NBER WORKING PAPER SERIES

OPTIMAL PRICE ADJUSTMENT WITH TIME-DEPENDENT COSTS

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Working Paper No. 1319

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
April 1984

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ABSTRACT

The purpose of this paper is to analyze an optimal pricing rule for the case in which the costs of price adjustment are time dependent, and where those costs depend positively on the magnitude of the percentage price change. By means of discrete time model, it is shown that the optimal response to the problem under consideration is to pre-set prices for each period at the end of the previous period. Within the period prices will adjust if the unexpected shock exceeds a threshold level. In such a case the new price is established at a level that is a weighted average of the pre-set level and of the equilibrium level that would have obtained in the absence of costs of contemporaneous price adjustment. Under certain conditions, which are derived in the paper, higher volatility of unexpected inflation might reduce relative price volatility.

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1. Introduction and Summary

In a recent paper Sheshinski and Weiss (1983) have analyzed the optimal price adjustment rule for a producer having market power and facing a fixed cost of price adjustment. Their contribution has demonstrated that the adjustment rule is of the (S, s) type, and that higher volatility of inflation would accompany a higher volatility of relative prices. The purpose of the present paper is to analyze optimal pricing rules for the case where the costs of price adjustment are time dependent, and where those costs depend positively on the magnitude of the percentage price change. Two economic considerations motivate these modifications. First, we expect that the cost of an unexpected price adjustment would exceed that of a price adjustment that is anticipated well ahead of time, because in the latter case more time is available to adjust plans. Next, as Rotemberg (1982) has argued "...under imperfect information consumers will tend to cater to firms with relatively stable price paths and avoid those firms which change their prices often and by large amounts...". Thus, the presumption is that consumers benefit from a more stable price structure. We capture this notion by imposing costs of price adjustment that depend on the magnitude of the price change.

Rotemberg (1982) and Sheshinski and Weiss (1983) postulate costs of price adjustment that are time independent. This paper modifies their analysis by assessing how time-dependent costs of price adjustment modify the pricing behavior of a producer. By means of a discrete time model, is shown that for the case analyzed in the paper optimal behavior requires pre-setting prices for each period at their certainty-equivalence level. Within the period, prices will adjust if the unexpected price-level change exceeds a threshold value. When this happens, the price moves to a level that corresponds to a weighted average of the pre-set prices and the price that would have obtained in the absence of costs of contemporaneous price adjustment. The weights and the threshold value justifying price adjustment have a simple representation as functions of the underlying parameters. Unlike the case analyzed by Sheshinski and Weiss (1983), under certain conditions higher volatility of unexpected inflation might reduce relative price volatility.[1] Necessary conditions are high volatility of inflation and small marginal costs of price adjustment.

The analysis also demonstrates that the elasticity of price volatility with respect to inflation volatility exceeds one. The reason for both results is that higher inflation volatility leads to more frequent contemporaneous price adjustment.

The paper is organized in the following manner. Section 2 describes the model, Section 3 derives the optimal pricing rule. Section 4 offers concluding remarks. The Appendix summarizes the notation used in the paper.

2. The Model

Starting with the case in which prices are fully flexible provides us with a benchmark for subsequent discussion. Flexible prices occur if the cost of contemporaneous price changes is nil, and if consumers attach no value to price stability. Consider in such an economy a producer of good x facing a demand for his product given by

$$(1) \qquad A - \frac{B}{2}\left(\frac{P_t}{\overline{P}_t}\right)$$

where $\bar{P}_t$ and P_t correspond to the price level and the price of good x in period t. The producer is facing a given price level. To simplify exposition, the analysis is conducted for the case where marginal costs of production are zero. The approach described in the paper can be applied also for the case where labor is a variable input.

Direct optimization reveals that the equilibrium price is given by:

$$(2) \qquad p_t^* = \bar{p}_t + \ln \frac{A}{B}$$

where lower case letters $(p, \bar{p})$ denote the logarithms of $P, \bar{P}$, and p^* stands for the equilibrium price. Let π_1^* denote real profits that correspond to the optimal price p_t^*. If the producer charges price p_t, instead of p^*, profits can be approximated by

$$(3) \qquad \pi(p_t) = \pi_1^* - \pi_2(p_t - p_t^*)^2$$

where π_2 corresponds to the absolute value of the second term of the Taylor expansion of real profits around p_t^*. From eq. 1 we obtain that:

$$(4) \qquad \pi_1^* = \pi_2 = \frac{A^2}{2B}$$

We would like to introduce price rigidity by using a modified version of Rotemberg's (1982) framework. Like Rotemberg, we assume two types of costs. One is a fixed cost per price change, denoted by c. The other is a cost that relates to the reputation of the seller. The presumption is that consumers benefit from a more stable price structure. We capture this notion by imposing a cost that is quadratic in the percentage change of prices. Unlike Rotemberg (1982) and Sheshinski and Weiss (1983), we assume that costs of price adjustment are time dependent. The presumption is that changes in prices that are known well ahead of time would impose lesser adjustment costs than unexpected, last minute changes because in the former case there is more time to adjust plans. This assumption is a natural extension of the notion that unexpected inflation imposes higher adjustment costs and welfare loss than anticipated inflation.

A simple way of introducing these considerations is to assume that price changes for period t are costless if they are made ahead of time (i.e., before period t). Price changes impose costs if they are made within the period (at time t), because they can not be foreseen (at time t-1).

In such an environment the producer pre-sets prices for period t at the end of period t-1, at a (logarithmic) level p_t^o. Under certain conditions he would change prices within the period to p_t'. Deriving optimal p_t^o and p_t' is the topic of Section 3. Let p_t denote the actual price of x in period t, and $R(p_t)$ the real profits that correspond to price p_t. We assume that

$$(5)\quad R(p_t) = \begin{cases} \pi(p_t^o) = \pi_1^* - \pi_2(p_t^o - p_t^*)^2 & \text{if } p_t = p_t^o \\ \pi(p_t') - c - \bar{c}(p_t' - p_t^o)^2 = \pi_1^* - \pi_2(p_t' - p_t^*)^2 - c - \bar{c}(p_t' - p_t^o)^2 & \text{if } p_t = p_t' \end{cases}$$

The specification of $R(p_t)$ is a key step in our discussion. If the price of x does not adjust within the period, it will deviate by $|p_t^o - p_t^*|$ from the price that maximizes real profits in a flexible equilibrium case. Thus, we can use equation 3 to find that profits are given by $\pi(p_t^o)$. Had the price adjusted within the period to p_t', under a flexible equilibrium profits would be $\pi(p_t')$. Because within-the-period price adjustment imposes costs, we should adjust profits $\pi(p_t')$ by the costs of price adjustment. Those costs are the fixed cost, c, and depend also on the magnitude of the percentage change. This is captured by the quadratic term $\bar{c}(p_t^o - p_t')^2$ where $|p_t^o - p_t'|$ corresponds to the percentage change of the price of x relative to its pre-set level. In the next Section we derive the properties of p_t^o, p_t'.

3. Optimal Pricing Policy

Optimal pricing policy should provide us with three rules: the optimal price to change if we decide to make a price adjustment within the period (p_t'); the switching conditions under which we would update prices within the period; and the optimal pre-set price (p_t^o). We derive the policy in three corresponding steps. First, for a given p_t^o we derive optimal p_t'. Next, for a given p_t^o and the corresponding optimal value of p_t' we find the conditions under which we would decide to update the price of x within the period (optimal switching rule). Lastly, subject to optimal p_t' and the optimal switching rule, we find the optimal value of p_t^o. We then use the optimal pricing rule to study the properties of the resulting variances of the absolute and the relative prices of x.

Consider first the case in which for a given p_t^o we decide to change the price of x within the period to p_t'. We would set p_t' so as to maximize the resulting profits, $R(p_t')$. This is achieved if (see equation 5)

(6) $$p'_t = \omega\, p^*_t + \omega_o\, p^o_t\,, \quad \text{where}$$

$$\omega = \frac{\pi_2}{\pi_2 + \bar{c}}\,, \quad \omega_o = \frac{\bar{c}}{\pi_2 + \bar{c}}\,.$$

The new price corresponds to a weighted average of the pre-set and the flexible prices. Higher costs of marginal price adjustment $(d\,\bar{c} > 0)$ would shift the resulting price closer to the pre-set level, whereas greater profits loss due to divergence form the flexible equilibrium would shift prices closer to the equilibrium.

Next, notice that we would adjust prices contemporaneously if

(7) $$R(p^o_t) < R(p'_t).$$

From equations 5-7 we obtain that this condition is satisfied if

(8) $$|p^o_t - p^*_t| > k,$$

where $k = \dfrac{\sqrt{c(\pi_2 + \bar{c})}}{\pi_2}$. Let us assume that the price level at time t is given by

(9) $$\bar{p}_t = g_t + \varepsilon_t\,, \quad E_{t-1}(\varepsilon_t) = 0.$$

E_{t-1} is the conditional expectation operator, using the information available at the end of period t-1. g_t is a non-stochastic term, corresponding to the expected price level at period t (expectation taking place at period t-1). Let δ_t denote

$$\delta_t = E_{t-1}\, p_t^* - p_t^o . \tag{10}$$

Our producer is facing a given price level. Thus, $E_{t-1}\, p_t^* = g_t + \ln \frac{A}{B}$, and p_t^o is set such as to maximize expected profits:

$$\max_{p_t^o} \; E_{t-1}(R_t). \tag{11}$$

Let $\rho(\varepsilon)$ denote the density function of ε. Using the switching rule obtained in equation 8 and the value of p'_t in equation 6, we get from equation 5 that expected profits are:

$$
\begin{aligned}
E_{t-1}(R_t) = \pi_1^* - \pi_2 \cdot \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} (\delta_t + \varepsilon)^2 \rho(\varepsilon)d\varepsilon \qquad (12)\\
- \int_{-\infty}^{\underline{\varepsilon}} [c + \omega \cdot \bar{c}\,(\delta_t + \varepsilon)^2]\rho(\varepsilon)d\varepsilon \\
- \int_{\bar{\varepsilon}}^{\infty} [c + \omega \cdot \bar{c}\,(\delta_t + \varepsilon)^2]\rho(\varepsilon)d\varepsilon
\end{aligned}
$$

where $\underline{\varepsilon} = -k - \delta_t$; $\bar{\varepsilon} = k - \delta_t$

We pre-set the price (p_t^o) so as to maximize expected profits. Notice that δ_t is a parameter whose value is set by the choice of p_t^o. Direct optimization of equation 12 reveals that whenever ε_t follows a symmetrical distribution, the optimal choice of p_t^o is at its "certainty equivalent" value, such that $\delta_t = 0$, i.e.

$$(13) \qquad p_t^o = E_{t-1}\, p_t^* .$$

Optimal choice of the pre-set price implies that the switching rule (equation 8) is given by

$$(8') \qquad p_t = \begin{cases} p_t^o & |\varepsilon_t| < k \\ p_t' & |\varepsilon_t| > k. \end{cases}$$

The price of x for period t is pre-set at the end of the previous period at its certainty-equivalence level (i.e., at the level that would maximize profits if the value of ε is zero). If the unexpected price level change exceeds a threshold value,[2] the price of x adjusts within the period to level p_t'. The threshold value depends positively on the cost of within-the-period price adjustment (both the fixed and the marginal cost). It depends negatively on a measure of the cost of price divergence from its optimal value (π_2).

We can now derive a measure of unexpected relative price adjustment. From equations 6, 8' we obtain that:

$$(14) \qquad p_t - \bar{p}_t - E_{t-1}(p_t - \bar{p}_t) = \begin{cases} -\varepsilon_t & \text{if} \quad |\varepsilon_t| < k \\ -\varepsilon_t\, \omega_o & \text{if} \quad |\varepsilon_t| > k. \end{cases}$$

If the price of x is pre-set, the shock to the relative price is equal to the aggregate price level shock. Allowing the price of x to adjust within the period cushions the shock to relative prices by

$$(15) \qquad 1 - \omega_o = \omega .$$

The greater is the cost of price divergence (π_2) and the lower the marginal cost of price adjustment, the greater the cushioning effect. To assess how higher unexpected inflation volatility affects relative price volatility let us impose further structure on the model by assuming that ε follows a normal distribution. Let $\phi(\varepsilon)$ and $\Phi(\varepsilon)$ denote the standard normal density function and cumulative distribution of ε, and V_y, σ_y denote the variance and standard deviation of y. From equation 14 we obtain that the volatility of relative prices is:

$$(16) \; V_{p-\bar{p}} = E_{t-1}[\{p_t - \bar{p}_t - E_{t-1}(p_t - \bar{p}_t)\}^2] = V_\varepsilon \cdot [H(z) + \{1 - H(z)\} \cdot (\omega_o)^2]$$

where $\quad z = k/\sigma_\varepsilon$, $H(z) = 1-2 \cdot \Phi(-z) - 2 \cdot z \cdot \phi(z)$.

A higher volatility of unexpected inflation $(dV_\varepsilon > 0)$ has two opposite effects. The direct effect increases relative price volatility at the rate of the increase in inflation volatility. At the same time, however, higher inflation volatility implies that the price of x would adjust more frequently within the period (i.e., more frequently $|\varepsilon_t| > k$). Because such an adjustment cushions the shock to relative prices, higher inflation volatility would indirectly work to reduce volatility. In terms of equation 16,

$dV_\varepsilon > 0$ would reduce the expression in the brackets [notice that $\frac{dH(z)}{dz} > 0$, and that $0 \leq H(z) \leq 1$]. From equation 16 we obtain that

$$(17) \qquad \frac{\partial V_{p-\bar{p}}}{\partial V_\varepsilon} = (\omega_o)^2 + \{H(z) - z^2 \cdot (-\phi'(z))\}\{1 - (\omega_o)^2\}.$$

It can be shown that[3]

$$(18) \qquad z^2 \cdot (-\phi'(z)) < H(z) \quad \text{if} \quad z > 1.36$$

$$z^2 \cdot (-\phi'(z)) > H(z) \quad \text{if} \quad z < 1.36.$$

Thus, for stable economies the effect of higher inflation volatility on relative price volatility is unambiguously positive, (i.e., if $\sigma_\varepsilon < k/1.36$). This is because the direct volatility effect dominates the cushioning effect of more frequent price adjustment. In terms of equation 17, both terms are positive. For economies where the volatility of inflation is high enough, however, the second term in equation 17 is negative (i.e., if $\sigma_\varepsilon > k/1.36$). Under certain conditions it might even dominate the first term, resulting in a region where a higher inflation volatility accompanies a lower relative price volatility. To obtain this result the cushioning effect ω of price adjustment should be adequately powerful. Inspection of equation 17 reveals that higher unexpected inflation volatility would reduce relative price volatility if and only if both the marginal cost of price adjustment are small enough, and the inflation rate is sufficiently volatile.[4]

Using equations 6, 8' we can obtain also that price volatility is given by:[5]

(19) $$V_{t-1}(p_t) = [1 - H(z)] \cdot V_\varepsilon \cdot (\omega)^2,$$

from which we can derive that

(20) $$\eta_{V_{t-1}(p_t),\, V_{t-1}(\bar{p}_t)} > 1$$

where $\eta_{y,z}$ denotes the elasticity of y with respect to z. Higher volatility of unexpected inflation would raise the volatility of the price of x at a higher rate because higher inflation volatility justifies more frequent price adjustment.

4. Concluding Remarks

Our discussion demonstrates that considering the possibility of time dependent costs of price adjustment allows us to focus on the cushioning effect of within-the period price adjustment. This effect reduces the magnitude of relative price volatility. Under certain conditions it might result in a drop in relative price volatility as a result of higher inflation volatility. These conditions seem restrictive enough to prevent aggregate analysis from revealing the importance of the cushioning effect. Our analysis suggests that the price behavior will vary across economies and sectors according to the degree of competition, the costs of price adjustment, the volatility of the inflationary process.

Footnotes

1. For a recent survey of the literature on relative price variability and inflation see Cukierman (1983). For an analysis of sticky prices due to time-independent fixed costs of adjustment see also Mussa (1981).

2. Such a pricing rule is similar to the ad hoc pricing rules applied previously in a macro context by Aizenman (1984) and McCallum (1977). Unlike those previous papers, in the present context the pricing rule is derived explicitly using an optimizing procedure.

3. Let us denote by a, b the points on the real plain defined by $(-z, \phi(z))$, $(z, \phi(z))$. Direct inspection reveals that $H(z)$ is the area bounded below the standard normal density function and above the line that passes points a and b. $z^2\phi'(-z)$ is the area of a triangle defined by the two tangents to $\phi(z)$ at points a and b, and by the line that passes points a, b. Due to the shape of the normal density function, $H > z^2 \cdot \phi'(-z)$ for large z, and $H < z^2 \cdot \phi'(-z)$ for small z. It turns out that $H = z^2 \cdot \phi'(-z)$ for $z \simeq 1.36$.

4. Notice that if $\bar{c} \to 0$, $\omega_o \to 0$.

5. $V_{t-1}(p_t)$ is the conditional variance of p, defined by $E_{t-1}[(p_t - E_{t-1}(p_t))^2]$. We use it as a volatility measure.

Appendix Notation

$\bar{P}_t$ = price level at time t

P_t = price of good x at time t.

W_t = = money wage at time t.

$\bar{p}$, p,w = logarithms of $\bar{P}$, P, W .

p_t^* = flexible equilibrium price.

π_t^* = real profits in a flexible equilibrium.

p_t^o = the pre-set price of good x for period t.

p_t' = the price of good x for period t if there is within the period price adjustment.

$R(p_t)$ = real profits that corresponds to price p_t.

E_t = expectation operator conditional on information available at time.

V_x = variance of x. As a measure of volatility of x we use a conditional variance:

$$V_{t-1}(x) = E_{t-1}[(x_t - E_{t-1}(x_t))^2]$$

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