the investment binaries (z_t), interpolation

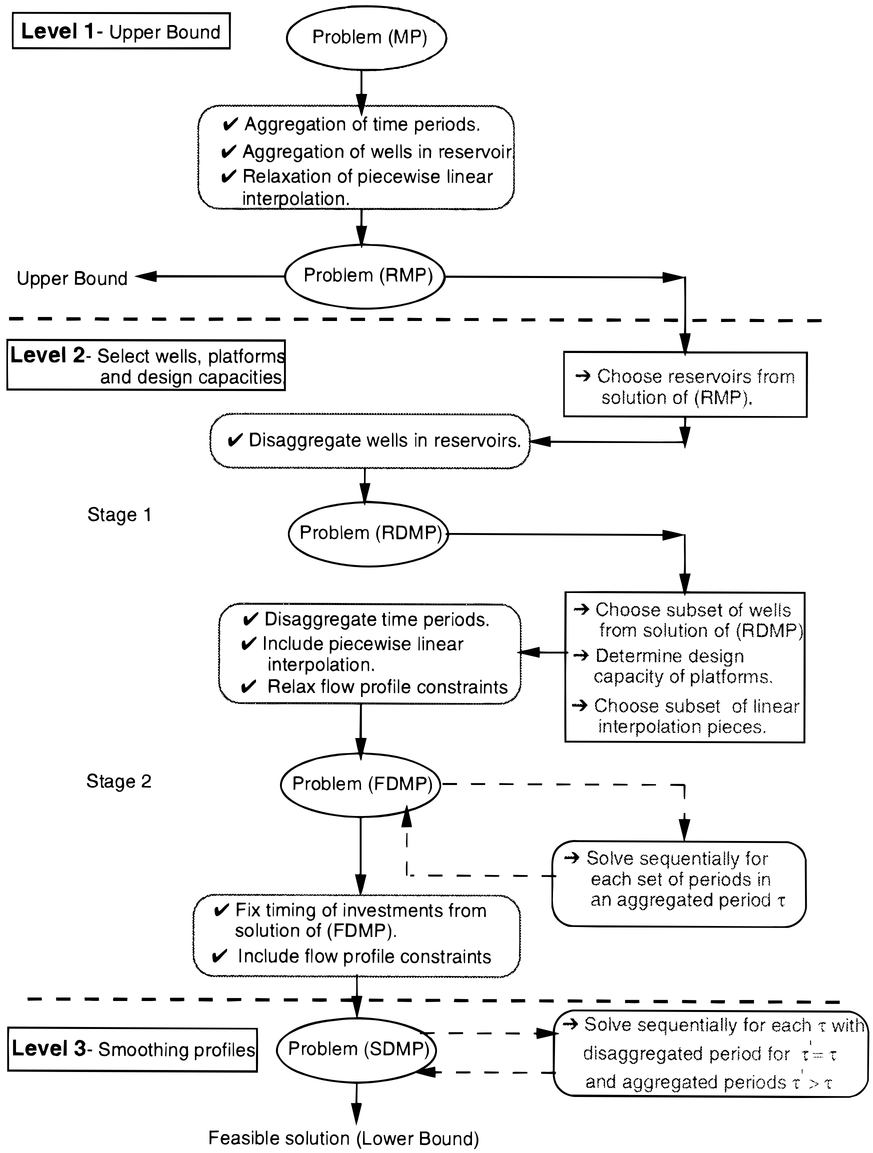

Figure 5. Decomposition algorithm flowchart.

binaries ($y_{j,t}$), and drilling rig binaries ($ZT_{k,t}$). The number of interpolation binaries is very large and scales with the number of wells, time periods, and cardinality of the set (J). A problem with a single field containing 10 reservoirs, 30 wells, and a planning horizon of 6 years with quarterly time periods would involve about 5000 binary variables.

This motivates the need to develop a sequential decomposition strategy that involves solving smaller subproblems that can generate a good solution to the planning problem. In the following section, a decomposition algorithm is proposed using aggregation of wells and time periods, followed by successive disaggregation. Although the proposed algorithm is not guaranteed to find the optimal solution, a relatively tight upper bound to the NPV can be obtained to assess the quality of the predicted solution.

3. Decomposition Algorithm

3.1. Description of Algorithm. The main steps of the proposed decomposition algorithm seek to arrive at a good solution that is not necessarily guaranteed to be optimal, but that can greatly reduce the computational requirements. An outline of the algorithm is as follows. The problem is divided into three major levels of solution and a broad overview of the main steps is presented in Figure 5.

3.1.1. Level 1: Upper Bounding. A relaxation of the original problem MP is solved to get an upper bound to the maximization problem. Note that a relaxation of the problem includes all feasible solutions of the problem MP; hence, the solution of a relaxed problem is at least as good as the solution of MP. For a maximization of the objective function, this represents an upper bound. The relaxation is obtained by using the following:

(1) Aggregation of time periods. Thus, each aggregated time period τ contains t_a time periods (see Figure 6).

(2) Aggregation of wells in a reservoir associated with the same well platform. Each well $w \in W_{WP,R}(r,\pi)$ is aggregated into a single well $\langle w \rangle \in W_{WP,R}(r,\pi)$ (see Figure 7). The parameters associated with those wells are aggregated to simulate aggregated flows from wells. The aggregated productivity index is obtained by adding the productivity index of each well in the aggregation. The pressure drop coefficients are reduced by a factor

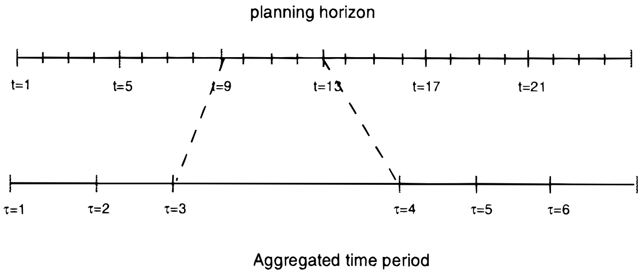

Figure 6. Aggregation of time periods.

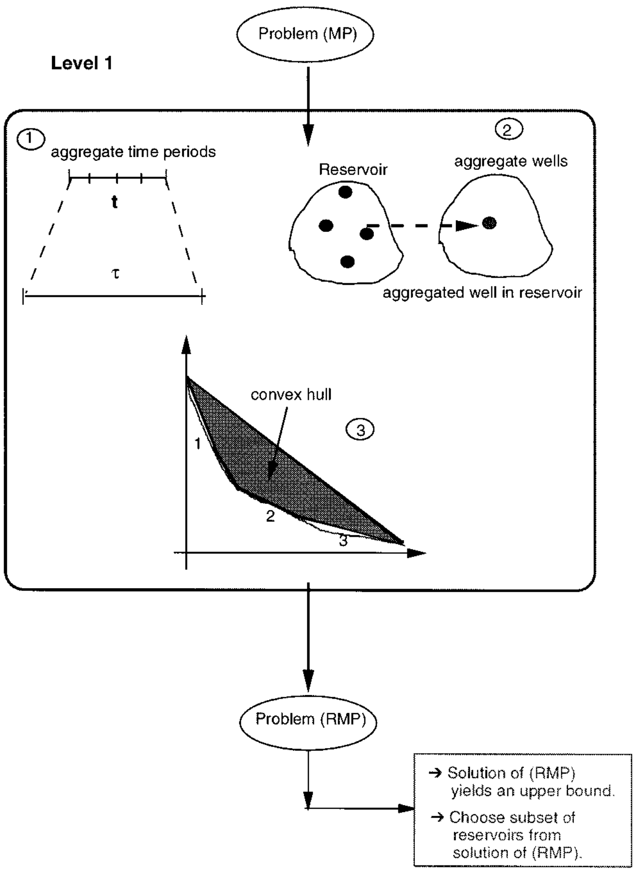

Figure 7. Decomposition algorithm—level 1.

of the number of wells in the aggregation, to determine equivalent pressure drop using aggregated flows in pipes.

(3) Relaxation of piecewise linear interpolation. The convex hull generated by the endpoints of each piece of the piecewise approximation is used to determine the value of interpolated variables (see Figure 7). This is equivalent to the relaxing constraint (A4) and setting $y_{j,t} = 1$.

The relaxed problem obtained from MP using the above steps is denoted by RMP. Figure 7 illustrates the steps involved in formulation of RMP from MP.

3.1.2. Level 2: Lower Bounding. In this level, a specific solution to MP is obtained by solving subproblems in different stages sequentially. The solution obtained from RMP is used to determine the *subset of reservoirs* $R' \subseteq R$ that will be developed. Thus, all those reservoirs not chosen in solution of RMP are excluded in R'. Additionally, a subset of the well platforms WP′ $\subseteq$ WP is chosen, which includes well platforms chosen in the solution of RMP (i.e., if an aggregated well $\langle w \rangle$ associated with that well platform is used). The problem RMP is now solved for the new sets R' and WP′, and the stages involved are the following:

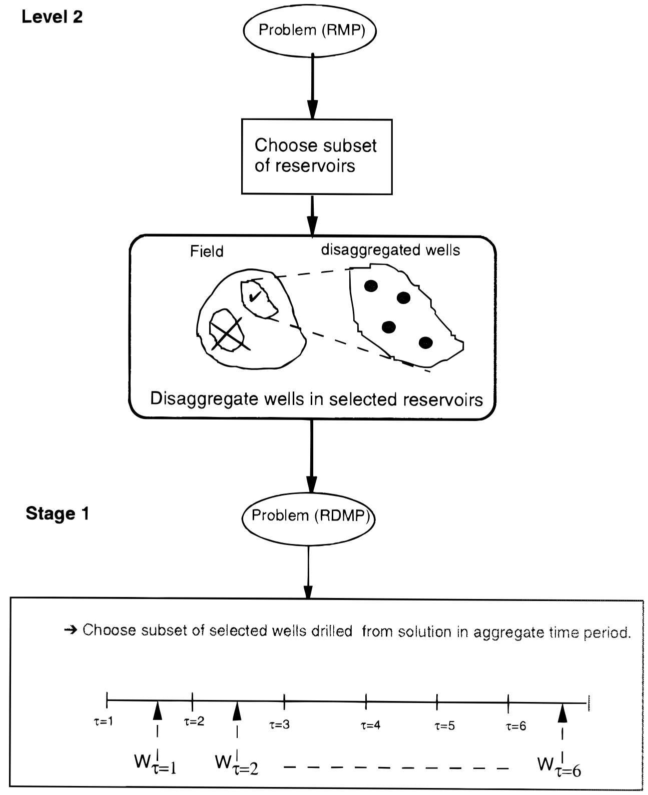

Figure 8. Decomposition algorithm—level 2, stage 1.

Stage 1. Disaggregating wells. For all $\langle w \rangle \in R'$, the wells are disaggregated. The problem RMP with disaggregated wells is denoted as RDMP.

The solution obtained from RDMP is used to determine the *choice of wells drilled over the planning horizon*. Thus, the space of binary variables is reduced by choosing a subset of wells W' from a solution of RDMP. All wells that are not utilized in the solution of RDMP are excluded from the planning problem. Figure 8 illustrates the steps involved in the formulation of RDMP from RMP. The solution of RDMP yields W_τ, which is the set of wells drilled in period τ.

Stage 2. Disaggregation of time periods, inclusion of piecewise linear interpolation, and relaxation of profile constraints (eq 28).

The problem FDMP is derived from RDMP by disaggregating the time periods, including all piecewise linear interpolation constraints (i.e., set $y_{j,t}$ as a binary variable and include eq A4, and relaxing the flow profile constraints across aggregate time periods. The problem FDMP is now solved in a forward rolling fashion sequentially for each $\tau = 1 \ldots T_a$, for the set of periods $t = (\tau - 1)t_a + t$, $t' = 1 \ldots t_a$.

Figure 9 illustrates the time periods for which FDMP is solved sequentially. Figure 10 illustrates the steps involved in the formulation of FDMP from RDMP. The choice of wells drilled within the aggregated time period is known from the solution of RDMP and is used as a basis for FDMP. Since FDMP contains only t_a time periods, it is a smaller problem and is not computationally as expensive.

The solution obtained from FDMP determines the choice of facilities and time periods when they are installed in disaggregated time period t. However, the

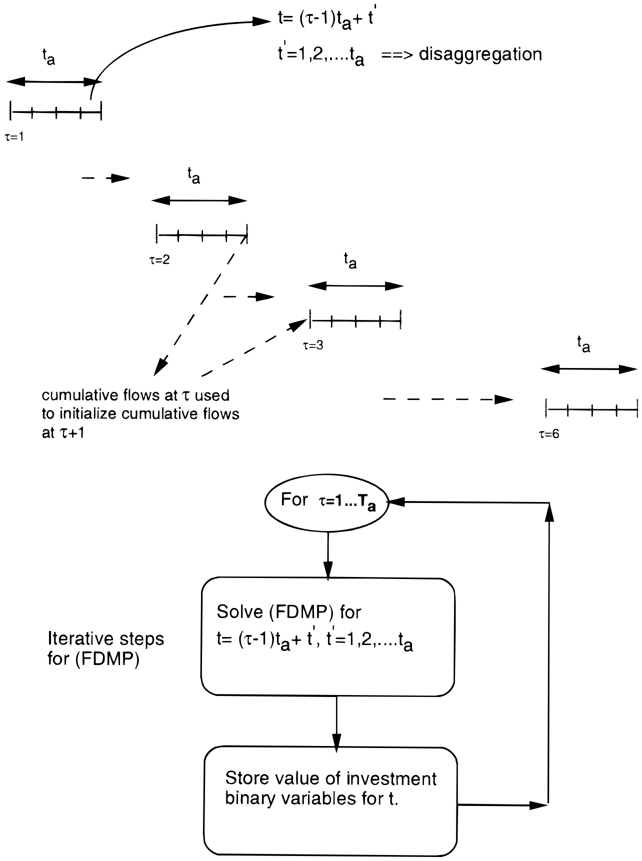

Figure 9. Solution of (FDMP) sequentially for each τ.

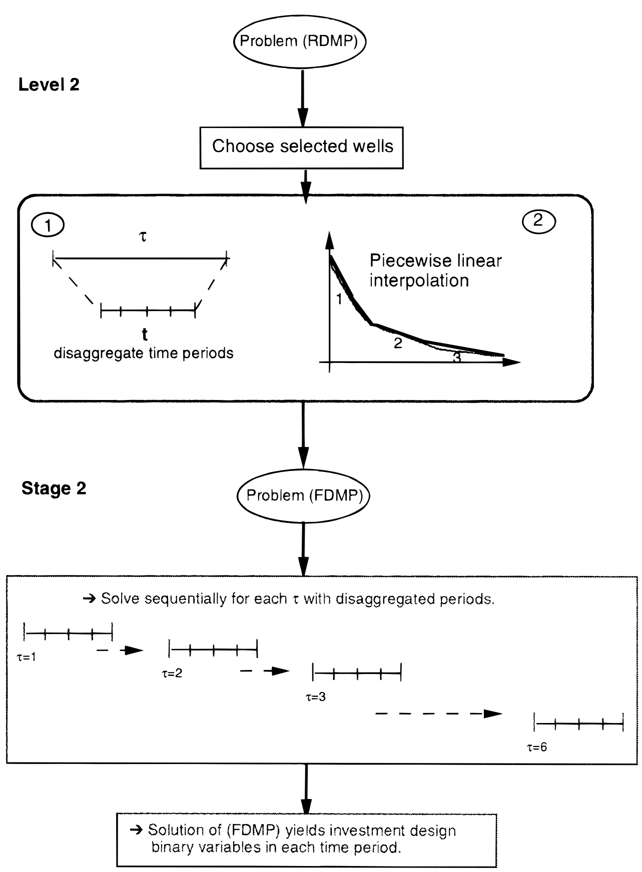

Figure 10. Decomposition algorithm—level 2, stage 2.

problem may not be a feasible solution to MP as the profile constraints were relaxed in FDMP.

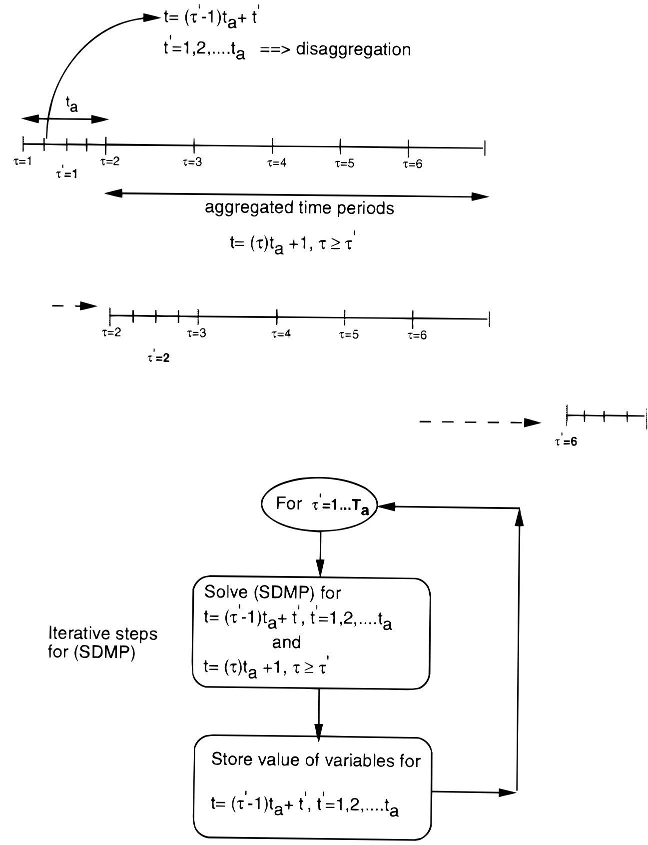

Figure 11. Solution of (SDMP) sequentially for each τ'.

3.1.3. Level 3: Smoothening of Profiles. If the profile constraints (eq 28) are not satisfied by the solution of FDMP, then the profile is smoothened as follows. The investment binaries $(z_t^p, z_t^{\pi,p}, z_t^{w,\pi,p})$ are fixed from the solution of FDMP. Thus, only the interpolation binary variables need to be determined to ensure feasibility of the profile constraints. The problem RDMP is now solved in a forward rolling fashion, where the profile constraints are also imposed. However, the solution of the problem with disaggregated time periods would lead to a very large problem with numerous interpolation binary variables. Therefore, problem SDMP is formulated from RDMP for fixed investment binaries, where the time periods are disaggregated in the first aggregated period only. Thus, the problem SDMP is solved for each $\tau' = 1 \ldots. T_a$ where $t = (\tau' - 1)t_a + t'$, $t' = 1 \ldots t_a$ (disaggregated periods for τ') and $t = \tau(t_a) + 1$, $\tau = \tau' \ldots T_a$ (aggregate periods for $\tau > \tau'$). Figure 11 illustrates the time periods for which SDMP is solved sequentially.

Figure 12 illustrates the steps involved in formulation of (SDMP) from (FDMP).

Thus, for all future aggregated periods, there is no disaggregation. The flow profile constraints are then for flows in disaggregated time periods for $\tau = \tau'$ and across aggregated time periods for higher values of $\tau > \tau'$. The solution obtained finally from the solution of SDMP for each τ gives the flow variables in each period t and represents the final solution to the problem MP. Note that the choice of aggregation of periods (e.g., 4 quarters into 1 year) is arbitrary. Increasing the number of periods aggregated (t_a) reduces the complexity of solving the relaxed problem, at the expense of increasing the relaxation gap between the solution of MP and RMP.

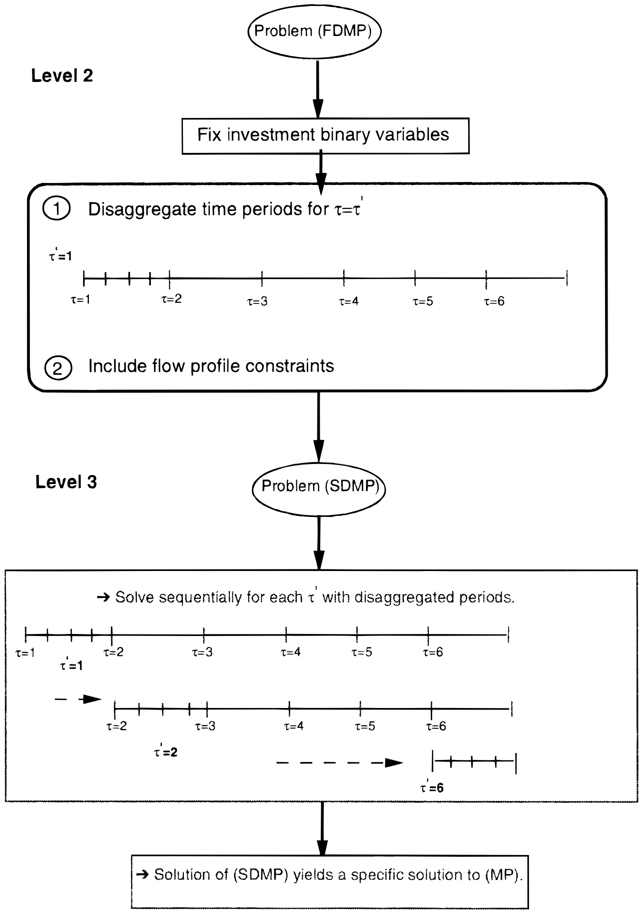

Figure 12. Decomposition algorithm—level 3.

3.2. Mathematical Formulation of Decomposition Algorithm. The problem MP may be concisely formulated as follows. The superscripts are removed from the variables to simplify the notation:

$$\text{MP:}\ \max \Psi = q(c_t,x_t,z_{k,t},e_t,Z_{k,t},ZT_{k,t}) \qquad (30)$$

subject to

$$m_1(x_t) = 0 \qquad (31)$$

$$m_2(x_t,v_t,g_t,\delta_t,\alpha,\beta) = 0 \qquad (32)$$

$$m_3(x_t,l_t,g_t,\rho) \le 0 \qquad (33)$$

$$m_4(x_t,\phi_t,y_{j,t},\lambda_{j,t}) = 0 \qquad (34)$$

$$m_5(x_t,z_t) \le 0 \qquad (35)$$

$$m_6(x_t,z_t,d_t,e_t) \le 0 \qquad (36)$$

$$m_7(z_t,Z_{k,t},ZT_{k,t}) \le 0 \qquad (37)$$

$$m_8(x_t,z_t) \le 0 \qquad (38)$$

$$\forall\ w \in W_{\text{WP}}(\pi),\ \pi \in \text{WP}(p),\ p \in \text{PP}$$

$$\forall\ k \in D,\ j \in J(r),\ t = 1\ ...\ T$$

The equivalence of equations shown in the above model are as follows. Equation 30 is equivalent to the objective function in eq 29. Equation 31 represents the mass balance equations in (1)−(3). Pressure balance equation (32) is equivalent to equations 4 and 5. Equation 33 includes the flow constraints in wells in eqs 6−8. All cumulative flow and linear interpolation constraints in (9)−(13) are represented by eq 34. Logical flow constraints (35) are equivalent to (14)−(22). Equation 36 represents the design equations (23)−(25). Equation 37 represents the drilling scheduling constraints in eqs 26 and 27. Finally, eq 38 is flow profile constraints in eq 28.

The effect of the aggregation of time periods and wells in reservoirs on the problem MP is explained as follows.

3.2.1. Property 1. Consider problem MP with the aggregation of time period τ, aggregation of wells in reservoirs, and relaxation of piecewise linear interpolation (fixing all $y_{j,t} = 1$ and relaxing eq A4). The resulting problem RMP is a relaxation of problem MP subject to the following conditions.

(1) The effective PI ($\langle$PI$\rangle$) of the aggregated well is the sum of the PIs of each well in the aggregation.

(2) The flow-pressure drop coefficients $\langle\alpha\rangle$ (and $\langle\beta\rangle$)is equal to $\alpha(\beta)$ divided by the number of wells in the aggregation.

(3) The discounted investment cost coefficients for each aggregate year τ is equal to the value of the discounted cost coefficient corresponding to the disaggregated period $t = (\tau)t_a$ (equivalent to a lagged cost coefficient).

The proof of this property is presented in Appendix C.

Any solution to MP that is feasible is a lower bound on Ψ and hence to its optimal value. After solving RMP, a feasible solution can be obtained by successive disaggregation. If the gap between the two solution is within a tolerance ϵ, then the solution obtained is within ϵ of the optimal solution to MP.

3.3. Decomposition Algorithm. The steps of the algorithm are as follows. The property presented in the previous section is used in level 1 of the algorithm to generate an upper bound to the solution of MP.

3.3.1. Level 1: Upper Bounding. Solve problem RMP to obtain an upper bound to Ψ where RMP is defined as

$$\text{RMP:}\ \max \Psi_{\text{RMP}} = q(c_\tau',\langle x_\tau\rangle,z_\tau,e_\tau,Z_{k,\tau},ZT_{k,\tau}) \qquad (39)$$

subject to

$$m_1(\langle x_\tau\rangle) = 0 \qquad (40)$$

$$m_2(\langle x_\tau\rangle,v_\tau,g_\tau,\delta_\tau,\langle\alpha\rangle,\beta) = 0 \qquad (41)$$

$$m_3(\langle x_\tau\rangle,\langle l_\tau\rangle,\langle g_\tau\rangle,\langle\rho\rangle) \le 0 \qquad (42)$$

$$m_4(\langle x_\tau\rangle,\phi_\tau,\bar{y}_{j,\tau},\lambda_{j,\tau}) = 0 \qquad (43)$$

$$m_5(\langle x_\tau\rangle,z_\tau) \le 0 \qquad (44)$$

$$m_6(\langle x_\tau\rangle,z_\tau,d_\tau,e_\tau) \le 0 \qquad (45)$$

$$m_7(z_\tau,Z_{k,\tau},ZT_{k,\tau}) \le 0 \qquad (46)$$

$$m_8(\langle x_\tau\rangle,z_\tau) \le 0 \qquad (47)$$

$$\forall\ \langle w\rangle \in W_{\text{WP},R}(r,\pi),\ r \in R(f),\ f \in F$$

$$\forall\ \pi \in \text{WP}(p),\ p \in \text{PP},\ j \in J(r),\ \tau = 1\ ...\ T_a$$

Note that RMP is defined for aggregate time periods and aggregate flows $\langle x_\tau \rangle$ based on the aggregation of wells. In addition, $y_{j,\tau} = 1 \ \forall \ j,\tau$ for piecewise linear interpolation, where the constraints A4 are relaxed in eq 43.

The solution Ψ_{RMP} is an upper bound to the solution of MP. From the solution of RMP, define the following subsets:

$$R' = \{r | \exists \ \tau \text{ s.t., } z_\tau^{w,\pi,p} = 1, \langle w \rangle \in W_{\text{WP},R}(r,p), \pi \in \text{WP}(\pi), p \in \text{PP}\}$$

$$\text{WP}' = \{\pi | \exists \ \tau \text{ s.t., } z_\tau^{\pi,p} = 1, \pi \in \text{WP}(p), p \in \text{PP}\}$$

3.3.2. Level 2. Lower Bounding. *Stage 1. Well Disaggregation.* Solve RDMP for the new set of reservoirs R' and well platforms WP′ with all the wells disaggregated in R'. The problem RDMP is defined as

$$\text{RDMP: } \max \Psi = q(c_\tau,x_\tau,z_\tau,e_\tau,Z_{k,\tau},ZT_{k,\tau}) \quad (48)$$

subject to

$$m_1(x_\tau) = 0 \quad (49)$$

$$m_2(x_\tau,v_\tau,g_\tau,\delta_\tau,\alpha,\beta) = 0 \quad (50)$$

$$m_3(x_\tau,l_\tau,g_\tau,\rho) \le 0 \quad (51)$$

$$m_4(x_\tau,\phi_\tau,\bar{y}_{j,\tau},\lambda_{j,\tau}) = 0 \quad (52)$$

$$m_5(x_\tau,z_\tau) \le 0 \quad (53)$$

$$m_6(x_\tau,z_\tau,d_\tau,e_\tau) \le 0 \quad (54)$$

$$m_7(z_\tau,Z_{k,\tau},ZT_{k,\tau}) \le 0 \quad (55)$$

$$m_8(x_\tau,z_\tau) \le 0 \quad (56)$$

$$\forall \ w \in W_{\text{WP}}(\pi), \pi \in \text{WP}'(p), p \in \text{PP}$$

$$\forall \ j \in J(r), r \in R', \tau = 1 \ldots T_{\text{a}}$$

The solution obtained from RDMP yields a subset of wells $W'_\tau(\pi)$ drilled in aggregate time period τ. The new set of wells is defined as

$$W'_r = \{w \in W_{\text{WP}}(\pi), \pi \in \text{WP}'(p), p \in \text{PP}, | \exists \ \tau \text{ s.t. } z_\tau^{w,\pi,p} = 1\}$$

$$\forall \ \tau = 1 \ldots T_{\text{a}}$$

The solution obtained from RDMP is also used to determine an upper bound on the cumulative flow of oil from a reservoir. The cumulative flow is a nondecreasing function of time, and from Figure 13, it is evident by observation that if

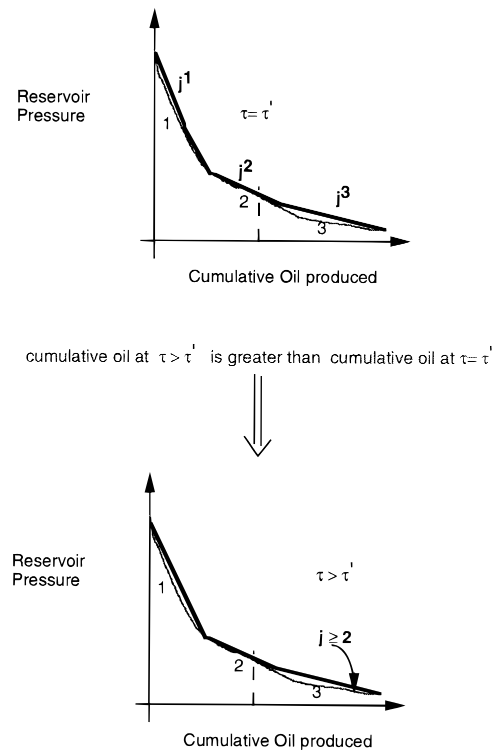

Figure 13. Selection of interpolation pieces.

$$\hat{x}_\tau^1 \Rightarrow y_{j,\tau} = 1 \qquad \text{for} \quad j = j^1$$

then

$$\hat{x}_\tau \ge \hat{x}_\tau^1 \Rightarrow y_{j,\tau} = 0 \qquad \text{for} \quad j < j^1$$

A new set $J_\tau(r)$ is defined as follows, to bound the cumulative oil flow from reservoirs in each period τ. Let

$$\hat{x}_\tau^{w,\pi,p} \Leftrightarrow y_{j,\tau} = 1 \qquad \text{for} \quad j = j_r^1$$

then, for each τ, define a new set

$$J_\tau(r) = \{j | j \le j_r^1, r \in R'\}$$

Clearly, this reduces the number of interpolation variables significantly in each period τ.

Stage 2. Disaggregation of Time Periods, with Piecewise Linear Interpolation. The original problem MP is now solved for the new set of wells W_τ, with disaggregated time periods and piecewise linear interpolation with $y_{j,t}$ as a binary variable. The resulting problem would, however, be too large due to the large number of interpolation binaries. To simplify the problem, the flow profile constraints in (38) are relaxed and the problem is solved independently of each set of periods $t = (\tau - 1)t_{\text{a}} + t'$, $t' = 1 \ldots t_{\text{a}}$ in aggregate period τ. Thus, FDMP is solved sequentially for each $\tau' = 1 \ldots T_{\text{a}}$, where FDMP is defined as

$$\text{FDMP: } \max \Psi = q(c_t,x_t,z_t,e_t,Z_{k,t},ZT_{k,t}) \quad (57)$$

subject to

$$m_1(x_t) = 0 \tag{58}$$

$$m_2(x_t,v_t,g_t,\delta_t,\alpha,\beta) = 0 \tag{59}$$

$$m_3(x_t,l_t,g_t,\rho) \le 0 \tag{60}$$

$$m_4(x_t,\phi_t,y_{j,t},\lambda_{j,t}) = 0 \tag{61}$$

$$m_5(x_t,z_t) \le 0 \tag{62}$$

$$m_6(x_t,z_t,d_t,e_t) \le 0 \tag{63}$$

$$m_7(z_t,Z_{k,t},ZT_{k,t}) \le 0 \tag{64}$$

$$\forall\ w \in W''_\tau(\pi),\ \pi \in \mathrm{WP}'(p),\ p \in \mathrm{PP}$$

$$\forall\ j \in J_{\tau'}(r),\ r \in R',\ t = (\tau' - 1)t_a + 1 \ldots \tau'(t_a)$$

The solution of FDMP yields values of all the investment binaries z_t, and drilling rig binaries $ZT_{k,t}$ in each disaggregated time period t. However, since the profile constraints were relaxed, it is possible that the flow profile may not be smooth and the solution may be infeasible in MP. In the next level, the flow profile is smoothened to get a feasible solution to MP.

3.3.3. Level 3. Smoothening of Profiles. It is necessary to solve the problem for all time periods t simultaneously in order to enforce the profile constraints. However, the number of binary interpolation variables would be too large to solve the problem in reasonable computation time.

To simplify the problem, the problem MP is now formulated for fixed values of $\bar{z}_t$ and $\bar{Z}_{k,t}$. The resulting problem SDMP is solved sequentially for each $\tau' = 1 \ldots T_a$ as follows:

(1) The time periods are disaggregated for period τ'. For all periods $\tau \ge \tau' + 1$ aggregated periods are used.

(2) On the basis of the solution of FDMP, define new sets $J_\tau'(r)$ as follows. Let

$$\hat{x}_\tau^{w,\pi,p} \Leftrightarrow y_{j,\tau} = 1 \qquad \text{for} \quad j = j_r^1$$

and

$$\hat{x}_{\tau+1}^{w,\pi,p} \Leftrightarrow y_{j,\tau} = 1 \qquad \text{for} \quad j = j_r^2$$

then, for each τ, define a new set

$$J'_\tau(r) = \{j | j_r^1 \le j \le j_r^2,\ r \in R'\}$$

This significantly reduces the number of interpolation binary variables in the problem SDMP. Also, the periods $\tau \ge \tau' + 1$ are aggregated, leading to a smaller number of interpolation binaries. Since the flows are defined for all aggregate time periods, the flow profile constraints may be written for all disaggregated time periods in period τ' and across aggregated time periods as well.

Thus, the model SDMP is solved as follows:

For each $\tau' = 1 \ldots T_a$ solve

$$\text{SDMP:}\ \max \Psi = q(c_t,x_t,\bar{z}_t,e_t,\bar{Z}_{k,t},\overline{ZT}_{k,t}) \tag{65}$$

subject to

$$m_1(x_t) = 0 \tag{66}$$

$$m_2(x_t,v_t,g_t,\delta_t,\alpha,\beta) = 0 \tag{67}$$

$$m_3(x_t,l_t,g_t,\rho) \le 0 \tag{68}$$

$$m_4(x_t,\phi_t,y_{j,t},\lambda_{j,t}) = 0 \tag{69}$$

$$m_5(x_t,\bar{z}_t) \le 0 \tag{70}$$

$$m_6(x_t,\bar{z}_t,d_t,e_t) \le 0 \tag{71}$$

$$m_8(x_t,\bar{z}_t) \le 0 \tag{72}$$

$$\forall\ w \in W'_{\tau'}(\pi),\ \pi \in \mathrm{WP}'(p),\ p \in \mathrm{PP}$$

$$\forall\ j \in J'_{\tau'}(r),\ r \in R'$$

$$\forall\ t = (\tau' - 1)t_a + t' \ldots \tau' + 1 \ldots T_a,\ t' = 1 \ldots t_a$$

The flow rates obtained from SMDP for each disaggregated period t in τ' is stored for each τ' and yields the solution to the MP. The objective function value Ψ is then calculated by evaluating the solution at fixed values of flow rate and values of investment and drilling rig binary variables.

4. Examples

4.1. Example 1. A small example problem is presented in this section to illustrate the performance of the algorithm. The offshore field in this problem consists of two reservoirs and four wells (two wells per reservoir) in one field, with a single well platform and production platform, a single drilling rig, and it is solved for a 6-year planning horizon with quarterly time periods (24 time periods). Note that this is only an illustrative problem to demonstrate algorithmic performance. Much larger problems can be solved in reasonable computation time using the proposed decomposition algorithm. The field data for this example is presented in Table 1. The piecewise interpolation data used for this example is shown in Table 2.

Table 1. Data for Example 1

	PI of wells[a]			
	R1W1	R1W2	R2W1	R2W2
PI (TBD/psi)	47.6	83.8	158.4	210.2
economic data				
inflation rate	4%			
discount rate	12%			
oil price	$22/barrel			
investment costs in millions of dollars				
	fixed cost	variable cost per TBD		
production platform	70	0.47		
well platform	10	0.19		
well costs				
R1W1	5.83			
R1W2	6.34			
R2W1	6.69			
R2W2	5.76			

[a] R1W1 denotes well 1 in reservoir 1.

Table 2. Interpolation Data for Example 1[a]

	reservoir 1			
	pressure (psi)	cum. oil (mill. barrels)	GOR	cum. gas (MMSCF)
1	1.0	0.0	0.888	0.0
2	0.885	0.123	0.739	0.094
3	0.629	0.737	0.729	0.555
4	0.539	0.860	0.949	0.847
5	0.501	0.934	1.0	0.971
6	0.486	1.0	0.963	1.0

	reservoir 2			
	pressure (psi)	cum. oil (mill. barrels)	GOR	cum. gas (MMSCF)
1	1.0	0.0	1.0	0.0
2	0.936	0.05	0.963	0.06
3	0.869	0.2	0.924	0.235
4	0.82	0.35	0.891	0.397
5	0.729	0.7	0.829	0.738
6	0.668	1.0	0.786	1.0

[a] Data scaled with respect to largest value.

The problem was first modeled using GAMS (Brooke et al., 1992) and solved using a full space MIP solver (CPLEX/GAMS). The problem involved 144 binary variables, 870 SOS variables, 3338 constraints, and 3079 continuous variables. The optimal solution of \$56.6 million was obtained in 3500 CPU s on a HP9000/700. The full space MIP was also solved by not declaring the binary variables as SOS variables, yielding a solution of \$47.9 million after 10 000 CPU s. Using the sequential decomposition algorithm, a feasible integer solution of \$56.1 million is obtained in less than 25 CPU s. The upper bound obtained in level 1 is \$61 million. The gap between the upper and lower bounds is less than 9%, suggesting a very good initial solution obtained from the decomposition algorithm. Also, the solution obtained using the decomposition algorithm is within 1% of the solution obtained using the full space solver. The computational expense for the decomposition algorithm is very small as a two-order of magnitude reduction was achieved in solution time. An important feature of this algorithm is that a measure of the quality of the solution is obtained from the calculated upper bound. The computational results are presented in Table 3. Note that the MILP problems that had to be solved with the proposed method are significantly smaller than the original problem.

Figure 14 shows the solution obtained, indicating the value of decision variables obtained from the planning model and that only two wells were drilled (R1W1 and R2W2). As indicated, the reservoirs are depleted within 5 years, and the capacity of the platforms is determined by the largest flow in all time periods. The platform facilities and wells are installed in the first time period as it maximizes the NPV when the oil is extracted as early as possible. It is interesting to note that the model chooses only one well in each reservoir. For the second reservoir, well 2 is chosen as it has a higher PI and lower cost. For the first reservoir, there is a trade-off between cost and the PI, and it chooses well 1 because even with the lower value of PI, the maximum flow rate is obtained as a result of a large driving force due to the reservoir pressure.

4.2. Example 2. A larger example problem, consisting of 1 field with 11 reservoirs and 29 wells was solved for a 24-time period, 6-year planning horizon. A potential choice of 2 well platforms associated with a single production platform was available, based on the location of the wells and their allocation to well platforms. A single drilling rig is available for drilling wells from the well platforms. The full space MILP problem, which involved 240 binary variables, 4880 SOS variables, 18 746 constraints, and 16 953 continuous variables, could not obtain a feasible integer solution using a full space MIP solver (CPLEX) even after 100 000 CPU s. The decomposition algorithm obtained a feasible integer solution of \$273 million within 6,600 CPU s. The solution was within 13% of the upper bound of \$310 million. The computational results for this example are presented in Table 4. The solution obtained indicates a requirement of only a single well platform, which results in a significant reduction in investment costs. In addition, only 13 wells are required in order to completely deplete the reservoirs, as compared to the conventional solution of drilling all 29 wells. This not only reduces the investment cost (by \$96 million), but also increases the availability of the drilling rig. The potential savings in investment cost clearly justifies the planning exercise for the development of oil fields. Figure 15 shows the flow profile obtained for 7 wells (out of 13 wells obtained from the solution) and for the well platform. The drilling period for the wells can be deduced from the flow profile, which is the earliest period when the scaled flow rate is non-zero.

5. Conclusions

A comprehensive multiperiod MILP model has been presented for the simultaneous capacity planning and scheduling of well and facility operations in offshore oil fields. The model incorporates a piecewise linear approximation to the reservoir performance, resource constraints, and surface constraints simultaneously in a single formulation. A sequential decomposition algorithm was proposed to solve the multiperiod MILP problem. Although the proposed algorithm is not guaranteed to find the optimal solution, it yields both

Table 3. Results for Example 1[a]

method	solution (\$ (millions))	equations	size: continuous variables	size: binary variables	SOS variables	time (CPU s) HP9000/700
full space	56.6	3338	3079	144	870	3500
		LP relaxation = 74.9				
proposed algorithm	56.1					25
level 1 (UB)	61.0	538	413	36	28	3
level 2						
stage 1		800	595	36	42	12
stage 2		397	257	24	56	4
level 3 (LB)	56.1	710	451	0	72	6

[a] 1 field, 2 reservoirs, 4 wells, 1 WP, 1 PP, and 1 drilling rig.

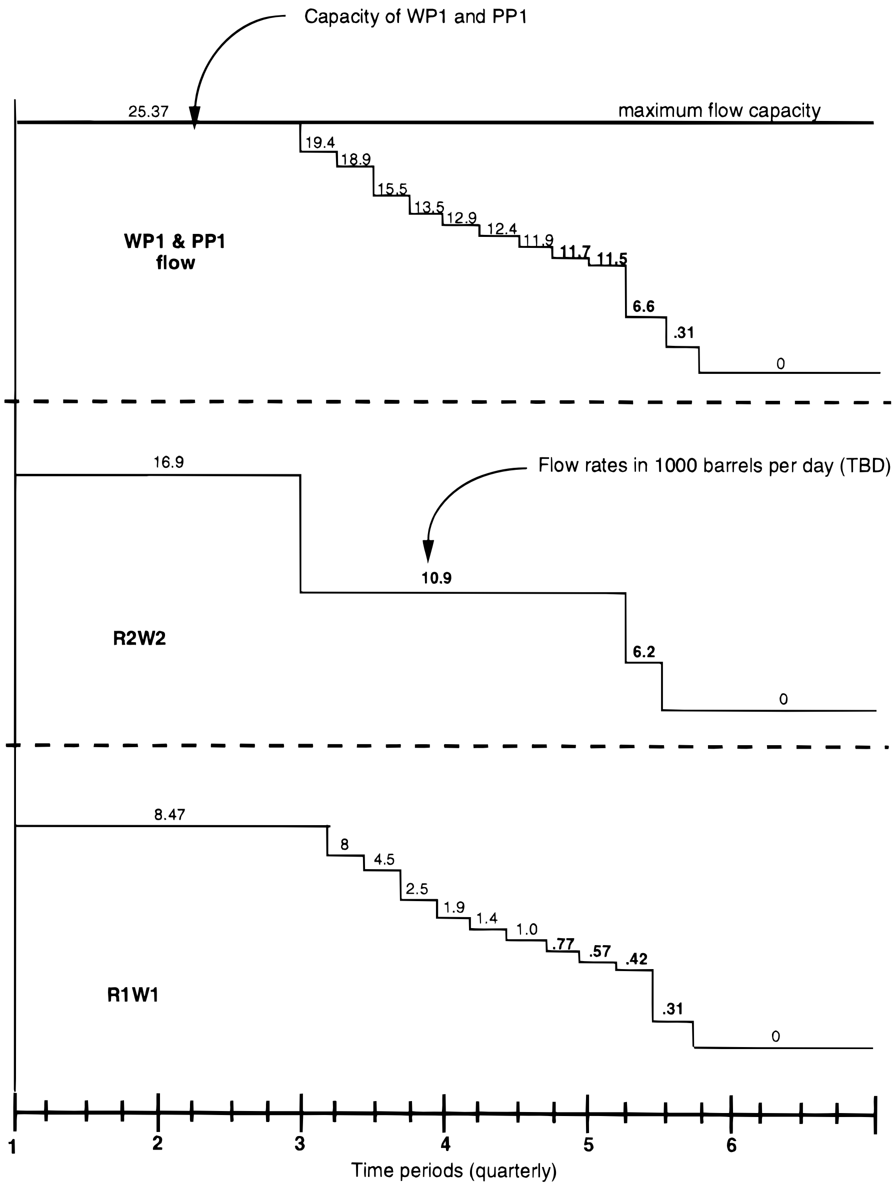

Figure 14. Example 1—solution.

Table 4. Results for Example 2[a]

method	solution ($(millions))	equations	size: continuous variables	size: binary variables	SOS variables	time (CPU s) HP9000/700
full space	*b*	18 746	16 953	240	4880	>100 000
		LP relaxation = 310				
proposed algorithm	273					6 600
level 1 (UB)	310	2899	2199	60	140	1000
level 2						
stage 1		3620	2642	36	175	4300
stage 2		1153	743	24	194	500
level 3 (LB)	273	2943	1583	0	273	800

[a] 1 field, 11 reservoirs, 29 wells, 2 WP, 1 PP, and 1 drilling rig. [b] No integer solution obtained.

a good feasible solution and an upper bound that provides a measure of the quality of the solution. Large industrial problems were solved using the proposed method in reasonable computation time, showing the practical applicability of this work as a planning tool for oil field development.

As for future work, as noted in the review, Devine and Lesso (1972) addressed the continuous two-dimensional facility location and allocation problem (without scheduling of well drilling and production planning). They presented a solution where they iteratively solved the problem using a two-stage algorithm, where they first fixed the well allocation and solved a location problem, followed by a solution of the allocation problem for a fixed location of wells. We see the method in this paper as a useful part of such a two-step algorithm, where the location of the platforms and wells are obtained from the solution of the location problem in

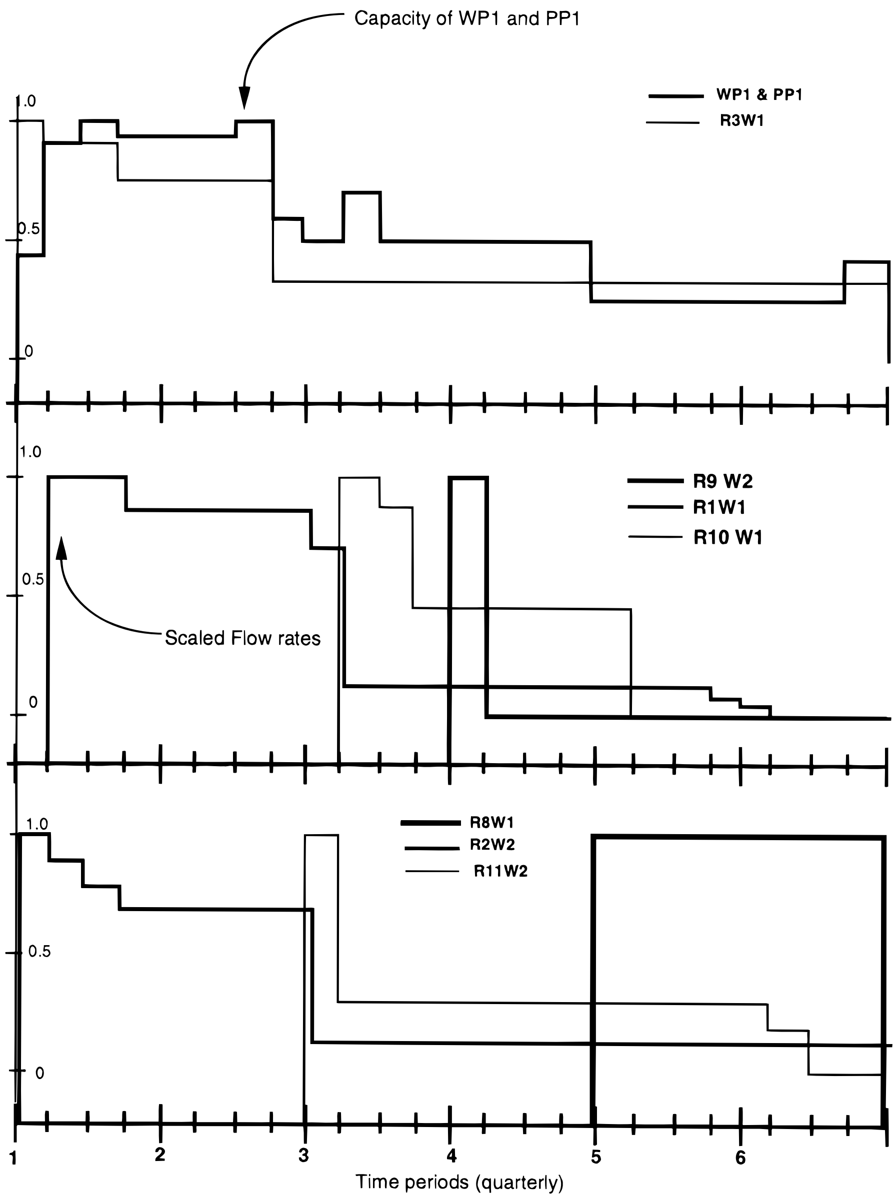

Figure 15. Example 2—scaled flow profile.

the first stage, and for a fixed location, the allocation and scheduling problem may be solved using the algorithm presented in this paper. Future work on the use of nonlinear equations as an alternative to piecewise linear interpolation is currently in progress.

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Appendix A. Piecewise Linear Interpolation

It is important to model the reservoir performance in order to capture the varying conditions in the reservoir. The reservoir pressure and GOR is a function of the cumulative amount of oil removed (see Figure 3) and this performance curve is nonlinear. In this work, it will be approximated using piecewise linear interpolation (see Nemhauser and Wolsey, 1988). The choice of the linear approximation piece used for interpolation is made using binary variables $y_{j,t}^{r,f}$. The constraints involved for linear interpolation are shown only for the pressure variables below, associated with a reservoir r (i.e., for eq 11). Note that $\tilde{v}_{j,t}^{r,f}$ is the value of $v_{j,t}^{r,f}$ at the end point of the jth piece used in linear interpolation and is prespecified from the reservoir performance curve. The equations are

$$v_t^{r,f} = \sum_{j \in J(R)} \lambda_{j,t}^{r,f} \tilde{v}_{j,t}^{r,f} \tag{A1}$$

$$\sum_{j \in J(r)} \lambda_{j,t}^{r,f} = 1 \tag{A2}$$

$$\lambda_{j,t}^{r,f} \leq y_{j,t}^{r,f} + y_{j-1,t}^{r,f} \tag{A3}$$

$$\sum_j y_{j,t}^{r,f} = 1 \tag{A4}$$

Equation A4 allows only one index j for which $y_{j,t} = 1$. Note that the variables y may be treated as SOS variables over the set j. From eq A3, $\lambda_{j,t}$ can be non-

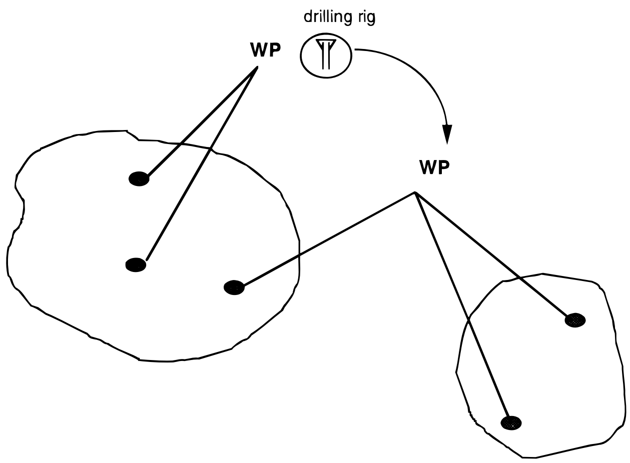

Figure 16. Use of drilling rig for drilling wells from well platforms.

zero for only two consecutive j. Thus, the corresponding jth piece is used for linear interpolation as all other $\lambda_{j,t} = 0$. Equation A1 determines the value of the interpolated variable as a convex combination of $\tilde{v}_{j,t}^{r,f}$ and $\tilde{v}_{j-1,t}^{r,f}$.

Appendix B. Drilling Rig Constraints

The following equations are used to model the resource constraints associated with drilling rigs. The main issues involved in the resource constraints are summarized below. Wells are drilled from the associated well platforms using drilling rigs (see Figure 16). However, there are only a limited number of drilling rigs (usually less than the number of well platforms) available in any time period for drilling a well. Besides the availability of the rig, the physical location of the rig on the well platform also determines which wells may be drilled in any time period. Thus, the scheduling of the movement of rigs between well platforms is essential to ensure its availability. The movement of drilling rigs across well platforms also lead to a fixed charge and a loss of time in the availability of the rig. This loss of time is accounted for in eq 26, where the number of wells drilled in a time period is a function of the number of moves of the rigs across well platforms.

The equations involved are as follows:

$$z_t^{w,\pi,p} \le \sum_{k \in D} zd_{k,t}^{\pi,p} \tag{B1}$$

$$\forall\ w \in W_{\mathrm{WP}}(\pi),\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t = 1\ ...\ T$$

Equation B1 states that if any well associated with well platform π is drilled in period t, then $\sum_k zd_{kt}$ must be 1 in that time period, implying at least one drilling rig is located on that well platform.

The number of moves of the drilling rig can be completely determined, if the first and last locations of the rig on well platforms are known in a time period. The number of moves of the rig across time periods can be determined from its first location in period $t + 1$ and the last location in period t. The moves within each time period is the minimum number of moves, which is determined from the number of well platforms on which the rig should be located in that time period.

Define binary variables $zf_{k,t}^{\pi,p}(zl_{k,t}^{\pi,p})$, which are equal to 1 if the kth rig is located on well platform π first (last) in period t. Thus, if in any period, if drilling rig k is located first on well platform π, then $zf_{k,t}(\pi,p) = 1$. Figure 17 shows an example of the values of the binary variables zf and zl associated with well platforms for a particular drilling schedule.

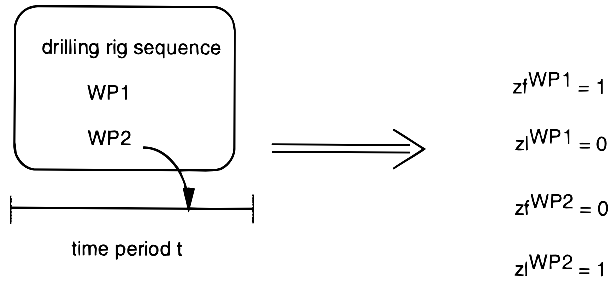

Figure 17. Illustration of values of drilling rig variables.

Clearly, the sum of zf and zl must be less than 1 when the rig is located on more than one well platform in the same time period. However, if all wells are drilled from the same well platform in a time period, then both zf and zl will be 1. To model both possibilities, a slack binary variable $s_{k,t}^p$ is used. Thus, $s_{k,t}^p$ takes a value of 1 (a slack) when only the πth platform is used for drilling using the kth rig in period t (equation B3).

The location of the rigs on the well platforms is modeled by the following equations:

$$zf_{k,t}^{\pi,p} + zl_{k,t}^{\pi,p} \le 1 + s_{k,t}^p \tag{B2}$$

Equation B2 states that the kth rig is located on the πth platform either first or last in that time period t (when $s_{k,t}^p = 0$). The slack variable takes a non-zero value if all wells are drilled from a single πth platform in that time period (eq B3). Thus, if both zf and zl are 1, then the slack is non-zero:

$$\left(\sum_{\pi \in \mathrm{WP}(p)} zd_{k,t}^{\pi,p} - 1\right) \le \Omega^{\mathrm{u}}(1 - s_{k,t}^p) \tag{B3}$$

The binary variables $zf_{k,t}^{\pi,p}$ and $zl_{k,t}^{\pi,p}$ are non-zero only if a well is drilled from well platform π:

$$zf_{k,t}^{\pi,p} \le zd_{k,t}^{\pi,p} \tag{B4}$$

$$zl_{k,t}^{\pi,p} \le zd_{k,t}^{\pi,p} \tag{B5}$$

Thus, eqs B4 and B5 indicate that $zf_{k,t}^{\pi,p}$ and $zl_{k,t}^{\pi,p}$ are zero when the kth drilling rig is not used to drill from platform π.

Finally, any move of rig k from period $t - 1$ to t is calculated from the value of zf_t and zl_{t-1}. The minimum number of moves required within a time period is then calculated from the number of well platforms on which rig k is located in that period.

$$zf_{k,t}^{\pi,p} - zl_{k,t-1}^{\pi,p} \le ZT_{k,t}^{\pi,p} \tag{B6}$$

$$\left(\sum_{\pi} zd_{k,t}^{\pi,p} - 1\right) \le Z_{k,t}^p \tag{B7}$$

$$\forall\ k \in D,\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t = 1\ ...\ T$$

Equation B6 determines the movement of the kth rig from platform π across time periods based on whether the rig is located first in period t on π and last in period $t - 1$. Equation B7 determines the movement of the rigs within a time period from the number of well platforms used for drilling in that period.

Appendix C. Proof of Property 1

Proof: Consider each of the modifications to MP individually.

1. Aggregation of Time Periods. For constraints involving variables (i.e., flow rates, pressure, etc.) independent of the length of time period Δt (i.e., eqs 1−8), aggregation of time periods is equivalent to *removing* all equations associated with time periods t where $t = (\tau - 1)t_a + t'$, $\forall\ t' = 2, 3...t_a$, $\tau = 1 \ldots T_a$.

$T_a = (T/t_a) =$ the number of aggregated time periods in the formulation.

Thus only periods $t = (\tau - 1)t_a + 1$ are included in the formulation.

Additionally, the constraint with cumulative flows (eq 9) can be treated as follows. Writing the cumulative flows equations (9) for periods t in aggregated period τ gives

$$\hat{x}_t^{w,\pi,p} = \hat{x}_{t-1}^{w,\pi,p} + x_{t-1}^{w,\pi,p}\Delta t \qquad t = (\tau - 1)t_a + 2 \tag{C1}$$

$$\hat{x}_t^{w,\pi,p} = \hat{x}_{t-1}{}^{w,\pi,p} + x_{t-1}{}^{w,\pi,p}\Delta t \qquad t = (\tau - 1)t_a + 3 \tag{C2}$$

$$\ldots$$

$$\hat{x}_t^{w,\pi,p} = \hat{x}_{t-1}^{w,\pi,p} + x_{t-1}^{w,\pi,p}\Delta t \qquad t = (\tau)t_a + 1 \tag{C3}$$

$$\forall\ w \in W_{WP}(\pi),\ \pi \in WP(p),\ p \in PP$$

Adding eqs C1−C3 yields

$$\hat{x}_{t''}^{w,\pi,p} = \hat{x}_{t'}^{w,\pi,p} + \sum_{t=t'}^{(\tau)t_a} x_t^{w,\pi,p}\Delta t \tag{C4}$$

$$t' = (\tau - 1)t_a + 1, \qquad t'' = (\tau)t_a + 1$$

Now, for aggregated time period τ the flow profile is nonincreasing. Therefore,

$$x_{t'}^{w,\pi,p} \geq x_t^{w,\pi,p} \qquad \forall\ t = t' + t''' \tag{C5}$$

$$t''' = 0, 1, \ldots (t_a - 1), \qquad t' = (\tau - 1)t_a + 1$$

By adding eq C5 for each t, and since $\Delta\tau = \Delta t(t_a)$, the cumulative flow at end of time period t is given by

$$\hat{x}_{t'}^{w,\pi,p}\Delta\tau \geq \sum_{t=t'}^{(\tau)t_a} x_t^{w,\pi,p}\Delta t, \qquad t' = (\tau - 1)t_a + 1 \tag{C6}$$

and this yields

$$\hat{x}_{t''}^{w,\pi,p} \leq \hat{x}_{t'}^{w,\pi,p} + x_{t'}^{w,\pi,p}\Delta\tau \tag{C7}$$

$$t'' = \tau(t_a) + 1, \qquad t' = (\tau - 1)t_a + 1$$

Since the aggregate problem only consists of periods $t = (\tau - 1)t_a + 1$, the cumulative flow in period $t = (\tau)t_a$ is overestimated. The oil flow rate has a positive coefficient in the objective function; therefore, an overestimation of the flow leads to an overestimation of the objective function value.

For equations involving binary variables (eqs 14−16), the terms z_t may be added for each time period t in aggregated period τ as follows:

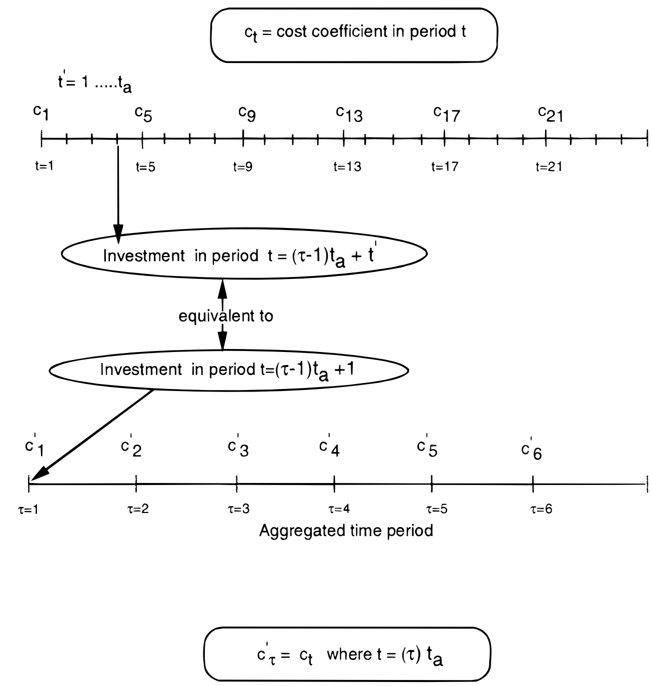

Figure 18. Lagged cost coefficients for investment binary variables.

$$\sum_{\tau}^{T_a} \sum_{t=(\tau-1)t_a+1}^{(\tau)t_a} z_t^{w,\pi,p} \leq 1 \tag{C8}$$

Define aggregated binary investment variables z_τ such that if an investment is made in any period t during an aggregated period τ, then it is made in the beginning of the period.

Thus,

$$\sum_{t=(\tau-1)t_a+1}^{(\tau)t_a} z_t^{w,\pi,p} = z_\tau^{w,\pi,p} \tag{C9}$$

For the linear equations (18)−(22), the sum of binary variables z_t within an aggregated period are replaced by z_τ.

Thus, the investment variables are used only for aggregated time periods. Since only equations for aggregated periods are in the formulation, this implies that any investment to be made in a period $t = (\tau - 1)t_a + t'$ for all $t' = 2, 3 \ldots t_a$, are made in period $t = (\tau - 1)t_a + 1$.

Thus,

$$c'_\tau = c_{t=\tau t_a} \tag{C10}$$

where the aggregated period τ corresponds to the disaggregated period $t = (\tau - 1)t_a + 1$.

Thus, by choosing the discounted coefficients with a lag of t_a time periods, the lagged investment cost coefficients c'_t underestimate the actual cost coefficients c_t (since $c'_t \leq c_{t'}\ \forall\ t' = t \ldots t + t_a$, and therefore the investment costs are underestimated, leading to an overestimation of the objective function value. Figure 18 illustrates the use of aggregate investment binary variables and lagged cost coefficients for aggregated periods.

Thus, inclusion of equations only for aggregated periods leads to a relaxation of the feasible region. The overestimation of the objective follows from using lagged cost coefficients and an overestimation of the cumulative flow amount.

2. Aggregation of Wells in Reservoirs. Since eqs 6−8 are linear, addition of equations associated with all wells $w \in W_{WP,R}(r,\pi)$ in the aggregation leads to variables associated with aggregated well $\langle w\rangle$ as shown below.

$$\langle x_t^{r,\pi,p}\rangle = (\sum_{w\in W_{WP,R}(r,\pi)} x_t^{w,\pi,p}) \tag{C11}$$

$$\langle x_t^{r,\pi,p}\rangle = \langle l_t^{w,\pi,p}\rangle + \langle g_t^{r,\pi,p}\rangle \tag{C12}$$

$$\langle l_t^{r,\pi,p}\rangle \le (\sum_{w\in W_{WP,R}(r,\pi)} \rho)P_{\max} \tag{C13}$$

$$\langle g_t^{r,\pi,p}\rangle \le \langle l_t^{w,\pi,p}\rangle \mathrm{GOR}_{\max} \tag{C14}$$

$$\forall\ r \in R(f),\ f \in F$$

$$\forall\ \pi \in \mathrm{WP}(p),\ p \in \mathrm{PP},\ t = 1 \ldots T$$

Thus, from eq C13, the equivalent PI of the aggregated well is the sum of the PI of wells in the aggregation.

In addition, the flows from the aggregated wells is assumed to be flowing in a single pipe, and the linear pressure drop coefficient is scaled by the number of wells in the aggregation to get an equivalent pressure drop. Let $N_w = |W_{WP,R}(r,\pi)|$ be the cardinality of the set $W_{WP,R}$. Thus, adding eq 4 for each $w \in W_{WP,R}(r,\pi)$, and using the fact that each well inside the same reservoir are at the same pressure, $v_t^{r,\pi,p}$ yields

$$N_w v_t^{\pi,p} = N_w v_t^{r,\pi,p} - \alpha\langle x_t^{r,\pi,p}\rangle - \beta\langle g_t^{r,\pi,p}\rangle - \langle \delta_t^{r,\pi,p}\rangle \tag{C15}$$

and this implies

$$v_t^{\pi,p} = v_t^{r,\pi,p} - \frac{\alpha}{N_w}\langle x_t^{r,\pi,p}\rangle - \frac{\beta}{N_w}\langle g_t^{r,\pi,p}\rangle - \frac{1}{N_w}\langle \delta_t^{r,\pi,p}\rangle \tag{C16}$$

Thus, an equivalent pressure drop coefficient may be used, where α and β may be scaled by N_w.

3. Relaxation of Piecewise Linear Interpolation. Clearly, fixing all $y_{j,t} = 1$ (see Appendix A), implies that the feasible region of the graph (see convex hull shown in Figure 7) includes all points which are convex combinations of $\tilde{v}_{j,t}^r$ for all j. This is in addition to the points that lie on the linear pieces to be used for interpolation. This represents a relaxation of the feasible space of MP.

Thus, the feasible region is relaxed in conjunction with an overestimation of Ψ, and therefore the solution to RMP is an upper bound to the solution of MP. QED.

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