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Optimal Pricing Strategies in Cognitive Radio Networks With Heterogeneous Secondary Users and Retrials

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ABSTRACT In a cognitive radio (CR) system, excessive access services for secondary users (SUs) lead to a substantial increase in congestion and the retrial phenomenon, both of which degrade the performance of CR networks, especially in overload conditions. This paper investigates the price-based spectrum access control policy that characterizes the network operator's provision to heterogeneous and delay-sensitive SUs through pricing strategies. Based on shared-use dynamic spectrum access (DSA), the SUs can occupy the dedicated spectrum without degrading the operations of primary users (PUs). The service to transmission of SUs can be interrupted by an arriving PU, while the interrupted SUs join a retrial pool called an orbit, later trying to use the spectrum to complete the service. In the retrial orbit, the interrupted SU competes fairly with other SUs in the orbit. Such a DSA mechanism is formulated as a retrial queue with service interruptions and general service times. Regarding the heterogeneity of delay-sensitive SUs, we consider two cases: the delay-sensitive parameter follows a discrete distribution and a continuous distribution, respectively. In equilibrium, we find that the revenue-optimal price is unique, while there may exist a continuum of equilibria for the socially optimal price. In addition, the socially optimal price is always not greater than the revenue-optimal price, and thus the socially optimal arrival rate is not less than the revenue-optimal one, which is contrary with the conclusion, i.e., the socially optimal and revenue-optimal arrival rates are consistent, drawn in the literature for homogeneous SUs. Finally, we present numerical examples to show the effect of various parameters on the operator's pricing strategies and SUs' behavior.

INDEX TERMS Cognitive radio network, dynamic spectrum access, retrial queue, optimal pricing, strategic behavior.

I. INTRODUCTION

Usage of the radiofrequency spectrum ranging from 3 KHz to 300 GHz is controlled by government agencies worldwide and certain sections of the spectrum are assigned to certain services or to legal licence holders. Most of the spectrum is operated based on on-and-off services. According to the Federal Communications Commission (FCC), the utilization rate of a majority of the assigned spectrum is rather low [1], [2]. Considering the scarcity of spectrum and the growing demand for wireless communication, the need for the coexisting heterogeneous wireless technologies is increasing. Since the cognitive radio (CR) network can adapt the transmitter parameters, such as transmission power and frequency band, to its operating environment, it facilitates the efficient use of the spectrum [3]. In CR networks, dynamic spectrum access (DSA) enables secondary users (SUs) to flexibly utilize dedicated spectrum which is used sporadically by primary users (PUs). That is, CR networks have the adaptability and capability to use the wireless spectrum opportunistically. Moreover, the potential of CR networks has been identified by research [4], standardization [5], [6], and policy [1], [7].

Among various DSA protocols in CR networks [8], [9], the opportunistic shared-use model, in which SUs can obtain

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TABLE 1. Comparison of literature on the pricing analysis with equilibrium strategies.

Contributions	[10]	[11]	[12]	[13]- [14]	[22]	[15]	[16]	[17]- [18]	This paper
Service discipline for SUs	FCFS	FCFS	FCFS	FCFS	FCFS	Retrial	Retrial	Retrial	Retrial
Number of channels	Single	Multiple	Single	Single	Single	Single	Single	Single	Single
General transmission time for PUs and SUs	–	–	–	–	√	–	√	–	√
SUs' heterogeneity	–	–	–	–	√	–	–	–	√
Revenue-optimal pricing strategy	–	√	–	–	√	–	–	–	√
Socially optimal pricing strategy	√	–	√	–	√	√	√	√	√
Multiple PUs	√	–	–	√	√	–	–	–	√
The position of interrupted SU	–	–	–	–	–	Stay in the service area	Stay in the service area	Retry again	Retry again

services from the spectrum leased by PUs without interfering with the operation of PUs, has been widely studied. In such an opportunistic shared-use model, SUs adopt decentralized access strategies. Excessive access requests of SUs lead to a substantial increase in congestion and the resulting retrial phenomenon degrades the performance of CR networks, especially in overload conditions. To alleviate congestion, an appropriate spectrum access control policy should be put forward. Thus, in this paper, we propose the price-based spectrum access control policy that characterizes the network operator's provision to heterogeneous and delay-sensitive SUs through pricing strategies adopting the opportunistic shared-use DSA. In this CR network, delay-sensitive SUs share a single PU band after paying an admission fee. Based on the utility function, each SU acts as a player making its spectrum access decision to maximize its own utility, which can be formulated as a non-cooperative game. From the perspective of the goal of optimization, the network operator can be a commercial planner or a social planner. The commercial planner's aim is to maximize its own revenue by employing a revenue-optimal price. For the social planner, a socially optimal price is proposed to maximize the social welfare of the network. In other words, the operator and SUs form a Stackelberg game, in which the operator is the leader and SUs are the followers.

Initially, to deal with the interaction between PUs and SUs, the exponential transmission times for PUs and SUs with service disruptions are employed to model the SUs' opportunistic accessing of the spectrum, where the service discipline is first-come-first-served (FCFS) [10]–[14]. However, this service rule may not be suitable in wireless systems with random access. Instead, to improve the throughput for SUs and minimize the probability of collision with packets being transmitted by SUs, the IEEE 802.11 protocol allows a back-off policy in CR systems; arriving SUs that find the PU band unavailable join a virtual waiting space and try to access the spectrum after a random amount of time (back-off time). In this sense, the retrial queueing models are more appropriate to characterize the back-off process of SUs when involved in a transmission collision. Wang and Li [15] used the M/M/1 retrial queue to explore the equilibrium joining strategies and socially optimal strategies of SUs with random access. To induce individually optimizing SUs to behave in a more socially optimal way, an appropriate admission fee is proposed to impose on SUs that join the orbit. Later, [16] extended the research to the general case where the service times follow general distributions, and different information situations were analyzed. Both of these work in [15] and [16] assumed the service of the interrupted SU can be cumulative and the interrupted SU has a higher priority than other SUs. But this assumption is not reasonable since the interrupted SU is designated a priority without any pay. Thus, with or without perfect sensing, Wang *et al.* [17], [18] relaxed such an assumption and they assumed the interrupted SU enters a retrial orbit to compete with other SUs fairly. In these two papers, under the assumption that SUs are homogeneous and the transmission times for the PU and SUs follow Exponential distributions, a pricing strategy maximizing the social welfare was proposed. When analyzing under perfect sensing [17], it is concluded by numerical examples that the profit-maximizing joining probability is in consistence with the socially optimal one. However, considering that each arriving SU carries a distinct job, their sensitivity to delay may differ. For example, some application types are insensitive to the delay and are willing to wait if they can eventually be served. However, many multimedia applications contain stringent delay requirements which cannot wait in reality. So in the present paper, we discuss a general case and characterize SUs' heterogeneity by the delay-sensitive parameter. Specifically, assuming the transmission times for PUs and SUs can be arbitrarily distributed, two situations, when the delay-sensitivity parameter follows a discrete or a continuous distribution, respectively, are investigated under perfect sensing. In addition, different from [15]–[18] which assumed that the PU band is licensed to only one PU, we allow for multiple PUs sharing the band. Table 1 summarizes these literatures on pricing analysis with equilibrium strategies.

To summarize, the contributions of this paper are three-fold. (1) **Model**: A novel CRN model, i.e, applying the M/G/1 retrial queue with service interruption in CR networks, is proposed. We characterize SUs' heterogeneity by the delay-sensitive parameter, which complements the equilibrium analysis for SUs with heterogeneous delay-sensitivity in [19]–[23] without a retrial mechanism. In our model, the interrupted SUs join a retrial orbit and retry independently for later service, in contrast to the assumptions in previous studies that they are lost or waiting in front of the server. In addition, relaxing the restriction identified in [17] that only one PU is licensed to access the band and consequently no PU ever waits in queue, we allow for multiple PUs that are transmitted by the FCFS discipline. (2) **Methodology**: When relaxing the assumption that SUs' and PUs' transmission times follow Exponential distributions, the Markovian property does not hold, and consequently, we adopt the method of supplementary variable to create a Markovian Process. The stability condition for such an M/G/1 retrial queue with service interruption is given. Several performance measures such as the steady state probabilities and SUs' mean waiting time are derived. Further, we extensively investigate the Stackelberg game between the operator (a commercial planner or a social planner) and SUs when the delay-sensitive parameter follows an arbitrary discrete or continuous distribution. Specially, the socially optimal price and the revenue-optimal price are both derived based on the game-theoretic analysis of the underlying price-based spectrum access control with strategic SUs. (3) **Managerial insights**: We show that the revenue-optimal price is unique, while there may exist a continuum of equilibria in the case of social welfare maximization. By comparing the revenue-optimal and socially pricing strategies under the discrete and continuous distributions of the delay-sensitive parameter, it is found the revenue-optimal price is not less than the socially optimal one. Furthermore, we carefully check the conclusion that the profit-maximizing joining probability coincides with the socially optimal one drawn in Wang *et al.* [17] for the homogeneous SUs, and find that this conclusion no longer holds when the delay-sensitivity of SUs is heterogeneous; that is, the profit-maximizing joining probability is not greater than the socially optimal one in the case with heterogeneous SUs.

The rest of this paper is organized as follows. The description of the system model and related queueing characteristics are given in Section II. To model the heterogeneity and delay-sensitivity of SUs, two scenarios, namely, the discrete distribution and continuous distribution, are considered in Sections III and IV, respectively. In each section, we study the operator's optimal pricing strategies and the corresponding equilibrium arrival rate of SUs. We find that the revenue-optimal price is not less than the socially optimal price, resulting in the revenue-optimal arrival rate being not greater than the socially optimal arrival rate. In Section V, numerical examples are presented to investigate the effects of various parameters on the operator and SUs. Finally, conclusions and some future research directions are given in Section VI.

II. MODEL

We consider a wireless network operator owning a single PU band that is licensed to legal PUs and shared by multiple SUs opportunistically, where PUs and SUs generate a Poisson arrival stream of demands with rates α and Λ, respectively. Arriving SUs (i.e., SU demands) can occupy the PU band without interfering with PUs after paying an admission fee. The arrived SU demands will lose immediately whenever the PU band is being used by another PU demand. Moreover, the service of the SU can be interrupted by arriving PUs (i.e., PU demands); that is, PUs have a higher priority than SUs. Upon the arrival of a PU demand, if the band is occupied by an SU demand, this SU demand will be squeezed out by the PU. Assume SUs have perfect detecting of PUs after joining the system and when being transmitted through the PU band, which is common in literature, such as [10]–[17], and we will try to relax this assumption in the future research. Furthermore, the service times have general distribution functions $G(x)$ with the Laplace-Stieltjes transform $g^*(s) = \int_0^\infty e^{-sy}dG(x)$ and the finite first two moments β_1, β_2 for PUs, and $F(x)$ with the Laplace-Stieltjes transform $f^*(s) = \int_0^\infty e^{-sx}dF(x)$ and the finite first two moments μ_1, μ_2 for SUs. We define $\mu(x)$ and $\beta(x)$ as the service completion rates: $\mu(x) = \frac{f(x)}{1-F(x)}$, $\beta(x) = \frac{g(x)}{1-G(x)}$, where $f(x)$ and $g(x)$ are the p.d.f of the service times for SUs and PUs. In many situations, the assumption of Poisson arrival and general transmission time holds. For example, packets in a data network arrive according to a Poisson process, while the packet length can be arbitrarily distributed. Readers can see more examples and explanations in Huang *et al.* [24] and the references therein.

If the PU band is available upon an SU's arrival, the SU can use the PU band immediately. Otherwise, it will join a retrial pool called an orbit to try its luck after a random amount of time, following a Poisson process with intensity θ. In addition, the SU that is in service and squeezed by PUs will join the retrial orbit to try again for service, and the service time for the SU is noncumulative. Since SUs finding the band unavailable try to access it sometime later independently, the retrial rate of the orbit is proportional to the number of SUs in the orbit. A PU can always occupy the band if it is idle or being used by an SU.

Based on the above definitions, the transmission collision of PUs can be modeled as an M/G/1 queue, whereas the transmission collision of SUs is characterized by an M/G/1 retrial queue with service interruptions. If we denote the distribution function and density function of busy period for PUs, which is defined as the period from the epoch a PU arrives at the band to the nearest epoch the band becomes idle again, by $B(\cdot)$ and $b(\cdot)$ respectively, then the Laplace transform of $B(\cdot)$ can be expressed as $b^*(s) = g^*[s + \alpha - \alpha b^*(s)]$ with the first two moments $\gamma_1 = \frac{\beta_1}{1-\alpha\beta_1}$, $\gamma_2 = \frac{\beta_2}{(1-\alpha\beta_1)^3}$ (see [25, Sec. 5.1.6] for more detail). Present the state of the system at time t by the pair $(I(t), N(t))$, where $N(t)$ and $I(t)$ denote the number of SUs in the orbit and the state of the PU band (0: idle; 1: serving SUs; 2: serving PUs). At time t, if $I(t) = 1$, we define $X(t)$ as the elapsed service time of the SU under service. If $I(t) = 2$, we define $Y(t)$ as the elapsed time of busy period for PUs and $\gamma(y) = \frac{b(y)}{1-B(y)}$. It then follows that the stochastic process $\{(I(t), N(t), X(t), Y(t)) : t \geq 0\}$ is Markovian.

Each SU is assumed to receive a reward of V utility units after being served and paying a service fee P. A waiting cost C per time unit exists when an SU remains in the

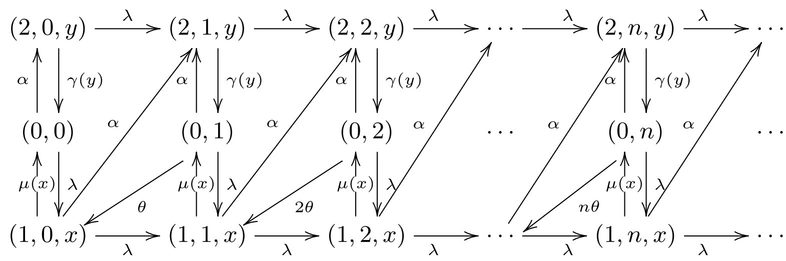

FIGURE 1. Transition rate diagram in the cognitive radio system.

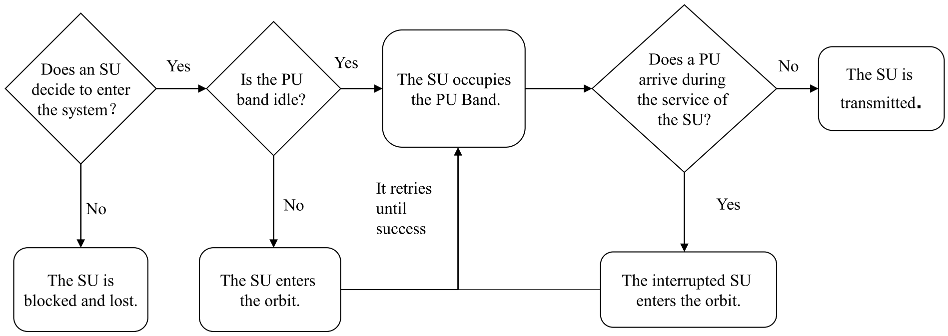

FIGURE 2. The network topology for the cognitive radio system.

TABLE 2. Comparisons between PUs and SUs.

Characteristics	PU	SU
Arrival process	Poisson process with rate α	Poisson process with rate Λ
Transmission time	Generally distributed	Generally distributed
Priority	Preemptive priority	No
Model for the transmission collision	M/G/1 queue	M/G/1 retrial queue with service interruption
Decision	All PUs join	Join or balk according to the expected utility

system, and SUs differ on the delay-sensitivities parameter. We consider both the discrete and continuous distributions of C in the following sections. Before joining the system, SUs have no information (i.e., the band status and the number of SUs in the retrial orbit, or the unobservable case as in [12]) about the system. Each SU can infer the long-term waiting time statistically by existing method [29] and respond to the expected utility by deciding whether to use the PU band or not. Assume that SUs are risk neutral and their decisions are irrevocable. In other words, neither the retrial of balking SUs nor the reneging of entering SUs is allowed. Recalling that PUs have permanent licenses to access the band and their services are not affected by the presence of SUs, the demand rate of PUs is constant, equal to α. Define the effective arrival rate of SUs by λ ($\lambda \leq \Lambda$), then the corresponding transition rate diagram and network topology are shown in Figures 1-2. All the differences between PUs and SUs are listed in Table 2. In addition, the main notations are summarized in Table 3.

TABLE 3. Notations.

Notations	Explanations
V	Service reward
C	Waiting cost per time unit
Λ	Potential arrival rate of SUs
α	Potential arrival rate of PUs
μ_1	First moment of the transmission time for SUs
μ_2	Second moment of the transmission time for SUs
β_1	First moment of the transmission time for PUs
β_2	Second moment of the transmission time for PUs
θ	Retrial rate of SUs
γ_1	First moment of the busy period for PUs
γ_2	Second moment of the busy period for PUs

In Figure 1, the state $(0, i)$ ($i \in \{0, 1, 2, \ldots\}$) means the PU band is idle and there are i SUs waiting in the retrial orbit; the state $(1, i, x)$ ($i \in \{0, 1, 2, \ldots\}$) represents an SU is being transmitted through the PU band with elapsed service time x and the number of SUs in the retrial orbit is i; and the state $(2, i, y)$ ($i \in \{0, 1, 2, \ldots\}$) denotes the band is occupied by PUs with elapsed time y of the busy period for PUs and there are i SUs in the retrial orbit. From state $(0, i)$, it may transform to state $(2, i, y)$ with rate α if a PU arrives at the band, or to state $(1, i-1, x)$ with rate $i\theta$ if an SU in the orbit retries successfully and thus the band is occupied by this SU with the number of SUs in the orbit reduced by one, or to state $(1, i, x)$ with rate λ if an SU arrives from outside and then the band is occupied by this new arriving SU. In a similar way, from state $(1, i, x)$, it may transform to state $(1, i+1, x)$ with rate λ if an SU arrives from outside and thus the number of SUs in the orbit increases by one, or to state $(2, i+1, y)$ with rate α if a PU arrives at the band and the interrupted SU enters the retrial orbit, or to state $(0, i)$ with rate $\mu(x)$ if the SU being served with the elapsed service time x is transmitted successfully and then the PU band becomes idle. Further, from state $(2, i, y)$, it may transform to state $(2, i+1, y)$ with rate λ if an SU arrives from outside and thus the number of SUs in the orbit increases by one, or to state $(0, i)$ with rate $\gamma(y)$ if PUs in the system with the elapsed time y of the busy period are transmitted successfully and consequently, the PU band becomes idle.

Remark 1: Note that when both the SU's service time and PUs' busy period follow Exponential distributions and the delay-sensitivity of SUs is homogeneous, our model degenerates to the one discussed in Wang *et al.* [17].

A. STEADY STATE SOLUTIONS

To investigate the steady state solution for such a system, we first give the stable condition below.

Assume an SU begins to be served at some epoch t_0, and a service cycle is defined as the length of time from the point t_0 to the nearest point $t_0+\tau$ when either an SU is transmitted successfully or the PU band is empty after the transmission of the PU which arrives during the SU is being transmitted (the preempted SU enters the orbit). Let ν_1 be the number of new SUs (arriving from outside) entering the system during the service cycle, and ν_2 be the number of old SUs (interrupted by PUs) entering the orbit during the service cycle (i.e., $\nu_2 = 0$ or 1).

In queueing terminology, for an SU under service, PUs' arrival can be regarded as a failure of the server from the perspective of SUs. The mean number of new SUs and old SUs entering the system in our model is equal to that in Falin [26]. The only difference is that when the band is busy (occupied by PUs or SUs), a new arriving SU in our model joins the retrial orbit, while the new one in Falin [26] joins a queue. Thus, we have $E(\nu_1) = \lambda(1-f^*(\alpha))(\gamma_1+\frac{1}{\alpha})$ (equal to $E\nu$ in the model of Falin [26]) and $E(\nu_2) = 1-f^*(\alpha)$ (equal to $E\mu$ in the model of Falin [26]).

We consider the time sequence t_k $(k = 1, 2, \ldots)$ when the process $(I(t), N(t))$ moves into the states $(0, i)$, $i \geq 0$. It means at time t_k $(k \geq 1)$ the PU band completes the service of PUs or an SU. Define N_j as the number of SUs in the orbit at time t_j and x_i as the mean drift from time t_j to t_{j+1}, that is, $x_i \equiv E(N_{j+1} - N_j \mid N_j = i)$.

Theorem 1: The Markov process is ergodic iff the condition $\lambda < \frac{\alpha f^*(\alpha)}{(1-f^*(\alpha))(1+\alpha\gamma_1)}$ is satisfied.

Proof: By the definition of x_i, i.e., $x_i \equiv E(N_{j+1} - N_j \mid N_j = i)$, we have

$$x_i = \frac{(\lambda+i\theta)(E(\nu_1)+E(\nu_2))}{\lambda+\alpha+i\theta} + \frac{\lambda\gamma_1\alpha - i\theta}{\lambda+\alpha+i\theta}. \quad (1)$$

As $i \to \infty$, we obtain

$$x = \lim_{i\to\infty} x_i = \lambda(1-f^*(\alpha))(\gamma_1+\frac{1}{\alpha}) - f^*(\alpha).$$

We first prove the Markov process is ergodic if $\lambda < \frac{\alpha f^*(\alpha)}{(1-f^*(\alpha))(1+\alpha\gamma_1)}$. It is readily to see that the Markov chain is irreducible and aperiodic. It remains to be proved that it is positive recurrent. We use [27, Th. 2] which states that an irreducible and aperiodic Markov chain is positive recurrent if $|x_i| < \infty$ for all i and $\lim_{i\to\infty}\sup x_i < 0$. Obviously, in our model, if $\lambda < \frac{\alpha f^*(\alpha)}{(1-f^*(\alpha))(1+\alpha\gamma_1)}$, these two inequalities hold.

Further, with regard to the necessity of this condition, according to the ergodic theorem of Markov chain which states all the stationary probabilities of an ergodic Markov chain are positive, we have the probability that the band is idle, i.e., P_0 (defined in Theorem 2), should be positive at steady state. It then follows that $\lambda < \frac{\alpha f^*(\alpha)}{(1-f^*(\alpha))(1+\alpha\gamma_1)}$ when the Markov process is ergodic.

Thus, the sufficient and necessary condition for the ergodicity of the Markov chain is $\lambda < \frac{\alpha f^*(\alpha)}{(1-f^*(\alpha))(1+\alpha\gamma_1)}$. □

Define $P_{0,i}(t)$, $P_{1,i}(t,x)dx$, $P_{2,i}(t,y)dy$ as the probabilities that the PU band is unoccupied, occupied by an SU with the elapsed service time between x and $x+dx$, occupied by the PU with the elapsed time of busy period for the PUs between y and $y+dy$ and the number of SUs in the orbit is i at time t, respectively. Let $M = \frac{\alpha f^*(\alpha)}{(1-f^*(\alpha))(1+\alpha\gamma_1)}$. Assume the stable condition $\lambda < M$ is fulfilled, and set $P_{0,i} = \lim_{t\to\infty} P_{0,i}(t)$ for $i \geq 0$; $P_{1,i}(x) = \lim_{t\to\infty} P_{1,i}(t,x)$ for $i \geq 0$ and $x \geq 0$; $P_{2,i}(y) = \lim_{t\to\infty} P_{2,i}(t,y)$ for $i \geq 0$ and $y \geq 0$.

From the transition rate diagram depicted before, we obtain the following equations at steady state

$$(\lambda+i\theta+\alpha)P_{0,i} = \int_0^\infty P_{1,i}(x)\mu(x)dx + \int_0^\infty P_{2,i}(y)\gamma(y)dy, \quad i \geq 0, \quad (2)$$

$$(\frac{d}{dx}+\mu(x)+\lambda+\alpha)P_{1,i}(x) = \lambda P_{1,i-1}(x), \quad i \geq 0, \quad (3)$$

$$(\frac{d}{dy}+\lambda+\gamma(y))P_{2,i}(y) = \lambda P_{2,i-1}(y), \quad i \geq 0, \quad (4)$$

together with the boundary conditions

$$P_{1,i}(0) = \lambda P_{0,i} + (i+1)\theta P_{0,i+1}, \quad i \geq 0, \quad (5)$$

$$P_{2,i}(0) = \alpha P_{0,i} + \alpha\int_0^\infty P_{1,i-1}(x)dx, \quad i \geq 0, \quad (6)$$

and the normalization equation

$$\sum_{i=0}^{\infty}\{P_{0,i} + \int_0^\infty P_{1,i}(x)dx + \int_0^\infty P_{2,i}(y)dy\} = 1, \quad (7)$$

where $P_{1,-1}(x) = 0$, $P_{2,-1}(y) = 0$.

To solve the above equations, we firstly define the following probability generating functions:

$$Q_0(z) = \sum_{i=0}^{\infty} P_{0,i} z^i,$$
$$Q_j(z,x) = \sum_{i=0}^{\infty} P_{j,i}(x) z^i, \quad j = 1, 2.$$

Multiplying both sides of equations (2)-(6) by z^i and summing over i, we have

$$\lambda Q_0(z) + \alpha Q_0(z) + z\theta Q_0'(z) = \int_0^\infty Q_1(z,x)\mu(x)dx + \int_0^\infty Q_2(z,y)\gamma(y)dy, \tag{8}$$
$$(\frac{\partial}{\partial x} + \mu(x) + \lambda + \alpha)Q_1(z,x) = z\lambda Q_1(z,x), \tag{9}$$
$$(\frac{\partial}{\partial y} + \lambda + \gamma(y))Q_2(z,y) = \lambda z Q_2(z,y), \tag{10}$$
$$Q_1(z,0) = \lambda Q_0(z) + \theta Q_0'(z), \tag{11}$$
$$Q_2(z,0) = \alpha Q_0(z) + \alpha z \int_0^\infty Q_1(z,x)dx. \tag{12}$$

By (11), we derive the solution to equation (9) as follows

$$Q_1(z,x) = (\lambda Q_0(z) + \theta Q_0'(z)) e^{(-\lambda(1-z)-\alpha)x} \overline{F}(x). \tag{13}$$

Similarly, by plugging equation (12) into (10), it follows that

$$Q_2(z,y) = \alpha Q_0(z) e^{-\lambda(1-z)y} \overline{B}(y) + \alpha z(\lambda Q_0(z) + \theta Q_0'(z)) \frac{1 - f^*(\lambda+\alpha-\lambda z)}{\lambda+\alpha-\lambda z} e^{-\lambda(1-z)y} \overline{B}(y). \tag{14}$$

Substituting equations (13)-(14) into (8) yields

$$Q_0(z) = c \cdot \exp\Big\{ -\int_z^1 \{\lambda + \alpha - \alpha b^*(\lambda - \lambda u) - \lambda\alpha u \frac{1 - f^*(\lambda+\alpha-\lambda u)}{\lambda+\alpha-\lambda u} \times b^*(\lambda-\lambda u) - \lambda f^*(\lambda+\alpha-\lambda u)\}/\{\theta \times f^*(\lambda+\alpha-\lambda u) + \alpha u\theta \frac{1 - f^*(\lambda+\alpha-\lambda u)}{\lambda+\alpha-\lambda u} \times b^*(\lambda-\lambda u) - u\theta\} du \Big\}, \tag{15}$$

where c is a constant.

Let $z \to 1$, then solving equations (13)-(15) subject to $Q_0(1) + \int_0^\infty Q_1(1,x)dx + \int_0^\infty Q_2(1,y)dy = 1$ gives that

$$c = \frac{1}{1+\alpha\gamma_1} + \frac{\lambda}{\alpha} - \frac{\lambda}{\alpha f^*(\alpha)}.$$

Since

$$Q_1(z) = \int_0^\infty Q_1(z,x)dx = (\lambda + \theta\frac{A_1}{A_2}) Q_0(z) \frac{1 - f^*(\lambda+\alpha-\lambda z)}{\lambda+\alpha-\lambda z}, \tag{16}$$

$$Q_2(z) = \int_0^\infty Q_2(z,y)dy = \alpha Q_0(z) \frac{1 - b^*(\lambda-\lambda z)}{\lambda-\lambda z} + \alpha z Q_1(z) \frac{1 - b^*(\lambda-\lambda z)}{\lambda-\lambda z}, \tag{17}$$

where

$$A_1 = \lambda + \alpha - \alpha b^*(\lambda-\lambda z) - \lambda\alpha z \frac{1 - f^*(\lambda+\alpha-\lambda z)}{\lambda+\alpha-\lambda z} \times b^*(\lambda-\lambda z) - \lambda f^*(\lambda+\alpha-\lambda z), \tag{18}$$
$$A_2 = \theta f^*(\lambda+\alpha-\lambda z) + \alpha z\theta \frac{1 - f^*(\lambda+\alpha-\lambda z)}{\lambda+\alpha-\lambda z} \times b^*(\lambda-\lambda z) - z\theta, \tag{19}$$

we obtain the probabilities that the band is under different states by inserting the value $z = 1$. Consequently, the probabilities that the band is idle, serving SUs or serving PUs are $P_0 = \frac{1}{1+\alpha\gamma_1} + \frac{\lambda}{\alpha} - \frac{\lambda}{\alpha f^*(\alpha)}$, $P_1 = \frac{\lambda(1-f^*(\alpha))}{\alpha f^*(\alpha)}$ and $P_2 = \frac{\alpha\gamma_1}{1+\alpha\gamma_1}$.

On the other hand, denote by $K(t)$ the number of SUs in the system at time t, then we have the generating function of $K(t)$, which is given by $Q(z) = Q_0(z) + zQ_1(z) + Q_2(z)$. Using Little's Law $E[W(\lambda)] = \frac{E[K]}{\lambda} = \frac{Q'(1)}{\lambda}$, the SU's mean waiting time in the system denoted by $E[W(\lambda)]$ can be derived as follows

$$E[W(\lambda)] = \{2(1+\alpha\gamma_1)^2(\alpha+\theta)(\alpha+\lambda)f^*(\alpha) - (\alpha(2\alpha(1+\alpha\gamma_1) + (2(1+\alpha\gamma_1)^2 - \alpha^2\gamma_2)\theta) + 2(1+\alpha\gamma_1)^2(\alpha+\theta)\lambda)(f^*(\alpha))^2 + 2\alpha(1+\alpha\gamma_1)^2\theta\lambda(f^*)'(\alpha)\}/\{2\alpha(1+\alpha\gamma_1) \cdot \theta f^*(\alpha)(-(1+\alpha\gamma_1)\lambda + (\alpha + \lambda + \alpha\gamma_1\lambda)f^*(\alpha))\}. \tag{20}$$

We summarize the above results as the following theorem.

Theorem 2: For the considered cognitive radio system, in steady state, we have:

(i) The probabilities that the PU band is idle, serving SUs or serving PUs are $P_0 = \frac{1}{1+\alpha\gamma_1} + \frac{\lambda}{\alpha} - \frac{\lambda}{\alpha f^*(\alpha)}$, $P_1 = \frac{\lambda(1-f^*(\alpha))}{\alpha f^*(\alpha)}$ and $P_2 = \frac{\alpha\gamma_1}{1+\alpha\gamma_1}$.

(ii) An SU's expected waiting time in the system is

$$E[W(\lambda)] = \{2(1+\alpha\gamma_1)^2(\alpha+\theta)(\alpha+\lambda)f^*(\alpha) - (\alpha(2\alpha(1+\alpha\gamma_1) + (2(1+\alpha\gamma_1)^2 - \alpha^2\gamma_2)\theta) + 2(1+\alpha\gamma_1)^2(\alpha+\theta)\lambda)(f^*(\alpha))^2 + 2\alpha(1+\alpha\gamma_1)^2\theta\lambda(f^*)'(\alpha)\}/\{2\alpha(1+\alpha\gamma_1) \cdot \theta f^*(\alpha)(-(1+\alpha\gamma_1)\lambda + (\alpha + \lambda + \alpha\gamma_1\lambda)f^*(\alpha))\}.$$

III. SCENARIO-I: DISCRETE DISTRIBUTION

Recall that a service fee P is collected at SUs' arrival. In a discrete distribution CR system, the operator of the PU band is a monopoly that knows how the price affects SUs' behavior and moves first to select the price to charge. Seeing the price posted by the operator and knowing all the system parameters, SUs can infer the long-term waiting time statistic by existing method [29] and respond to the expected utility. Thus, in the game theory context, all the potential SUs can be seen as players in a simultaneous move and non-cooperative game, where their strategies are joining or balking and their aim is to maximize their own profit. In SUs' strategies, we try to look for a Nash equilibrium. Let $U(s_{tagged}, s_{others})$ be the utility of a tagged SU who adopts strategy s_{tagged}, when others follow strategy s_{others}. For the later statement to be clearer, the definition of Nash equilibrium is given here.

Definition 3 [30]: In the n-player normal game, the strategy $(s_1, s_2, \cdots, s_n)$ is a Nash equilibrun if for each player j, s_j is (at least tied for) player j's best response to the strategies specified for the $n-1$ other players, $(s_1, \cdots, s_{j-1}, s_{j+1}, \cdots, s_n)$.

That is, a strategy s_e is a Nash equilibrium if it is the best response against itself, i.e., $U(s_e, s_e) > U(s, s_e)$ for every $s \in \phi$ (ϕ is the set of SU available actions). Suppose there are multiple SU types (denote the number of SU types by N) with delay-sensitive parameters C_i, $i \in \{1, 2, \ldots, N\}$ and $C_i < C_j$ if $i < j$. The corresponding probability is p_i ($i \in \{1, 2, \ldots, N\}$) and $\sum_{i=1}^{N} p_i = 1$. Then the potential arrival rate of C_i-SUs ($i \in \{1, 2, \ldots, N\}$) is $\Lambda_i = \Lambda p_i$. If we denote the C_i-SUs' effective arrival rate by λ_i, the effective arrival rate to the network is $\lambda = \sum_{i=1}^{N} \lambda_i$. Recall that an SU receives a reward V after paying the service fee P and being transmitted. Thus, the expected utility of a C_i-SU is defined as the benefit of joining the system minus the cost including the service fee and expected waiting cost:

$$U_i(\lambda) = V - P - C_i E[W(\lambda)].$$

It can be concluded that $\lambda_i = \Lambda_i$ if $\lambda_j > 0$ ($j > i$) because of $C_i < C_j$ for $j > i$. That means for $j > i$, if some C_j-SUs join the system, C_i-SUs must all join. In addition, to rule out the case where no SU would enter the system even if the PU band is free and available, we assume that $V - C_1\, E[W(0)] > 0$ holds.

A. SUS' STRATEGIES

Denote the equilibrium arrival rate of the network by λ_e^d, then we have

$$\lambda_1 = \min\{\Lambda_1, \lambda_e^d\},$$

$$\lambda_i = \min\{\Lambda_i, (\lambda_e^d - \sum_{j=1}^{i-1} \Lambda_j)^+\}, \quad i \in \{2, 3, \ldots, N-1\},$$

$$\lambda_N = (\lambda_e^d - \sum_{j=1}^{N-1} \Lambda_j)^+.$$

To derive the equilibrium solutions, we first investigate the monotonicity of $E[W(\lambda)]$ with respect to λ below.

Lemma 4: In such a cognitive radio network system, a joining SU's mean waiting time in the system is monotonously increasing in λ.

Proof: Since the stable condition for such a system is $\lambda < M$, that implies

$$\lim_{\lambda \to M^-} E[W(\lambda)] = +\infty.$$

In addition, we derive that the denominator of $E[W(\lambda)]$ expressed in equation (20) is positive under the steady-state condition, and it approaches to zero when $\lambda \to M^-$. So the sign of the numerator of $E[W(\lambda)]$ should be positive as $\lambda \to M^-$.

As $\lambda \to M^-$, the numerator of $E[W(\lambda)]$ is $\frac{\alpha f^*(\alpha) M_1}{f^*(\alpha)-1}$, where

$$M_1 = [-1 + f^*(\alpha)]\{2(1+\alpha\gamma_1)^2(\alpha+\theta) + \alpha[-2\gamma_1(1 + \alpha\gamma_1) + \alpha\gamma_2]\theta f^*(\alpha)\} - 2\alpha(1+\alpha\gamma_1)\theta (f^*)'(\alpha), \tag{21}$$

and $0 < f^*(\alpha) < 1$. It then follows that $M_1 < 0$.

Furthermore, differentiating $E[W(\lambda)]$ with respect to λ leads to

$$\begin{aligned}\frac{dE[W(\lambda)]}{d\lambda} &= -M_1/\{2\theta[-\lambda - \alpha\gamma_1\lambda + \alpha f^*(\alpha) + \lambda f^*(\alpha) \\ &\quad + \alpha\gamma_1\lambda f^*(\alpha)]^2\} \\ &> 0, \end{aligned} \tag{22}$$

which indicates $E[W(\lambda)]$ is increasing in λ. □

Following the monotonicity of $E[W(\lambda)]$, we can now establish the uniqueness and existence of SUs' equilibrium strategies.

Theorem 5: In such a cognitive radio network system where there are N types of SUs with delay sensitive parameters C_i ($i \in \{1, 2, \ldots, N\}$) and $C_i < C_j$ if $i < j$, there exists a unique equilibrium arrival rate λ_e^d such that:

(1) If $V - P \le C_1 E[W(0)]$, then $\lambda_e^d = 0$.
(2) If $C_1 E[W(0)] < V - P < C_1 E[W(\Lambda_1)]$, then λ_e^d is the unique solution to $V = P + C_1 E[W(\lambda)]$ for $\lambda \in (0, \Lambda_1)$.
(3) If $C_1 E[W(\Lambda_1)] \le V - P \le C_2 E[W(\Lambda_1)]$, then $\lambda_e^d = \Lambda_1$.
(4) If $C_i E[W(\sum_{j=1}^{i-1} \Lambda_j)] \le V - P \le C_i E[W(\sum_{j=1}^{i} \Lambda_j)]$, then λ_e^d is the unique solution to $V = P + C_i E[W(\lambda)]$ for $\lambda \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j)$, $i \in \{2, \ldots, N\}$.
(5) If $C_i E[W(\sum_{j=1}^{i} \Lambda_j)] \le V - P \le C_{i+1} E[W(\sum_{j=1}^{i} \Lambda_j)]$, then $\lambda_e^d = \sum_{j=1}^{i} \Lambda_j$ for $i \in \{2, \ldots, N-1\}$.
(6) If $C_N E[W(\sum_{j=1}^{N} \Lambda_j)] \le V - P$, then $\lambda_e^d = \sum_{j=1}^{N} \Lambda_j$.

Proof: The increasing property of $E[W(\lambda)]$ in λ gives that the expected utility of C_j-SUs, i.e., $U_j(\lambda) = V - P - C_j E[W(\lambda)]$, decreases with respect to λ.

For C_1-SUs, if $U_1(0) \le 0$, the best response of SUs against $\lambda = 0$ is to balk. So $\lambda = 0$ is the best response against itself, that is, $\lambda_e^d = 0$ is the unique equilibrium strategy.

If $U_1(\Lambda_1) < 0 < U_1(0)$, because of the continuity and monotonicity of $U_1(\lambda)$, there exists a unique solution λ_e^* to the equation $U_1(\lambda) = 0$ in the interval $\lambda \in (0, \Lambda_1)$. For any strategy $\lambda \in (0, \lambda_e^*)$, the best response for a tagged SU is to join, so such a strategy cannot be an equilibrium. Similarly, for any strategy $\lambda \in (\lambda_e^*, \Lambda_1)$, the best response is balking, so such a strategy cannot be an equilibrium, too. Whereas, for the strategy λ_e^*, any strategy $\lambda \in [0, \Lambda_1]$ is a best response for a tagged SU. Therefore, in this case, $\lambda_e^d = \lambda_e^*$ is the unique equilibrium strategy.

If $U_1(\Lambda_1) \geq 0$ and $U_2(\Lambda_1) \leq 0$, the best response for the tagged C_1-SU is to join.

For $i \in \{2, 3, \ldots, N\}$, recall that $C_{i-1} < C_i$ and the C_i-SUs may enter the network only when all C_{i-1}-SUs choose joining. Thus, when $U_i(\sum_{j=1}^{i-1} \Lambda_j) > 0$, in equilibrium, some or all C_i-SUs choose to join. The equilibrium analysis for C_i-SUs is similar with that for C_1-SUs, and we omit it here. □

B. OPTIMAL PRICING BY THE OPERATOR

Having derived SUs' equilibrium arrival rate, in this subsection we turn our attention to analyze the decision of the operator of the PU band, who wants to earn a profit by charging entering SUs. Based on different objectives, the operator can be viewed as a commercial planner that aims to maximize its own revenue or a social planner if its objective is to maximize the social welfare of the system.

Recalling that the operator moves first to select a price and then SUs decide whether to join the system or not after knowing the price charged, it is as if the operator and all the potential SUs are playing a two-stage Stackelberg game, where the operator is the leader and SUs are the followers. Denote the operator's revenue and the social welfare of the network per time unit by R and S, respectively. The two-stage Stackelberg game is then defined as follows:

$$\text{First stage: } \max_{P \geq 0} R(P)(\text{or } S(P)), \tag{23}$$

$$\text{Second stage: } \max_{\Lambda \geq \lambda \geq 0} U_i(\lambda) = V - P - C_i E[W(\lambda)], \quad i = 1, 2, \ldots, N. \tag{24}$$

Based on SUs' equilibrium arrival rate given in Theorem 5, we now adopt backward induction to investigate the operator's optimal pricing strategies. Since the operator's revenue is zero if no SU chooses to join (i.e., $\lambda_e^d = 0$), in what follows we only focus on the cases where $\lambda_e^d \neq 0$ fulfills.

1) REVENUE-OPTIMAL PRICING

When charging a price P, the operator's objective is to maximize its own revenue, that is, $\max_{0 \leq P \leq V} R(P) = \lambda_e^d(P)P$, where $\lambda_e^d(P)$ is SUs' equilibrium arrival rate at price P defined in Theorem 5. Observing that we cannot derive closed form solutions for λ_e^d in some cases, it is difficult to obtain the optimal price to maximize the operator's revenue. Thus, we try to simplify this problem by seeking an equivalent one.

Assume $\sum_{j=1}^{0} \cdot = 0$, then we first consider the cases when $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j)$ $(i \in \{1, 2, \ldots, N\})$ so that λ_e^d is the unique root to $U_i(\lambda) = 0$. Because of the monotonicity of function $U_i(\lambda)$, there must exist a one-to-one mapping between P and λ_e^d in this situation. Next consider the cases $\lambda_e^d = \sum_{j=1}^{i} \Lambda_j$, $i \in \{1, 2, \ldots, N\}$, where in order to reach the maximum of the revenue, the operator would increase the price to drain the C_i-SUs'. So in equilibrium we have $U_i(\lambda) = 0$, $i \in \{1, 2, \ldots, N\}$. Namely, there always exists a one-to-one mapping between P and λ_e^d, and it is equivalent to seek the optimal arrival rate.

Hence, the objective function can be rewritten as

$$\begin{aligned} \max_{\lambda_e^d} R(\lambda_e^d) &= \lambda_e^d(P)P \\ &= \lambda_e^d(V - C_i E[W(\lambda_e^d)]), \end{aligned} \tag{25}$$

if $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j]$, $i \in \{1, 2, \ldots, N\}$, and the second-order derivative of $R(\lambda_e^d)$ with respect to λ_e^d is

$$\frac{\partial^2 R(\lambda_e^d)}{\partial (\lambda_e^d)^2} = -2C_i E'[W(\lambda_e^d)] - \lambda_e^d C_i E''[W(\lambda_e^d)]. \tag{26}$$

As

$$\begin{aligned} \frac{\partial^2 E[W(\lambda_e^d)]}{\partial (\lambda_e^d)^2} &= \frac{(1+\alpha\gamma_1)(-1+f^*(\alpha))M_1}{\theta[-(1+\alpha\gamma_1)\lambda + (\alpha + \lambda + \alpha\gamma_1\lambda)f^*(\alpha)]^3} \\ &> 0, \end{aligned} \tag{27}$$

it can be obtained that $\frac{\partial^2 R(\lambda_e^d)}{\partial (\lambda_e^d)^2} < 0$; that is, $R(\lambda_e^d)$ is concave in λ_e^d. For $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j]$, $i \in \{1, 2, \ldots, N\}$, let λ_e^{Rdi} be the solution to $\frac{\partial R(\lambda_e^d)}{\partial \lambda_e^d} = 0$, if it exists.

Thus, the equilibrium arrival rate which makes the manager's revenue maximize is obtained as follows

$$\lambda_e^{Rd} = \arg\max_{\lambda_e^d \in \{\lambda_e^{Rdi}, \sum_{j=1}^{i} \Lambda_i : i \in \{1,2,\ldots,N\}\}} R(\lambda_e^d).$$

For any $i \in \{1, 2, \ldots, N\}$, if $\lambda_e^{Rd} = \lambda_e^{Rdi}$ or $\sum_{j=1}^{i} \Lambda_i$, the corresponding price is $P^{Rd} = R - C_i E[W(\lambda_e^{Rd})]$.

We summarize the above analysis in the following theorem.

Theorem 6: When the SUs' delay-sensitive parameter follows a discrete distribution, the unique revenue-optimal price is

$$P^{Rd} = R - C_i E[W(\lambda_e^{Rd})],$$

where $\lambda_e^{Rd} = \arg\max_{\lambda_e^d \in \{\lambda_e^{Rdi}, \sum_{j=1}^{i} \Lambda_i : i \in \{1,2,\ldots,N\}\}} R(\lambda_e^d)$ and λ_e^{Rdi} is the solution to equation $\frac{\partial R(\lambda_e^d)}{\partial \lambda_e^d} = 0$ for $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j]$, $i \in \{1, 2, \ldots, N\}$, if it exists.

2) SOCIALLY OPTIMAL PRICING

For a social planner, the social welfare is defined as the sum of operator and the SU surplus. The SU surplus per time unit at a given price is $SS = \sum_{i=1}^{N} \lambda_i U_i(\lambda_e^d)$. The operator's surplus is the same as the operator's revenue and so it is expressed as $OS = \lambda_e^d P$. So the social welfare function of the network per unit of time is

$$S(\lambda_e^d) = \sum_{j=1}^{i-1} \Lambda_j [V - C_j E[W(\lambda_e^d)]] + (\lambda_e^d - \sum_{j=1}^{i-1} \Lambda_j) \times [V - C_i E[W(\lambda_e^d)]], \quad (28)$$

where $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j]$, $i \in \{1, 2, \ldots, N\}$, defined in Theorem 5, is a function of price P. Note that the price charged for entering SUs is just an internal variable transferring from SUs to the operator. So for the remainder of this subsection, we turn to consider the arrival rate.

By taking the second-order derivative of $S(\lambda_e^d)$ with respect to λ_e^d, we have

$$\frac{\partial^2 S(\lambda_e^d)}{\partial (\lambda_e^d)^2} = -\sum_{j=1}^{i-1} \Lambda_j C_j E''[W(\lambda_e^d)] - 2C_i E'[W(\lambda_e^d)] - (\lambda_e^d - \sum_{j=1}^{i-1} \Lambda_j) C_i E''[W(\lambda_e^d)] < 0. \quad (29)$$

So $S(\lambda_e^d)$ is concave with respect to λ_e^d for any $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j]$, $i \in \{1, 2, \ldots, N\}$.

Similar to the case of revenue-optimal pricing, the equilibrium arrival rate which makes the social welfare maximize is given by

$$\lambda_e^{Sd} = \arg\max_{\lambda_e^d \in \{\lambda_e^{Sdi}, \sum_{j=1}^{i} \Lambda_i : i \in \{1,2,\ldots,N\}\}} S(\lambda_e^d),$$

where for $i \in \{1, 2, \ldots, N\}$, λ_e^{Sdi} is the solution to equation $\frac{\partial S(\lambda_e^d)}{\partial \lambda_e^d} = 0$ for $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j)$, if it exists. Sequently, the socially optimal pricing strategy is given in what follows.

Theorem 7: When the SUs' delay-sensitive parameter follows a discrete distribution, the socially optimal price exists: (1) If $\lambda_e^{Sd} \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j)$ ($i \in \{1, 2, \ldots, N\}$), the corresponding price is unique and given by

$$P^{Sd} = R - C_i E[W(\lambda_e^{Sd})];$$

(2) If $\lambda_e^{Sd} = \sum_{j=1}^{i} \Lambda_j$ ($i \in \{1, 2, \ldots, N\}$), any price in the following interval can be socially optimal:

$$P^{Sd} \in (\max\{R - C_{i+1} E[W(\sum_{j=1}^{i} \Lambda_j)], 0\}, R - C_i E[W(\sum_{j=1}^{i} \Lambda_j)]).$$

Having characterized the optimal pricing strategies of the operator as a commercial planner or a social planner, we now turn to compare them in the following corollary.

Corollary 8: The revenue-optimal price is not less than the socially optimal one when there are N types of SUs with different delay-sensitive parameters.

Proof: Since $E[W(\lambda)]$ is increasing in λ, the equilibrium arrival rate is non-increasing in the admission fee P. So in the following we just need to prove that the revenue-optimal arrival rate is not greater than the socially optimal one.

Differentiating $R(\lambda_e^d)$ and $S(\lambda_e^d)$ with respect to λ_e^d yield that

$$\frac{\partial R(\lambda_e^d)}{\partial \lambda_e^d} = V - C_i E[W(\lambda_e^d)] - C_1 \lambda_e^d E'[W(\lambda_e^d)],$$

and

$$\frac{\partial S(\lambda_e^d)}{\partial \lambda_e^d} = V - C_i E[W(\lambda_e^d)] - C_i \lambda_e^d E'[W(\lambda_e^d)] + \{C_i \sum_{j=1}^{i-1} \Lambda_j - \sum_{j=1}^{i-1} \Lambda_j C_j\} E'[W(\lambda_e^d)],$$

for $\lambda_e^d \in (\sum_{j=1}^{i-1} \Lambda_j, \sum_{j=1}^{i} \Lambda_j]$, $i \in \{1, 2, \ldots, N\}$. It is obvious that $\frac{\partial S(\lambda_e^d)}{\partial \lambda_e^d} \geq \frac{\partial R(\lambda_e^d)}{\partial \lambda_e^d}$ because $C_j < C_i$ if $j < i$. Thus, we divide our proof into two cases according to the different revenue-optimal arrival rates.

The first one is $\lambda_e^{Rd} = \lambda_e^{Rdi}$ ($i \in \{1, 2, \ldots, N\}$), which implies that $\frac{\partial R(\lambda_e^d)}{\partial \lambda_e^d}|_{\lambda_e^d = \lambda_e^{Rd}} = 0$. It then follows that $\frac{\partial S(\lambda_e^d)}{\partial \lambda_e^d}|_{\lambda_e^d = \lambda_e^{Rd}} \geq 0$. Thus, we can easily conclude that $\lambda_e^{Sd} \geq \lambda_e^{Rd}$ in this case.

Similarly, in the second case where the revenue-optimal arrival rate satisfies $\lambda_e^{Rd} = \sum_{j=1}^{i} \Lambda_j$ ($i \in \{1, 2, \ldots, N\}$), we have $\frac{\partial R(\lambda_e^d)}{\partial \lambda_e^d}|_{\lambda_e^d = \lambda_e^{Rd}} \geq 0$, leading to $\frac{\partial S(\lambda_e^d)}{\partial \lambda_e^d}|_{\lambda_e^d = \lambda_e^{Rd}} \geq 0$ also holds. So we come to the same conclusion. □

The conclusion in the above corollary seems natural. From the service provider's perspective, the server plays the role of a monopolist. To maximize its own profit, the provider will raise the price to extract SUs' benefit to zero. However, the social planner is indifferent to the utility of SUs and the monopolist. Thus, the price set by the monopolist is an upper bound of that chosen by the social planner.

IV. SCENARIO-II: CONTINUOUS DISTRIBUTION

Now assume that the SUs' delay-sensitive parameters are distributed on $[C_l, C_h]$ with distribution function $H(\cdot)$ and density function $h(\cdot)$. Obviously, sometimes $\lambda_e^c = 0$ or $\lambda_e^c = \Lambda$ is an equilibrium. Otherwise, there exists a parameter C_e with which marginal SUs are indifferent between joining and balking in equilibrium. In other words, in equilibrium, SUs with delay sensitive parameter $C \leq C_e$ would join the system, otherwise not. Thus, the equilibrium arrival rate of the system is $\lambda(C_e) = \Lambda H(C_e)$. In addition, $V - C_l E[W(0)] > 0$ is assumed to fulfill to ensure some SUs enter the system.

A. SUS' STRATEGIES

In virtue of the indifference of marginal SUs, we have

$$V - P = C_e E[W(\lambda(C_e))]. \quad (30)$$

According to Lemma 4, the right hand of equation (30) increases as C_e grows. It reaches its minimum at $C_e = C_l$, and maximum at $C_e = C_h$. Then the equilibrium solutions are summarized in the following theorem.

Theorem 9: In such a cognitive radio network system where SUs' delay-sensitive parameters are distributed on $[C_l, C_h]$, there exists a unique equilibrium arrival rate λ_e^c.

(1) If $V - P \leq C_l E[W(\lambda(C_l))]$, then $\lambda_e^c = 0$.
(2) If $C_l E[W(\lambda(C_l))] < V - P < C_h E[W(\lambda(C_h))]$, then C_e is the unique solution to $V = P + CE[W(\lambda(C))]$ for $C \in (C_l, C_h)$ and $\lambda_e^c = \Lambda H(C_e)$.
(3) If $V - P \geq C_h E[W(\lambda(C_h))]$, then $\lambda_e^c = \Lambda$.

B. OPTIMAL PRICING BY THE OPERATOR

Having characterized SUs' equilibrium behavior, we are now in a position to investigate the optimal pricing strategies from the points of revenue maximizing and social welfare maximizing.

1) REVENUE-OPTIMAL PRICING

To maximize the operator's revenue, the optimization problem to find the optimal price and corresponding arrival rate can be formulated as

$$\max_{0 \leq P \leq V} R(P) = \lambda_e^c(P) P \quad (31)$$

$$\text{Subject to } \lambda_e^c = \begin{cases} 0, & \text{if } P \geq V - C_l E[W(\lambda(C_l))]; \\ \lambda_e^*, & \text{if } V - C_h E[W(\lambda(C_h))] < P < V \\ & \quad - C_l E[W(\lambda(C_l))]; \\ \Lambda, & \text{if } P \leq V - C_h E[W(\lambda(C_h))], \end{cases} \quad (32)$$

where $\lambda_e^* = \Lambda H(C_e)$ and C_e is the unique solution to equation $U(C) = V - P - CE[W(\lambda(C))] = 0$ for $C \in (C_l, C_h)$.

The main focus of this optimization problem is the case where $V - C_h E[W(\lambda(C_h))] < P < V - C_l E[W(\lambda(C_l))]$. Because of the one-to-one mapping between P and λ_e^c, the objective function is transformed to the following one

$$\max_{C_l \leq C_e \leq C_h} R(C_e) = \lambda(C_e)(V - C_e E[W(\lambda(C_e))]) \triangleq \lambda(C_e) U_0(C_e).$$

The second-order derivative of $R(C_e)$ with respect to C_e is $R''(C_e) = 2\lambda'(C_e)U_0'(C_e) + \lambda(C_e)U_0''(C_e) + \lambda''(C_e)U_0(C_e)$. Because

$$\lambda'(C_e) = \Lambda h(C_e) > 0, \quad (33)$$

$$\lambda''(C_e) = \Lambda h'(C_e), \quad (34)$$

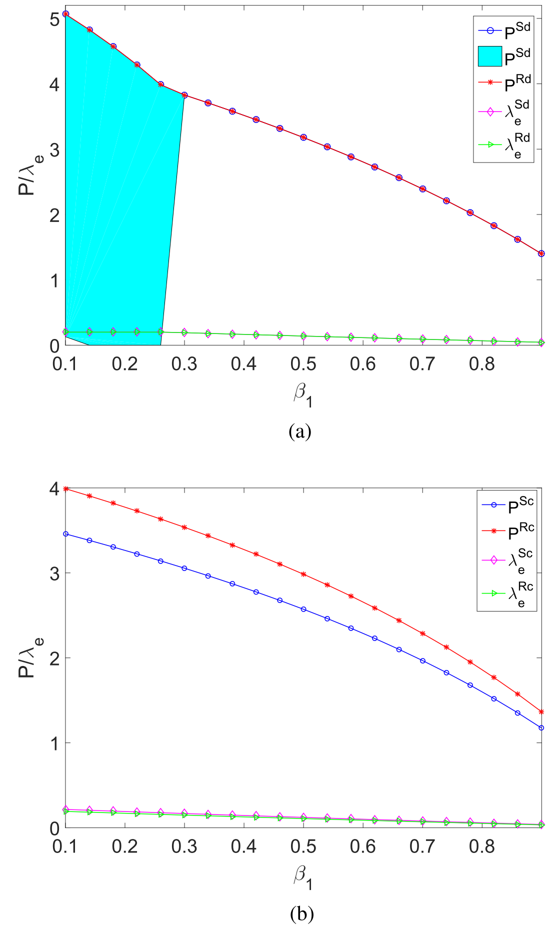

FIGURE 3. Optimal pricing strategies and equilibrium arrival rate vs. β_1 for $R = 10$, $C_1 = 1$, $C_2 = 2$, $C_l = 1$, $C_h = 2$, $\Lambda = 0.8$, $\Lambda_1 = 0.2$, $\Lambda_2 = 0.6$, $\alpha = 0.5$, $\theta = 2$, $\beta_2 = 3$, $f^*(\alpha) = 0.5$, $(f^*)'(\alpha) = -1$. (a) Two SU types. (b) Continuous distribution.

$$U_0'(C_e) = -E[W(\lambda(C_e))] - C_e \frac{\partial E[W(\lambda)]}{\partial \lambda} \lambda'(C_e) < 0, \quad (35)$$

$$U_0''(C_e) = -2\frac{\partial E[W(\lambda)]}{\partial \lambda}\lambda'(C_e) - C_e \frac{\partial E[W(\lambda)]}{\partial \lambda}\lambda''(C_e)) - C_e \frac{\partial^2 E[W(\lambda)]}{\partial \lambda^2}(\lambda'(C_e))^2, \quad (36)$$

the sign of $h'(C_e)$ is uncertain, which leads to that we can not determine the sign of $U''(C_e)$ and thus the uncertainty of $R''(C_e)$. Therefore, we solve this model in what follows under a specific distribution, that is, the uniform distribution on $[C_l, C_h]$, which is common in [19]–[21].

Under the uniform distribution, it follows that $h'(C_e) = 0$ and thus $R''(C_e) < 0$. So $R(C_e)$ is concave and then the revenue-optimal pricing strategy is followed.

Theorem 10: When the SUs' delay-sensitive parameter is uniformly distributed on $[C_l, C_h]$, the revenue-optimal price uniquely exists.

(1) If there exists $C_e^{Rc} \in (C_l, C_h)$ satisfying $R'(C_e^{Rc}) = 0$, the optimal price is

$$P^{Rc} = V - C_e^{Rc} E[W(\lambda(C_e^{Rc}))].$$

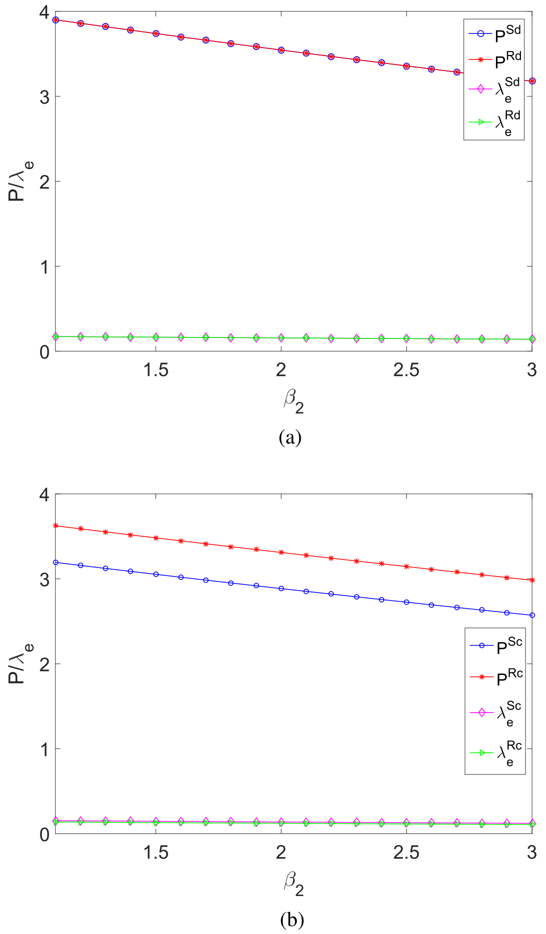

FIGURE 4. **Optimal pricing strategies and equilibrium arrival rate vs. β_2 for $R = 10$, $C_1 = 1$, $C_2 = 2$, $C_l = 1$, $C_h = 2$, $\Lambda = 0.8$, $\Lambda_1 = 0.2$, $\Lambda_2 = 0.6$, $\alpha = 0.5$, $\theta = 2$, $\beta_1 = 0.5$, $f^*(\alpha) = 0.5$, $(f^*)'(\alpha) = -1$. (a) Two SU types. (b) Continuous distribution.**

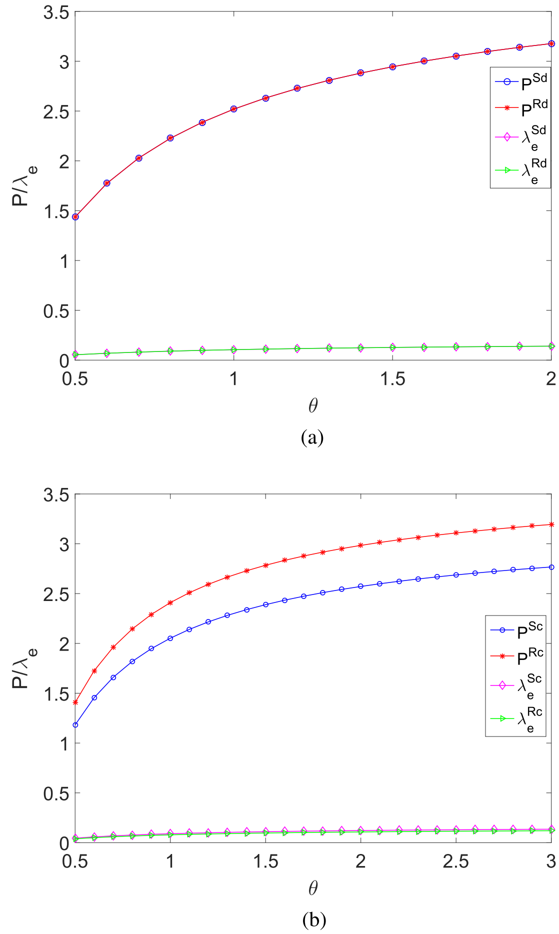

FIGURE 5. **Optimal pricing strategies and equilibrium arrival rate vs. θ for $R = 10$, $C_1 = 1$, $C_2 = 2$, $C_l = 1$, $C_h = 2$, $\Lambda = 0.8$, $\Lambda_1 = 0.2$, $\Lambda_2 = 0.6$, $\alpha = 0.5$, $\beta_1 = 0.5$, $\beta_2 = 3$, $f^*(\alpha) = 0.5$, $(f^*)'(\alpha) = -1$. (a) Two SU types. (b) Continuous distribution.**

(2) Otherwise, the optimal price is

$$P^{Rc} = V - C_h E[W(\lambda(C_h))].$$

2) SOCIALLY OPTIMAL PRICING

From the perspective of a social planner, the social welfare can be regarded as the aggregate utility obtained by all entering SUs. Thus, the social welfare of the network at price P is

$$S(P) = \Lambda \int_{C_l}^{C_e(P)} (V - CE[W(\lambda(C_e(P)))])h(C)dC. \quad (37)$$

In a same manner, to maximize the social welfare, seeking the optimal pricing strategy maximizing the social welfare is equivalent to searching the optimal marginal delay-sensitive parameter. Under the uniform distribution, the optimization of equation (37) can be rewritten as

$$\max_{C_e \in [C_l, C_h]} S(C_e) = \Lambda V \frac{C_e - C_l}{C_h - C_l} - \Lambda E[W(\lambda(C_e))] \frac{C_e^2 - C_l^2}{2(C_h - C_l)}. \quad (38)$$

Based on Lemma 4 and equations (27), (33), it is easily derived that $\frac{\partial^2 S(C_e)}{\partial C_e^2} < 0$ and thus the socially optimal pricing strategy is at hand.

Theorem 11: When the SUs' delay-sensitive parameter is uniformly distributed on $[C_l, C_h]$, there exist socially optimal pricing strategies as follows.

(1) If there exists $C_e^{Sc} \in (C_l, C_h)$ satisfying $\frac{\partial S(C_e)}{\partial C_e} = 0$, the socially optimal price is

$$P^{Sc} = V - C_e^{Sc} E[W(\lambda(C_e^{Sc}))].$$

(2) Otherwise, any price satisfying the following inequality can maximize the social welfare

$$P^{Sc} \leq V - C_h E[W(\lambda(C_h))].$$

Furthermore, the revenue-optimal price is not less than the socially optimal one according to the fact $\frac{\partial S(C_e)}{\partial C_e} \geq \frac{\partial R(C_e)}{\partial C_e}$.

Corollary 12: When the delay-sensitive parameter follows a uniform distribution, the revenue-optimal price is not less than the socially optimal one.

In the above, we only give the detailed analysis for the optimal pricing strategies when the delay-sensitive parameter

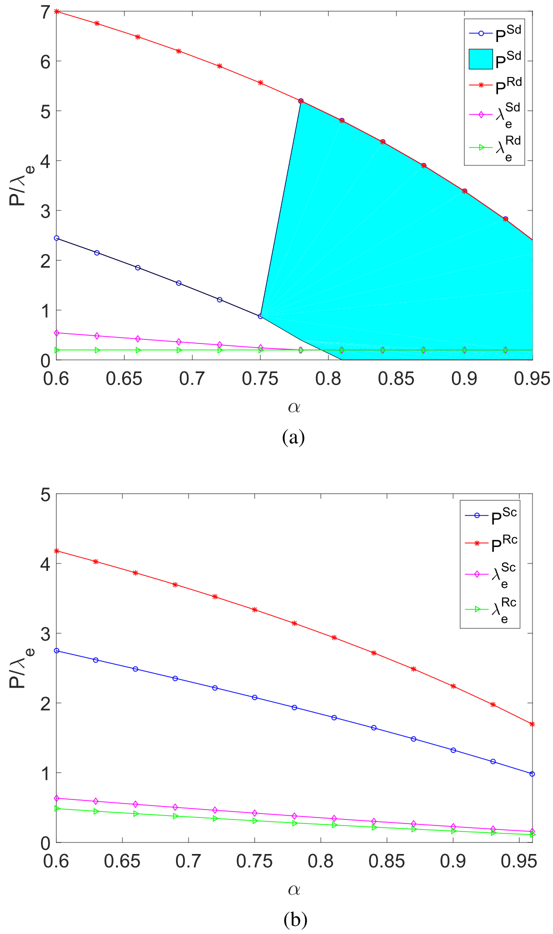

FIGURE 6. **Optimal pricing strategies and equilibrium arrival rate vs. α for $R = 10$, $C_1 = 1$, $C_2 = 2$, $C_l = 1$, $C_h = 2$, $\Lambda = 0.8$, $\Lambda_1 = 0.2$, $\Lambda_2 = 0.6$, $\mu = 3$, $\theta = 2$, $\beta_1 = 0.5$, $\beta_2 = 3$. (a) Two SU types. (b) Continuous distribution.**

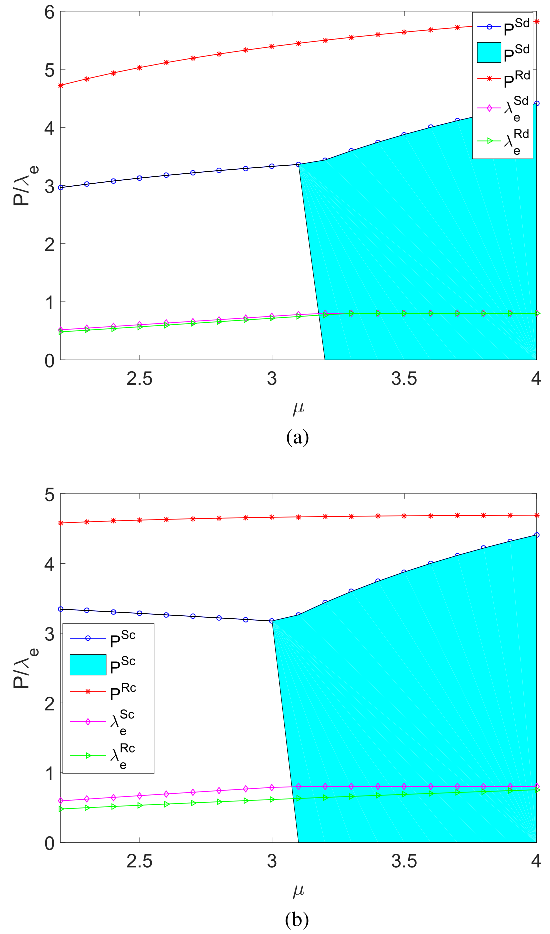

FIGURE 7. **Optimal pricing strategies and equilibrium arrival rate vs. μ for $R = 10$, $C_1 = 1$, $C_2 = 2$, $C_l = 1$, $C_h = 2$, $\Lambda = 0.8$, $\Lambda_1 = 0.2$, $\Lambda_2 = 0.6$, $\alpha = 0.5$, $\theta = 2$, $\beta_1 = 0.5$, $\beta_2 = 3$. (a) Two SU types. (b) Continuous distribution.**

is uniformly distributed. Recalling the conclusion that the revenue-optimal price is not less than the socially optimal price holds when N types of SUs follow a discrete distribution, we can surmise that this conclusion still applies under any continuous distribution by letting N approach infinity.

V. NUMERICAL EXAMPLES

In this section, we present numerical examples to show the effects of several system parameters on the pricing strategies and SUs' behavior, and examine whether the conclusion that the socially optimal joining probability coincides with the revenue-optimal one when the delay-sensitivity of SUs is homogeneous drawn in Wang *et al.* [17] holds or not for the case with heterogeneous SUs.

Assume that in the case of continuous distribution SUs' delay-sensitive parameters follow a uniform distribution and two types of SUs exist in the discrete distribution case. With regard to the sensitivity of SUs' arrival rate, from Figures 3-5, we observe that SUs' arrival rate is non-increasing with the first two moments of PUs' service time β_1, β_2, and non-decreasing with the retrial rate θ, which can be explained as follows. Firstly, the increase in PUs' expected service time means the band needs more time to serve PUs. Due to the priority of PUs, that causes SUs' mean waiting time to grow and thus SUs have less incentive to enter the system. Secondly, the growth in the second moment β_2 leads to a bigger variance (i.e., higher volatility in the service time), which results in SUs' mean waiting time rising. So SUs are reluctant to join the system. Lastly, a larger retrial rate implies that the probability of an SU occupying the band successfully increases, thus decreasing its mean waiting time and motivating more SUs to join the system.

Next, we explore the sensitivity of optimal pricing strategies with respect to the system parameters. It is known that when the arrival rate decreases, there are two ways to improve the revenue. One is to increase the fee to compensate for the loss caused by the decreasing arrival rate, and the other is to decrease the fee to slow the decreasing trend in the arrival rate. Similarly, if the arrival rate increases, the operator can increase the price, or it may lower the price to make the arrival rate grow faster. We observe that the changing trends in the revenue-optimal and socially optimal prices are the same as that of the arrival rate. This means that, if more SUs enter the system, to obtain more profit, the manager or the social planner will raise the price. In contrast, facing a decreasing arrival rate, the manager or the social planner will cut the price to attract more SUs, thus slowing the decreasing arrival rate. We find colored areas in some figures, which means that any pricing strategy in that area can maximize the social welfare, which coincides with the theoretical analysis when the equilibrium arrival rate equals Λ_1 or $\Lambda_1 + \Lambda_2$. Note that for those figures with colored areas, the changing trend of optimal pricing strategies is the same as that of the arrival rate is referred to the changing trend before and after the colored area.

On the other hand, when investigating the effect of PUs' arrival rate and SUs' service time distribution on the pricing strategies and SUs' arrival rate, in Figures 6-7, we assume SUs' service time follows an Exponential distribution with parameter μ. Clearly, in equilibrium, the revenue-optimal price, the socially optimal price, and SUs' arrival rate are increasing in service rate μ and decreasing in arrival rate α, except for that of the socially optimal price in the case of continuous distribution with respect to μ. The reason is that the mean service time equal to $\frac{1}{\mu}$ is decreasing in service rate, while the probability that an SU in service is interrupted increases with respect to PUs' arrival rate α.

Furthermore, from Figures 3-7, one can see that the revenue-optimal price is not less than the socially optimal price. Accordingly, the revenue-optimal arrival rate is not greater than the socially optimal rate, consistent with the theoretical analysis, but contrary with that derived in Wang *et al.* [17] for homogeneous SUs.

VI. CONCLUSION

This paper has provided an analysis of the optimal pricing strategies of the network operator and the non-cooperative joining behavior of SUs in a cognitive radio system with a single PU channel utilized by multiple PUs and multiple SUs, where the transmission times for SUs and PUs follow general distributions and the disrupted SU enters a retrial orbit to try again.

We show that a unique Nash equilibrium exists in the non-cooperative game where SUs are possibly the different players and decide whether to join the system or not based on the utility function capturing their delay-sensitive heterogeneity. From the perspective of the operator who can act as a commercial planner aiming to maximize its own revenue or a social planner aiming to maximize the social welfare, we investigate the optimal pricing strategies. We find that the revenue-optimal price is unique, while there may exist multiple socially optimal pricing strategies locating in an interval. Furthermore, the revenue-optimal price is not less than the socially optimal price, leading to the arrival rate under the revenue-optimal price being not greater than the rate under the socially optimal price. This finding is consistent with the reality but contrary with the conclusion drawn in the literature for homogeneous SUs; that is, the socially optimal arrival rate coincides with the revenue-optimal one. The reason for that is the commercial planner is selfish, whereas the social planner is indifferent when faced with the utility of SUs and commercial planner.

An interesting future research topic is to investigate the effects of sensing failures on the operator's pricing strategies and SUs' joining behavior. In addition, in our paper, although the collision of PUs is modelled by an M/G/1 queueing system, the PUs are assumed to join the system without balking. So the analysis for PUs' balking behavior is another future research direction.

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