This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2007 proceedings.

Optimal Precoder for Rate$\leqslant$1 Space-Time Block Codes

Alireza Ghaderipoor and Chintha Tellambura
Department of Electrical and Computer Engineering
University of Alberta, CANADA
Email: {ghaderi, chintha}@ece.ualberta.ca

***Abstract*— Despite primary space-time coding where the channel state information (CSI) is available at the receiver only, the capacity and performance of multiple-input multiple-output (MIMO) systems can be increased significantly when a complete or partial CSI is available at the transmitter. Recently, limited feedback methods including antenna subset selection and *unitary precoding* have been proposed for orthogonal space-time codes where a partial knowledge of the channel is available at the transmitter via an error-free, zero-delay feedback channel. In this paper, we propose a general structure matrix rather than a unitary one for precoding. By maximizing the signal-to-noise ratio (SNR) per received symbol, we find the optimal precoder for general space-time codes with rate$\leqslant$1 symbol per channel use. The performance of the optimal scheme is analytically evaluated. Next, we extend the result for limited feedback systems. Simulation results show that the proposed precoder outperforms the previous work.**

I. Introduction

Multiple-input multiple-output (MIMO) wireless channels, created by deploying antenna arrays at both the transmitter and receiver, promise high capacity and high-quality wireless communication links [1], [2]. To fully exploit the benefits of MIMO channels, space-time modulation and receiver algorithms are required to provide a sensible performance and complexity tradeoff. Space-time block codes (STBC) with rate$\leqslant$1 symbol per channel use are of interest when the number of receive antennas may be one or more, particularly in the downlink of mobile systems. Orthogonal STBCs (OSTBCs) [3] are a class of STBCs that guarantee full diversity and simple decoding.

Primary schemes proposed for exploiting multiple antennas at the transmitter and/or receiver commonly assume that by applying pilots or training sequences, the receiver can estimate the channel gains accurately, but this information is not available at the transmitter. However, in communication systems that experience a slow fading environment, complete or partial knowledge of the channel may be available at the transmitter. Channel state information (CSI) at the transmitter may be exploited in two ways: antenna subset selection and precoding. The optimum precoder matrix can be obtained based on the eigen structure of the channel matrix [4]. Due to the bandwidth limits on feedback channel, however, full CSI is not always available at the transmitter. Therefore, precoding techniques using limited feedback are of interest [5].

In [5], the authors propose a codebook of unitary precoders derived from Grassmannian subspace packing for limited feedback systems. The codebook is known to both the transmitter and receiver and for each channel realization, only the index of the appropriate matrix (precoder) is sent back to the transmitter. The precoder structure is originally proposed in [6] for differential unitary space-time modulation (DUSTM) which consists of a diagonal matrix and a rectangular sub-matrix of the Discrete Fourier Transform (DFT) matrix. The diagonal terms are some points on the unit circle in the complex plain where their angles are defined by some integers that should be optimized.

In this paper, when CSI is available at the transmitter, we relax the precoder matrix from being unitary matrix to a general structure matrix. We extend the precoder design for all rate$\leqslant$1 STBCs. Considering the power constraint at the transmitter, we maximize the received signal-to-noise ratio (SNR) for each transmitted symbol to find the optimal precoder. We show that any precoding for STBCs with rate$\leqslant$1 symbol per channel use (e.g. [5]) is not optimal. In fact, we show that the optimal precoding for any STBCs with rate$\leqslant$1 symbol per channel use is reduced to *transmit beamforming* of the transmit signals individually, by the weighting vector equal to the corresponding right singular vector of the largest singular value of the channel matrix. Due to the use of a general matrix, the proposed precoding method outperforms the unitary precoding proposed in [5] for OSTBCs. To show this, we analytically derive the exact bit error rate (BER) of the system and compare it with the performance of the previous work. Finally we extend the results for limited feedback systems. Simulation results show that our proposed precoding method outperforms the previous limited feedback precoder for OSTBCs [5].

II. System Model

Consider a narrow-band, flat fading communication system with N_t transmit and N_r receive antennas (MIMO(N_t, N_r)) where the channel is fixed at least for T symbol periods. Each transmission takes T channel uses where the linear transformation between the transmit and receive antennas can be modeled as

$$\mathbf{X} = \sqrt{\rho}\,\mathbf{HFS} + \mathbf{V} \tag{1}$$

where the matrix $\mathbf{X} \in \mathcal{C}^{N_r \times T}$ is the complex received matrix, $\mathbf{S} \in \mathcal{C}^{M \times T}(M \leqslant \min(N_t, N_r, T))$ is the transmitted matrix, $\mathbf{F} \in \mathcal{C}^{N_t \times M}$ is the precoder matrix, $\mathbf{V} \in \mathcal{C}^{N_r \times T}$ is the additive

1-4244-0353-7/07/$25.00 ©2007 IEEE

noise matrix, $\mathbf{H} \in \mathcal{C}^{N_r \times N_t}$ is the channel matrix and ρ is the total transmit power at each signaling interval. Entries of $\mathbf{H}$ and $\mathbf{V}$ are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance, $\mathcal{CN}(0,1)$.

For each transmission, according to the input information, Q signals, $\{s_1, \ldots, s_Q\}$, are chosen from a signal constellation (for example pulse amplitude modulation (PAM) or quadratic amplitude modulation(QAM)) with unit average energy. The transmit STBC matrix $\mathbf{S}$ with rate Q/T symbol per channel use is constructed to be sent over M virtual transmit antennas. $\mathbf{S}$ satisfies the power constraint $\mathrm{E}\{\mathrm{tr}(\mathbf{S}^*\mathbf{S})\} = 1$ where $\mathrm{E}\{\cdot\}$, $\mathrm{tr}(\cdot)$ and * denote the expectation, matrix trace and Hermitian respectively. $\mathbf{S}$ is precoded by $\mathbf{F}$ and sent over N_t transmit antennas. To ensure that the transmit power on each signaling interval is ρ, the precoder matrix should satisfy the following power constraint:

$$\mathrm{tr}(\mathbf{F}^*\mathbf{F}) = 1 \tag{2}$$

It can be shown [7] that for any rate$\leqslant$1 STBC, the optimum performance is obtained when the code is orthogonal, i.e. $\mathbf{S}^*\mathbf{S} = (1/T)\mathbf{I_T}$. Considering $\mathbf{HF}$ as the equivalent channel, the virtual MIMO(N_r, M) system is decoupled to the following single-input single-output (SISO) model:

$$\tilde{s}_q = \sqrt{\rho}\left(\frac{T}{Q}\|\mathbf{HF}\|^2\right) s_q + z_q \quad , \quad q = 1, \ldots, Q \tag{3}$$

where z_q is the additive white noise with $\mathcal{CN}\left(0, \frac{T}{Q}\|\mathbf{HF}\|^2\right)$ distribution and $\|\cdot\|$ denotes the Frobenius norm. Based on (3), the optimum achievable SNR (γ) for each transmitted symbol is

$$\gamma = \left(\frac{T}{Q}\|\mathbf{HF}\|^2\right)\rho. \tag{4}$$

III. Optimal Precoder

To find the optimal precoder, we assume that the space-time code used by the system is orthogonal. We will show that this assumption does not affect the final results. In fact, we maximize the SNR of the OSTBCs as the optimal SNR bound for general STBCs.

It is obvious from (4) that the optimal precoder is obtained when $\|\mathbf{HF}\|^2$ is maximized. Consider the ordered singular value decomposition (svd) of $\mathbf{H}$ and $\mathbf{F}$ as follows:

$$\mathbf{H} = \mathbf{V}_L \mathbf{\Sigma} \mathbf{V}_R^* \quad , \quad \mathbf{F} = \mathbf{U}_L \mathbf{A} \mathbf{U}_R^*$$

where $\mathbf{V}_L \in \mathcal{U}^{N_r \times N_r}$, $\mathbf{V}_R \in \mathcal{U}^{N_t \times N_t}$, $\mathbf{U}_L \in \mathcal{U}^{N_t \times M}$, $\mathbf{U}_R \in \mathcal{U}^{M \times M}$ are all unitary matrices, i.e. $\mathcal{U}^*\mathcal{U} = \mathbf{I}$, and $\mathbf{\Sigma} \in \mathcal{R}_+^{N_r \times N_t}$ and $\mathbf{A} \in \mathcal{R}_+^{M \times M}$ are diagonal matrices with decreasing order, i.e. $\sigma_i > \sigma_{i+1}$. Since $\mathbf{V}_L$ and $\mathbf{U}_R$ are both square unitary matrices, therefore

$$\begin{aligned}\|\mathbf{HF}\|^2 &= \|\mathbf{V}_L \mathbf{\Sigma} \mathbf{V}_R^* \mathbf{U}_L \mathbf{A} \mathbf{U}_R^*\|^2 \\ &= \|\mathbf{\Sigma} \mathbf{V}_R^* \mathbf{U}_L \mathbf{A}\|^2\end{aligned} \tag{5}$$

It has been shown [4]-[5] that the optimum $\mathbf{U}_L$ is $\bar{\mathbf{V}}_R$ which is a unitary matrix constructed by the first M columns of $\mathbf{V}_R$. Thus,

$$\begin{aligned}\|\mathbf{HF}\|^2 &= \|\mathbf{\Sigma} \mathbf{V}_R^* \bar{\mathbf{V}}_R \mathbf{A}\|^2 \\ &= \|\mathbf{\Sigma} \begin{bmatrix} \mathbf{I}_M \\ \mathbf{0}_{(N_t - M) \times M} \end{bmatrix} \mathbf{A}\|^2 \\ &= \|\bar{\mathbf{\Sigma}} \mathbf{A}\|^2\end{aligned}$$

where $\bar{\mathbf{\Sigma}}$ is the ordered diagonal matrix constructed by the first M largest singular values of the channel matrix $\mathbf{H}$. Therefore, the optimal precoder is obtained from the following optimization problem:

$$\max_{a_i, i=1,\ldots,M} \sum_{i=1}^{M} a_i^2 \sigma_i^2 \tag{6}$$

subject to the power constraint in (2) or equivalently

$$\sum_{i=1}^{M} a_i^2 = 1. \tag{7}$$

It can be easily shown that the optimum answer to the maximization problem in (6) subject to (7), is $a_1 = 1$ and $a_i = 0$ for $i = 2, \ldots, M$.

It means that the optimal precoder sends all signals $\{s_1, \ldots, s_Q\}$ over the maximum singular value of the channel matrix with the full transmit power. Therefore, with optimal precoding, independent of the original STBC used by the system, the transmission rate is increased to one symbol per channel use ($T = Q = M = 1$) and the performance of the system is equivalent to the performance of the corresponding OSTBC. Finally, by optimal precoding, the equivalent system model for each transmit signal is

$$\tilde{s} = \sqrt{\rho}\, \sigma_1^2\, s + z \tag{8}$$

where $z \sim \mathcal{CN}(0, \sigma_1^2)$, $\sigma_1 = \sigma_{max}(\mathbf{H})$, and consequently $\gamma_{opt} = \rho\sigma_1^2$. This system model shows that the optimal precoding for any space-time code with rate$\leqslant$1 is reduced to transmit beamforming of the transmit signals individually, by the weighting vector equal to the corresponding singular vector of the largest singular value of the channel matrix $\mathbf{H}$.

IV. Exact BER Analysis for Optimal Precoding

Performance analysis of transmit beamforming has been studied in literature before. In [8], the cumulative density function (cdf) and probability density function (pdf) of σ_1^2 is presented in the form of generalized hypergeometric functions, generalized Marcum Q-functions, modified Bessel functions and incomplete gamma functions, when the channel gains are correlated $\mathcal{CN}(\mu, \sigma^2)$. They use the cdf and pdf to calculate the *outage probability* in a system with transmit beamforming. In [9] and [10], the BER analysis is presented only for BPSK modulation while the expressions in [9] are in the form of hypergeometric functions with matrix arguments. In general, hypergeometric functions are very slow converging functions and therefore numerically hard to compute [9].

In this section, we present the exact closed-form expression of the BER performance for the system model in (8) when the transmitted signal, s, is selected from a PAM or QAM constellation and a Gray code mapping is used. Our expressions do not involve with hypergeometric functions and consist of some summations with finite indexes.

A. Exact BER expression for PAM and QAM signallings

Assume the transmitted signal s in (8) is selected from a $I \times J$ rectangular QAM constellation with unit average energy and a Gray code mapping. I and J denote the number of in-phase and quadrature amplitudes respectively. We define

$$P_{I|\sigma_1^2}(k) = \sum_{i=0}^{(1-2^{-k})I-1} \beta_I(k,i) Q\left(\sqrt{\eta(i)\sigma_1^2}\right) \tag{9}$$

where

$$\beta_I(k,i) = (-1)^{\lfloor \frac{i \cdot 2^{k-1}}{I} \rfloor} \cdot \left(\frac{2}{I}\right)\left(2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{I} + \frac{1}{2} \right\rfloor\right),$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{u^2}{2}}\, du \quad , \quad \eta(i) = \frac{6(2i+1)^2 \rho}{I^2 + J^2 - 2}$$

and $\lfloor x \rfloor$ denotes the largest integer to x. Now the average BER of the $I \times J$ rectangular QAM conditioned on σ_1^2 is expressed as [11]

$$P_{b|\sigma_1^2} = \frac{1}{\log_2(I \cdot J)} \left(\sum_{k=1}^{\log_2 I} P_{I|\sigma_1^2}(k) + \sum_{l=1}^{\log_2 J} P_{J|\sigma_1^2}(l) \right). \tag{10}$$

Note that (10) reduces to the BER of a BPSK signal for $I = 2$ and $J = 1$, an I-array PAM signal for $J = 1$, and an M-array square QAM signal for $I = J = \sqrt{M}$. Thus, the exact BER is obtained by averaging (10) over the distribution of σ_1^2. Interested readers may refer to [11] for more details on the derivation of (9) and (10).

B. Distribution of the largest singular value of $\mathbf{H}$

Assume $m = \min(N_t, N_r)$, $n = \max(N_t, N_r)$ and the Hermitian matrix $\mathbf{W} \in \mathcal{C}^{m \times m}$ defined as

$$\mathbf{W} = \begin{cases} \mathbf{H}^*\mathbf{H} & \text{if } N_t \leqslant N_r \\ \mathbf{H}\mathbf{H}^* & \text{if } N_t > N_r \end{cases}.$$

The nonzero singular values of $\mathbf{H}$ are corresponded to the eigenvalues of $\mathbf{W}$ by $\lambda_i = \sigma_i^2, i = 1, \ldots, m$. Therefore, $\lambda_1 = \lambda_{max}(\mathbf{W})$ and σ_1^2 have the same distribution.

When $\mathbf{H}$ is a complex random matrix of i.i.d. elements with $\mathcal{CN}(0,1)$ distribution, $\mathbf{W}$ is called a complex Wishart random matrix. By defining

$$K_{m,n}^{-1} = \prod_{k=1}^{m} \Gamma(m-k+1)\Gamma(n-k+1),$$

the joint pdf of the ordered eigenvalues $\lambda_1 \geqslant \cdots \geqslant \lambda_m$ of $\mathbf{W}$ is expressed as [12]

$$f_{\boldsymbol{\lambda}}(\boldsymbol{\lambda}) = K_{m,n}\, e^{-\sum \lambda_i} \prod \lambda_i^{n-m} \prod_{i<j} (\lambda_i - \lambda_j)^2 \tag{11}$$

where $\Gamma(\cdot)$ is the Gamma function and the unlabelled sum and products run from $i = 1$ to m. By integrating (11) over $\lambda_2, \ldots, \lambda_m$, the pdf of λ_1 is expressed as

$$\begin{aligned} f_{\lambda_1}(\lambda_1) = {} & K_{m,n}\, e^{-\lambda_1} \lambda_1^{n-m} \times \\ & \int_0^{\lambda_1} e^{-\lambda_2} \lambda_2^{n-m} (\lambda_1 - \lambda_2)^2 \int_0^{\lambda_2} \cdots \\ & \int_0^{\lambda_{m-1}} e^{-\lambda_m} \lambda_m^{n-m} \prod_{k=1}^{m-1} (\lambda_k - \lambda_m)^2\, d\lambda_m \cdots d\lambda_2 \end{aligned} \tag{12}$$

Calculation of (12) is a complicated process in general, but for MIMO systems is tractable since m is relatively small.

1) Case $m = 1$: In this case, $\mathbf{W}$ is only a random number with central Chi-square distribution, variance one and n degrees of freedom, $\chi(1,n)$. Since $K_{1,n} = \Gamma(n)$, from (11) it is easy to see that

$$f_{\lambda_1}(\lambda_1) = \frac{1}{\Gamma(n)}\, e^{-\lambda_1} \lambda_1^{n-1}. \tag{13}$$

2) Case $m = 2$: In this case $K_{2,n} = \Gamma(n)\Gamma(n-1)$. From (12) we have

$$\begin{aligned} f_{\lambda_1}(\lambda_1) = {} & \frac{1}{\Gamma(n)\Gamma(n-1)}\, e^{-\lambda_1} \lambda_1^{n-2} \times \\ & \int_0^{\lambda_1} e^{-\lambda_2} \lambda_2^{n-2} (\lambda_1^2 - 2\lambda_1\lambda_2 + \lambda_2^2)\, d\lambda_2 . \end{aligned}$$

Since [13]

$$\int_0^x \frac{1}{\Gamma(n)} u^{n-1} e^{-u}\, du = 1 - e^{-x} \sum_{i=0}^{n-1} \frac{x^i}{i!}$$

and by some manipulations we obtain

$$\begin{aligned} f_{\lambda_1}(\lambda_1) = {} & \frac{1}{\Gamma(n)}\, e^{-\lambda_1} \lambda_1^{n-2} \Big[n(n-1) - 2(n-1)\lambda_1 + \lambda_1^2 \Big] - \\ & \frac{1}{\Gamma(n)}\, e^{-2\lambda_1} \lambda_1^{n-2} \Big[n(n-1) + (n-1)(n-2)\lambda_1 \\ & + \sum_{i=2}^{n} \left(\frac{n(n-1)}{i(i-1)} - 2\frac{(n-1)}{(i-1)} + 1 \right) \frac{\lambda_1^i}{i!} \Big]. \end{aligned} \tag{14}$$

3) Case $m > 2$: In general, it is easy to show that

$$f_{\lambda_1}(\lambda_1) = \sum_{t=1}^{m} e^{-t\lambda_1} G_t(\lambda_1) \tag{15}$$

where

$$G_t(\lambda_1) = \sum_{j=0}^{D_t} a_{t,j}\, \lambda_1^j \tag{16}$$

denotes the corresponding polynomial coefficient of $e^{-t\lambda_1}$ and D_t is the degree of $G_t(\cdot)$. Although there is no straightforward expression or algorithm, symbolic mathematic programs (e.g.

Maple, Mathematica) can be used to easily solve (12) and compute $a_{t,j}$'s. For instance, for $m = 3$ and $n = 4$ we have:

$$G_1(\lambda_1) = 6\lambda_1 - 8\lambda_1^2 + \frac{9}{2}\lambda_1^3 - \lambda_1^4 + \frac{1}{12}\lambda_1^5$$

$$G_2(\lambda_1) = -12\lambda_1 + 4\lambda_1^2 + \lambda_1^3 - \lambda_1^4 - \frac{1}{12}\lambda_1^5 - \frac{1}{12}\lambda_1^6$$

$$G_3(\lambda_1) = 6\lambda_1 + 4\lambda_1^2 + \frac{1}{2}\lambda_1^3$$

C. Averaging $P_{b|\lambda_1}$ over the distribution of λ_1

From (9) and (10), it is clear that to find $P_b = E_{\lambda_1}\{P_{b|\lambda_1}\}$, we need to calculate $P_I(k) = E_{\lambda_1}\{P_{I|\lambda_1}(k)\}$ (note that $\lambda_1 = \sigma_1^2$). From (9) we have

$$P_I(k) = \sum_{i=0}^{(1-2^{-k})I-1} \beta_I(k,i)\, E_{\lambda_1}\Big\{Q\Big(\sqrt{\eta(i)\lambda_1}\Big)\Big\}$$

Therefore, by considering the general distribution of λ_1 (15) and (16), we obtain

$$P_I(k) = \sum_{i=0}^{(1-2^{-k})I-1} \beta_I(k,i) \sum_{t=1}^{m}\sum_{j=0}^{D_t} a_{t,j} \cdot \int_0^\infty \lambda_1^j\, e^{-t\lambda_1} Q\Big(\sqrt{\eta(i)\lambda_1}\Big)\, d\lambda \qquad (17)$$

It is well known that [14]

$$\int_0^\infty t^L\, x^{L-1}\, e^{-tx} Q\big(\sqrt{\alpha x}\big)\, dx = \phi(L,t,\alpha)$$

where

$$\phi(L,t,\alpha) = \Gamma(L)\left(\frac{1-\mu}{2}\right)^L \sum_{r=0}^{L-1}\binom{L-1+r}{r}\left(\frac{1+\mu}{2}\right)^r$$

and

$$\mu = \sqrt{\frac{\alpha}{2t+\alpha}}\ .$$

Finally, from (17) we obtain

$$P_I(k) = \sum_{i=0}^{(1-2^{-k})I-1} \beta_I(k,i) \sum_{t=1}^{m}\sum_{j=0}^{D_t} \frac{a_{t,j}}{t^{j+1}}\ \phi\left(j+1, t, \eta(i)\right) \qquad (18)$$

and consequently the average BER of the $I \times J$ rectangular QAM signal transmitted through the system model in (8) is expressed as

$$P_b = \frac{1}{\log_2(I \cdot J)}\left(\sum_{k=1}^{\log_2 I} P_I(k) + \sum_{l=1}^{\log_2 J} P_J(l)\right). \qquad (19)$$

Fig. 1 shows the BER performance of the optimal precoding for a MIMO system with $N_t = 4$ transmit antennas and $N_r =$ 1, 2 and 3 receive antennas. The transmit signal is selected from a 16-QAM constellation. In this figure, solid lines are the results from (19) and the symbolic points are from simulations which verify our analytical results.

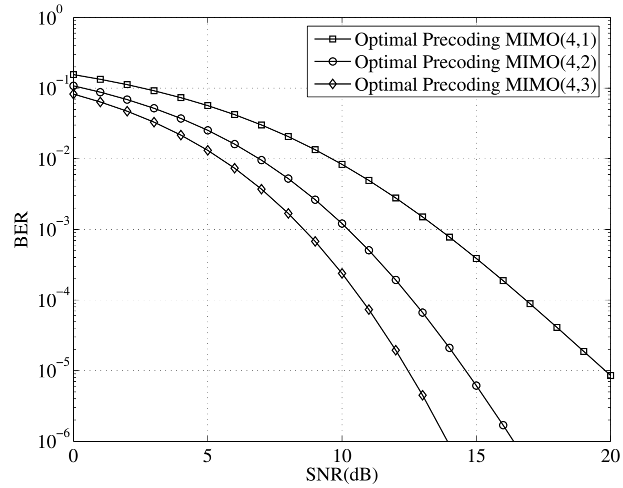

Fig. 1. Bit error rate of optimal precoding. Solid lines are analytic results and symbolic points are from simulation.

V. Limited Feedback Precoding

In Section III, we showed that the optimal precoding for any arbitrary STBC with rate$\leqslant 1$ symbol per channel use, is to send the signals one-by-one through the sub-channel corresponding to the largest singular value of the channel matrix. To this end, we require the $\bar{\mathbf{v}}_R$ at the transmitter for precoding where $\bar{\mathbf{v}}_R$ is the right eigenvector of the channel matrix $\mathbf{H}$, corresponding to its largest singular value. Therefore, the optimal precoding is applicable when the transmitter knows the channel (or $\bar{\mathbf{v}}_R$) perfectly. This happens in slow fading environments or when the system uses time division duplex (TDD) technique for transmission where the transmitter and receiver use the same channel.

In some applications, while complete CSI is not available for the transmitter, a limited–bandwidth feedback channel is available to send partial CSI from the receiver to the transmitter. For this limited feedback systems, we use a set (codebook) $\mathcal{F}$ of $L = 2^{N_b}$ precoders that are pre-known to the transmitter (N_b is the number of feedback bits). For a given $\mathbf{H}$, the only feedback parameter is $\mathcal{I}$ which is the index of $\mathbf{f}_{\mathcal{I}} \in \mathcal{F}$, obtained from the following optimization problem:

$$\mathbf{f}_{\mathcal{I}} = \arg\max_{\mathbf{f}_i \in \mathcal{F}} \|\mathbf{H}\mathbf{f}_i\|^2 \qquad (20)$$

Note that since $M = 1$ for optimal precoder, therefore $\sigma^2_{\max}(\mathbf{H}\mathbf{f}_i) = \|\mathbf{H}\mathbf{f}_i\|^2$.

The remaining problem is to generate the codebook $\mathcal{F}$. The members of $\mathcal{F}$ should be designed based on the following *distortion* minimization problem [5]:

$$E_{\mathbf{H}}\left\{\min_{\mathbf{f}\in\mathcal{F}}\left(\|\mathbf{H}\bar{\mathbf{v}}_R\|^2 - \|\mathbf{H}\mathbf{f}\|^2\right)\right\} \qquad (21)$$

It has been shown [5] that the minimization in (21) leads to the maximization of the *chordal distance* between any pairs of precoders in $\mathcal{F}$. The chordal distance is defined as

$$d_{chord}(\mathbf{f}_i, \mathbf{f}_j) = \frac{1}{\sqrt{2}}\left\|\mathbf{f}_i\mathbf{f}_i^* - \mathbf{f}_j\mathbf{f}_j^*\right\|,\ 1 \leqslant i \neq j \leqslant L. \qquad (22)$$

Similar to the codebook structure proposed in [6] that can be very easily implemented, we construct the codebook $\mathcal{F}$ as follows

$$\mathcal{F} = \{\mathbf{e}, \Phi\mathbf{e}, \dots, \Phi^{L-1}\mathbf{e}\}$$

where $\mathbf{e} = \frac{1}{\sqrt{N_t}}[\overbrace{1, \dots, 1}^{N_t}]^t$ and Φ is a diagonal matrix given by

$$\Phi = \mathrm{diag}\{e^{j\frac{2\pi}{L}u_1}, e^{j\frac{2\pi}{L}u_2}, \cdots, e^{j\frac{2\pi}{L}u_{N_t}}\}$$

where $0 \leqslant u_i < L$ are the design parameters and should be optimized as follows

$$\mathbf{u} = \arg\max_{\{u_i\}} \min_{1\leqslant l<L} d_{chord}(\mathbf{e}, \Phi^l\mathbf{e})$$

where $\mathbf{u} = [u_1, u_2, \dots, u_{N_t}]^t$.

In [6], it is assumed that $\{u_i\}$'s are integer and then by exhaustive or random search, the optimum $\mathbf{u}$ is found. Recently in [15], the authors relax $\{u_i\}$'s to be real numbers and use the genetic algorithm to find the optimum solution. They show that the precoders obtained from genetic algorithm outperform the precoders extracted from exhaustive search.

Fig. 2 shows the optimal and limited feedback BER performance of a MIMO system with $N_t = 4$ transmit antennas and $N_r = 2$ receive antennas when $N_b = 5$ bits of information is feedbacked or equivalently the codebook $\mathcal{F}$ consists of 32 precoders. For comparison, the BER of Alamouti's code [16] with no precoding ($N_t = M = 2$) and the optimal and limited feedback BER of precoded Alamouti code ($M = 2$) [5] are included. The transmitted signals are chosen from 16-QAM constellation. Fig. 2 clearly shows that the precoding with $M = 1$ is the optimum case when the STBC code rate is less or equal to one symbol per channel use. This figure shows that even limited feedback precoding with $M = 1$ outperforms the optimal precoding for Alamouti's code proposed in [5].

VI. Conclusion

In this paper, we introduced a new method for precoding of STBCs with rate$\leqslant 1$ symbol per channel use. We relaxed the precoder matrix to be a general matrix rather than a unitary one. We showed that the optimal precoding for rate$\leqslant 1$ STBCs is to send the data symbols one-by-one through the sub-channel corresponding to the largest singular value of the channel matrix. By employing the singular value distribution of the channel matrix, the exact BER performance of the optimal precoder was derived. Genetic–algorithm–based limited feedback precoder design was developed. Our simulation results show that our proposed precoding outperforms the previous precoding method for OSTBCs in [5].

VII. Acknowledgement

We would like to thank iCORE Wireless Communications Laboratory and Alberta Ingenuity Fund for supporting our research.

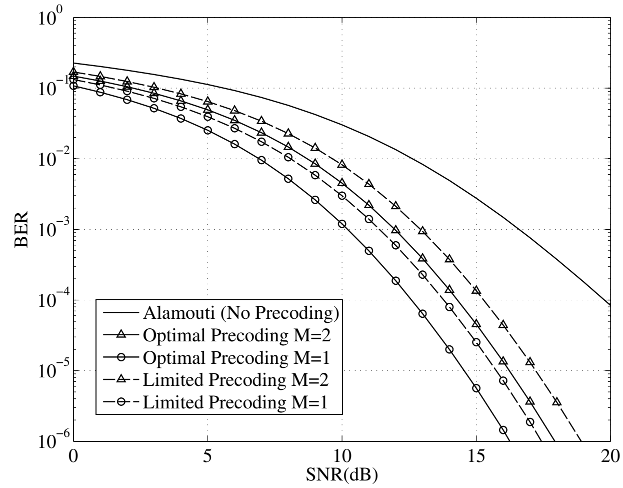

Fig. 2. Bit error rate of MIMO(4,2) for 16-QAM signaling with $N_b = 5$ bits limited feedback precoding.

References

[1] E. Telatar, "Capacity of multiantenna Gaussian channels," *AT&T Bell Laboratories, Tech. Memo.*, 1995.

[2] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 311–335, March 1998.

[3] V. Tarokh, H. Jafarkhani, and A. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456–1467, 1999.

[4] A. Scaglione, P. Stoica, S. Barbarossa, G. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Processing*, vol. 50, no. 5, pp. 1051–1064, 2002.

[5] D. Love and R. J. Heath, "Limited feedback unitary precoding for orthogonal space-time block codes," *IEEE Trans. Signal Processing*, vol. 53, no. 1, pp. 64–73, 2005.

[6] B. Hochwald and W. Sweldens, "Differential unitary space-time modulation," *IEEE Trans. Commun.*, vol. 48, no. 12, pp. 2041–2052, Dec. 2000.

[7] A. Ghaderipoor and C. Tellambura, "Optimal limited feedback technique for small size and low rate MIMO systems," in *proc. Int. Wireless Commun. and Mobile Computing Conf. (IWCMC)*, Vancouver, BC, Canada, July 3–6, 2006.

[8] M. Kang and M.-S. Alouini, "Largest eigenvalue of complex Wishart matrices and performance analysis of MIMO MRC systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 3, pp. 418–426, Apr. 2003.

[9] A. Grant, "Performance analysis of transmit beamforming," *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 738–744, Apr. 2005.

[10] L. Garth, P. Smith, and M. Shafi, "Exact symbol error probabilities for SVD transmission of BPSK data over fading channels," in *proc. IEEE Int. Conf. on Commun. (ICC)*, vol. 4, May 16–20, 2005, pp. 2271–2276.

[11] K. Cho and D. Yoon, "On the general BER expression of one- and two-dimensional amplitude modulations," *IEEE Trans. Commun.*, vol. 50, no. 7, pp. 1074–1080, 2002.

[12] A. Edelman, "Eigenvalues and condition numbers of random matrices," Ph.D. dissertation, Department of Mathematics, MIT, May 1989.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 5th ed. Academic Press, Inc., 1994.

[14] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[15] A. Ghaderipoor, M. Hajiaghayi, and C. Tellambura, "Unitary matrix design for differential space-time modulation and limited feedback precoding," in *proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Commun. (PIMRC)*, Helsinki, Finland, September 11-14, 2006.

[16] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communication," vol. 16, pp. 1451–1458, Oct. 1998.