Optimal Pricing Policy with Recommender Systems

Dirk Bergemann
Department of Economics,
Yale University

Deran Ozmen
Department of Economics,
Yale University

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Abstract

We look at one of the informational novelties introduced by the existence of internet market, namely "the recommender systems". A recommender system is a system employed by some internet sellers, which collects data from all previous customers about their experiences and makes inferences from this data to recommend a product to an active customer. The recommender system can also be interpreted as a peer-to-peer system where the seller provides a platform for the buyers to share their experiences. We interpret the role of a recommender system as reducing uncertainty for the customers, which creates some additonal surplus to be distributed between the customers and sellers employing such systems. We differentiate the customers with respect to the extremity of their preferences, which also implies different valuations for decreased uncertainty. We show that an internet seller employing such as system can extract a non-negligable share of this surplus from the customers through higher prices in the presence of a competitive fringe without recommender systems. However optimal pricing by the seller with the system leads to a less than full market share, since the seller finds it optimal to leave out the buyers with moderate taste to the fringe. Thus the optimal pricing mechanism does not employ the recommender system at the efficient level, in other words there is under utilization. We also find that the overall under-utilization might entail over-utilization of the system for some products and under-utilization for others.

Keywords: Internet; recommender system; uncertainty; information; optimal pricing; efficiency

JEL classification: D4, D70, D8, L15

1 Introduction

The new developments in computer technology and the increased volume of internet sales highlight some differences between internet and brick-and-mortar markets. The center of interest of this paper is one such difference, the fact that the internet sales pave way to a large accumulation of data about the customers and products, which was not feasible in the standard brick-and-mortar markets. An internet seller has access to technology which helps him monitor all the browsing and the shopping his customers make and even ask for their feedbacks. Therefore, internet sellers can build large databases that consists of personalized data on all their customers, the customers' past purchases and the feedback from those purchases.

In this paper we analyze one particular usage of the accumulated information, "the recommender systems". A recommender system is a software program which utilizes the formerly mentioned database to make statistical inferences about which product a customer would like best. The best example of such a system is employed by Amazon.com. Once a customer makes a purchase there, the next time she logs on to Amazon.com, a recommendation pops up on the screen for her. Such a recommendation is the result of the data processing of a recommender system, where the data consists of previous purchases of and the feedback left by each customer. We can also see the recommender system as a peer-to-peer system where the seller provides a platform for the buyers to share their experiences.

From an economist's point of view, the recommender systems represent an informational linkage that creates additional surplus through reducing uncertainty that the customers face when making a choice between different products. The source of uncertainty is two-fold: uncertainty about one's own taste and uncertainty about the products. We will focus on the latter source in the on-line market for horizontally differentiated products, where the difference between customers' tastes also play a role. In this setting the recommender system helps customers to make better informed choices and thus creates some additional surplus.

Although various computer engineers are working on technical issues about the subject and as economists Shapiro and Varian (1999) and Vulkan (2003) informally discuss the role of recommender systems, economics literature has provided us with very few formal models to answer the questions related to this subject. The only formal model in this area was introduced by Avery, Resnick, Zeckhauser (1999), which takes a mechanism design point of view towards the problem. In this paper we take the mechanism as given and we introduce one of the first formal theoretic models that describes the interaction between a seller with a recommender system and customers in the presence of a competitive fringe.

The main question we would like to answer is regarding the division of this created surplus between the seller and the customers. The websites which provide such services usually have slightly higher prices than the others, which could be interpreted as the fee for the information the sellers sell through the recommender system. Therefore, optimal pricing is the main focus of the paper. Our results show that a seller with a recommender system can indeed extract a non-negligible share of the value created through prices higher than the prices charged by sellers without recommender systems.

We show that the optimal pricing problem of the seller is the dual of the problem about what share of the market to capture and how to distribute the buyers over the different products so that enough information could be gathered on each product. At this stage of the problem there is an obvious "coordination" element coming from the fact that as the seller has more customers, he will be able to make better recommendations and thus attract more customers. However, there is also what we interpret as a "within competition effect" which is due to the fact that if there are a lot of customers buying one particular product at a certain time, others would be willing to delay the purchase of that product and be directed to other products for the time. These two effects together determine what the seller's optimal distribution of customers is over different products.

We also ask the question whether the resulting allocation of buyers to different products and sellers as a result of the optimal pricing scheme is efficient or not. We find that the optimal pricing mechanism does not employ the recommender system at the efficient level, in other words there is under utilization. We also find that the seller's overall under-utilization of the system might entail over utilization of the system for some products and under utilization for others.

The road map is as follows: We will first describe the model in section two and then as a benchmark case we will discuss what happens both in terms of efficiency and equilibrium in this setting in the absence of a recommender system in section three. In sections four and five we will look at the efficient solution and the equilibrium respectively. And finally we will conclude comparing the two and discussing the contributions of the recommender system to society.

2 The Model

There is one seller with recommender system, denoted by M, and a competitive fringe with no such system, denoted by F, in the on-line market for a particular product. Within the market, there are two different types of the product, type -1 and type 1. We will denote the type of the product by $x \in \{-1, 1\}$. There is a continuum of buyers in $[-1, 1]$, where each buyer is characterized by his preference $\theta \in [-1, 1]$. θ is distributed uniformly in $[-1, 1]$. The gross utility value a buyer of type θ derives from a type x product is specified as

$$u(\theta, x) = V - (\theta - x)^2 \tag{1}$$

There are two periods with flow of products and there is uncertainty about their types. In period one two versions of the product arrive at all sellers. These versions are differentiated only with respect to the priors attached to them. The prior for one version attaches probability $\frac{1}{2} - \varepsilon$ to it being type 1 and the prior for the other version attaches probability $\frac{1}{2} + \varepsilon$ to it being type 1, where $\varepsilon \in \left[0, \frac{1}{2}\right]$. We will label these versions by l and h and the probabilities by α_l and α_h respectively. We will consider ε as the "initial information".In period two a new version, which we will label by m, arrives with prior $\alpha_m \in \{\alpha_l, \alpha_h\}$ at all sellers. In period 1, neither the buyers nor the sellers know what α_m will exactly be, but they attach $\frac{1}{2}$ probability to α_m being α_h and α_l.

Marginal cost of all versions for all sellers equals c. We assume that the price for each version in the competitive fringe equals c. There is no discounting. Each buyer buys at most one product each period. Moreover a buyer has to buy different versions of the product each period. We also assume that the per period outside utility for each buyer is smaller than $V - c - 2$, so each buyer is actually willing to buy some version each period from the fringe.

Given this setting, the recommender system is described as follows: Let $x_i \in \{-1, 1\}$ be the type of version $i \in \{l, h\}$. Also let μ_i denote the measure of buyers who buy version $i \in \{l, h\}$ from seller M in period one. Between periods one and two, for each $i \in \{l, h\}$ the seller M receives a random signal $y_i(x_i) \in \{\emptyset, -1, 1\}$ where $y_i(x_i) = \emptyset$ with probability $1 - \mu_i$ and $y_i(x_i) \in \{-1, 1\}$ with probability μ_i. Conditional on $y_i(x_i) \in \{-1, 1\}$, $y_i(x_i) = x_i$ with probability $\frac{1}{2} + \gamma$ and $y_i(x_i) = \{-1, 1\}/x_i$ with probability $\frac{1}{2} - \gamma$, where $\gamma \in \left[0, \frac{1}{2}\right]$. We will interpret γ as the precision of the signal when received. Clearly the probability of receiving a signal on a version increases in the measure of buyers buying that version. This captures the effect that as a seller has more customers, the recommender system will have more input. Given this random structure, the recommender system is a pre-committed direct mechanism that computes the posteriors $\alpha_i'(y_i)$ and reports them only to the buyers who have bought from him in period one. We will use $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}'$ to denote $\{\alpha_l, \alpha_m, \alpha_h\}$ and $\{\alpha_l', \alpha_m', \alpha_h'\}$ where $\alpha_m' = \alpha_m$.

In period one, seller M announces prices for each version, i.e. $\mathbf{p} = (p_l, p_h) \in \mathbf{R}^2$. We will also use p_{is} to denote the price of version i at seller s. The search cost is zero for all buyers, thus buyers log onto all websites, observe all prices and each buyer simultaneously chooses a version to buy $i \in \{l, h\}$ and a seller to buy from $s \in \{M, F\}$.

In period two, seller M announces prices for each version, $(p_l', p_m', p_h') . \in \mathbf{R}^3$ and reveals the recommendations to the regular buyers. Buyers simultaneously choose a version to buy $i' \in \{j, m\}$ where j will denote $\{l, h\}/i$, and a seller to buy from $s' \in \{M, F\}$ given the information they observe. Note that since the search cost is zero a buyer can get the recommendation from seller M and yet buy from the fringe.

3 Benchmark Case: No Recommender System:

As a benchmark case we will first analyze the efficient solution and the subgame perfect equilibria in the case with no recommender system. Notice that this case is equivalent to our model if $\gamma = 0$ or $\varepsilon = \frac{1}{2}$.

Proposition 1 *There exists a unique equilibrium and it generates the efficient allocation. It is characterized by marginal cost pricing in both periods for all versions. Given the pricing in the first period all buyers with $\theta \geqslant 0$ buy version h and all buyers with $\theta < 0$ buy version l.*

When there is no recommender system seller M's products are identical to those of the fringe and thus we get standard Bertrand solution. Since the only thing the first period distribution of buyers affect is their own utilities in the first period, efficiency allocates them with respect to their first period preferences which turns out to coincide with the equilibrium allocation. Given the benchmark case, we can now analyze efficiency and equilibrium in the presence of the recommender system.

4 Efficiency with the recommender system

It is an interesting question to ask what the efficient first period distribution of buyers over different versions should be given the fact that buyers' purchases generate a trade-off between two effects: A direct effect on their utility and an indirect informational effect through the recommender system as explained above.

We will introduce some new definitions that will be used very widely throughout the paper.

Definition 1 (BALANCE) *A distribution of buyers with (μ_h, μ_l) is balanced if $\mu_h = \mu_l$ and unbalanced if $\mu_i > \mu_j$ for some $i \in \{l, h\}$ where $\frac{\mu_i}{\mu_j}$ is the level of unbalance.*

We can think of the recommender system as a potential way to create endogenous differentiation between the two versions which are exogenously differentiated by ε. In particular, if the distribution is unbalanced favoring one version, the levels of information gathered on the two versions will be different and thus the versions will be differentiated even further when the posteriors are computed. The question we would like to answer is whether it is efficient to create such endogenous differentiation through an unbalanced distribution and if so, which buyers should benefit from this unbalance. Regarding this last point we introduce the following definition.

Definition 2 (SHIFT) *Take a distribution of buyers with (μ_i, μ_j) where $\mu_i \geqslant \mu_j$ for some $i \in \{l, h\}$. It is "unshifted" if the set of buyers buying versions l and h respectively are positive measure line segments of the form $[-1, .]$ and $[., 1]$. It is "shifted towards* *moderates" if the set of buyers buying version j consists of two positive measure segments L_1, L_2 of the forms $[-1,.]$, $[.,1]$ with $\|L_1\| \geqslant \|L_2\|$ if $j = l$ and $\|L_1\| \leqslant \|L_2\|$ if $j = h$, where $\frac{\min\{\|L_2\|,\|L_1\|\}}{\max\{\|L_2\|,\|L_1\|\}}$ is the degree of the shift. If $\frac{\|L_2\|}{\|L_1\|} = 1$, we say the distribution is "centered". If the distribution satisfies the criteria of one of these definitions except the set of buyers buying i consists of two segments separated only by another segment around zero, we say it satisfies the definition "with an interruption".*

If a distribution is shifted towards moderates, it is the buyers with extreme tastes that benefit more from the endogenous differentiation created by the unbalanced distribution. In other words we can think of it as the buyers with moderate tastes experiencing a version very intensely and the extremists of both types getting really good recommendations thanks to them, whereas if a distribution is unshifted it usually sacrifices the extremists of one type for the other.

With the definitions in mind we will give some formal characterizations. The per period expected gross utility for a buyer with type θ from buying version i under probability system $\boldsymbol{\alpha}$ is

$$E_{\boldsymbol{\alpha}} u(\theta, x_i) = V - \theta^2 - 2\theta + 2\alpha_i\theta \tag{2}$$

Let us suppose for efficiency purposes that all second period prices equal c and given the recommendations, second period utility is maximized with respect to the remaining choice set $\{j, m\}$. Then what matters from a first period point of view for a buyer with $\theta \geqslant 0$ is the *expected maximized posterior* which is given by

$$\bar{\alpha}_{iM}(\mu_j) = E\left(\max_{i' \in \{j,m\}} \alpha'_{i'}(y_{i'}(x_{i'})) \mid \mu_j\right) \tag{3}$$

if the first period purchase is made from seller M and thus the buyer has access to recommendation and by

$$\bar{\alpha}_{iF}(\mu_j) = \bar{\alpha}_{iF} = E\left(\max_{i' \in \{j,m\}} \alpha_{i'}\right) \tag{4}$$

if the first period purchase is made from the fringe.

Then two-period value function for a buyer with type $\theta \geqslant 0$ conditional on the first period choices (i, s) and the prices is

$$\overline{U}(\theta, i, s, \mu_l, \mu_h, \mathbf{p}) = \begin{pmatrix} 2V - 2\theta^2 - 4\theta + \\ 2\theta(\alpha_i + \bar{\alpha}_{is}(\mu_j)) \\ -p_{is} - c \end{pmatrix} \tag{5}$$

For buyers with $\theta < 0$, everything follows the same except we replace "max" operator with "min" in the expressions above and we get the value function $\underline{U}(\theta, i, s, \mu_l, \mu_h, \mathbf{p})$ by replacing $\overline{\alpha}_{is}(\mu_j)$ with the *expected minimized posterior* $\underline{\alpha}_{is}(\mu_j)$.

The value function employs some nice properties which will help characterize the efficient solution and the equilibrium.

Lemma 1 *For all $i \in \{l, h\}$ and all $\mu_j \geqslant 0$*

1. *$\bar{\alpha}_{iM}(\mu_j) \geqslant \bar{\alpha}_{iF}$ and $\underline{\alpha}_{iM}(\mu_j) \leqslant \underline{\alpha}_{iF}$*
2. *$\frac{\partial \bar{\alpha}_{iM}(\mu_j)}{\partial \mu_j} = -\frac{\partial \underline{\alpha}_{iM}(\mu_j)}{\partial \mu_j} > 0$*
3. *for all $\gamma \in \left[\frac{1}{2}, 1\right]$ and $\varepsilon \in \left[\frac{1}{2}, 1\right]$, $\frac{\partial^2 \bar{\alpha}_{iM}(\mu_j)}{\partial \mu_j \partial \gamma} \geqslant 0$ and $\frac{\partial^2 \bar{\alpha}_{iM}(\mu_j)}{\partial \mu_j \partial \varepsilon} \leqslant 0$*
4. *the two-period value function is supermodular in θ and $\bar{\alpha}_{iM}$, $\underline{\alpha}_{iM}$.*

The first two points in the lemma makes it obvious that all buyers should buy from seller M for efficiency, since more information is only better. Point three will help us determine when having an unbalanced distribution is better. As the distribution becomes unbalanced, the utility of one group of buyers increases at the expense of the other group. Points three implies that the gain from an unbalanced distribution is higher when information is more valuable. And finally point four reveals the fact that extremists' gain from information is more than the gain of moderates and this will determine whether the distribution will be unshifted or shifted towards the moderates. The following proposition combines all these effects and reveals the efficient allocation.

Proposition 2 (EFFICIENCY) *The efficient solution is such that seller M has full market share and there exists a unique $\eta^* > 7$ such that ;*

1. *for $\frac{\gamma}{\varepsilon} \leqslant \eta^*$, the unique efficient distribution of buyers is balanced and unshifted,*
2. *for $\infty > \frac{\gamma}{\varepsilon} > \eta^*$, there are two symmetric efficient distributions of buyers that are unbalanced and shifted towards moderates but not centered.*

Figure 1 illustrates the efficient distribution given in the proposition. To give an interpretation to the proposition it is important to analyze the term $\frac{\gamma}{\varepsilon}$. A high $\frac{\gamma}{\varepsilon}$ indicates either high informativeness or a low initial information, both of which make the information added to the system by the recommender system more valuable. Thus, the proposition implies that for

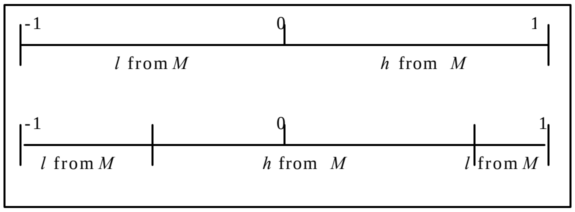

Figure 1: The efficient distribution of buyers for $\frac{\alpha}{\varepsilon} \leqslant \eta^*$ above and one of the two symmetric efficient distributions for $\frac{\alpha}{\varepsilon} > \eta^*$ below

low levels of $\frac{\alpha}{\varepsilon}$, the efficient allocation is no different than the efficient allocation in the benchmark case with no recommender system because the contribution of the recommender system is not big enough to change the trade-offs. However, for $\frac{\alpha}{\varepsilon}$ large enough, information created by the recommender system becomes valuable enough so that the efficient distribution becomes unbalanced. The unbalance favors extremists over moderates since extremists' value for more information is higher.

Proposition 3 *For $\frac{\alpha}{\varepsilon} > \eta^*$, as $\frac{\alpha}{\varepsilon}$ increases, each efficient distribution becomes more unbalanced and is shifted more towards the moderates. As $\frac{\alpha}{\varepsilon} \to \infty$, the distribution becomes centered.*

Proposition three implies that as information becomes more valuable it pays off to increase the degree of unbalance and put a higher burden on moderates.

5 The Equilibrium

We will be looking for subgame perfect equilibria with a certain refinement which will be introduced shortly. Our first observation is regarding the second period problem.

Lemma 2 *The minimum second period price in the market in any subgame perfect equilibria equals marginal cost for each version.*

This lemma is due to the fact that a buyer can get the recommendation from seller M and yet buy from the fringe given the recommendation. Thus the services all sellers provide are identical in period two and the competition is at the Bertrand level. The interesting part of the problem is the first period prices.

We first look at the subgame played between the buyers after seller M announces $\mathbf{p}$. Let $(\widehat{\mu}_l, \widehat{\mu}_h)$ represent the belief of a buyer about's others actions after observing $\mathbf{p}$. Value function now is the same as in equation (5) except (μ_l, μ_h) is replaced by $(\widehat{\mu}_l, \widehat{\mu}_h)$. Then let $(i^*(\theta, \mathbf{p}, \widehat{\mu}_l, \widehat{\mu}_h), s^*(\theta, \mathbf{p}, \widehat{\mu}_l, \widehat{\mu}_h))$ represent type θ buyer's optimal version and seller choices that maximize the value function, given the prices and belief of the buyer. In equilibrium the beliefs of the buyers about the others' behavior must be correct. This is a fixed point problem where the $(\mu_l(\mathbf{p}), \mu_h(\mathbf{p}))$ that equals the vector of measures implied by the optimal choices $\{i^*(\theta, \mathbf{p}, \mu_l(\mathbf{p}), \mu_h(\mathbf{p})), s^*(\theta, \mathbf{p}, \mu_l(\mathbf{p}), \mu_h(\mathbf{p}))\}_{\theta \in [0,1]}$ is the fixed point. There might be multiple fixed points due to the coordination element inherent in the problem. The next lemma provides a basis for refinement.

Lemma 3 *The subgame perfect equilibria in the subgame following the announcement of any $\mathbf{p} \in \mathbf{R}^2$ are strictly pareto ranked and an increase in the pareto rank coincides with a strict increase in the market share of seller M for both versions.*

Definition 3 *A subgame perfect equilibrium of the whole game is "coordinative" if for all $\mathbf{p} \in \mathbf{R}^2$ it induces a pareto dominant equilibrium in the subgame followed by the announcement of $\mathbf{p}$ in the first period.*

Although there might be multiple equilibria, due to standard communication arguments, we will refine the equilibrium concept such that the pareto dominant equilibria is picked in the subgame followed by announcement of any $\mathbf{p}$.

Thus, from now on we focus on the fixed point $(\mu_l(\mathbf{p}), \mu_h(\mathbf{p}))$ that constitutes the highest measures of buyers buying versions h and l from seller M when buyers act optimally as a response to $\mathbf{p}$. Now, we can write the profit of seller M as a function of $\mathbf{p}$ as follows:

$$\pi_M(\mathbf{p}, \mu_h(\mathbf{p}), \mu_l(\mathbf{p})) = \begin{pmatrix} \mu_h(\mathbf{p})(p_h - c) + \\ \mu_l(\mathbf{p})(p_l - c) \end{pmatrix}$$

Let us consider the trade-off involved in increasing p_h for example. As in a usual monopolist problem the market share for h decreases. However the decrease in μ_h triggers another effect such that the utility of the purchasers of version l decreases since they will get less information on h next period. But this means μ_l also decreases and it becomes a snowball effect feeding back into the cycle. The optimal prices (p_h^*, p_l^*) for seller M maximize the expression above by offsetting this trade-off and generate the optimal market shares $(\mu_h(\mathbf{p}^*), \mu_l(\mathbf{p}^*))$. The next proposition illustrates how these are attained for different levels of $\frac{\alpha}{\varepsilon}$.

Proposition 4 (EQUILIBRIUM) *In all coordinative subgame perfect equilibria seller M has less than full market share where the set of buyers buying from the fringe is a positive measure line segment around 0. There exists a unique $\eta_1 \in (5,7]$ and $\eta_2 > 7$ such that*

1. *if $\frac{\gamma}{\varepsilon} \leqslant \eta_1$, the equilibrium is unique and is characterized by prices $p_i^*(\gamma,\varepsilon) - c = \frac{1}{18}\max\left\{\frac{1}{2}\gamma\left(1-4\varepsilon^2\right),(\gamma-\varepsilon)\right\}$ for all $i \in \{l,h\}$, which gives rise to a balanced and unshifted distribution where the set of buyers buying versions l and h from seller M are respectively $\left[-1,-\frac{1}{3}\right]$ and $\left[\frac{1}{3},1\right]$;*

2. *if $\eta_1 < \frac{\gamma}{\varepsilon} \leqslant \eta_2$, there are two symmetric equilibria which are characterized by prices $p_i^*(\gamma,\varepsilon) > c$ for all $i \in \{l,h\}$ and the distribution of buyers is unbalanced and unshifted with an interruption where the market share of the fringe is strictly less than $\frac{1}{3}$;*

3. *if $\infty > \frac{\gamma}{\varepsilon} > \eta_2$, there are two symmetric equilibria which are characterized by prices $p_i^*(\gamma,\varepsilon) > c$ for all $i \in \{l,h\}$ the distribution of buyers is unbalanced and shifted towards moderates, but not centered, with an interruption where the market share of the fringe is strictly less than $\frac{1}{3}$.*

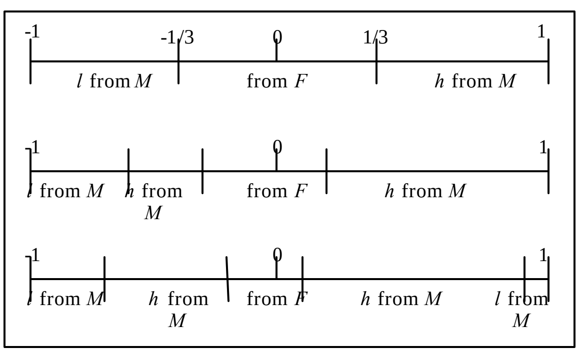

Figure 2: From top to bottom, the equilibrium distribution of buyers for $\frac{\gamma}{\varepsilon} \leqslant \eta_1$, one of the two symmetric equilibrium distributions for $\eta_2 \geqslant \frac{\gamma}{\varepsilon} > \eta_1$ and one of the two symmetric symmetric equilibrium distributions for $\frac{\gamma}{\varepsilon} > \eta_2$

The first thing to observe from the proposition is that seller M is able to put a mark-up over marginal cost in the first period just because he will be selling a differentiated product in the second period, differentiation being a lower uncertainty. Figure 2 illustrates what the equilibrium distribution of buyers look like for different values of $\frac{\gamma}{\varepsilon}$. First thing to observe is that the seller finds it optimal to leave sufficiently moderate preferenced buyers to the fringe, since they have a low willingness to pay for increased information. Similar to the efficient allocation, for low levels of $\frac{\gamma}{\varepsilon}$, since the potential contribution of the recommender system is not valuable enough the seller prefers a balanced distribution. As $\frac{\gamma}{\varepsilon}$ increases, the seller finds it optimal to form an unbalanced distribution since the informational gains of doing so increases the profits. As $\frac{\gamma}{\varepsilon}$ increases further, the seller finds it optimal to favor extremists over moderates since they have a higher willingness to pay for increased information compared to moderates.

Proposition 5 *In any coordinative subgame perfect equilibria, the profits of seller M increases in γ and decreases in ε and*

1. *if $\frac{\gamma}{\varepsilon} \leqslant \eta_1$ the measures of buyers buying either version from either seller do not change with γ or ε but the prices for both versions increase(decrease) with the same rate in $\gamma(\varepsilon)$;*

2. *if $\eta_1 < \frac{\gamma}{\varepsilon}$ and $\mathbf{p}^*(\gamma,\varepsilon)$ is such that $\mu_i(\mathbf{p}^*) > \mu_j(\mathbf{p}^*)$ for some $i \in \{l,h\}$, the relative price $\frac{p_i^*(\gamma,\varepsilon)}{p_j^*(\gamma,\varepsilon)}$ decreases, the total market share of seller M and the degree of unbalance increases in $\frac{\gamma}{\varepsilon}$ and the equilibrium is shifted more towards moderates as $\frac{\gamma}{\varepsilon}$ increases. As $\frac{\gamma}{\varepsilon} \to \infty$, the distribution becomes centered.*

The proposition shows that overall, as ε decreases or γ increases the profits increase. The increase in profit is achieved solely through direct exploitation of increased willingness to pay, i.e. increased prices,when the level of informativeness is low. However as the signals become more informative or as the initial information gets lower, the seller actually gets even higher profits by increasing his market share through one particular version, since the increased measure of buyers for one version feeds into the profits as a further increase in the price for the other version. Thus when the informativeness is high, a higher speed of increase in profits is attained through keeping the price of one version relatively low compared to the other version and increasing the market share.

The next question we would like to answer is how the efficient allocation compares to the equilibrium allocation. The first obvious difference is that the equilibrium leads to under-utilization of the recommender system since it leaves out some buyers to the fringe.

The reason for this is that the seller internalizes only the gains in the utility of the marginal buyer and not the total gain to all buyers when he attempts to increase his market share. The next proposition reveals how the distribution of buyers differ between the two.

Proposition 6 *The efficient distributions of buyers always imply full market share whereas the equilibrium distributions are always less than full market share. For all* $(\gamma, \varepsilon) \in \left\{(\gamma, \varepsilon) \mid \frac{\gamma}{\varepsilon} \leqslant \eta_1\right\}$, *both the equilibrium and efficient distributions of buyers is balanced. For all* $(\gamma, \varepsilon) \in \left\{(\gamma, \varepsilon) \mid \frac{\gamma}{\varepsilon} > \eta_1\right\}$ *each equilibrium distribution of buyers is more unbalanced than the corresponding efficient distribution.*

The last proposition reveals another difference between the equilibrium allocations and the efficient allocations. Even though the seller does not utilize the recommender system to the full extent, the system is over-utilized for one version and under-utilized for the other. The seller increases his share of the market for one version beyond the optimal level because he does not internalize the loss in the utilities of all buyers buying that version, but he only internalizes the loss in the utility of the marginal buyer, through lower prices. This creates a discrepancy between the efficient allocation and the equilibrium

6 Conclusion and Research in Progress

Our results have confirmed the fact that a seller with a recommender system can charge for this service through high prices, because by reducing uncertainty for the buyers he creates additional surplus to be distributed. However, the seller finds it optimal to capture a less than full market share. In particular, the optimal pricing results in segmentation in the market. The buyers who utilize the recommender system the most by paying a high premium are the ones who have relatively extreme tastes, in other words higher valuation for decreased uncertainty. The seller extracts some of this surplus from the extreme-taste buyers. The buyers with relatively moderate tastes prefer not to use the recommender system to its full extent by either buying from elsewhere or including themselves in the experimentation group. Since there might be some buyers left out, the level of information accumulation implied by optimal pricing is not at the highest possible level, therefore we can say that there is under utilization of the system from an efficiency point of view. Moreover, we found that this overall under-utilization might entail over-utilization for some products and under-utilization for others. In other words, it is certain that the recommender system increases overall welfare by creating additional value, but due to the seller's incentives it is not employed in the socially optimal way..

The next step is taking the model to a dynamic level, where the buyers enjoy the choice of making their purchases at different points in time. For the system to function it needs input from the buyers. This means the first input, i.e. the first buyers, will be especially valuable for the seller and thus we would expect them to enjoy a premium in the form of lower prices. As more and more buyers make purchases and leave feedback, the information that the seller has becomes more valuable and thus we would expect the price of information to increase over time.

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