Optimal Release of Information By Firms

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ABSTRACT

This paper provides a positive theory of voluntary disclosure by firms. Previous theoretical work on disclosure of new information by firms has demonstrated that releasing public information will often make all shareholders worse off, due to an adverse risk-sharing effect. This paper uses a general equilibrium model with endogenous information collection to demonstrate that there exists a policy of disclosure of information which makes all shareholders better off than a policy of no disclosure. The welfare improvement occurs because of explicit information cost savings and *improved* risk sharing. This provides a positive theory of precommitment to disclosure, because it will be unanimously voted for by stockholders and will also represent the policy that will maximize value ex ante. In addition, it provides a "missing link" in financial signalling models. Apart from the effects on information production analyzed in this paper, most existing financial signalling models are inconsistent with a firm taking actions which facilitate future signalling because release of the signal makes all investors worse off.

PUBLICLY TRADED FIRMS EXPEND resources to release information to outside security holders on an almost continual basis, even in the absence of regulatory requirements to do so. Firms enter into commitments to continue to release such information; e.g., such commitments are common in bond covenants. This paper develops an equilibrium model to explain such an expenditure by firms and shows that in many circumstances this public information makes all traders better off. The welfare improvement occurs in part because some traders would acquire costly information in the absence of the public announcement, while all abstain from information collection given the announcement. In addition, releasing public information is shown to improve risk sharing.

There has been much analysis of mechanisms which act to signal or release information to traders. Dividend policy and capital structure policy have been analyzed as signals in, e.g., Bhattacharaya [3], Miller-Rock [20], and Ross [23]. No general explanation has been put forth which rationalizes an ex ante desire by traders for such information.[1] Ross [23] demonstrates that a particular observable variable can release information, but the desirability of such information release is not studied. Miller-Rock [20] presents a model which implies

[1] Harris-Raviv [15] shows that in a dynamic model where a firm's stockholders cannot diversify, signalling information may provide a welfare improvement. This is only for certain parameter values, however, and the paper does not claim to have provided a theory of the demand for the release of information.

that information release makes traders worse off, but the information is released despite this because firms cannot precommit to refrain from releasing information. None of the existing signalling models analyze traders' incentives to acquire information. Our analysis of such costly acquisition provides an explanation of why stockholders would want firms to signal or release information, which is a necessary prerequisite to many explanations of how they signal information.

On most dates a firm is neither buying nor selling its own shares, and all trade is in the secondary exchange market and involves only outside investors. The model developed here provides an explanation of the establishment of a policy of releasing information even on dates when the firm is not selling new securities to the public. On dates when the firm is selling new securities, there is a well-known adverse selection explanation of why firms must use signalling or release information. The explanation is presented in Leland-Pyle [18]: the firm which is better informed than investors must convince potential investors that it is not selling shares simply because it knows that the price is too high (see also Myers-Majluf [21]). In addition, one can interpret recent results in auction theory developed in Milgrom-Weber [19] as providing another explanation of disclosure on dates when the firm is selling securities. They show that precommitment to unconditionally releasing information can increase the expected selling price in an auction.

A simple model is developed which focuses on the equilibrium effects of an information release policy. A pure exchange economy is assumed, which implies that information cannot influence the production decisions of firms. It also implies that traders have no direct use for the information in formulating any personal production plans. Despite this, we provide conditions when all traders are better off as the result of a policy of publicly disclosing information. A brief discussion of ways of extending this result to a production economy is presented in Section III.

There has been previous discussion of the effect of disclosure on private information acquisition (e.g., Diamond [6], Fama-Laffer [9], Gonedes [10], and Hakansson [12, 13]), but the effect has not been studied in an equilibrium model with rational expectation formation and risk aversion (Diamond [6] presents an analysis with risk-neutral traders). A related analysis of the arguments for and against government regulation of disclosure is presented in Ross [24]. There has also been previous analysis of the effects of disclosure on investor welfare in pure exchange settings where the effects on private information acquisition are absent. This literature includes Hakansson, Kunkel, and Ohlson [14], Hirshleiffer [17], and Trueman [25]; see also the references in Verrecchia [27]. These studies of disclosure focus on the effects on risk sharing. One effect analyzed is that early release of information can reduce risk sharing because it leaves traders with suboptimal portfolio weights which they would have adjusted if given the chance to trade before the information was disclosed (the classic example in a nonfinancial setting is finding out who in society has cancer before individuals can contract for life insurance). This negative value of information does not appear to be especially significant in financial asset markets because almost continual trade is allowed. A second effect, discussed in Hakansson, Kunkel, and Ohlson [14],

occurs if the new information influences the optimal allocation of risky assets across traders. In this case, an opportunity for using new information can make traders better off in some circumstances. We analyze a model which assumes that traders have identical constant absolute risk aversion preferences, a class of preferences for which the optimal risk sharing given homogeneous information is not influenced by new information. This assumption is made to focus on the welfare effect of information release on private information acquisition as well as for tractability.

The model is of a single period, two-asset exchange market with one safe and one risky asset. The risky asset is called a firm, and the economy is large: we analyze an economy where the number of traders is infinite so each is atomistic and behaves competitively. The model is solved for a noisy rational expectations equilibrium, as developed in Diamond-Verrecchia [7] and Hellwig [16]. Traders have the opportunity to acquire costly information about the firm's return, and they can select an information disclosure policy for the firm. The information acquisition model is similar to Grossman-Stiglitz [11] and Verrecchia [26], but differs from both. Traders all have access to an information production technology which allows each to observe, at a cost, $c > 0$, a piece of information which is correlated with the return of the firm, but conditional on the true return of the firm, the pieces of information are independent of each other.

The main result of the paper is a characterization of the optimal public information release policy for the firm. Under an assumption that the firm's cost per unit of precision of producing information is no higher than that faced by traders, we show that there is an information release policy which increases the ex ante expected utility of all traders, as compared with a policy of not releasing information. Releasing public information reduces the incentive to acquire private information. There are two components to the beneficial effect of releasing low cost information to the public. The first component is the savings of real resources which would be devoted to private information acquisition if public information were not released. The second component is an improvement in risk sharing compared to the situation where information is not released. This arises because the public information makes traders' beliefs more homogeneous and reduces the magnitude of speculative positions which informed traders take.

An even better arrangement would be an agreement among traders to all refrain from acquiring any private information or a tax on information acquisition. This would provide the benefits mentioned in the previous paragraph without any disclosure of public information. This is not feasible because the act of acquiring private information is not observable by other traders. Each trader would have a huge incentive to cheat and acquire information if he (or she) thought he would be the only informed trader. Releasing public information makes it incentive compatible to refrain from acquiring private information.

The information release policy makes all traders better off. Given this unanimity result, we have a positive theory of why firms precommit to regularly release information. If the security holders of the firm vote, all will prefer the optimal disclosure policy to no disclosure. Alternatively, if one imagines the firm initially being sold to outside security holders on a date at which there is

symmetric information to all, and if the "previous" owner of the firm chooses an optimal selling mechanism, he maximizes his revenue from selling the firm by precommitting it to the optimal information release policy. In contrast, if the firm has no disclosure policy, the individually rational decisions made by traders result in strictly lower welfare.

The balance of the paper is organized as follows: Section I develops the Diamond-Verrecchia [7], Hellwig [16], and Verrecchia [27] models, and characterizes private information production when there is no public information released. In Section II, a characterization is presented of the impact of public information on the production of private information, and on the welfare of traders. Section III discusses some possible extentions and generalizations. Section IV concludes the paper.

I. Rational Expectations Equilibrium and Individual Information Production

A version of the noisy competitive rational expectations model with diverse information developed by Diamond-Verrecchia [7], Hellwig [16], and Verrecchia [26] is used to characterize the equilibrium private demand for information to be used for trading purposes. We present a brief description of the model here; the referenced papers provide the details. The characterization is used for evaluating the effects of public information release on investor welfare and the incentives for such release.

Traders are risk-averse and have constant absolute risk aversion preferences (exponential utility) with a common level of risk tolerance, denoted by $r > 0$. There are two assets. One is a riskless asset which returns one unit of the single consumption good (its current price serves as numeraire). The other asset is risky; its return is represented by the random variable $\tilde{u}$, whose realization is u. Each trader's prior beliefs are that $\tilde{u}$ has a normal distribution with mean Y_0 and precision (i.e., inverse of variance) $h_0 > 0$. There are T traders indexed by $t = 1$, $\cdots$, T. We examine limiting results for a large economy in which $T \to \infty$, and each trader is an insignificant part of the total market. This assumption justifies each trader's competitive conjecture that his demand and thus his information do not affect the equilibrium price of the risky asset.

In addition to his prior information, each trader can acquire costly information about the risky asset's return. At a physical cost of $c > 0$, each trader can acquire a piece of information about $\tilde{u}$. The costly information available to trader t is the return, u, plus some error, i.e., it is the random variable, $\tilde{y}_t$, defined as

$$\tilde{y}_t = \tilde{u} + \tilde{\varepsilon}_t,$$

where $\tilde{\varepsilon}_t$ is normal with mean zero and precision $s > 0$. For all t, $\tilde{\varepsilon}_t$ is independent of the errors, $\tilde{\varepsilon}_j$, of all other traders, $j \neq t$. The act of acquiring information is not observable by other traders.

In addition to any information acquired by traders, there is another stochastic influence on the market. This is a factor other than information which causes prices to vary. An example would be a stochastic life-cycle motive for trade which

perturbs many traders' demand curves. The factor, which we term noise, results in equlibrium prices revealing only a part of aggregate information collected (see the discussion in Grossman-Stiglitz [11] and Diamond-Verrecchia [7]). The noise is formally represented by uncertainty about the per capita supply of risky assets. Because prices depend on both supply and demand, the uncertainty about supply prevents the price from revealing the position of the demand curve. This prevents all of the information which conditions the demand curve from being revealed.

Trader t is endowed with $\bar{B}_t$ riskless assets and x_t risky assets, where x_t is the realization of the random variable, $\tilde{x}_t$. The $\tilde{x}_t$ are mutually independent normal random variables with mean zero and variance $T \cdot V > 0$ and are independent of all of the information random variables, $\tilde{y}_t$. Per capita supply of the riskless and risky assets are given by $\bar{B} \equiv (1/T) \sum_{t=1}^{T} \bar{B}_t$ and $\tilde{X} \equiv (1/T) \sum_{t=1}^{T} \tilde{x}_t$, respectively. The random variable, $\tilde{X}$, has a normal distribution with mean zero and variance $V > 0$ because each individual endowment has a variance of $T \cdot V$. Each trader's $\tilde{x}_t$ has correlation with $\tilde{X}$ of $T^{-1/2}$. As $T \to \infty$, this correlation approaches zero and becomes uncorrelated with $\tilde{X}$. As a result, a trader's endowment provides no information about the per capita endowment, $\tilde{X}$. Each trader observes his own endowment (but not any other trader's endowment) before any decisions or policies are made. Each trader chooses whether to acquire information and at that point knows only his own endowment and the information release policy of the firm. Then, he observes the realization of the information he purchases (if any), formulates his demand, and trades in a competitive market. Finally, he consumes the proceeds realized from his chosen portfolio.

A. Equilibrium

Given each trader's decision whether to acquire costly information, the concept of equilibrium is a rational expectations competitive equilibrium (see Diamond-Verrecchia [7] for a formal definition). Each trader takes as given the equilibrium price of risky assets in terms of the riskless numeraire, and takes as given the joint probability distribution of $\tilde{y}_t$, $\tilde{X}$, $\tilde{u}$, and P, respectively; his information (if any); per capita endowment of risky assets; return of the risky asset; and the equilibrium price. Note that $\tilde{P}$ is an endogenous variable: a rational expectations equilibrium is a conjectured joint distribution which is self-fulfilling when traders choose optimal demand functions on the basis of the conjecture. This captures the idea that the endogenous information in prices is used correctly. The equilibrium price is the value which equates supply and demand on the basis of optimal demands conditioned on the true statistical properties of each trader's private information and all other observable variables.

In this model, the information acquisition decision is endogenous as well. Each trader makes a conjecture about the joint distribution of $(\tilde{y}_t, \tilde{X}, \tilde{u}, \tilde{P})$ and on the basis of this conjecture, each optimally decides whether to acquire information. This yields a fraction of traders, $\lambda \in [0, 1]$, who acquire costly information, and in equilibrium this implies a joint distribution which is identical to the original conjecture. This is a concept of equilibrium which allows analysis of the full effect of changes in the supply of public and private information on the equilib-

rium price distribution, expenditures on information, and on the expected utility of traders.

We begin by stating some results from previous work which provide closed-form representation for the equilibrium distribution of the risky asset's price. A result from Hellwig [16] is stated as Lemma 1. It provides a characterization of an equilibrium price for a given, exogenous, value of λ (the fraction of informed traders). An equilibrium price is a linear function of the liquidating value of the risky asset, $\tilde{u}$, and of the realized per capita supply of risky assets, $\tilde{X}$. Traders cannot observe $\tilde{u}$ or $\tilde{X}$ directly, but do observe $\tilde{P}$, and Lemma 1 provides a self-fulfilling belief about the joint distribution of $\tilde{P}$, $\tilde{u}$, and $\tilde{X}$.

LEMMA 1. *The rational expectations competitive price, $\tilde{P}$, for a given value of λ, converges in probability as $T \to \infty$ to*[2]

$$\tilde{P} = \alpha Y_0 + \beta \tilde{u} - \gamma \tilde{X}$$

where

$$\alpha = \frac{rVh_0}{(rVh_0 + r\lambda sV + r^3(\lambda s)^2)}$$

$$\beta = \frac{r\lambda sV + r^3(\lambda s)^2}{(rVh_0 + r\lambda sV + r^3(\lambda s)^2)}$$

$$\gamma = \frac{V + r^2\lambda s}{(rVh_0 + r\lambda sV + r^3(\lambda s)^2)}.$$

Proof: See Proposition 5.2 of Hellwig [16, p. 492], and note that in Hellwig's notation, $A^* = r$ and $B^* = r\lambda s$. Q.E.D.

B. Ex Ante Expected Utility for a Given Value of λ

As an input to the analysis of equilibrium information acquisition decisions, we state as Lemma 2 a result from Verrecchia [27], which provides a closed-form expression for the expected utility of a trader for a given exogenous value of λ, and an exogenously specified acquisition decision. The interpretation of this expression for expected utility is that it is conditional on a trader's own endowment (a trader knows his own endowment and does not treat it as random). It treats $\tilde{u}$, $\tilde{P}$, $\tilde{Y}_t$, and $\tilde{X}$, as random variables, and provides ex ante expected utility calculated before these random variables are observed. This is an appropriate measure of the expected utility of a trader and will later serve as an input into the information acquisition decision because the decision to buy information is made before these random variables are observed.

In Lemma 2, the information acquisition decision is specified exogenously. A new bit of notation must be specified to allow for the two possible decisions: acquire or do not acquire information.

Let the pair $(\hat{c}, \hat{s})$, respectively, denote the chosen expenditure on information

[2] This is the unique equilibrium of this linear form; the existence of nonlinear equilibria is currently unknown.

and the chosen precision of private information observed by a trader. There are two possible values for the pair $(\hat{c}, \hat{s})$. If a trader acquires private information, then $(\hat{c}, \hat{s}) = (c, s)$. If a trader acquires no private information and remains uninformed, $(\hat{c}, \hat{s}) = (0, 0)$. Although $\hat{c}$ and $\hat{s}$ represent the values for a particular trader, we do not subscript them, nor the individual endowment of risky assets, x_t, with the index for an individual trader, t, simply to avoid messy notation.

LEMMA 2. *An individual trader's ex ante expected utility, given his endowment of risky assets, x, for a given fraction of informed traders, λ, and information acquisition decision, $(\hat{c}, \hat{s})$, is given by*

$$U(\hat{s}, \hat{c}; \lambda) = - \left(\frac{h_0}{\frac{(rVh_0)^2 + h_0 V(V + r^2\lambda s)^2}{(rVh_0 + r\lambda s V + r^3\lambda^2 s^2)^2}\left(\hat{s} + h_0 + \frac{(rs\lambda)^2}{V}\right)} \right)^{1/2}$$

$$\cdot \exp\left(\frac{-\bar{B} - xY_0 + \hat{c}}{r} + \frac{1}{2}\left(\frac{x}{r}\right)^2\left[h_0 + \frac{(rVh_0)^2}{V(V + r^2\lambda s)^2}\right]^{-1}\right).$$

Proof: See Verrecchia [27], Lemma 2, and note that with λ given, $E[rs] = r\lambda s$, and $E[r] = r$. Q.E.D.

C. Equilibrium Information Acquisition When None is Released

If a trader chooses to buy information and expects that a fraction, λ, of the other traders will do so as well, his expected utility is $U(\hat{s} = s, \hat{c} = c; \lambda)$. If the trader chooses not to buy information, his expected utility is $U(\hat{s} = 0, \hat{c} = 0; \lambda)$. For a given value of λ, if $U(\hat{s} = s, \hat{c} = c; \lambda) > U(\hat{s} = 0, \hat{c} = 0; \lambda)$, a trader will choose to become informed (and choose to become uninformed if the reverse inequality holds). The equilibrium fraction of informed traders, $\lambda \in [0, 1]$, is the value such that when traders optimize, given a conjecture about the fraction, the value which each conjectures is self-fulfilling. For example, $\lambda = 0$ is an equilibrium if, for *all* traders, $U(\hat{s} = s, \hat{c} = c; \lambda = 0) < U(\hat{s} = 0, \hat{c} = 0; \lambda = 0)$. Lemma 3 provides a complete characterization of the equilibrium value of λ, under the assumption that the firm does not release any public information.

LEMMA 3. *The equilibrium value of λ is*

(a) $\lambda = 0$ if $e^{2c/r} - 1 \geq \dfrac{s}{h_0}$

(b) $\lambda = 1$ if $e^{2c/r} - 1 \leq \dfrac{s}{h_0 + \dfrac{(rs)^2}{V}}$

(c) $\lambda = \dfrac{\sqrt{V}}{r\hat{s}} \sqrt{\dfrac{\hat{s}}{e^{2c/r} - 1} - h_0} \in (0, 1)$ otherwise.

Proof: The condition for $\lambda = 0$ is that all traders prefer to be uninformed when

all others are, or $U(\hat{s} = 0, \hat{c} = 0; 0) > U(\hat{s} = s, \hat{c} = c; 0)$, which provides the condition for part (a). For $\lambda = 1$ to be equilibrium, each trader must want to be informed when all others are as well, or $U(\hat{s} = s; \hat{c} = c; 1) > U(\hat{s} = 0; \hat{c} = 0; 1)$, providing condition (b). For an interior solution $\lambda \in (0, 1)$, $U(\hat{s} = s, \hat{c} = c; \lambda) = U(\hat{s} = 0, \hat{c} = 0; \lambda)$ or $e^{2c/r} - 1 = \dfrac{s}{h_0 + (r\lambda s)^2/V}$. Note that $\dfrac{s}{h_0 + (r\lambda s)^2/V}$ is a decreasing, continuous function of λ, therefore, if the conditions for (a) and (b) both fail, there exists $\lambda \in (0, 1)$ which makes this an equality, and its value is given in part (c).   Q.E.D.

The intuition behind this result is clear: the value of information to a trader depends on its precision both absolutely and relative to other traders. In case (a) ($\lambda = 0$), the information is sufficiently costly (high c) and the precision of prior information is sufficiently high (high h_0) such that it will not pay to acquire it, even if no one else does and even though it gives an informational advantage over all other traders. In case (b) ($\lambda = 1$), the information is of sufficiently low cost and prior information is of such low precision that it will pay to acquire it even if everyone else has information of the same precision. This is because, with noise, an uninformed trader is at an informational disadvantage relative to informed traders and will be willing to buy cheap, precise information to remove the *disadvantage*, even though it gives an advantage over no trader. If the costs are between these two extremes, then the fraction of informed traders, λ, adjusts until the expected utility net of information costs for a trader is equal if informed or uninformed, and a trader is equally well off given either decision. Figure 1 shows the equilibrium value of λ for points in (s, c) space, given values of h_0, V, and r.

II. Release of Public Information

We now examine the effects of the firm establishing a policy of disclosing information to the public. The firm is assumed to be able to precommit to releasing public information of a given precision. Figure 2 provides the time sequence of events. It is important that traders know the firm's information release policy when they each decide whether to acquire private information. The exact time that traders observe the public information is not critical; instead of point C in Figure 2, it could just as well be between points E and F.[3]

The firm follows an announced policy of releasing a public signal which we denote by the random variable, $\tilde{Y}$, with realization Y. $\tilde{Y}$ provides information about $\tilde{u}$, the value of the risky asset. $\tilde{Y}$ is given by

$$\tilde{Y} = \tilde{u} + \tilde{\zeta}$$

where $\tilde{\zeta}$ has a normal distribution, with mean zero and precision $\Delta > 0$, and is

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[3] It is not essential that traders know the realization of public information when choosing whether to acquire information, but at least they must know the announcement is coming. This is because (as shown in the proof of Lemma 4) the acquisition decision does not depend on the conditional mean of the risky asset's return, but only on the conditional precision, which is known before observing the actual release.
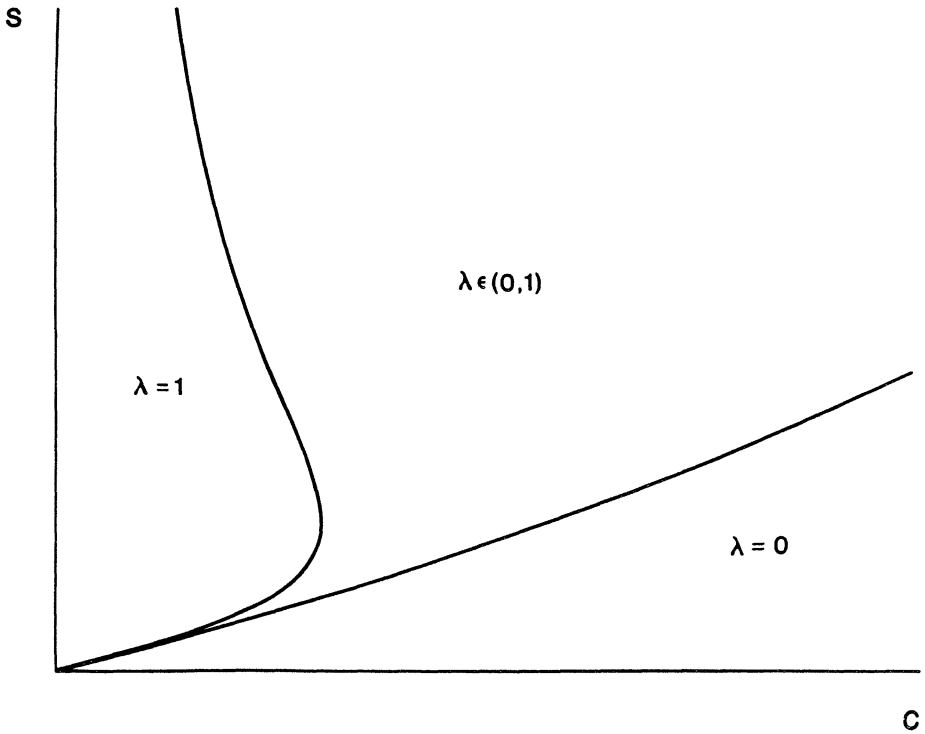
S

$\lambda \in (0,1)$

$\lambda = 1$

$\lambda = 0$

C

Figure 1. Determinants of λ; The Fraction of Informed Traders

A	B	C	D	E	F	G
Receive own endowment	Learn firm's information release policy	Observe Public Information	Choose Whether to Acquire Private Information	Observe Private Information	Trade	Consume
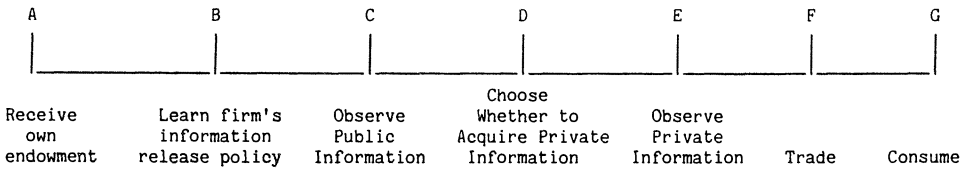
Figure 2. Time Sequence of Events

independent of all other exogenous random variables in the model, $\tilde{u}$, $\tilde{\varepsilon}_t$, and $\tilde{x}_t$.

The release of public information increases the precision of all traders' information, whether or not they acquire private information. Lemma 4 describes how the release of public information influences an individual trader's information acquisition decision and how public information affects the equilibrium value of λ.

LEMMA 4.

(1) *The optimal information acquisition decision (in equilibrium) for a trader who observes the public information $\tilde{Y} = \tilde{u} + \tilde{\zeta}$ (where $\tilde{\zeta}$ has precision Δ and mean zero) is identical to that of the trader in a market in which he and all other traders have prior information of precision $h_0 + \Delta$ (rather than h_0).*

(2) *The equilibrium value of* λ *is given by substituting* $h_0 + \Delta$ *for* h_0 *in the expressions in Lemma* 3.

Proof:

(1) Releasing public information, $\tilde{Y}$, increases the conditional precision of $\tilde{u}$ for all traders by Δ: normal distribution theory (see Degroot [5, p. 54]) implies that the conditional precision for uninformed traders is $h_0 + \Delta$; for informed traders, it is $h_0 + \Delta + s$ (because $\tilde{\varepsilon}_t$ and $\tilde{\zeta}$ are independent). Releasing $\tilde{Y}$ also makes the conditional mean of $\tilde{u}$ vary (it becomes $Y_0 + \dfrac{h_0^{-1}}{h_0^{-1} + \Delta^{-1}}(Y - Y_0)$ rather than Y_0). However, the conditional mean, Y_0, does not influence $\dfrac{U(\hat{s} = s, \hat{c} = c; \lambda)}{U(\hat{s} = 0, \hat{c} = 0; \lambda)}$, because it enters only through the term $\exp(-xY_0/r)$ which factors out of the numerator and denominator. Individual decisions when $\tilde{Y}$ is released are equivalent to those in an economy with prior precision $h_0 + \Delta$, because the optimal pair $(\hat{s}, \hat{c})$ is identical in the two economies.

(2) Because all individual decisions in the presence of public information are equivalent to a prior precision of $\tilde{u}$ equal to $h_0 + \Delta$, the proof of Lemma 3 follows, substituting $h_0 + \Delta$ for h_0 in all expressions.   Q.E.D.

Together, Lemma 3 and Lemma 4 characterize the effect of public information on private information production. There are three cases. In case (a), where no one acquires any information even in the absence of public release, the equilibrium remains one with no private acquisition. In case (c), where a proper subset of traders acquire information when there is no public announcement, the fraction λ of traders who acquire information is reduced, reducing the aggregate expenditure on information. In case (b), where *all* traders acquire costly information when there is no public release, the effect depends on the magnitude of Δ, the incremental precision of the public announcement. If Δ is small, then it may have no impact on private information production. This corresponds to a situation where all traders would acquire information when there is no release, even if its costs were increased. If the announcement's precision exceeds the critical value $\Delta = \dfrac{s}{e^{2c/r} - 1} - h_0 - \dfrac{(rs)^2}{V}$, then it reduces private information acquisition. In summary, an announcement of public information of sufficient precision will reduce private information production, unless there is initially no such private information production to reduce.

We can now characterize the smallest precision of public information which will result in the elimination of incentives for private production of information when $\lambda \in (0, 1)$ in the absence of a release. The case of $\lambda = 1$ is discussed at the end of this section, and when $\lambda = 0$, there is no private information even without a release.

LEMMA 5. *If, in the equilibrium without release of information,* $\lambda \in (0, 1)$, *then the smallest precision,* Δ, *of public information which results in no private acquisition of information is the incremental level of precision above* h_0, *the prior*

precision, which would have been obtainable from observing the price alone in the equilibrium without release. This level of precision is $\Delta = (rs\lambda)^2/V$, or in terms of exogenous parameters, $\Delta = \dfrac{s}{e^{2c/r-1}} - h_0$.

Proof: The precision of a trader's posterior distribution of $\tilde{u}$ conditional on the public signal is $h_0 + \Delta$. If $\lambda \in (0, 1)$ in the absence of a public signal, then from Lemma 3 we know

$$e^{2c/r} - 1 = \frac{s}{h_0 + \dfrac{(r\lambda s)^2}{V}}.$$

Let $\hat{h}_0$ denote the precision available to a trader without acquiring private information. If $\hat{h}_0 \geq h_0 + (r\lambda s)^2/V$, rather than the initial value $\hat{h}_0 = h_0$, then $\lambda = 0$ becomes the equilibrium fraction of informed traders given a public signal, $\hat{Y}$, as case (a) of Lemma 3 shows. Any value of $\Delta < (rs\lambda)^2/V$ is inconsistent with the conditions for $\lambda = 0$ given in Lemma 3, because an announcement of that precision yields $\hat{h}_0 < h_0 + (r\lambda s)^2/V$. Q.E.D.

Having characterized the information release policies which eliminate the incentives for private information production, we turn now to analyze their optimality. Public information has at least two types of effects on the expected utility of traders. It can change the equilibrium amount of public and private information obtained by traders, both of which influence the distribution of asset prices. In addition to changing the price distribution, it can change the total expenditure on information, depending on the relative costs of producing public information by the firm and of private information by traders.

Under very general conditions, the firm will have a cost advantage in producing the public information described in Lemma 2, as compared with the total costs of all investors who produce private information when there is no release. This is because the amount of information obtainable from price alone is less than the aggregate precision using all information produced by traders, because of the noise which prevents full relevation and complete aggregation. Unless the firm has a large information cost disadvantage relative to investors, its information costs will be much lower because it needs to produce much less information. In addition, it may be reasonable to assume that the firm has a cost advantage in producing information about itself. In many situations the firm might be assumed to have no incremental cost of producing information over the cost of producing information used for production and control.

Lemma 6 shows that even if one assumes that the firm's costs are similar to a trader's, the firm's costs are negligible relative to the aggregate of all traders.

LEMMA 6. *Assume the firm can produce any amount of information at a constant cost per unit of precision, c/s, equal to the cost per unit of precision of each trader. Then the firm's cost of producing information which eliminates private information acquisition is negligible relative to the aggregate expenditure on information when $\lambda \in (0, 1)$.*

Proof: With no release, the total private expenditure on information is $T \lambda c$, where T is the total number of traders. The total cost of producing public information of precision $(r\lambda s)^2/V$ is $c(r\lambda)^2 s/V$. For the large T of a public firm, the firm's information costs are negligible relative to $T \lambda c$. Further, if $V > (r\lambda)^2 s$, the firm's total costs are less than the per capita costs of the private information produced without any release.   Q.E.D.

The interpretation of Lemma 6 is that the per investor costs of producing public information of precision $\Delta = (rs\lambda)^2/V$ is essentially zero for a large firm. Under any conditions when this cost is essentially zero, Proposition 1 provides a very strong characterization of the optimal information release policy.

Conditional on a realization, Y, of public information, the expectation of $\tilde{u}$ is

$$Y_0 + \frac{h_0^{-1}}{h_0^{-1} + \Delta^{-1}}(Y - Y_0),$$ and the precision of $\tilde{u}$ is $h_0 + \Delta$. As a result, each

trader's expected utility (conditional on Y) can be written using the expression

in Lemma 2 by first replacing the expectation Y_0 with $Y_0 + \dfrac{h_0^{-1}}{h_0^{-1} + \Delta^{-1}}(Y - Y_0)$,

then replacing the precision h_0 with $h_0 + \Delta$, and finally substituting the new equilibrium of λ. The ex ante expected utility of a trader in a firm in which public information will be announced is then the expectation of this conditional expected utility, taking the expectation with respect to possible realizations of information, $\tilde{Y}$. This final expectation reflects the fact that additional information does not reduce the risk of the asset, but simply provides earlier resolution of uncertainty.

PROPOSITION 1.   *If the per trader cost of producing public information of precision* $\Delta = (rs\lambda)^2/V$ *is zero, and if* $\lambda \in (0, 1)$, *then releasing information of this precision is the unique Pareto optimum: all investors have higher ex ante expected utility with this policy than with any other.*

Proof: The technical aspects are in the Appendix. The expected utility of each trader t, net of information costs, is the same in equilibrium if each becomes informed or uninformed, $U_t(\hat{s} = s, \hat{c} = c; \lambda) = U_t(\hat{s} = 0, \hat{c} = 0; \lambda)$, because $\lambda \in (0, 1)$: call this level of expected utility $\bar{U}_t$. Focus on the expression for expected utility if a trader chooses to be uninformed and spend zero on information. If, instead, the firm releases public information, his share of costs will similarly be zero. In the Appendix, it is shown that the trader's expected utility is greater (than $\bar{U}_t$) if there is a public release of precision $\Delta = (rs\lambda)^2/V$, which eliminates private information acquisition. This public release leads all traders to choose to refrain from additional information acquisition (see Lemma 4), implying that each trader's expected utility is greater than $\bar{U}_t$ when this public information is released, and that all are better off than in the absence of a public release.

To see that this is the unique optimum level of precision to announce, note that public information of lower precision will yield a positive fraction of informed traders, and one can apply the above argument with this positive fraction in place of the original λ to show that all traders can be made better off. The reason that precision greater than $\Delta = (rs\lambda)^2/V$ is suboptimal also allows us to charac-

terize the case where $\lambda = 0$ in the absence of a release. Public information with greater precision cannot reduce private information acquisition below zero, and thus does not further influence acquisition. The announcement simply provides all traders with better information, and it is well known (see, e.g., Hakansson, Kunkel, and Ohlson [14]) that with constant absolute risk aversion and pure exchange, increased public information reduces risk sharing and makes investors worse off (see Appendix for proof in terms of this model). Information release may leave traders indifferent, only if initial endowments provided optimal risk sharing, but this case is ruled out by the stochastic endowments.   Q.E.D.

Proposition 1 characterizes the firm's optimal information release policy when it has costs per unit of precision less than or equal to those faced by traders. The private incentive to acquire costly private information when there is no release differs from the social optimum because the private information reduces the amount of risk sharing among investors because informed investors take large "speculative" positions on the basis of their information. For this reason, if a freely enforceable agreement for all traders to refrain from private information acquisition were available, it would make them better off. In fact, an enforceable agreement for all traders to refrain from information production would lead to Pareto superior allocations compared to even the optimal public release policy because even public information can reduce risk sharing somewhat. Such an agreement is not easily enforceable, because if other traders are uninformed, each has a private incentive to acquire information, and private information production cannot be observed by other traders. The optimal release policy in this case is to release the smallest amount of information which will eliminate private acquisition. When comparing changes in levels of information obtained by all investors (rather than improving one's own information holding others' constant), *less* information is preferred to more. The value of public release is that it homogenizes information and eliminates the use of resources to produce information. All traders are made better off: those who would have been informed; those who would have been uninformed; potential short sellers; and even traders who were not endowed with any shares of the firm. A key to this unanimity is the fact that the firm needs to produce much less information than the aggregate of all traders, implying that the cost of the firm producing the information is negligible, and no relevant questions of how the costs are shared across investors arise.

The cost savings are an important component of the welfare improvement. In addition, the removal of the trader-specific risk associated with trading on private information is important. The corollary to Proposition 1 states that even when the firm has a very large cost disadvantage, the release of information may possibly make traders better off. The cost disadvantage considered assumes that the firm's cost of producing a single signal with precision $(rs\lambda)^2/V$ is as large as the aggregate cost of *every* trader acquiring a signal of precision s (not simply the fraction λ of traders who would acquire information in the absence of a public announcement).

COROLLARY TO PROPOSITION 1.   *Assume that $\lambda \in (0, 1)$ and that the part of the cost of producing information for a single release of precision $(r\lambda s)^2/V$ paid by a*

trader is c, implying that there are no cost savings relative to private information acquisition. Then, for some parameter values, traders are better off and for other values are worse off with the public release. Two sufficient conditions for beneficial public release are available. If noise is sufficiently large ($V \to \infty$), then all traders are better off with this costly public release, or if the absolute value of a trader's endowment of risky assets is sufficiently large ($|x_t| \to \infty$), then that trader is better off with this costly public release.

    Proof: See Appendix.

    This corollary shows that the cost savings are not the entire explanation for the beneficial effects of public release of information because a welfare improvement is possible without cost savings. The explanation for the sufficiency of large noise is that this implies that the price contains very little information, and traders' beliefs can be very different if they acquire private information. The possibility of having very different beliefs translates into the possibility of traders undertaking very large speculative positions in the risky asset. This implies very unequal risk sharing. In sharp contrast to the analysis without the possibility of information acquisition, the release of public information here improves risk sharing, by making information more homogeneous.

    One way to explain this improved risk sharing is to consider what would happen if the realization of traders' endowments, x_t for $t = 1, \cdots, T$, happened to be Pareto optimal. The full information Pareto optimal allocation of risk here is for each trader to bear an equivalent amount of risk, and therefore, $x_t = x$ for all t would represent a Pareto optimum. If this were the realization of endowments, and some traders acquire private information, then traders would trade away from this allocation (to a non-Pareto optimal allocation) with probability one. Recall that traders observe only their own endowment and the per capital average is stochastic, so individual rationality does not rule out this trade.

    The importance of the absolute value of the initial endowment is due to the effect of homogenizing information on the variability of the market price of the risky asset. Homogenizing information can reduce the variability of the market price. The unconditional mean endowment is zero, and as a result the net trade at the variable prices is large when a trader's endowment differs greatly from its expected value of zero.

A. Information Which All Traders Acquire: The Case of $\lambda = 1$

    We have not yet analyzed the optimal information release policies when $\lambda = 1$ and all traders acquire costly information. Lemma 3 shows that when information has very low cost per unit of precision and when there is substantial noise, then everyone will acquire information; $\lambda = 1$ if and only if the conditions from part (b) of Lemma 3 hold, or

$$e^{2c/r} - 1 \le \frac{s}{h_0 + \dfrac{(rs)^2}{V}}.$$

If this is an equality, we have an interior solution with $\lambda = 1$: all traders are

indifferent between being informed or not when each conjectures that all others will be informed. A strict inequality implies a corner solution where all prefer to be informed. A public announcement of low precision Δ will not influence any trader's information acquisition if

$$e^{2c/r} - 1 \leq \frac{s}{h_0 + \dfrac{(rs)^2}{V} + \Delta} .$$

Releasing an amount of information which does not reduce λ makes traders worse off (similar to the $\lambda = 0$ case). Releasing any amount which reduces the fraction of traders acquiring information to a positive number less than one is suboptimal, because Proposition 1 shows that a further release (to imply no privately informed traders) would make traders better off. The two possible optimal release policies are to release no public information and to release just enough to eliminate private acquisition: which policy is better depends on parameter values.

The result of Proposition 1 that costless public release makes traders strictly better off applies to parameter values which imply an "interior solution" with $\lambda = 1$ (conditions when Lemma 3, part (b) holds as an equality). The expected utility of a trader given a public release of precision just sufficient to eliminate private information acquisition, and expected utility given no release, are both continuous functions of c, the cost of information. Expected utility given public release is a strictly *increasing* function of c, given all other parameters, over the interval $[0, c^*]$, where c^* is the cost level which implies $\lambda = 1$ as an interior solution. This is because the only effect on welfare of reducing c below c^* is to increase the precision, $\Delta = s/(e^{2c/r} - 1) - h_0$, of public information which must be released to eliminate private acquisition. Expected utility when there is no public release is strictly decreasing in c, given other parameters, for $c \in [0, c^*]$, because $\lambda = 1$ for all c in that interval, and the only effect on welfare of increasing c is to increase all traders' information costs.

The results in the previous paragraph allow the following characterization of the optimal release policy. There is a value of the cost of information, $\bar{c} \in (0, c^*)$, such that for $c \in (0, \bar{c})$ the optimal policy is no release and for $c \in (\bar{c}, c^*)$ the optimal policy is to release public information.

III. Extensions and Generalizations

The analysis has used many strong assumptions to allow a full characterization of the effects of releasing information. Although it is not possible to show what the results would be without these assumptions, some insights are available into the robustness of the results and possible extensions. Based on results in Admati [1] which analyzes models without information acquisition like Diamond-Verrecchia [7], but with many assets, it appears clear that our results would carry over to a many-firm model if the information which each could disclose was firm-specific and did not influence information production decisions of the stockholders of other firms.

A. Normality

The assumption of normal distributions in this model is made primarily for tractability and does not appear critical to the results. However, one feature of the assumed joint normality of asset returns and public and private information, while quite general, is not universal. This is the feature that releasing more public information reduces the value of acquiring a given piece of private information. The public information and private information in this model are substitutes rather than complements. Analysis in Milgrom-Weber [19] suggests that this property is closely related to public and private information being affiliated random variables. Roughly, this means that they are positively correlated. An example in which the release of public information might increase the value of private information (be complementary types of information) would be an announcement that a firm's value was in an extreme tail of its distribution, without specifying which tail. This would signal an increased value of sampling estimates of the firm's value. This paper's results depend on public and private information being substitutes rather than complements.

B. Heterogeneity

The result that *all* traders are made better off with public release does depend on some degree of homogeneity of traders, although not as much homogeneity as is assumed here is needed. There is literature focusing on how heterogeneity can lead to differences of opinion on disclosure policy (e.g., Hakansson [12, 13]). There is a degree of heterogeneity of traders already present in the model—some choose to be informed and some uninformed when $\lambda \in (0, 1)$ in the absence of release. Allowing either of two cost levels for information, c_1 for some traders and $c_2 > c_1$ for others will not change our results, so long as the costs are not too different. For example, if most of the traders face low costs and find it worthwhile to acquire information, the public disclosure will generally result in a Pareto improvement because the improved risk sharing more than compensates the low-information-cost traders for giving up their cost advantage. If most traders face high costs, and in the absence of public release only the few low-cost traders would acquire information, then release will generally make the low-information-cost traders worse off. This is because there is very little room for improving risk sharing when only a few traders would have been informed, so there is little social gain to compensate the few low-cost traders for their private loss of an informational cost advantage. Similar arguments show that sufficient heterogeneity of risk tolerances will lead to disagreement over the optimal disclosure policy: very risk-tolerant traders will benefit little from the improved risk sharing caused by public information, and they will be the ones who would be willing to pay the most for private information because their high risk tolerance implies that they will undertake large absolute value of trades based on private information.

There is one group of traders who deserve comment because they face very low cost access to information—insiders. Insiders who would have been allowed to trade on the basis of their inside information in the absence of public release would, by the above argument, be made worse off by disclosure. We have not

analyzed insiders in this model, but the model does have some implications for the study of insider trading. Insiders, unlike other traders with low cost information, can be compensated directly for losing their informational advantage. They can be prohibited from trading or required to disclose before trading. Although this appears to be a feasible contracting arrangement between stockholders and managers, it appears to be rare. The model suggests that informed insider trading will not have a very large impact on traders if the number of insiders is small, implying that λ is close to zero. If this is the case, allowing a few insiders to trade before disclosing their information might not have a very large impact on the welfare of other traders. If insiders were permitted to sell (or could not be stopped from selling) their information to many outsiders instead of simply trading for themselves and then disclosing it to the public, then the existence of insiders would dramatically change the analysis in this paper. The point of these comments is to show that the behavior of insiders will not necessarily change our conclusions.

C. Discretion

We assume that the firm can precommit to a policy of releasing information, e.g., by setting a date within a quarter that it announces its earnings. For certain types of news, the arrival of potential reports is stochastic, and there may be a problem of firm discretion in making announcements. For example, in Miller-Rock [20], firms want to release good news but withhold bad news. Traders are aware of this incentive and are consequently not fooled, but without an ability to precommit, firms may still have incentives to try to fool investors by using costly methods to convey information, as in Miller-Rock [20]. For some analysis of discretionary release, see Verrecchia [28]. These models cannot explain why firms do not do their best to precommit not to release information because traders are made strictly worse off by the information release policy. Adding this paper's results on the beneficial effects of public disclosure on private information acquisition to the analysis of discretionary release suggests an explanation of why costly release policies persist, such as informative dividend policy or signalling by delaying convertible bond calls, and why we rarely observe firms trying to commit to noninformative policies. This would be an interesting area for future work.

D. Production

A very interesting extension of this model would allow information to have value in formulating the firm's production decisions.[4] In this case, additional information acquired directly by the firm, or inferred from the informative equilibrium price, will improve the distribution of output. Some results on optimal disclosure given this structure are available without additional analysis. If firm

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[4] Allen [2], independently of this paper, develops a framework for modeling the use of private information for production decisions and concludes that private information can be socially useful. The model in that paper is not structured to address information production or disclosure by firms— its only source of information is private information acquisition by traders.

managers have access to an information technology which is at least as good as that faced by traders and if there is no major incentive problem between managers and outside security holders which is complicated by private information, then all acquisition ought to be centralized with the manager. This is because the price aggregates information imperfectly because of noise; transmitting a given amount of information to the manager is more costly if information is produced on the outside (see Lemma 5). In this case, the only reason for disclosure is its effect on private information acquisition by outside security holders, which duplicates the manager's information, and the results of this paper follow exactly.

If outside security holders have access to some information at costs substantially lower than managers, then it may be beneficial for traders to acquire large amounts of information. In fact, the amount which they acquire in the absence of a release may be less than the amount which maximizes the value of the firm's output. In this case, there will be reasons to commit to policies which encourage outside information production by traders. It is not clear that this case is of practical importance, but it is worthy of further analysis, as are cases where there are some types of information for which the manager has a cost advantage and others a cost disadvantage.

E. Proprietary Information

The outcome of a disclosure policy motivated by the considerations in the model would be a security market in which most traders do not acquire information which is available to many traders at a positive cost. The aggregate expenditure on information acquisition would be greatly reduced by the firm's disclosure policy. A literal interpretation of the results would lead to the empirically unsupported result that no security analysis is undertaken. Firms release somewhat less information than the model predicts. In part, this reflects the proprietary nature of some information: releasing certain information may hurt the firm's competitive position and result in higher costs of disclosure than modeled here. This would suggest that private information acquisition should be most profitable for proprietary information.[5]

The large number of security analysts currently employed is also at odds with a literal interpretation of the results. The existence of proprietary information may again be part of the explanation, but one may also interpret much of the work of such analysts as processing of public information in an accessible form. In this view, the beneficial role of disclosure also includes a reduction in such analysts' costs of operation. Competition would tend to pass such savings along to traders. More disclosure should also make analysts' forecasts more similar, providing the risk-sharing benefits analyzed in the text.

IV. Conclusions

The analysis has demonstrated that there is an important role for disclosure of public information even when traders have no direct use for the information

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[5] It is interesting that partial disclosure of proprietary information may serve to signal implications for valuation of undisclosed information, see Bhattacharaya-Ritter [4]. There may also be a link between disclosure of proprietary and nonproprietary information (see Dye [8]).

other than for adjusting their portfolios. The release serves to change the incentives for production of private information by traders, and the public release protects traders from themselves, because in the absence of the release, a trader's individual welfare can be improved, given the decisions of other traders, by acquiring private information, but collectively they are worse off with the private acquisition. The results predict that firms will establish policies for releasing information, and that it will be the sort of information which traders could have acquired at some finite cost. If traders could never acquire information at any cost, then it need not be disclosed, because disclosing it does not influence their information production decisions.

When one allows for the various complications outlined above, one is still left with the central conclusion of this model. The firm itself represents a natural coalition for maximizing the welfare of its security holders. A policy of releasing information can make all (or most) security holders better off. However, individual traders operating noncooperatively in their own individual interest would find it profitable to acquire more information and reduce welfare as a result. Information production firms (like expanded bond rating companies) who contract directly with only a small subset of traders would not necessarily choose the optimal policies because they would maximize the value of the information sold. The Pareto optimal policies do not maximize the amount of the information which might be sold regarding the firm. In fact, they minimize the amount. The firm as a coalition tends to be the entity which contracts with such rating companies, because the firm as a coalition can be given the proper incentives. Other entities may validate or even produce public information, but the model's prediction is that it is the firm who mandates and pays for the release of public information.

Appendix

Proof of Proposition 1: The expected utility of a trader when there is no public release is given in Lemma 2. As a first step toward calculating expected utility given a public release, when $\lambda \in (0, 1)$ or $\lambda = 1$ as an interior solution, we write the expected utility of a trader conditional on a particular realization of public information, $\tilde{Y} = Y$, of precision $\Delta \geq \dfrac{(rs\lambda)^2}{V}$ (which eliminates private information production). As described in the text, this involves substitution in the Lemma 2 expression of the conditional values for the unconditional values of mean (Y_0) and precision (h_0) of the distribution of $\tilde{u}$, and substituting the new equilibrium value of λ, which is zero. Conditional on $\tilde{Y} = Y$, expected utility is given by

$$
-\left(\frac{h_0 + \Delta}{\left(1 + \dfrac{V}{r^2(h_0 + \Delta)}\right)(h_0 + \Delta)} \right)^{1/2}
$$

$$
\cdot \exp\left(\frac{-x\left\{ Y_0 + \dfrac{h_0^{-1}}{h_0^{-1} + \Delta^{-1}}(Y - Y_0) \right\} - B}{r} + \frac{1}{2}\left(\frac{x}{r}\right)^2\left[h_0 + \Delta + \frac{r^2(h_0 + \Delta)^2}{V} \right]^{-1} \right)
$$

where $\Delta \geq \dfrac{(rs\lambda)^2}{V}$ and the λ in this inequality is the value in the absence of release. Taking the expectation with respect to the normal random variable $\tilde{Y}$ yields the unconditional expected utility of a trader with public release:

$$-\left(\left(\frac{h_0 + \Delta}{\left(1 + \dfrac{V}{r^2(h_0 + \Delta)}\right)(h_0 + \Delta)}\right)^{1/2}\right.$$

$$\cdot \exp\left(\frac{-xY_0 - B}{r} + \frac{1}{2}\left(\frac{x}{r}\right)^2\left[h_0 + \Delta + \frac{r^2(h_0 + \Delta)^2}{V}\right]^{-1}\right)$$

$$\left.\cdot\left(E\left[\exp\left(\left(-\frac{x}{r}\right)\cdot\left(\frac{h_0^{-1}}{h_0^{-1} + \Delta^{-1}}\right)\cdot(\tilde{Y} - Y_0)\right)\right]\right)\right).$$

Evaluating the expectation with respect to the normal random variable, $\tilde{Y} - Y_0$, using the property of its moment generating function, yields

$$E\left[\exp\left(\left(-\frac{x}{r}\right)\cdot\left(\frac{h_0^{-1}}{h_0^{-1} + \Delta^{-1}}\right)\cdot(\tilde{Y} - Y_0)\right)\right] = \exp\left(\frac{1}{2}\right)\left(\frac{x}{r}\right)\left(\frac{2(h_0^{-1})^2}{h_0^{-1} + \Delta^{-1}}\right).$$

This allows the following closed-form expression for the expected utility of a trader given the public release with $\Delta \geq \dfrac{(rs\lambda)^2}{V}$:

$$-\left(\left(\frac{h_0 + \Delta}{\left(1 + \dfrac{V}{r^2(h_0 + \Delta)}\right)(h_0 + \Delta)}\right)^{1/2}\right.$$

$$\left.\cdot \exp\left(\frac{-xY_0 - B}{r} + \frac{1}{2}\left(\frac{x}{r}\right)^2\left\{\left[h_0 + \Delta + \frac{r^2(h_0 + \Delta)^2}{V}\right]^{-1} + \frac{(h_0^{-1})^2}{h_0^{-1} + \Delta^{-1}}\right\}\right)\right) \equiv R.$$

It is immediate that $R(\Delta)$ is a strictly decreasing function of Δ, over the domain $\Delta \in \left[\dfrac{(r\lambda s)^2}{V}, \infty\right)$; the region in which $\lambda = 0$. This is the traditional negative value of this public information on risk sharing with exponential utility. For the balance of this analysis of public information release, $\Delta = \dfrac{(r\lambda s)^2}{V}$ will be assumed.

Because the per trader costs of public information are negligible, the cost of information of a trader is identical to that of an uninformed trader in the $\lambda \in (0, 1)$ economy without public release. To establish that, for all $\lambda \in (0, 1)$, all traders are better off with the public release of precision $\Delta = \dfrac{(r\lambda s)^2}{V}$, we must show for $\lambda \in (0, 1)$ that

$$R\left(\Delta = \frac{(r\lambda s)^2}{V}\right) > U(\hat{s} = 0, \hat{c} = 0; \lambda). \tag{A1}$$

We have already demonstrated that without release and with $\lambda \in (0, 1)$, $U(\hat{s} = 0, \hat{c} = 0; \lambda) = U(s = \hat{s}, c = \hat{c}; \lambda)$. Define

$$\Delta = \frac{(rs\lambda)^2}{V} \quad \text{(for the balance of the Appendix)}$$

$$R_1 \equiv \frac{h_0 + \Delta}{\left(1 + \dfrac{V}{r^2(h_0 + \Delta)}\right)(h_0 + \Delta)},$$

$$R_2 \equiv \left[h_0 + \Delta + \frac{r^2(h_0 + \Delta)^2}{V}\right]^{-1} + \frac{(h_0^{-1})^2}{h_0^{-1} + \Delta^{-1}}$$

$$L_1 \equiv \frac{h_0}{\dfrac{(rVh_0)^2 + h_0 V(V + r^2\lambda s)^2}{(rVh_0 + r\lambda s(V + r^2\lambda s))^2}(h_0 + \Delta)}$$

$$L_2 \equiv \left[h_0 + \frac{(rVh_0)^2}{V(V + r^2\lambda s)^2}\right]^{-1} \quad \text{and}$$

$$G \equiv \exp\left(\frac{-xY_0 - B}{r}\right).$$

(A1) is rewritten as

$$-(R1)^{1/2} \cdot \exp\left(\frac{1}{2}\left(\frac{x}{r}\right)^2 \cdot R_2\right) \cdot G > -(L_1)^{1/2} \cdot \exp\left(\frac{1}{2}\left(\frac{x}{r}\right)^2 \cdot L_2\right) \cdot G. \qquad \text{(A2)}$$

To establish (A2), it is sufficient that $(L_1)^{1/2} > (R_1)^{1/2}$ and $L_2 > R_2$. To establish $(L_1)^{1/2} > (R_1)^{1/2}$, we write this as (where we drop the subscript on h_0),

$$\left(\frac{h}{\dfrac{(rVh)^2 + hV(V + r^2 \lambda s)^2}{(rV + r \lambda s(V + r^2 \lambda s))^2}\left(h + \dfrac{(r \lambda s)^2}{V}\right)}\right)^{1/2}$$

$$> \left(\frac{h + \dfrac{(r \lambda s)^2}{V}}{1 + \dfrac{V}{h + \dfrac{(r \lambda s)^2}{V}}\left(h + \dfrac{(r \lambda s)^2}{V}\right)}\right)^{1/2}$$

All terms are positive: preserve the inequality by squaring both sides, multiply

both by $h + \dfrac{(r\,\lambda\,s)^2}{V}$ and define $A = (V + r^2\,\lambda\,s)$ and $B = r\,\lambda\,s$, yielding

$$\frac{h}{\dfrac{(rVh) + hVA^2}{(rVh + BA)^2}} > \frac{h + \dfrac{B^2}{V}}{1 + \dfrac{\dfrac{V}{\left(h + \dfrac{B^2}{V}\right)r^2}}{}}.$$

Rewrite this as

$$\frac{h[r^2V^2h^2 + B^2A^2 + 2BArVh]}{r^2V^2h^2 + hVA^2} > \frac{\left(h + \dfrac{B^2}{V}\right)r^2\left(h + \dfrac{B^2}{V}\right)}{\left(h + \dfrac{B^2}{V}\right)r^2 + V}.$$

Cross multiply, substitute for A and B, and cancel identical terms yielding

$$r^2\lambda^2s^2V^4 + r^6\lambda^4s^4V^2 + 2r^4s^3\lambda^3V^3 + 2r^2\lambda s V^4h > 0,$$

which holds because all terms are positive, establishing the original inequality. To establish $L_2 > R_2$ (where we again drop the subscript on h_0 and define $B \equiv r\,\lambda\,s$), write this as

$$\left[h + \frac{r^2Vh^2}{(V+rB)^2}\right]^{-1} > \left[h + \frac{B^2}{V} + \left(h + \frac{B^2}{V}\right)^2\frac{r^2}{V}\right]^{-1} + \frac{\dfrac{B^2}{V}}{\dfrac{B^2}{V}h + h^2}.$$

Rewrite this as

$$\frac{(V+rB)^2}{h(V+rB)^2+r^2Vh^2} > \frac{V^3}{hV^3+B^2V^2+h^2r^2V^2+r^2B^4+2hr^2B^2} + \frac{B^2}{B^2h+Vh^2}$$

$$= \frac{2V^3B^2h+V^4h^2+B^4V^2+r^2B^2h^2+r^2B^6+2hr^2B^4V}{(hV^3+B^2V^2+h^2r^2V^2+r^2B^4+2hr^2B^2V)(B^2+Vh^2)}.$$

Multiplying both sides by h, cross-multiply (all terms are positive), and cancel identical terms, yielding

$$2hr^3B^5V^2 + 2h^3r^3BV^4 + 2hr^3B^5V^2 + 2h^2r^3B^3V^3 > 0$$

and the original inequality is established.

Proof of Corollary: The expected utility of a trader given the public release of precision $\Delta = \dfrac{(r\,\lambda\,s)^2}{V}$, at cost c to the trader is given by

$$-R_1^{1/2}\cdot\exp\left(\frac{1}{2}\left(\frac{x}{r}\right)^2\cdot R_2\right)\cdot H \equiv R^*$$

where $H \equiv \exp((-\,xY_0 - B - c)/r)$.

A trader is better off with release if and only if

$$R^* > \bar{U} = U(\hat{s} = s, \hat{c} = c; \lambda).$$ (A3)

From Lemma 2,

$$U(\hat{s} = s, \hat{c} = c; \lambda) = -L_3^{1/2} \cdot \exp\left(\frac{1}{2}\left(\frac{x}{r}\right)^2 \cdot L_2\right) \cdot H,$$

where

$$L_3 \equiv \frac{h_0}{\dfrac{(rVh_0)^2 + h_0 V(V + r^2 \lambda s)^2}{(rVh_0 + r \lambda s(V + r^2 \lambda s))^2} (h_0 + \Delta + s)}.$$

We proved above that $L_2 > R_2$, and we know $H > 0$. However, for some parameter values $L_3 < R_1$, and this allows cases where (A3) is false. For example, let $c = h_0 = r = V = 1$, $s = 2$, $\lambda = \frac{1}{2}$, and $x = 0$. Then $R^* = -\frac{9}{10}e^{-3}$ while $U(\hat{s} = s, \hat{c} = c; \hat{\lambda} = \lambda) = -\frac{9}{20}e^{-3}$.

A sufficient condition for $L_3 > R_1$ and for (A3) to hold is $V \to \infty$. Rewrite L_3 by multiplying by $\dfrac{h_0}{h_0}$, then multiplying both sides of $L_3 > R_1$ by V^{-2}, yielding

$$\frac{rVh_0 + r \lambda s(V + r^2 \lambda s)^2}{\left(r^2 h_0 + \dfrac{(V + r^2 \lambda s)^2}{V}\right)\left(h_0 + \dfrac{(r \lambda s)^2}{V} + s\right)} > \frac{1}{\left(\dfrac{1}{V} + \dfrac{1}{\left[h_0 + \dfrac{(r \lambda s)^2}{V}\right]r^2}\right)}.$$

The left-hand side tends to ∞ as $V \to \infty$, which the limit of right-hand side as $V \to \infty$ is $h_0 r^2$.

Finally, for any finite value of parameters, $L_3 > 0$ and finite, therefore (A3) is true for sufficiently large values of x^2.

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