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Optimal Product Attributes in Single Choice Models

Single Choice Models for Product Positioning

Recently, the optimal determination of new product attributes has attracted considerable attention [1,3,4,5,6,7]. It is assumed that ideal as well as real product perceptions of individual consumers can be represented in a carthesian attribute space by multidimensional scaling procedures. Furthermore, it is assumed that purchase probabilities for individual products vary with the distance of the product from an ideal position. In the single choice model, purchase probability equals one for the product closest to the ideal, and it equals zero for all other products [2]. This is not unplausible on empirical grounds.[1)]

These assumptions being given, a manufacturer may want to determine attribute levels for a new product to be introduced into the market such that some objective function, i.e. sales maximisation, is optimised.

The product positioning problem as stated above starts from the following data:

J : a set of mutually independent attributes which are considered as potentially relevant to constitute brand preferences;
the attribute space is $\mathbb{R}^{\bar{j}}$, where $\bar{j}$ gives the number of elements (cardinality) in J,

K : a set of customers, where $\bar{k}$ gives the number of customers in K,

I : a set of known products or brands,

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1) A comparison of the single choice model and the probabilistic model by Shocker and Srinivasan [1] is given in a recent paper by Albers and Brockhoff [7].

c_{kj} : an estimate of the k-th ($k \varepsilon K$) customer's perception of an ideal product expressed by the attribute level of the j-th ($j \varepsilon J$) attribute,

e_{ij} : an estimate of the perception of the known brands $i \varepsilon I$, expressed by the attribute level of the j-th ($j \varepsilon J$) attribute,

s_{kj} : a salience, measuring the relative importance of each attribute $j \varepsilon J$ to the customer $k \varepsilon K$,

r_k : potential demand of the k-th customer ($k \varepsilon K$).

According to the assumptions of the single choice model and the measurement of distances by the weighted Minkowski-metric, a customer chooses the new product characterised by the attribute levels y_j ($j \varepsilon J$) , if

$$(1) \qquad \left| \left[\sum_{j \varepsilon J} s_{kj} \; | \; c_{kj} - y_j |^m \right]^{\frac{1}{m}} \right| \leq d_k \qquad (k \varepsilon K),$$

where

$$(2) \qquad d_k = \mathrm{Min}\{ \left| \left[\sum_{j \varepsilon J} s_{kj} \; | \; c_{kj} - e_{ij} |^m \right]^{\frac{1}{m}} \right| \; i \; \varepsilon \; I \} \qquad (k \varepsilon K).$$

After introducing binary variables:

$$(3) \qquad x_k = \begin{cases} 1, \text{ if (1) holds,} \\ 0, \text{ if (1) does not hold,} \end{cases} \qquad (k \varepsilon K),$$

the optimal position for the new product can be determined by the following optimisation problem:

$$(4) \qquad \sum_{k \varepsilon K} r_k \cdot x_k \Rightarrow \mathrm{Max!}$$

subject to:

$$(5) \qquad \left| \left[\sum_{j \varepsilon J} s_{kj} \; (|c_{kj} - y_j|)^m \right]^{\frac{1}{m}} \right| - d_k \leq (1 - x_k) \cdot M \qquad (k \varepsilon K),$$

$$(6) \qquad x_k \; \varepsilon \; \{0,1\} \qquad (k \varepsilon K),$$

where M is a finite upper bound on the left-hand-side of (5).

This mixed integer nonlinear programming problem has real variables y_j ($j \varepsilon J$), and binary variables x_k ($k \varepsilon K$). Special solution procedures have been supplied by Albers and Brockhoff (3) for the case $s_{kj} = 1$ ($k \varepsilon K$, $j \varepsilon J$) and $r_k = 1$ ($k \varepsilon K$), by Albers [6] and by Zufryden [5] for the more general cases presented above. A concept for the multi-period and stochastic cases has been developed by Brockhoff [4].

Comments on Zufryden's Model ZIPMAP

Objectives

Zufryden states that besides (4), other objective functions might be possible (p. 65)[5]. However, if these functions would become nonlinear, the resulting problem might easily become nontractable by analytical optimisation approaches. This would be the case for profit maximisation, where either price would be considered as an attribute or cost would be dependent on product positions.

Solvability

According to Jeroslow [7], Zufryden considers the problem (4) s.t. (5) and (6) to be too difficult to be solved analytically. Therefore, he developes a two stage approach called ZIPMAP, which solves the original problem via approximation. The approximation is based on the substitution of parallelotopes for (1), which linearises the problem. However, the reference made to Jeroslow [7] does not cover the case under consideration. Jeroslow's impossibility theorem applies to those problems only, where the integer variables occur in the nonlinear terms of the constraints. This is not the case in (5). In fact, the specific structure of the problem gives rise to a special purpose algorithm that capitalises on the separation of the real and integer variables in the constraints (5). This is shown by Albers [6].

ZIPMAP as a Matching or a Multidimensional Knapsack Problem

The ZIPMAP procedure which should help solve large problems encompasses a matching problem (in its constraints (6)) with $\bar{k}$ variables and up to $\bar{k}(\bar{k}-1)/2$ constraints or a multidimensional knapsack problem (constraints (7) or (9), respectively) with $\bar{k}$ variables and $(\bar{k}-1)$ constraints or $\bar{k}$ variables and $(\bar{k}-1)\cdot p + \bar{k}$ constraints, respectively (p: number of inscribed parallelotopes). In the case of a large number of customers ($\bar{k}$) and a high level of approximation (p), these formulations may well lead to relatively high CPU-time. This may be concluded from the computational experience reported in Garfinkel-Nemhauser [9] (p. 383) and Salkin-de Kluyver (p. 138)[10].

Determination of the Parallelotopes

The preference set of a customer containing all positions for a new product which cause the customer to choose the new product has, geometrically, the shape of a hyperellipsoid which results from formula (1). The ZIPMAP procedure approximates these hyperellipsoids by parallelotopes orthogonal to their axes. These solids have the property that pairwise intersections of n parallelotopes imply n-wise intersection. Zufryden [5] proposes at least two general possibilities for the construction of the parallelotopes:

- an outer parallelotope that circumscribes the hyperellipsoid,
- one or more interior parallelotopes that inscribe the hyperellipsoid.

Zufryden does not prescribe the specific way of approximation. It remains up to the user. To complete the description of ZIPMAP we propose a general formula for the determination of the lower and upper boundaries (b^{l}_{kgj} and b^{u}_{kgj}) of the parallelotope of customer k for attribute dimension j according to the g-th type of approximation:

(7) $$b^{l}_{kgj} = c_{kj} - \left|\frac{w_{gj}}{s_{kj}}\right|^{\frac{1}{m}} \cdot d_k \qquad (k\varepsilon K,\ j\varepsilon J,\ g\varepsilon G)$$

(8) $$b^{u}_{kgj} = c_{kj} + \left|\frac{w_{gj}}{s_{kj}}\right|^{\frac{1}{m}} \cdot d_k \qquad (k\varepsilon K,\ j\varepsilon J,\ g\varepsilon G)$$

where

G : set of required approximations

w_{gj} : parameter or weight to represent the type of approximation $g\varepsilon G$ within attribute dimension $j\varepsilon J$.

In the case of an outer parallelotope, we let

(9) $$w_{1j} = 1 \qquad (j\varepsilon J).$$

In the case of interior parallelotopes, we require:

(10) $$\sum_{j\varepsilon J} w_{gj} = 1 \qquad (g\varepsilon G)$$

(11) $$w_{gj} \geq 0 \qquad (g\varepsilon G,\ j\varepsilon J)$$

The sufficiency of these conditions can be seen easily. The parallelotope is tangent to the hyperellipsoid, if

(12) $$\left|\left[\sum_{j\varepsilon J} s_{kj}\ |\ c_{kj} - b^{l}_{kgj}|^{m}\right]^{\frac{1}{m}}\right| = d_k \qquad (k\varepsilon K,\ g\varepsilon G)$$

Substituting (7) into (12) we get

(13) $$\left|\left[\sum_{j\varepsilon J} w_{gj}(d_k)^{m}\right]^{\frac{1}{m}}\right| = d_k \qquad (k\varepsilon K,\ g\varepsilon G).$$

Now, we realize that (13) holds if we require (7), (8), (10), and (11).

[illegible]

- n_1 for one outer parallelotope must be greater than or equal to n for the PROPOSAS-solution. This is due to the fact that the outer parallelotopes have a larger volume than the original hyperellipsoids. Table 1 also shows, however, that the centre point from the intersection of the n_1 parallelotopes gives, in general, poor solutions to the original problem (see column n_2 for one outer parallelotope).

- n_1 and n_2 for one and more interior parallelotope(s) must be less than or equal to n for the PROPOSAS-solution, which is due to the smaller volume of the interior parallelotopes as compared with the original hyperellipsoids. Table 1 also shows that $n_2 \geq n_1$. This is due to the fact that the centre point (Q) of the intersection of a maximum number (n_1) of parallelotopes may be located within n_2 hyperellipsoids where $n_2 - n_1$ hyperellipsoids are approximated by parallelotopes that do not intersect the other n_1 parallelotopes. This is demonstrated in fig. 1.

- Figure 1 about here -

Comparing the number of buyers of the new product n with n_1 as detected by the algorithms PROPOSAS and ZIPMAP for interior parallelotopes, respectively, we can derive the following relationships (from table 1):

- The degree of approximation n_1/n decreases with increasing number of attribute dimensions. This is obvious because the more attribute dimensions the higher is the degree of freedom for the hyperellipsoids to intersect eachother pairwise.

- The degree of approximation n_1/n increases with increasing metric parameter m. This is due to the fact that with $m \rightarrow \infty$ the preference sets approach parallelotopes. This result was expected by Zufryden [5]. It should be noted that the poorest approximation results from the often used city-bloc metric (m = 1.0).

- The degree of approximation n_1/n increases with increasing number of inscribed parallelotopes. For one interior parallelotope we find the degree ranging from 50% up to 90%, whereas 5 interior parallelotopes improve the result to the range from 70% up to 100%.

Comparison with Respect to CPU-time

Comparing the computing times (CPU-times) required for both algorithms (see table 2) one can realise that those for ZIPMAP are, in general, smaller than those for PROPOSAS if only up to two parallelotopes are used for approximation in ZIPMAP. Otherwise the CPU-times tend to grow larger than those in PROPOSAS. Other problem characteristics are codeterminants of computing time. This is shown by the problems 10, 11, and 12 that do not follow the trend. However, one should remember the results on the degree of approximation. Suboptimal solutions that are equal to the "optimal" ZIPMAP solutions can be derived by PROPOSAS within small fractions of the CPU-times that are required for determining the optimal solution.

- Table 2 about here -

Comparison with Respect to Storage Requirements

The use of as many interior parallelotopes as possible in order to improve the degree of approximation is restricted by the required internal storage capacity. Given an in-core storage capacity of 20.000 variables for the matrix of pairwise intersections (number of elements = $\frac{1}{2}\cdot\bar{k}(\bar{k}-1)\cdot p^2$, where p: number of interior parallelotopes) we can apply the less interior parallelotopes the more customers have to be considered. This is demonstrated in table 3.

No. of customers $\bar{k}$	20	30	40	50	60	70	80	90	100	110
Maximal No. of Interior Parallelotopes p	10	6	5	4	3	2	2	2	2	1

Tab. 3 : Restrictions for the number of interior parallelotopes with respect to the number of customers if storage capacity is restricted

From that we may conclude that for realistic problems with $\bar{k} \geq 100$ we only can use at most 2 interior parallelotopes for approximation at the assumed level of storage capacity.

Conclusion

We conclude that ZIPMAP provides an interesting approximation to optimal solutions for the new product positioning problem assuming a single choice model. However, the relatively low degree of approximation is not compensated by reasonable savings of CPU-time. Rather, the CPU-times for ZIPMAP are only slightly better than those for PROPOSAS. As PROPOSAS provides optimal solutions to the same class of problems as ZIPMAP, PROPOSAS must be considered to be superior.

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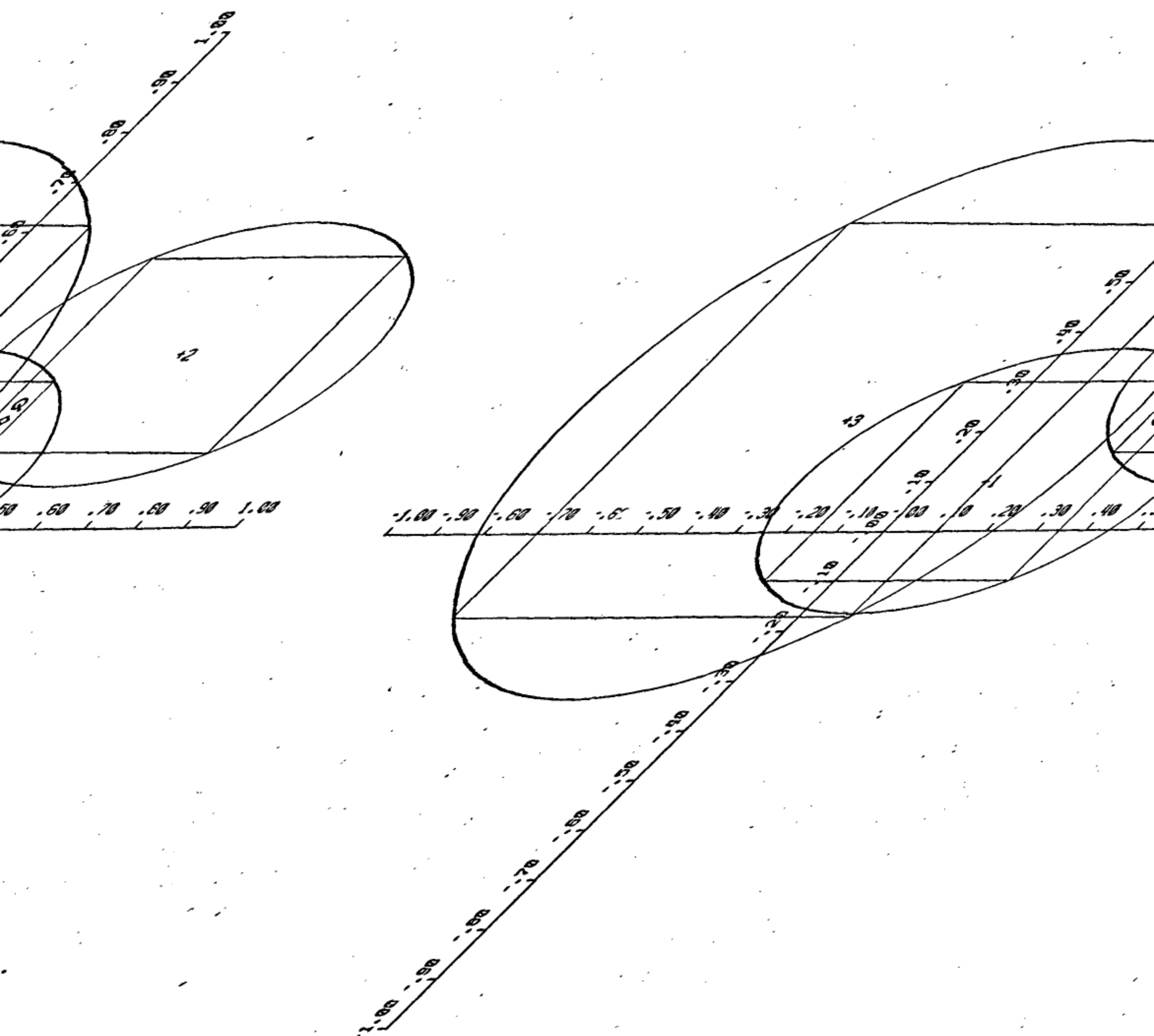

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Fig. 1: Geome
ellipso

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