

## Optimal Scheduling of Smart Homes Energy Consumption with Microgrid

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**Abstract**—Microgrid is taken as the future Smart Grid, and can work as a local energy provider to domestic buildings and reduce energy expenses. To further lower the cost, a Smart Homes idea is suggested. Smart Homes of the future will include automation systems and could provide lower energy consumption costs and comfortable and secure living environment to end users. If the energy consumption tasks across multiple homes can be scheduled based on the users' requirement, the energy cost and peak demand could be reduced. In this paper the optimal scheduling of smart homes' energy consumption is presented using mixed-integer linear programming. In order to minimize a one-day forecasted energy consumption cost, operation and electricity consumption tasks are scheduled based on different electricity tariffs, electricity task time window and forecasted renewable energy output. A case study of thirty homes with their own microgrid indicates the possibility of cost saving and asset utilization increase.

**Keywords:** *planning/scheduling; smart homes; microgrid; mixed-integer linear programming; optimization*

### I. INTRODUCTION

The future electricity distribution system will be meshed and intelligent and better known as Smart Grid, which includes advanced digital meters, distribution automation, communication systems and distributed energy resources. The desired Smart Grid functionalities include self-healing, optimizing asset utilization and minimizing operations and maintenance expenses [1]. As Smart Grid of the future, microgrid is considered to enable widespread inclusion of renewable resources, distributed storage and demand response programs in distribution [2].

Several studies have considered how to design the capacity of a microgrid system to minimize the annual cost [3]–[5]. A computer program that optimizes the equipment arrangement of each building linked to a fuel cell network and the path of the hot-water piping network under the cost minimization objective has also been developed [6]. Energy management systems and optimal scheduling of microgrid have been produced to generate an optimum operation plan for a microgrid [7]–[9]. Hawkes and Leach [10] presented a linear programming cost minimization model for the high level system design and corresponding unit commitment of generators and storages within a microgrid. Compared with centralized generation, the sensitivity analysis of results to variations in energy prices indicates a microgrid can offer an

economic proposition. This model can provide both the optimized capacities of candidate technologies as well as the operating schedule.

This paper considers a residential building with its own microgrid, distributed energy resources and automation system. The idea of the smart home originated from the concept of home automation, which provides some common benefits to the end users, which include lower energy costs, provision of comfort and security and provision of home-based health care and assistance to elderly or disabled users [11]. Several works have been proposed to achieve the energy conservation and management perspectives [12]–[16] but these scheduling optimization models only consider operation scheduling based on the given energy profile. Derin and Ferrante [17] described a model that concerns both operation scheduling and electricity consumption tasks order scheduling. However, their results indicate relatively a high computation time to schedule only three electricity consumption tasks.

Scheduling tasks subject to limited resources is a well known problem in many areas of the process industry and of other fields [18]–[22] but there are differences for the scheduling of electrical appliances. In this work, the objective is to minimize the total one-day-ahead expense of the Smart Homes' energy consumption, including the operation and energy cost. Both operation of the distributed energy generation technologies and domestic electrical appliances are scheduled based on the electricity price at each time interval, the renewable energy output forecast, subject to constraints on the starting and ending time for each appliance.

The rest of this paper is organized as follows: the mathematical model is shown in Section II, the case study is presented in Section III, the optimal results are given in Section IV, and the discussion and conclusions are given in Section V.

### II. MATHEMATICAL MODEL

The smart homes power consumption scheduling problem is formed as a mixed integer linear programming model. The daily power consumption tasks are scheduled based on their given operation time window (earliest starting time and latest ending time), and the objective is to minimize the daily power cost and shave the power consumption peak. It assumes the smart building system has its own microgrid and some decentralized resources (such as wind generator,

CHP generator, boiler, thermal storage and electrical storage) to provide the basic electricity and also it has a grid connection to obtain electricity during peak hours or sell electricity to the grid when there is surplus electricity generation. Since this model only provides the optimal scheduling for one day, equipment capacity selection is not considered and all the equipment capacities are given. The electricity price profile from the grid is given and may vary according to time of day. The model also assumes the weather forecast can provide 24 hour wind speed data.

A list of the notation used in the MILP model is given below:

#### Indices

$i$	tasks
$j$	homes in the smart building
$t$	time interval

#### Parameters

$a_t$	electricity price at time $t$ (£/kWh <sub>e</sub> )
$A$	wind generator blade area (m <sup>2</sup> )
$C_i$	power consumption capacity of task $i$
$C_{CHP}$	CHP generator capacity (kW <sub>e</sub> )
$C_{WIND}$	wind generator capacity (kW <sub>e</sub> )
$C_B$	boiler capacity (kW <sub>th</sub> )
$C_{ELE}$	electrical storage capacity (kWh <sub>e</sub> )
$C_{THS}$	thermal storage capacity (kWh <sub>th</sub> )
$CH_{ELE}$	electrical storage charge limit (kW <sub>e</sub> )
$CH_{THS}$	thermal storage charge limit (kW <sub>th</sub> )
$DI_{ELE}$	electrical storage discharge limit (kW <sub>e</sub> )
$DI_{THS}$	thermal storage discharge limit (kW <sub>th</sub> )
$E_{j,i}$	latest ending time of task $i$ in home $j$
$H_t$	head demand at time $t$ (kW <sub>th</sub> )
$m_{ELE}$	cost per unit input (maintenance) for electrical storage unit (£/kWh <sub>e</sub> )
$m_{THS}$	cost per unit input (maintenance) for thermal storage unit (£/kWh <sub>th</sub> )
$m_{WIND}$	wind generator maintenance cost (£/kWh <sub>e</sub> )
$n_E$	electricity selling price (£/kWh <sub>e</sub> )
$n_G$	price of natural gas (£/kWh)
$P_{j,i}$	processing time of task $i$ in home $j$
$S_{j,i}$	earliest starting time of task $i$ in home $j$
$v_t$	wind speed (m/s)
$w_i$	output from wind generator at time $t$ (kW <sub>e</sub> )
$\alpha$	CHP generator electrical efficiency
$\beta$	boiler efficiency
$\gamma$	CHP heat-to-power ratio
$\delta$	Time interval duration (hour)
$\rho$	air density (kg/m <sup>3</sup> )
$\eta_{ELE}$	electrical storage charge/discharge efficiency
$\eta_{THS}$	thermal storage charge/discharge efficiency
$\eta_W$	wind generator efficiency

#### Positive Variables

$Ex_t$	electricity exported to the grid at time $t$ (kW <sub>e</sub> )
$f_t$	thermal storage discharge rate at time $t$ (kW <sub>th</sub> )
$g_t$	thermal storage charge rate at time $t$ (kW <sub>th</sub> )
$In_t$	electricity imported from the grid at time $t$ (kW <sub>e</sub> )

$ISe$	initial state of electrical storage (kWh <sub>e</sub> )
$ISt$	initial state of thermal storage (kWh <sub>th</sub> )
$Q$	daily electricity cost of a home (£)
$Se_t$	electricity in storage at time $t$ (kWh <sub>e</sub> )
$St_t$	heat in storage at time $t$ (kWh <sub>th</sub> )
$wc_t$	output from CHP generator at time $t$ (kW <sub>e</sub> )
$x_t$	output from boiler at time $t$ (kW <sub>th</sub> )
$y_t$	electrical storage discharge rate at time $t$ (kW <sub>e</sub> )
$z_t$	electrical storage charge rate at time $t$ (kW <sub>e</sub> )

#### Binary Variables

$Te_{j,i,t}$	1 if task $i$ in home $j$ ends at time $t$ , 0 otherwise
$Ts_{j,i,t}$	1 if task $i$ in home $j$ starts at time $t$ , 0 otherwise
$w_{j,i,t}$	1 if task $i$ in home $j$ is done at time $t$ , 0 otherwise

The constraints imposed on the optimization are:

#### A. Capacity constraints:

The output from each equipment should not be over its designed capacity,

CHP generator:

$$wc_t \leq C_{CHP} \quad \forall 1 \leq t \leq T \quad (1)$$

Boiler:

$$x_t \leq C_B \quad \forall 1 \leq t \leq T \quad (2)$$

Electrical storage:

$$Se_t \leq C_{ELE} \quad \forall 1 \leq t \leq T \quad (3)$$

Thermal storage:

$$St_t \leq C_{THS} \quad \forall 1 \leq t \leq T \quad (4)$$

#### B. Energy storage constraints

Electricity stored in the electrical storage at time  $t$  is equal to the amount stored at  $t - 1$  plus the electricity charged minus the electricity discharged.

$$Se_t = Se_{t-1} + z_t \eta_{ELE} \delta - y_t \delta / \eta_{ELE} \quad \forall 1 \leq t \leq T \quad (5)$$

Electricity stored must return to the initial state at the end of the day (no nett accumulation over the whole day),

$$Se_0 = Se_T = ISe \quad (6)$$

The rates of discharge or charge of electricity cannot exceed the electrical storage discharge and charge limits:

$$y_t \leq DI_{ELE} \quad \forall 1 \leq t \leq T \quad (7)$$

$$z_t \leq CH_{ELE} \quad \forall 1 \leq t \leq T \quad (8)$$

Heat stored in the thermal storage at time  $t$  is equal to the amount stored at  $t - 1$  plus the heat charged minus the heat discharged.

$$St_t = St_{t-1} + g_t \eta_{THS} \delta - f_t \delta / \eta_{THS} \quad \forall 1 \leq t \leq T \quad (9)$$

Stored heat must return to the initial state at the end of the day,

$$St_0 = St_T = IS_t \quad (10)$$

The rates of discharge and charge of heat cannot exceed the thermal storage discharge and charge limits:

$$f_t \leq DI_{THS} \quad \forall 1 \leq t \leq T \quad (11)$$

$$g_t \leq CH_{THS} \quad \forall 1 \leq t \leq T \quad (12)$$

### C. Wind generator output

The electricity output from the wind generators is calculated from the wind power generation equation:

$$wi_t = 0.5 \rho A v_t^3 \eta_w \quad \forall t \quad (13)$$

### D. Energy balances

The electricity consumed during each time period is supplied by the wind generator, CHP generator, electricity received from the electrical storage and grid minus electricity sent to the electrical storage and grid.

$$\sum_j \sum_i w_{j,i,t} C_i = wi_t + wc_t + y_t - z_t + In_t - Ex_t \quad \forall 1 \leq t \leq T \quad (14)$$

The heat consumed during each time period is supplied by the CHP generator, boiler, heat received from the thermal storage minus heat sent to the electrical storage.

$$H_t = wc_t \gamma + x_t + f_t - g_t \quad \forall 1 \leq t \leq T \quad (15)$$

### E. Starting time and finishing time

The starting time of each task cannot be earlier than the given earliest starting time,

$$\sum_{t \geq S_{j,i}} Ts_{j,i,t} = 1 \quad \forall j, i \quad (16)$$

The finishing time of each task cannot be later than the latest ending time,

$$\sum_{t \leq E_{j,i}} Te_{j,i,t} = 1 \quad \forall j, i \quad (17)$$

If a task starts at time  $t$ , it must end at time  $t$  plus its processing time,  $P_{j,i}$ ,

$$Ts_{j,i,t} = Te_{j,i,t+P_{j,i}} \quad \forall j, i, 1 \leq t \leq T - P_{j,i} \quad (18)$$

All tasks must operate continuously from their starting time to their ending time.

$$w_{j,i,t} = w_{j,i,t-1} + Ts_{j,i,t} - Te_{j,i,t} \quad \forall j, i, 1 \leq t \leq T \quad (19)$$

### F. Objective function

The objective function is to minimize the total daily electricity cost, which includes: the operation and maintenance cost of the CHP generator, wind generator, electrical storage and thermal storage; the cost of electricity purchased from the grid; the revenue from electricity sold to the grid. Since the equipment capacities are fixed, their capital costs are independent of the schedule and are therefore not considered.

$$Q = \sum_t [(wc_t n_G / \alpha + wi_t m_{WIND} + In_t a_t - Ex_t n_E + x_t n_G / \beta + y_t m_{ELE} + f_t m_{THS}) \delta] \quad (20)$$

## III. CASE STUDY

The case study building system has 30 homes with the following energy suppliers:

- one CHP generator with a capacity of 40kW<sub>e</sub>, its electrical efficiency is 35%, heat to power ratio is 1.3, and natural gas cost is 2.7p/kWh;
- one wind farm with a capacity of 10kW<sub>e</sub> and a maintenance cost of 0.5p/kWh<sub>e</sub>;
- one boiler with capacity of 85kW<sub>th</sub> and natural gas cost is 2.7p/kWh;
- one electrical storage unit with a capacity of 10kW<sub>e</sub>h; the charge/discharge efficiency is 95%, discharge limit and charge limit are both 10kW<sub>e</sub>, and the maintenance cost is 0.5p/kWh<sub>e</sub>;
- one thermal storage unit with a capacity of 20kW<sub>th</sub>h; the charge/discharge efficiency is 98%, discharge limit and charge limit are both 20kW<sub>th</sub>, and the maintenance cost is 0.1p/kWh<sub>th</sub>;
- a grid connection (allowing import and export of electricity when operating parallel to the grid); the electricity price at different times is collected from Balancing Mechanism Reporting System [23] and shown in Fig.1 and when electricity is sold to the grid, it is 1p/kWh<sub>e</sub>;

Each time interval is half an hour, so in total there are 48 time intervals. The total heat demand profile is generated for a building with floor area of 2500m<sup>2</sup> on a sample winter day using CHP Sizer Version 2 Software [24]. For the electricity demand, each home has 12 basic tasks that consume electricity as shown in Table I. These tasks are available to

be scheduled according to the given earliest starting time, latest finishing time, their respective processing times and power requirements [25].

In this case study, there are 10 identical wind generators, with an efficiency of 47%. The blade diameter is 1.6m and the wind speed is generated from a Weibull distribution using MATLAB with a mean velocity of 7m/s. The cut-in and cut-out wind speeds are respectively 5m/s and 25m/s, and the nominal wind speed is 12m/s. The wind generators do not produce any power when the wind speed is under the cut-in speed or above the cut-out speed. When the wind speed is above the nominal wind speed, the power output is at the maximum output, which is equal to the output produced at the nominal wind speed. Between cut-in and nominal wind speed, the wind generator power output varies according to equation 13.

#### IV. RESULTS

##### A. Results with earliest starting time

The starting time of the case study is from 8am and the ending time is 8am the next morning. The earliest starting case (a scheduling heuristic) means all the domestic electricity appliances are turned on at their given earliest starting time. It is similar to common living habits: for example, the washing machine would be turned on as soon as people want to do some washing, most probably when people leave home for work in the morning. The optimal heat balance resulting from this case is shown in Fig. 2 and the optimal electricity balance is shown in Fig. 3. Equipment operation time from each technique is scheduled accordingly to minimize the total operation cost. The electrical storage is used to store electricity when there is excess electricity; it is mainly for utilizing the wind generator output more efficiently. There is no excess electricity sold to the utility grid. With the earliest starting time schedule, the peak hours are mainly during the evening when occupants are back from work. During this time, about 51% of the total electricity is imported from the utility grid. The total cost is £144.6.

##### B. Results with time window

When a time window is allowed, the domestic tasks as well as the equipment operation time are scheduled in order to minimize the total electricity cost (equation 20). Tasks, such as interior lighting and fridge, have fixed electricity consumption time period and have no other alternatives. But tasks with flexible operation time can be scattered as much as possible to avoid electricity consumption peak and utilize electricity generated from local generators as much as possible. But when electricity is cheaper from grid, it will be imported instead of generating from generators. The optimal heat balance is shown in Fig. 4 and the optimal electricity balance is shown in Fig. 5. There are only two peak periods: early in the morning and in the evening. The other time periods have a flat electricity consumption. Only 30% of the total electricity is bought from the utility grid, electricity being mainly provided by the local distributed resources, and there is no electricity sold back to the utility grid. The total cost is £117.5, which is 18.7% lower than the earliest starting time case. The solution was obtained using CPLEX 11.0.1 in GAMS 22.7 on a PC with an Intel Core 2 Duo, 2.99 GHz CPU and 3.25GB of RAM. The model involved 36,770 equations with 53,510 continuous and 52,920 discrete variables. This case required 7.5 CPU s to solve.

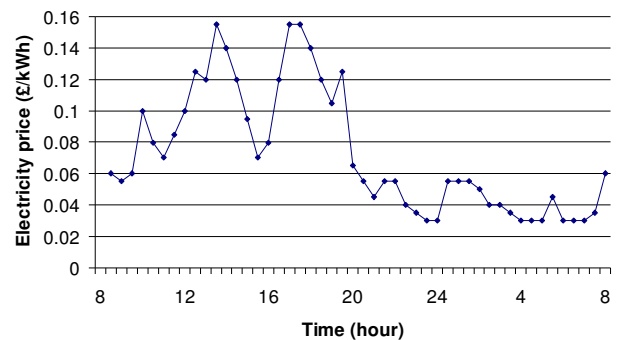


Figure 1. Electricity tariff (Jan 10<sup>th</sup>, 2010)

TABLE I. ELECTRICITY CONSUMPTION TASK

	Task	Power (kW)	Earliest starting time (hour)	latest Finishing time (hour)	Duration (hour)
1	Dish Washer	1	9	17	3
2	Washing machine	1	9	12	1.5
3	Spin Dryer	3	13	18	1
4	Cooker Hob	3	8	9	0.5
5	Cooker Oven	5	18	19	0.5
6	Cooker Microwave	1.7	8	9	0.5
7	Interior lighting	0.84	18	24	6
8	Laptop	0.1	18	24	2
9	Desktop	0.3	18	24	3
10	Vacuum cleaner	1.2	9	17	0.5
11	Fridge	0.3	0	24	24
12	Electrical car	3.5	18	8	3

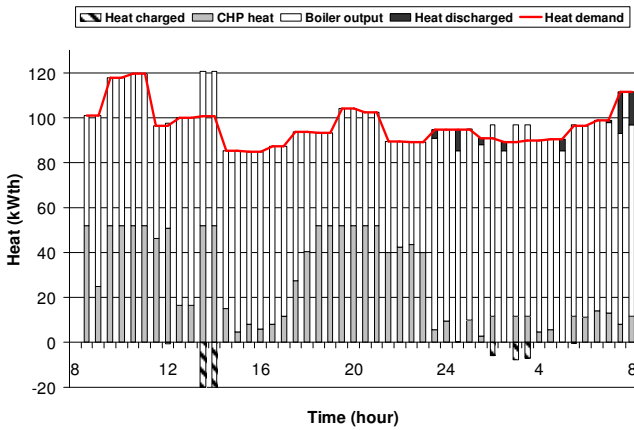


Figure 2. Heat balance with the earliest starting time

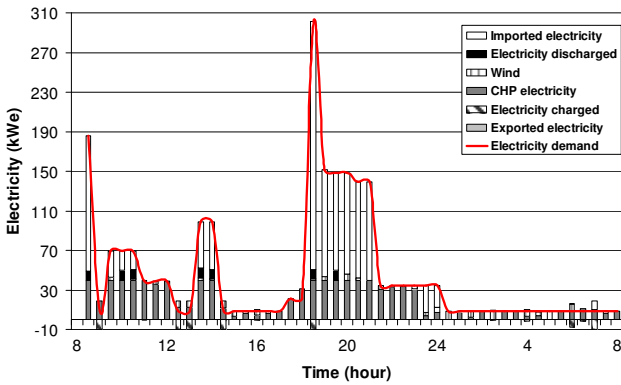


Figure 3. Electricity balance with the earliest starting time

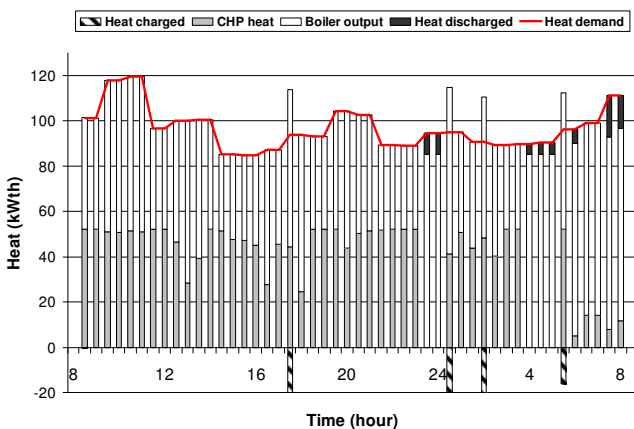


Figure 4. Heat balance with time window

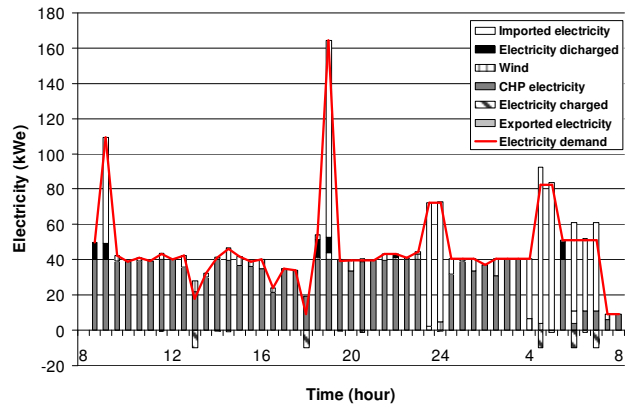


Figure 5. Electricity balance with time window

V. DISCUSSION AND CONCLUSIONS

In the case study, twelve domestic tasks from thirty homes are scheduled. Compared with the case where the tasks all start at their earliest possible starting time, the electricity consumption peak was decreased from 290kW to 165kW and the electricity consumption pattern became flatter. This peak shaving has the benefit of releasing the burden on the central grid and reducing the expense of upgrading the current grid infrastructure to fulfill increasing energy demand. The peak shaving also depends on the family living habits: for instance, they may prefer doing the washing during night time and would specify their own time window for the task. On the other hand, when the domestic task scheduling is implemented in real life, it could also affect people's behaviour.

Given the fixed heat demand profile used in these case studies, scheduling the tasks results in a more balanced heat supply than the earliest starting time case. This can be seen in Fig. 4, which shows that the CHP is providing more heat more steadily than in the earliest starting time case

The tasks shown are assumed to have constant power consumption over the processing time but this is not always realistic: e.g., washing machines have different power consumption during different stages of each program. Future scheduling models will need to account for tasks with time-varying power consumption.

The smart homes energy consumption scheduling problem is formulated as a mixed integer linear programming model. According to the case study for a winter day, it could provide 18.7% savings compared with the earliest starting time case (our current living habit). With the objective function, the tasks can be scheduled to coincide with the cheapest electricity price while keeping their operation within the desired time window. There are further benefits more when power is supplied by the wind generator. The power output from the wind generator is not constant and varies from hour to hour according to the weather conditions and other constraints. But when a weather forecast can be used to predict the power output during the next 24 hours, or even longer, the domestic tasks

can be scheduled to use the output of the wind generators when it is available, providing further savings for the customers. The scheduling formulation also increases the asset utilization, e.g., in the case study, the CHP generator was used more efficiently and became the main supplier of electricity and heat.

In the area of Smart Grids, it is considered that there is two-way communication between the power supplier and the customers. The power will be distributed according to the demand and supply. Traditional methods provide the customers only peak price hours while the Smart Grid could provide real-time electricity pricing. The process of price recalculating is similar to real-time traffic monitoring, aiming for a balance between supply and demand. This model considers the problem from the point of view of the customers, with a fixed electricity price profile. In the future, it might be possible to include this model as part of a full Smart Grid model where the electricity price is determined as part of the optimization, along with the scheduling of tasks.

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