Optimal risk sharing for law invariant monetary utility functions

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Abstract

We consider the problem of optimal risk sharing of some given total risk between two economic agents characterized by law-invariant monetary utility functions. We first prove existence of an optimal risk sharing allocation which is in addition increasing in terms of the total risk. We next provide an explicit characterization in the case where both agents' utility functions are comonotone. The general form of the optimal contracts turns out to be given by a sum of options (stop-loss contracts, in the language of insurance) on the total risk. In order to show the robustness of this type of contracts to more general utility functions, we introduce a new notion of strict risk aversion conditionally on lower tail events, which is typically satisfied by the semi-deviation and the entropic utility functions. Then, in the context of an AV@R-agent facing an agent with strict monotone preferences and exhibiting strict risk aversion conditional on lower tail events, we prove that optimal contracts again are European options on the total risk.

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