

Optimal Power Allocation for Smart-Grid Powered Point-to-Point Cognitive Radio System

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Abstract—This paper proposes an optimal power allocation analysis for a point-to-point wireless system when powered by a smart grid. We propose to minimize the total power consumption cost while ensuring individual and total throughput constraints. The power cost is computed based on different dynamic pricing models of the power consumption. Analytical solutions are derived for each pricing model. The derived solutions are shown to be modified versions of the water-filling solution. Water-filling based algorithms are proposed for the resource allocation with each pricing model. Performance comparison and pricing effect are shown through simulations.

Index Terms—Energy consumption awareness, spectrum access efficiency, smart grid systems.

I. INTRODUCTION

Power consumption has become one of the key design requirements of communication systems since the emergence of the concept of ‘green communications’ [1]. In fact, modern wireless systems are urged to cope with the new regulations and reduce their power consumption and CO_2 impact. On the other hand, smart grids are shown to be the future of energy delivery due to their high abilities to dynamically adapt the prices in order to prevent energy concentration and flatten the peak loads. It holds the promise to create a distributed energy delivery network and enhance the capacity and the efficiency of the grid by means of two-way communications between end-users and power plants, as well as by the inclusion and use of various types of renewable energy sources [2]. Therefore, smart grids are envisioned to provide a real-time power-usage pricing, thereby creating an opportunity for wireless systems to adjust their power consumption accordingly to reduce their power consumption’s cost. Given that, modern wireless systems should adapt their transmission schemes to profit from opportunities given by smart grids.

On the other hand, the concept of cognitive radios (CR) has emerged as a key solution for the current spectrum scarcity caused by the static spectrum allocation policies [3], [4]. It allows an efficient use of the spectrum by allowing the coexistence of license-exempt users, called secondary users, with legacy users, called primary users. Due to its potentials, CR has created a significant research interest, ranging from spectrum sensing methods [5], [6] to resource allocation methods [4]. With the emergence of smart grids, resource allocation problems in CR become more pertinent due to the new opportunities given by dynamic power pricing. Therefore, it is important to revisit the resource allocation protocols

proposed for CR networks by taking into consideration the power provided from smart grids.

In smart grids, CR has been seen as the best alternative to address the communication challenge between the different components of the grid due to its spectrum efficiency capabilities [7]. In our work, we are envisaging to profit from smart grid power pricing opportunities to enhance cognitive radio energy efficiency. To this end, optimizing resource allocation for wireless systems when enabled by smart grid power has been investigated in the literature. In [8], the authors considered an LTE system when powered by a smart grid system that includes different retailers. The resource allocation for the operator tried to minimize the power consumption through the shut off of the non-used base stations and optimize the procured power from the retailers. On the other hand, other works considered a pricing scheme for the allocation of the available resources [9]. For instance, the authors proposed a Stackelberg game to allocate the power among a set of users while a price on the interference was imposed by the primary users which provide some revenue. Dynamic pricing was also a key component that affects the consumption behavior and may encourage the shift of the power consumption. Different research works have considered this scheme. For example, in [10], the authors proposed a usage based dynamic pricing by considering different threshold that models the peak loads and the normal load and tries to influence the users’ behavior.

In this work, we propose a resource allocation analysis for a point-to-point wireless system when powered by the smart grid. Different pricing models are studied. The objective is to minimize the total power consumption cost of the wireless system while ensuring minimum individual and total throughput thresholds and respecting interference constraints towards licensed users. Even-though the targeted power gains are not important to generate profit in the case of classic wireless systems due to small power consumption, this work can be of paramount importance for large wireless systems with high power consumption and high targeted throughput (60 Ghz communication, free space optical communication) which are envisaged to replace back-haul connections based on wired links until now. We are targeting analytical solutions to allow analysis of the system’s performance and obtained gains. Obtained results can be employed later in decision algorithms for multiple service providers.

The rest of this paper is organized as follows. Section II describes the system model. In Section III, we formulate our

resource allocation problem and present the smart grid dynamic power pricing models used in this work. In Section IV, we analyze the resource allocation problems and propose the power allocation solutions for the different pricing models. Simulation-based analysis is presented in Section V. Finally, conclusions are presented in Section VI.

II. SYSTEM MODEL

We consider a cognitive radio system composed of one secondary user noted CU and one primary user noted PU. We assume an underlay interference sharing mode where the CU is allowed to share all the available channels with the PU under the constraint of not exceeding the interference temperature I^{th} threshold at the PU receiver. We suppose N channels are available for the CU transmission. In our work, we do not specify the nature of these channels but it can be applied to different multi-channel schemes (time, frequency, space, or physical division). The gains of the direct channels of the CU are denoted by $\{g_1^{(c)}, \dots, g_j^{(c)}, \dots, g_N^{(c)}\}$, while the interference channels from the cognitive transmitter to the primary receiver are denoted by $\{g_1^{(p)}, \dots, g_j^{(p)}, \dots, g_N^{(p)}\}$.

III. PROBLEM FORMULATION

A. Power Allocation Problem

The objective of our work is to determine the optimal allocated power P_j at each channel j that minimizes the cost of the power consumption while guaranteeing a minimum total throughput r_T^{th} as well as an individual minimum throughput r_j^{th} at each channel j and respecting the interference constraint at the PU. Mathematically, the optimization problem is written as follows

$$\min_{P_1, \dots, P_N} \sum_{j=1}^N c_j(P_1, \dots, P_N) \quad (1a)$$

$$\text{s.t.} \quad \sum_{j=1}^N r_j(P_j) \geq r_T^{\text{th}} \quad (1b)$$

$$r_j(P_j) \geq r_j^{\text{th}} \quad \forall j \in \{1 \dots N\}, \quad (1c)$$

$$P_j g_j^{(p)} \leq I^{\text{th}} \quad \forall j \in \{1 \dots N\}, \quad (1d)$$

where $c_j(P_1, \dots, P_N)$ is the cost of the power consumed at the j -th channel and $r_j(P_j)$ is the throughput obtained through the j -th channel, written using the Shannon expression as

$$r_j(P_j) = \log_2 \left(1 + \frac{P_j g_j^{(c)}}{N_0} \right), \quad (2)$$

where N_0 is the power spectral density of the noise.

One of the application of these constraints, the N channels represent different paths for the CU transmission (different time slots, different antennas, different users). r_j^{th} will be used as an individual throughput per path that needs to be guaranteed as a minimum required service per path while r_T^{th} will be the total needed throughput of the wireless system.

In this optimization problem, depending on the values of the individual throughput threshold r_j^{th} , the total throughput

threshold r_T^{th} , and the interference thresholds I^{th} , we distinguish these two special cases:

- If $\sum_{j=1}^N r_j^{\text{th}} \geq r_T^{\text{th}}$, then the total throughput constraint is not useful since in this case it will be automatically guaranteed.
- If $r_j \left(\frac{I^{\text{th}}}{g_j^{(p)}} \right) < r_j^{\text{th}}$ or $\sum_{j=1}^N r_j \left(\frac{I^{\text{th}}}{g_j^{(p)}} \right) < r_T^{\text{th}}$, the problem is infeasible as the maximum power allowed in each channel to guarantee respecting the interference constraint does not allow to reach the minimum required throughput.

B. Power Pricing Model

We assume that the smart grid is governed by a policy affecting the unit price in real time function of the market price and the user's consumption. We denote by $\mu_j(P_1, \dots, P_N)$ the unit price function of the power consumed at the j -th channel (the unit price function is a channel dependent since the channels can represent time slots in the case of time division or different physical locations or even different service providers). The power cost per channel is then deduced as

$$c_j(P_1, \dots, P_N) = \mu_j(P_1, \dots, P_N) P_j. \quad (3)$$

IV. PROBLEM ANALYSIS WITH DIFFERENT PRICING MODELS

Given a convex cost function of the power $c_j(P_1, \dots, P_N)$, the problem (1) is a convex optimization problem. Thus, we propose to alternatively solve its dual problem using the Karush-Kuhn-Tucker (K.K.T) conditions as duality gap is zero under the Slater condition [11]. For our problem the Slater condition is satisfied when the problem is feasible. Mathematically, these Slater conditions are required:

$$r_j \left(\frac{I^{\text{th}}}{g_j^{(p)}} \right) \geq r_j^{\text{th}}, \quad \forall j \in \{1, \dots, N\} \quad (4)$$

$$\sum_{j=1}^N r_j \left(\frac{I^{\text{th}}}{g_j^{(p)}} \right) \geq r_T^{\text{th}}. \quad (5)$$

The dual problem of the category problem (1) can be written as follows

$$\begin{aligned} \mathcal{L} & \left(\{P_j\}_{j=1}^N, \lambda_0, \{\lambda_j\}_{j=1}^N, \{\nu_j\}_{j=1}^N \right) \\ & = \sum_{j=1}^N c_j(P_1, \dots, P_N) + \lambda_0 \left(r_T^{\text{th}} - \sum_{j=1}^N r_j(P_j) \right) \\ & + \sum_{j=1}^N \lambda_j \left(r_j^{\text{th}} - r_j(P_j) \right) + \sum_{j=1}^N \nu_j \left(P_j g_j^{(p)} - I^{\text{th}} \right), \quad (6) \end{aligned}$$

where λ_0 , $\{\lambda_j\}_{j=1}^N$, and $\{\nu_j\}_{j=1}^N$ are the K.K.T multipliers.

After simplifications, the K.K.T constraints are written as follows

$$\sum_{i=1}^N \frac{\partial c_i(P_1, \dots, P_N)}{\partial P_j} = \frac{g_j^{(c)} \lambda_0 / N_0}{(1 + P_j g_j^{(c)} / N_0) \log 2} \quad \forall j \in \{1 \dots N\} \quad (7)$$

$$\lambda_0 \left(r_T^{\text{th}} - \sum_{j=1}^N r_j(P_j) \right) = 0 \quad (8)$$

$$P_j \geq \frac{2^{r_j^{\text{th}} - 1}}{g_j^{(c)} / N_0} \quad \forall j \in \{1 \dots N\} \quad (9)$$

$$P_j \leq \frac{I^{\text{th}}}{g_j^{(p)}} \quad \forall j \in \{1 \dots N\} \quad (10)$$

$$\lambda_0 \geq 0. \quad (11)$$

Hence, the optimal power allocation is deduced function of the power cost function and the channel gains as follows

$$P_j = \left[\frac{\lambda'_0}{\sum_{i=1}^N \frac{\partial c_i(P_1, \dots, P_N)}{\partial P_j}} - \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}, \quad (12)$$

where λ'_0 is a constant proportional to λ_0 determined such that the total throughput constraint is saturated (i.e., $\sum_{j=1}^N r_j(P_j) = r_T^{\text{th}}$), while P_j^- and P_j^+ defined as

$$\begin{cases} P_j^- = \frac{2^{r_j^{\text{th}} - 1}}{g_j^{(c)} / N_0} \\ P_j^+ = \frac{I^{\text{th}}}{g_j^{(p)}} \end{cases} \quad (13)$$

Note that in (12), we use the following notation

$$\begin{bmatrix} x \\ x \end{bmatrix}^{x^\pm} = \begin{cases} x^+ & \text{if } x > x^+, \\ x^- & \text{if } x < x^-, \\ x & \text{otherwise.} \end{cases} \quad (14)$$

A. Constant Unit Price

In this section, we assume the simplest model of a constant unit price function for all channels $\mu_j(P_1, \dots, P_N) = \mu$, $\forall j$, then power price consumption is given by $c_j(P_j) = \mu P_j$. Hence, the power expression in (12) is simplified to

$$P_j = \left[\lambda'_0 - \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}. \quad (15)$$

The throughput per channel is written as

$$r_j = \left[\log_2 \left(\frac{\lambda'_0 g_j^{(c)}}{N_0} \right) \right]_{r_j^-}^{r_j^+}, \quad (16)$$

with r_j^- and r_j^+ defined as

$$\begin{cases} r_j^- = r_j(P_j^-) = r_j^{\text{th}} \\ r_j^+ = r_j(P_j^+) = \log_2 \left(1 + \frac{I^{\text{th}} g_j^{(c)}}{N_0 g_j^{(p)}} \right). \end{cases} \quad (17)$$

In this case, λ'_0 can be deduced analytically as

$$\lambda'_0 = \left(\frac{2^{r_T^{\text{th}}}}{\prod_{j \in S_c} 2^{r_j^-} \prod_{j \in S_p} 2^{r_j^+} \prod_{j \notin \{S_c \cup S_p\}} (g_j^{(c)} / N_0)} \right)^{\frac{1}{N - (|S_c| + |S_p|)}}, \quad (18)$$

with S_c and S_p defined as

$$\begin{cases} S_c = \left\{ j \in \{1, \dots, N\} \text{ such that } \lambda'_0 - \frac{N_0}{g_j^{(c)}} < P_j^- \right\} \\ S_p = \left\{ j \in \{1, \dots, N\} \text{ such that } \lambda'_0 - \frac{N_0}{g_j^{(c)}} > P_j^+ \right\}. \end{cases} \quad (19)$$

We observe that we obtain a similar equation as the well-known water-filling expression used in resource allocation algorithms over multichannel systems [12]. This is due to the constant unit price function which makes our optimization problem mathematically equivalent to a total power minimization problem under individual and total rate constraints.

B. Channel Dependent Unit Price

In this section, we assume that the unit price depends on the channel without dependence on the power consumed $\mu_j(P_1, \dots, P_N) = \mu_j$, then the allocated power expression is reduced to

$$P_j = \left[\lambda'_0 - \mu_j \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}, \quad (20)$$

where the water-level λ'_0 is expressed as

$$\lambda'_0 = \left(\frac{2^{r_T^{\text{th}}}}{\prod_{j \in S_c} 2^{r_j^-} \prod_{j \in S_p} 2^{r_j^+} \prod_{j \notin \{S_c \cup S_p\}} \frac{(g_j^{(c)} / N_0)}{\mu_j}} \right)^{\frac{1}{N - (|S_c| + |S_p|)}}, \quad (21)$$

with S_c and S_p defined as

$$\begin{cases} S_c = \left\{ j \in \{1, \dots, N\} \text{ such that } \frac{\lambda'_0}{\mu_j} - \frac{N_0}{g_j^{(c)}} < P_j^- \right\} \\ S_p = \left\{ j \in \{1, \dots, N\} \text{ such that } \frac{\lambda'_0}{\mu_j} - \frac{N_0}{g_j^{(c)}} > P_j^+ \right\}. \end{cases} \quad (22)$$

Here also, we obtain a water-filling expression with the modification that the channel unit price power will affect the power allocated in each channel. The allocated power is obtained as the difference between the water-level and the product of the unit price by the noise-to-signal ratio instead of the noise to signal ratio only in ordinary water-filling.

C. Power Consumption Dependent Unit Price

In this section, we assume that the unit price depends on the consumed power by that channel. In this scenario, we consider that the unit price function depends on the consumed power in that channel only. This case is obtained when each channel unit price is determined independently from the other channels' power consumption. Generalization to interdependent channel consumption will be studied later. We consider the following function for the unit price

$$\mu_j(P_1, \dots, P_N) = \mu_j(P_j), \quad (23)$$

Inserting (23) in (12), we obtain the following equation to solve for the allocated power per channel

$$P_j = \left[\frac{\lambda'_0}{P_j \frac{\partial \mu_j(P_j)}{\partial P_j} + \mu_j(P_j)} - \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}. \quad (24)$$

Then, the problem of solving the power allocation per channel P_j is written as finding the zero of a function in a finite interval, $P_j^- \leq P_j \leq P_j^+$, as follows

$$g(P_j) = \left(P_j + \frac{N_0}{g_j^{(c)}} \right) \left(P_j \frac{\partial \mu_j(P_j)}{\partial P_j} + \mu_j(P_j) \right) - \lambda'_0 = 0. \quad (25)$$

The expression of λ'_0 is obtained by solving the total throughput constraint which is transformed into finding the zero of the function $f(\lambda'_0)$ for $\lambda'_0 \geq 0$ with

$$f(\lambda'_0) = r_T^{\text{th}} - \left[\sum_{j \in S_c} r_j^- + \sum_{j \in S_p} r_j^+ + \sum_{j \notin \{S_c \cup S_p\}} r_j(P_j) \right] = 0, \quad (26)$$

with S_c and S_p defined as

$$\begin{cases} S_c = \left\{ j \in \{1, \dots, N\} \text{ such that } \frac{\lambda'_0}{P_j \frac{\partial \mu_j(P_j)}{\partial P_j} + \mu_j(P_j)} - \frac{N_0}{g_j^{(c)}} < P_j^- \right\} \\ S_p = \left\{ j \in \{1, \dots, N\} \text{ such that } \frac{\lambda'_0}{P_j \frac{\partial \mu_j(P_j)}{\partial P_j} + \mu_j(P_j)} - \frac{N_0}{g_j^{(c)}} > P_j^+ \right\}. \end{cases} \quad (27)$$

Although in this case we do not obtain a strictly speaking water-filling expression, a similar algorithm can be developed where λ'_0 will represent a fictive water-level as it remains constant for all channels. The pseudo water-filling expression can be deduced from (24) as follows

$$P_j = \left[\frac{\lambda'_0}{\hat{\mu}_j} - \frac{N_0}{g_j^{(c)}} \right]_{P_j^-}^{P_j^+}, \quad (28)$$

with $\hat{\mu}_j = P_j \frac{\partial \mu_j(P_j)}{\partial P_j} + \mu_j(P_j)$ is the effective power cost in the j -th channel. Note that this is not a water-filling equation as $\hat{\mu}_j$ depends on the allocated power P_j but it only allows to analyze the allocated power function to the channel gains and the price coefficients. Thus, we obtain a system of non-linear coupled equations (26) and (25). An iterative approach allow us to determine this water-level and thus obtain the optimal power allocation per channel by solving at each step consecutively (26) and (25) until convergence. This algorithm has the same convergence speed as the regular water-filling algorithm. The only difference is that the water-level is determined analytically in regular water-filling while it is obtained numerically by solving (26) in this case.

V. SIMULATION RESULTS

We consider a cognitive user randomly located in a cell with a radius $d_0 = 1$ Km. We assume that the CU is equipped with a smart meter that could provide it with (instantaneous) unit pricing in real-time. We consider $N = 10$ channels. The total required throughput $r_T^{\text{th}} = 50$ Mbps while the individual required throughput per channel is $r_j^{\text{th}} = 1$ Mbps, $\forall j$. We consider a Rayleigh fading channel model. The interference threshold is fixed to be equal to the noise floor $I^{\text{th}} = N_0 = -120$ dBm. For the pricing coefficients, unless stated differently in the figures' legend, we consider

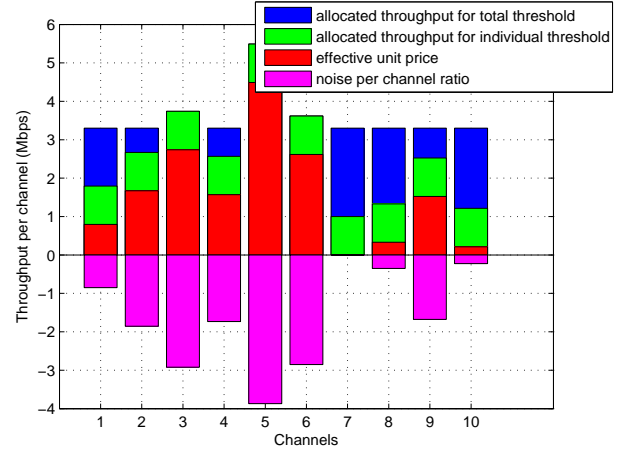


Fig. 1. Snapshot of allocated power per channel with power dependent pricing model.

the following values. For the constant unit price model, we use $\mu = 1$. For the channel dependent unit price model, μ_j are generated from a random variable uniformly distributed between 0 and 1. For the power dependent unit price model, we use $\mu_j(P_j) = aP_j + b$ with $a = 1$ and $b = 0.1$.

In Fig. 1, we plot the allocated throughput per channel for a sample generation of channel gains. We observe the behavior of the allocated power per channel and verify that it is similar to a water-filling but with a fictive water-level since the unit power price is not fixed but it depends on the power as explained earlier.

In Fig. 2, we compare the obtained total power cost for the proposed different pricing models when taking into consideration pricing dynamics to the assumption of static pricing (simple power minimization). The total cost is presented for different average channel gains. We first observe that as the channel gain increases, the total power cost decreases for the different pricing models since less power is needed to achieve the targeted throughput. Comparing the total power cost when considering dynamic pricing to the power optimization with the assumption of constant pricing, we observe the incurred gain due to the employed algorithms. Obviously, there is no gain with the constant pricing model since in this case the power consumption minimization and the power cost minimization are equivalent.

In Fig. 3, we plot the obtained the total power cost function of the channels' average gain for different values of the pricing coefficients to observe their effect on the total cost. Obviously, the total power cost is increasing proportionally to the increase of the pricing coefficients. The constant pricing model corresponds to the linearly dependent pricing model with $b = 0$ that is why when b is small enough, the obtained cost with the two models becomes very close.

In Fig. 4, we plot the total power cost function of the number of channels. As the number of channels increases, the total power cost decreases as there are more opportunities for the system to guarantee the required throughput with lower power

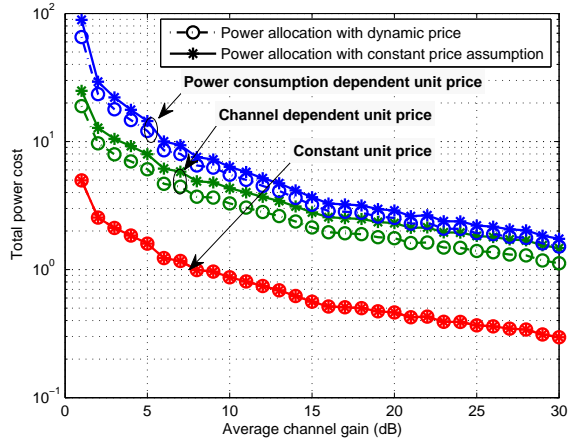


Fig. 2. Total power consumption cost function of the average channel gain.

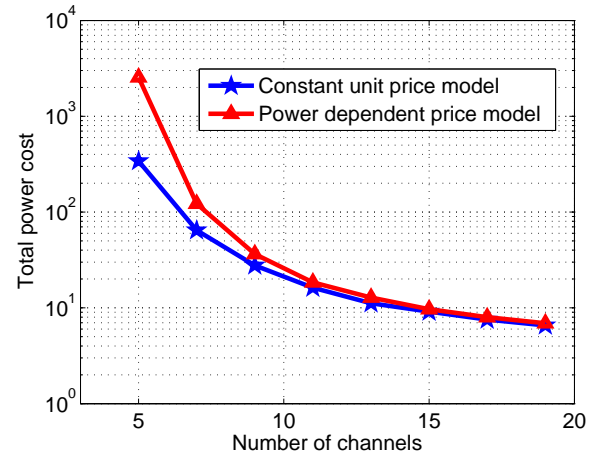


Fig. 4. Total power consumption cost function of the number of channels.

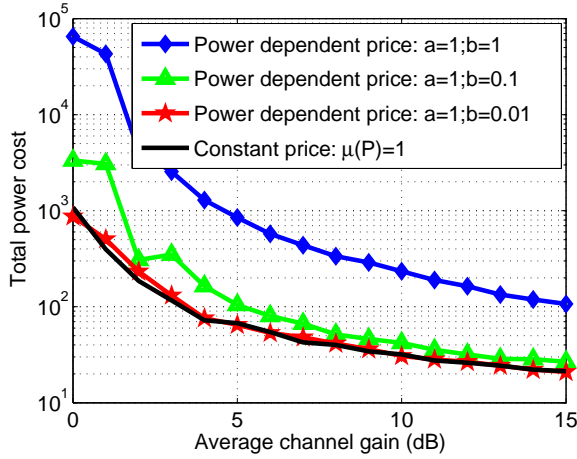


Fig. 3. Total power consumption cost function of the pricing coefficients.

required. And for this reason, we observe that the total cost became nearly equal with the two pricing models for high number of channels as the power per channel became very small. This is in part due also to the independence between channels in the unit price function. It will be interesting to propose a power allocation when the unit price is affected by all channels consumption and to see the effect of increasing the number of channels on the total price.

VI. CONCLUSION

This paper proposes a resource allocation scheme for a smart grid-enabled cognitive radio user. The smart grid allows the user to obtain real time power pricing policy. This information is exploited by the user to minimize its power consumption cost through intelligent power allocation. Analytic expressions of the allocated power are developed for different cost functions and low-cost algorithms are presented for the power allocation. Simulation results show the gain that the cognitive system achieved by profiting from the dynamic power pricing through the proposed power allocation scheme.

VII. ACKNOWLEDGMENT

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