Optimal Scheduling of Capture Times in a Multiple Capture Imaging System

Ting Chen and Abbas El Gamal

Information Systems Laboratory
Department of Electrical Engineering
Stanford University
Stanford, California 94305, USA

ABSTRACT

Several papers have discussed the idea of extending image sensor dynamic range by capturing several images during a normal exposure time. Most of these papers assume that the images are captured according to a uniform or an exponentially increasing exposure time schedule. Even though such schedules can be justified by certain implementation considerations, there has not been any systematic study of how capture time schedules should be optimally determined. In this paper we formulate the multiple capture time scheduling problem when the incident illumination probability density function (pdf) is completely known as a constrained optimization problem. We aim to find the capture times that maximize the average signal SNR. The formulation leads to a general upper bound on achievable average SNR using multiple capture for any given illumination pdf. For a uniform pdf, the average SNR is a concave function in capture times and therefore well-known convex optimization techniques can be applied to find the global optimum. For a general piece-wise uniform pdf, the average SNR is not necessarily concave. The cost function, however, is a Difference of Convex (D.C.) function and well-established D.C. or global optimization techniques can be used.

Keywords: Signal-to-Noise Ratio(SNR), Dynamic Range(DR), multiple capture, high speed imaging, image sensor

1. INTRODUCTION

High frame rate CMOS image sensors with non-destructive readout capabilities have been recently demonstrated.[1,2] As discussed in many papers,[3,4] this capability can be used to enhance the sensor dynamic range. The idea is to capture several images during a normal exposure time − shorter integration time images capture the brighter areas of the scene while longer integration time images capture the darker areas of the scene. This method has been shown to achieve better SNR than other schemes such as logarithmic sensors and well capacity adjusting.[5]

One important problem in the implementation of multiple capture that has not received much attention is the selection of the number of captures and their time schedule to achieve a desired image quality. Several papers[4,5] have assumed exponentially increasing capture time schedules, while others[6,7] have assumed uniformly spaced capture time schedules. Even though such schedules can be justified by certain implementation considerations, there has not been any systematic study of how optimal capture time schedules can be determined. By finding optimal schedules, one can achieve the image quality requirements with less captures, which is desirable since reducing the number of captures reduces the imaging system computational power, memory, and power consumption requirements as well as the noise generated by the multiple readouts.

To determine the capture time schedule, scene illumination information is needed. Such information may not be available before taking one or more captures of the scene. Thus in general an "online" scheduling algorithm that determines the time for the next capture based on updated information about the scene illumination gathered during previous captures may be needed. Finding such optimal online schedules appears intractable. Instead, in this paper, we investigate the time capture scheduling problem assuming

Correspondence: Email: tingchen@isl.stanford.edu, abbas@isl.stanford.edu; Telephone: 650-725-9696; Fax: 650-723-8473

Sensors and Camera Systems for Scientific, Industrial, and Digital Photography Applications III, Morley M. Blouke, John Canosa, Nitin Sampat, Editors,
Proceedings of SPIE Vol. 4669 (2002) · 0277-786X/02/$15.00

complete scene illumination information. This can be viewed as an "offline" version of the problem. Optimal solutions to the offline scheduling problem can provide guidance for developing online scheduling heuristics as well as performance bounds on such heuristics.

In the following section we provide background on the image sensor pixel model assumed, define SNR and dynamic range, and briefly discuss the multiple capture method. In Section 3, we formulate the multiple capture time scheduling problem when the incident illumination probability density function (pdf) is completely known. We use average pixel signal SNR as an objective function since it generally provides good indication of image quality. To simplify the analysis, we assume that read noise is much smaller than shot noise and thus can be ignored. With this assumption the simple Last-Sample-Before-Saturation (LSBS) algorithm for synthesizing high dynamic range image from multiple captures is optimal with respect to SNR.[6] We use this formulation to establish a general upper bound on achievable average SNR using multiple capture for any given illumination pdf. In Section 4 we show that for a uniform pdf, the average SNR is a concave function in capture times and therefore the global optimum can be easily found using well-known convex optimization techniques. In Section 5 we show that for a general piece-wise uniform pdf, the average SNR is not necessarily concave. The cost function, however, is a Difference of Convex (D.C.) function and well-established D.C. or global optimization techniques can be used.

2. BACKGROUND

We assume image sensors operating in direct integration, *e.g.*, CCDs and CMOS PPS, APS, and DPS. Figure 1 depicts a simplified pixel model and the charge $Q(t)$ versus time t for such sensors. During capture, each pixel converts incident light into photocurrent i_{ph}. The photocurrent is integrated onto a capacitor and the charge $Q(T)$ (or voltage) is read out at the end of exposure time T. Dark current i_{dc} and additive noise corrupt the photocharge. The noise can be expressed as the sum of three independent components, (i) shot noise $U(T) \sim \mathcal{N}(0, q(i_{ph} + i_{dc})T)$, where q is the electron charge, (ii) readout circuit noise $V(T)$ (including quantization noise) with zero mean and variance σ_V^2, and (iii) reset noise and FPN C with zero mean and variance σ_C^2. Thus the output charge from a pixel can be expressed as

$$Q(T) = \begin{cases} (i_{ph} + i_{dc})T + U(T) + V(T) + C, & \text{for } Q(T) \leq Q_{sat} \\ Q_{sat}, & \text{otherwise,} \end{cases}$$

where Q_{sat} is the saturation charge, also referred to as *well capacity*. The SNR can be expressed as

$$\mathrm{SNR}(i_{ph}) = \frac{(i_{ph}T)^2}{q(i_{ph} + i_{dc})T + \sigma_V^2 + \sigma_C^2} \qquad \text{for } i_{ph} \leq i_{max}, \tag{1}$$

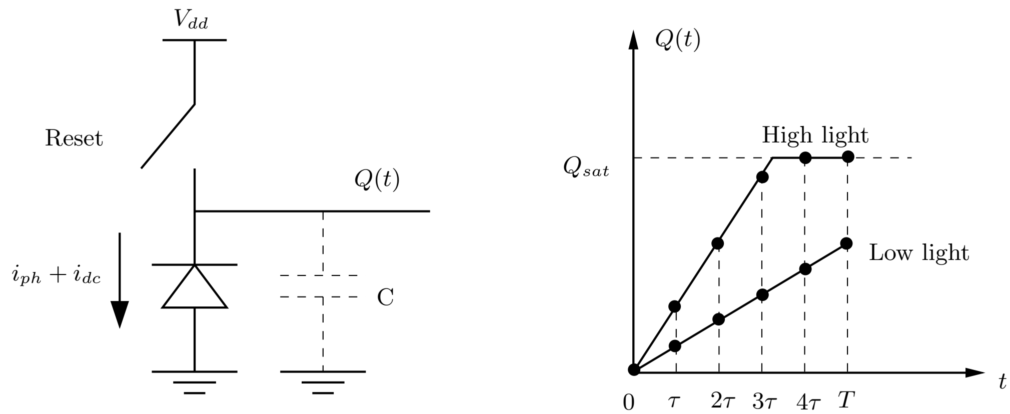
where $i_{max} \approx Q_{\mathrm{sat}}/T$ refers to the maximum non-saturating photocurrent. Note that SNR increases with i_{ph}, first at 20dB per decade when reset, FPN and readout noise dominate, then at 10dB per decade when shot noise dominates. SNR also increases with T. Thus it is always preferable to have the longest possible exposure time. However, saturation and motion impose practical upper bounds on exposure time.

Sensor dynamic range[8] is defined as the ratio of the maximum non-saturating photocurrent $i_{\max}$ to the smallest detectable photocurrent $i_{\min} = \frac{q}{T}\sqrt{\frac{1}{q} i_{dc} T + \sigma_V^2 + \sigma_C^2}$. Dynamic range can be extended by capturing several images during exposure time without resetting the photodetector.[3,4] Using the Last-Sample-Before-Saturation (LSBS) algorithm[4] dynamic range can be extended at the high illumination end as illustrated in Figure 1(b). Liu *et. al.* , have shown how multiple capture can also be used to extend dynamic range at the low illumination end using weighted averaging. Their method reduces to the LSBS algorithm when only shot noise is present.[6]

3. PROBLEM FORMULATION

We assume complete knowledge of the scene induced photocurrent pdf * and seek to find the capture time schedule $\{t_1, t_2, ..., t_N\}$ for N captures that maximizes the average SNR with respect to the given pdf $f_I(i)$

*This is equivalent to knowing the scene illumination pdf for a known sensor response.



(a) (b)

Figure 1. (a) Photodiode pixel model, and (b) Photocharge $Q(t)$ vs. Time t under two different illuminations. Assuming multiple capture at uniform capture times $\tau, 2\tau, \ldots, T$ and using the LSBS algorithm, the sample at T is used for the low illumination case, while the sample at 3τ is used for the high illumination case.

(see Figure 2). We assume that the pdf is zero outside a finite length interval $(i_{\min}, i_{\max})$. For simplicity we ignore all noise terms except for shot noise due to photocurrent. Let i_k be the maximum non-saturating photocurrent for capture time t_k, $1 \leq k \leq N$. Thus

$$t_k = \frac{Q_{\text{sat}}}{i_k},$$

and determining $\{t_1, t_2, ..., t_N\}$ is equivalent to determining the set of currents $\{i_1, i_2, ..., i_N\}$. The SNR as a function of photocurrent is now given by

$$\text{SNR}(i) = \frac{Q_{\text{sat}} i}{q i_k}$$

for $i_{k+1} < i \leq i_k$ and $1 \leq k \leq N$. To avoid saturation we assume that $i_1 = i_{\max}$.

Under these assumptions, the **capture time scheduling problem** is as follows:

Given $f_I(i)$ and N, find $\{i_2, ..., i_N\}$ that maximizes the average SNR

$$E\left(SNR(i_2, ..., i_N)\right) = \frac{Q_{\text{sat}}}{q} \sum_{k=1}^{N} \int_{i_{k+1}}^{i_k} \frac{i}{i_k} f_I(i)\, di, \tag{2}$$

subject to: $0 \leq i_{\min} = i_{N+1} < i_N < \ldots < i_k < \ldots < i_2 < i_1 = i_{\max} < \infty$.

Upper bound: Note that since we are using the LSBS algorithm, $\text{SNR}(i) \leq \frac{Q_{\text{sat}}}{q}$ and thus for any N,

$$\max E\left(\text{SNR}(i_1, i_2, ..., i_N)\right) \leq \frac{Q_{\text{sat}}}{q}.$$

This provides a general upper bound on the maximum achievable average SNR using multiple capture. Now, for a single capture with capture time corresponding to $i_{\max}$, the average SNR is given by

$$E\left(\text{SNR}_{\text{SC}}\right) = \frac{Q_{\text{sat}}}{q} \int_{i_{\min}}^{i_{\max}} \frac{i}{i_{\max}} f_I(i)\, di = \frac{Q_{\text{sat}} E(I)}{q i_{\max}},$$

where $E(I)$ is the expectation (or average) of the photocurrent i for given pdf $f_I(i)$. Thus for a given $f_I(i)$, multiple capture can increase average SNR by no more than a factor of $i_{\max}/E(I)$.

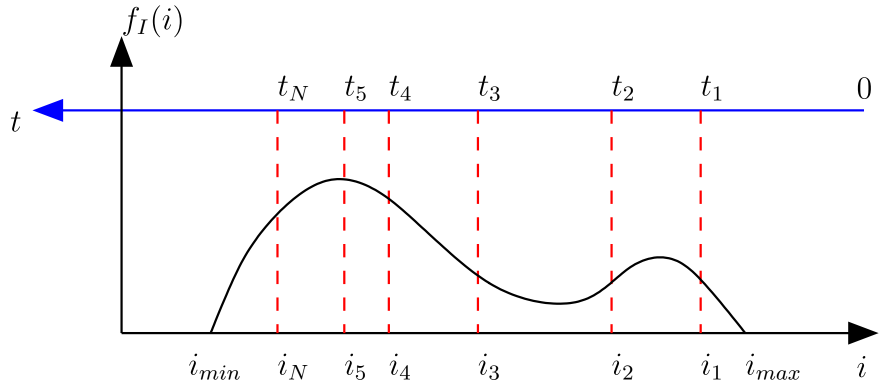


Figure 2. Photocurrent pdf showing capture times and corresponding maximum non-saturating photocurrents. Note that the positions of capture times are not drawn in scale.

4. OPTIMAL SCHEDULING FOR UNIFORM PDFS

In this section we show how our scheduling problem can be optimally solved when the photocurrent pdf is uniform.

In this case, the scheduling problem becomes:

Given a uniform photocurrent illumination pdf over the interval $(i_{\min}, i_{\max})$ *and* N*, find* $\{i_2, ..., i_N\}$ *that maximizes the average SNR*

$$E\left(SNR(i_2, ..., i_N)\right) = \frac{Q_{\text{sat}}}{q(i_{\max} - i_{\min})} \sum_{k=1}^{N} (i_k - \frac{i_{k+1}^2}{i_k}), \tag{3}$$

subject to: $0 \leq i_{\min} = i_{N+1} < i_N < \ldots < i_k < \ldots < i_2 < i_1 = i_{\max} < \infty$.

Note that for $2 \leq k \leq N$, the function $(i_k - \frac{i_{k+1}^2}{i_k})$ is concave in the two variables i_k and i_{k+1} (which can be readily verified by showing that the Hessian matrix is negative semi-definite). Since the sum of concave functions is concave, the average SNR is a concave function in $\{i_2, ..., i_N\}$. Thus the scheduling problem reduces to a convex optimization problem with linear constraints, which can be optimally solved using well known convex optimization techniques such as gradient/sub-gradient based methods. Table 4 provides examples of optimal schedules for 2 to 10 captures assuming uniform photocurrent pdf over $(0, 1]$. Note that the optimal capture times are quite different from the commonly assumed uniform or exponentially increasing time schedules. Figure 3 compares the optimal average SNR to the average SNR achieved by uniform and exponentially increasing schedules. To make the comparison fair, we assumed the same maximum exposure time for all schedules. Note that using our optimal scheduling algorithm, with only 10 captures, the $E(\text{SNR})$ is within 14% of the upper bound. This performance cannot be achieved with the exponentially increasing schedule and requires over 20 captures to achieve using the uniform schedule.

5. SCHEDULING FOR PIECE-WISE UNIFORM PDFS

In the real world, not too many scenes exhibit uniform illumination statistics. On the other hand, the optimization problem for general pdfs appears to be quite intractable. In this section we investigate the scheduling problem for piece-wise uniform illumination pdfs. Since any pdf can be approximated by a piece-wise uniform pdf, solutions for piece-wise uniform pdfs can provide good approximations to solutions of the general problem. Such approximations are illustrated in Figures 4 and 5. The empirical illumination pdf of the scene in Figure 4 has two non-zero regions corresponding to direct illumination and the dark shadow regions, and can be reasonably approximated by a 2 segment piece-wise uniform pdf. The empirical pdf of the scene in Figure 5, which contains large regions of low illumination, some moderate illumination regions,

Capture Scheme	Optimal Exposure Times (t_k/t_1)									
	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}
2 Captures	1	2	–	–	–	–	–	–	–	–
3 Captures	1	1.6	3.2	–	–	–	–	–	–	–
4 Captures	1	1.44	2.3	4.6	–	–	–	–	–	–
5 Captures	1	1.35	1.94	3.1	6.2	–	–	–	–	–
6 Captures	1	1.29	1.74	2.5	4	8	–	–	–	–
7 Captures	1	1.25	1.61	2.17	3.13	5	10	–	–	–
8 Captures	1	1.22	1.52	1.97	2.65	3.81	6.1	12.19	–	–
9 Captures	1	1.20	1.46	1.82	2.35	3.17	4.55	7.29	14.57	–
10 Captures	1	1.18	1.41	1.71	2.14	2.76	3.73	5.36	8.58	17.16

Table 1. Optimal capture time schedules for a uniform pdf over interval $(0, 1]$.

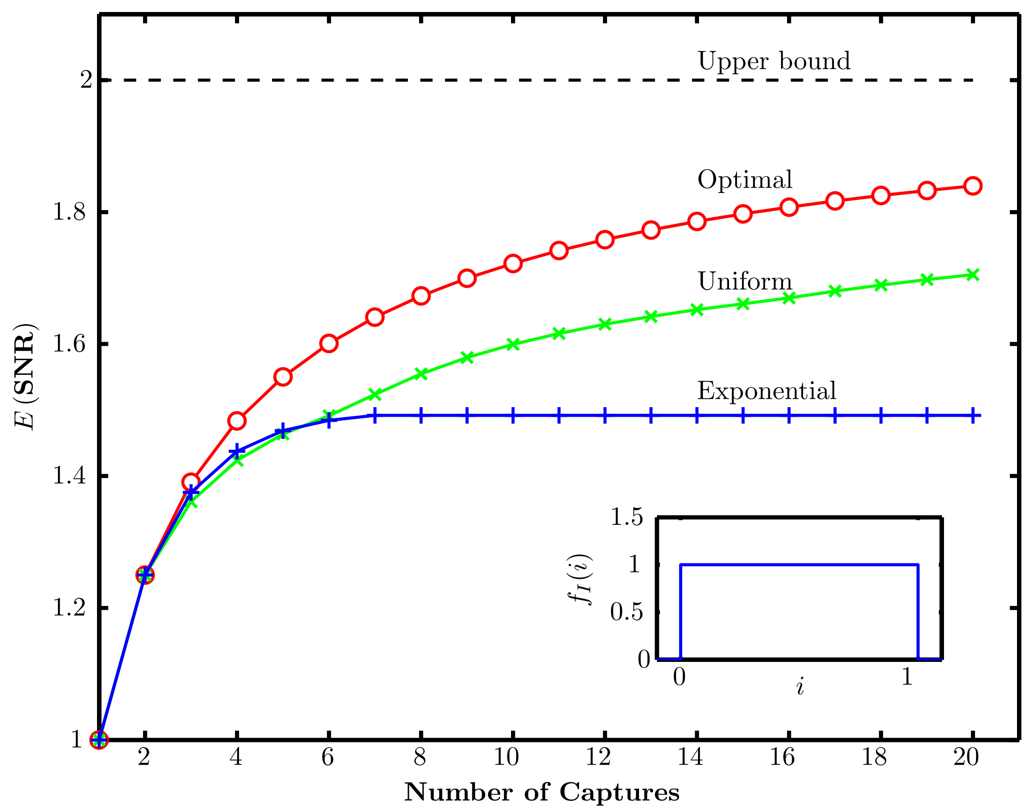


Figure 3. Performance comparison of optimal schedule, uniform schedule, and exponential (with exponent = 2) schedule. $E\,(\mathrm{SNR})$ is normalized with respect to the single capture case with $i_1 = i_{max}$.

and small very high illumination regions is approximated by a 3 segment piece-wise uniform pdf. Of course better approximations of the empirical pdfs can be obtained using more segments, but as we shall see, solving the scheduling problem becomes more complex as the number of segments increases.

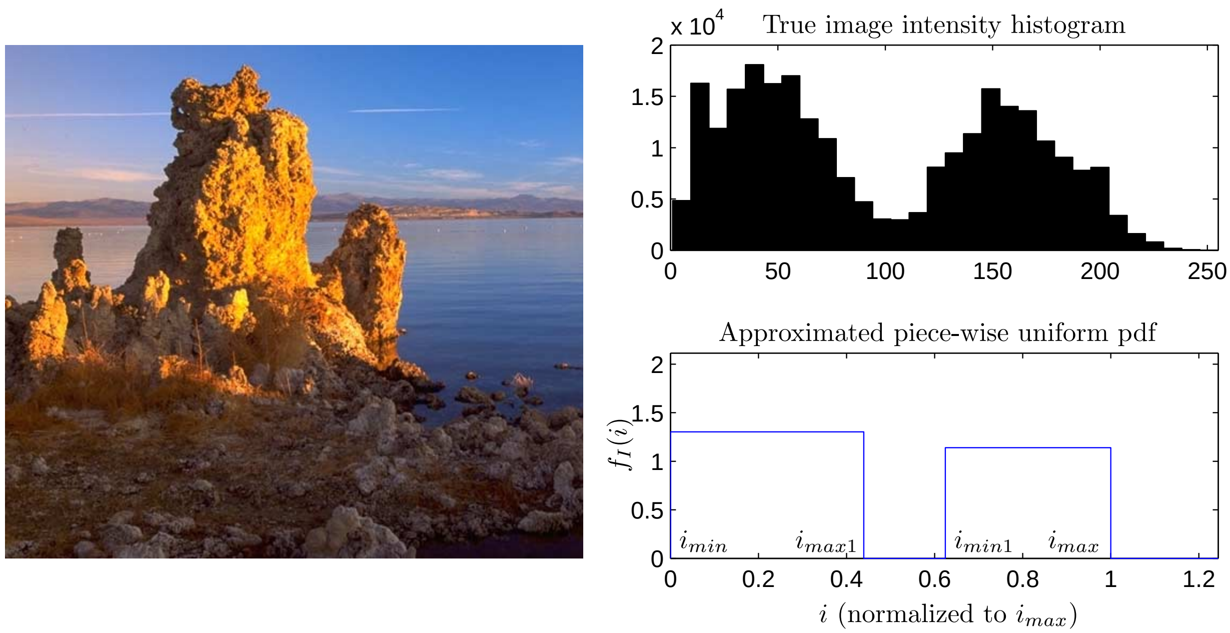


Figure 4. An image with approximated two-segment piece-wise uniform pdf

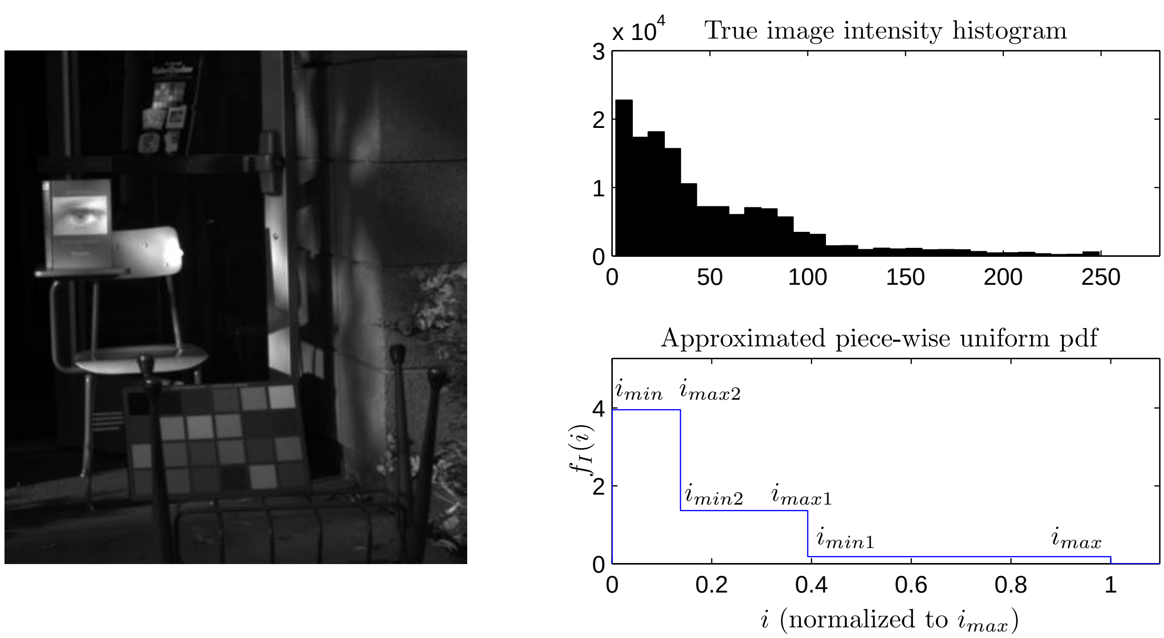


Figure 5. An image with approximated three-segment piece-wise uniform pdf

First consider the scheduling problem for a two-segment piece-wise uniform pdf. We assume that the pdf is uniform over the intervals (i_{min}, i_{max1}), and (i_{min1}, i_{max}). Clearly, in this case, no capture should be assigned to the interval (i_{max1}, i_{min1}), since one can always do better by moving such a capture to i_{min1}. Now, assuming that k out of the N captures are assigned to segment (i_{min1}, i_{max}), the scheduling problem becomes:

Given a 2-segment piece-wise uniform pdf with k captures assigned to interval (i_{min1}, i_{max}) and $N-k$ captures to interval (i_{min}, i_{max1}), find $\{i_2, ..., i_N\}$ that maximizes the average SNR

$$E\left(SNR(i_2,...,i_N)\right) = \frac{Q_{\text{sat}}}{q}\left(c_1\sum_{j=1}^{k-1}(i_j - \frac{i_{j+1}^2}{i_j}) + c_1(i_k - \frac{i_{\text{min1}}^2}{i_k}) + c_2\frac{i_{\text{max1}}^2 - i_{k+1}^2}{i_k} + c_2\sum_{j=k+1}^{N}(i_j - \frac{i_{j+1}^2}{i_j})\right), \tag{4}$$

where the constants c_1 and c_2 account for the difference in the pdf values of the two segments,

subject to: $0 \le i_{\min} = i_{N+1} < i_N < \ldots < i_{k+1} < i_{\max1} \le i_{\min1} \le i_k < \ldots < i_2 < i_1 = i_{\max} < \infty$.

The optimal solution to the general 2-segment piece-wise uniform pdf scheduling problem can thus be found by solving the above problem for each k and selecting the solution that maximized the average SNR.

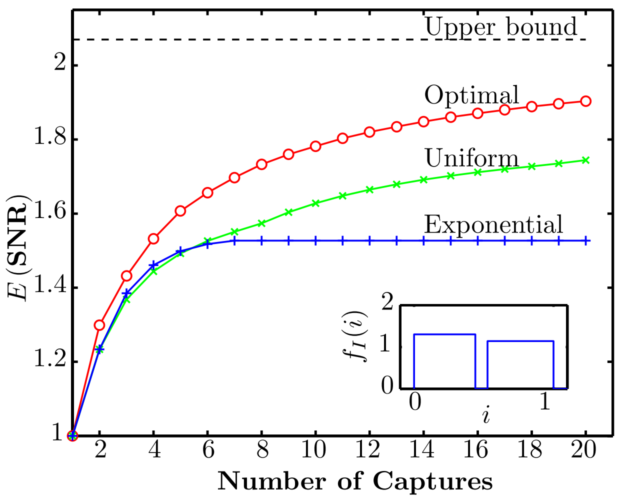
Simple investigation of the above equation shows that $E\left(\text{SNR}(i_2,...,i_N)\right)$ is concave in all the variables except i_k. Certain conditions such as $c_1 i_{min1}^2 \ge c_2 i_{max1}^2$ can guarantee concavity in i_k as well, but in general the average SNR is not a concave function. A closer look at equation (4), however, reveals that $E\left(\text{SNR}(i_2,...,i_N)\right)$ is a Difference of Convex (D.C.) function,[9,10] since all terms involving i_k in equation (4) are concave functions of i_k except for $c_2 i_{max1}^2/i_k$, which is convex. This allows us to apply well-established D.C. optimization techniques (*e.g.*, see [9,10]). It should be pointed out, however, that these D.C. optimization techniques are not guaranteed to find the global optimal.

In general, it can be shown that average SNR is a D.C. function for any M-segment piece-wise uniform pdf with a prescribed assignment of the number of captures to the M segments. Thus to numerically solve the scheduling problem with M-segment piece-wise uniform pdf, one can solve the problem for each assignment of captures using D.C. optimization, then choose the assignment and corresponding "optimal" schedule that maximizes average SNR.

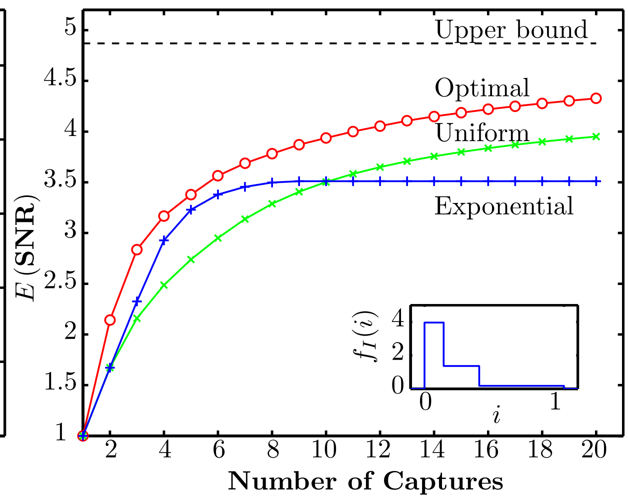
One particularly simple yet powerful optimization technique that we have experimented with is Sequential Quadratic Programming (SQP) with multiple randomly generated initial conditions. Figures 6(a) and (b) compare the solution using SQP with 10 random initial conditions to the uniform schedule and the exponentially increasing schedule for the two piece-wise uniform pdfs of Figures 4 and 5. Due to the simple nature of our optimization problem, we were able to use brute-force search to find the globally optimal solutions, which turned out to be identical to the solutions using SQP. Note that unlike other examples, in the 3-segment example, the exponential schedule initially outperforms the uniform schedule. The reason is that with few captures, the exponential assigns more captures to the large low and medium illumination regions than the uniform.

6. DISCUSSION

The paper presented the first systematic study of optimal selection of capture times in a multiple capture imaging system. Previous papers on multiple capture have assumed uniform or exponentially increasing capture time schedules justified by certain practical implementation considerations. It is advantageous in terms of system computational power, memory, power consumption, and noise to employ the least number of captures required to achieve a desired dynamic range and SNR. To do so, one must carefully select the capture time schedule to optimally capture the scene illumination information. To develop understanding of the scheduling problem and as a first step towards developing online algorithms, we formulated the offline scheduling problem, *i.e.*, assuming complete prior knowledge of scene illumination pdf, as an optimization problem where average SNR is maximized for a given number of captures. Ignoring read noise and FPN and using the LSBS algorithm, our formulation leads to a general upper bound on the average SNR for any illumination pdf. For a uniform illumination pdf, we showed that the average SNR is a concave function in capture



(a)



(b)

Figure 6. Performance comparison of the optimal schedule, uniform schedule, and exponential (exponent = 2) schedule for the scenes in Figures 4 and 5. $E\,(\mathrm{SNR})$ is normalized with respect to the single capture case with $i_1 = i_{max}$.

times and therefore the global optimum can be found using well-known convex optimization techniques. For a general piece-wise uniform illumination pdf, the average SNR is not necessarily concave. Average SNR is, however, a D.C. function and can be solved using well-established D.C. or global optimization techniques.

The offline scheduling algorithms we discussed can be directly applied in situations where enough information about scene illumination is known in advance. It is not unusual to assume the availability of such prior information. For example all auto-exposure algorithms used in practice, assume the availability of certain scene illumination statistics. Our results can also be used to develop heuristic online algorithms that can perform better than the offline algorithm with only partial information of scene illumination statistics. In situations where read noise and FPN is too high to neglect, our results may not be completely satisfactory, since the LSBS algorithm is no longer optimal and dynamic range can also be extended at the low illumination end as shown by Liu *et. al.* .[6] Our upper bound on average SNR of Q_{sat}/q still holds, however.

ACKNOWLEDGEMENTS

The work reported in this paper was partially supported by the Programmable Digital Camera (PDC) Project by HP, Agilent, Kodak and Canon. The authors would like to thank the members of the PDC project for suggestions that greatly improved the presentation of the paper.

REFERENCES

1. S. Kleinfelder, S. Lim, X. Liu, and A. El Gamal, "A 10,000 Frame/s 0.18um CMOS Digital Pixel Sensor with Pixel-Level Memory," in *2001 ISSCC Digest of Technical Papers*, pp. 88–89, (San Francisco, CA), February 2001.
2. D. Handoko, S. Kawahito, Y. Todokoro, and A. Matsuzawa, "A CMOS Image Sensor with Non-Destructive Intermediate Readout Mode for Adaptive Iterative Search Motion Vector Estimation," in *2001 IEEE Workshop on CCD and Advanced Image Sensors*, pp. 52–55, (Lake Tahoe, CA), June 2001.
3. O. Yadid-Pecht and E. Fossum, "Wide intrascene dynamic range CMOS APS using dual sampling," *IEEE Trans. on Electron Devices* **44**(10), pp. 1721–1723, 1997.
4. D. Yang, A. El Gamal, B. Fowler, and H. Tian, "A 640x512 CMOS image sensor with ultra-wide dynamic range floating-point pixel level ADC," *IEEE J. Solid-State Circuits* **34**(12), pp. 1821–1834, 1999.

5. D. Yang and A. El Gamal, "Comparative Analysis of SNR for Image Sensors with Enhanced Dynamic Range," in *Proceedings of SPIE*, vol. 3649, pp. 197–211, (San Jose, CA), January 1999.
6. X. Liu and A. El Gamal, "Photocurrent Estimation from Multiple Non-destructive Samples in a CMOS Image Sensor," in *Proceedings of SPIE*, vol. 4306, pp. 450–458, (San Jose, CA), January 2001.
7. D. Handoko, S. Kawahito, Y. Takokoro, M. Kumahara, and A. Matsuzawa, "A CMOS Image Sensor for Focal-plane Low-power Motion Vector Estimation," in *Symposium of VLSI Circuits*, pp. 28–29, June 2000.
8. A. El Gamal, *EE392b Classnotes: Introduction to Image Sensors and Digital Cameras*, http://www.stanford.edu/class/ee392b, Stanford, California, 2001.
9. R. Horst and H. Tuy, *Global optimization: deterministic approaches*, Springer, New York, 1996.
10. R. Horst, P. M. Pardalos, and N. V. Thoai, *Introduction to global optimization*, Kluwer Academic, Boston, Massachusetts, 2000.