

Journal paper

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## **Optimal Scheduling of Trains on a Single Line Track**

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## **ABSTRACT**

This paper describes the development and use of a model designed to optimise train schedules on single line rail corridors. The model has been developed with two major applications in mind, namely: as a decision support tool for train dispatchers to schedule trains in real time in an optimal way; and as a planning tool to evaluate the impact of timetable changes, as well as railroad infrastructure changes. The mathematical programming model described here schedules trains over a single line track. The priority of each train in a conflict depends on an estimate of the remaining crossing and overtaking delay, as well as the current delay. This priority is used in a branch and bound procedure to allow an optimal solution to reasonable size train scheduling problems to be determined efficiently. The use of the model in an application to a 'real life' problem is discussed. The impacts of changing demand by increasing the number of trains, and reducing the number of sidings for a 150 kilometre section of single line track are discussed. It is concluded that the model is able to produce useful results in terms of optimal schedules in a reasonable time for the test applications shown here.

# 1 Introduction

## 1.1 Background

The model described in this paper is designed to be used in two main modes, namely: as a decision support tool for train dispatchers to schedule trains in real-time in an optimal way; and as a planning tool to evaluate the impacts of timetable changes, as well as railroad infrastructure changes on train arrival times and train delays. The model is primarily designed to optimally schedule trains on single line railroads. Under such conditions, trains operate over a single track and can only overtake and cross each other at specific locations referred to here as sidings. This type of operation is common throughout the world. Most developing countries have networks which are mainly single line track.

In Australia, the average length of haul for interstate freight movements is around 1500 kilometres. The bulk of this interstate network consists of single line track. The extent of that network is shown in Figure 1. The train dispatching function outside urban areas is currently performed using mainly manual methods. The dispatcher resolves train conflicts on a time-distance graph using experience and knowledge of prevailing conditions. The criteria for conflict resolution includes the priority of each train, current train delays, and expected remaining overtake and crossing delay. This criteria is used to develop an improved lower bound. When used with the depth first search Branch and Bound procedure, optimal solutions to large problems can be found.

The main objective for developing the model described here is to provide the operator with a tool which helps him/her to perform the train dispatching function. The model is not designed to replace the dispatcher. The operator's experience and knowledge of local conditions, will continue to be used. Train dispatching decisions, which to a certain extent involve human as well as technical factors, will require human intervention to resolve problems. However, with such an optimisation model available, the operator is able to quickly update a schedule as unplanned events occur. The new optimal schedule offered by the model may not be fully implementable for practical reasons. However, the gap between the optimum and the practically feasible schedule, can be readily assessed. The penalty for not being able to implement the optimum schedule, in terms of operating cost and travel time reliability, can be evaluated against the practical factors which prevent implementation of the optimum schedule.

A second major use of the model relates to the planning of railroad operations. Such planning can be conveniently divided into two components, namely: short to medium term train planning; and railroad infrastructure planning associated with train operations. The model can be used to evaluate the implications of changes to a timetable in terms changed train departures, additional trains, and changes in train speeds. The optimum scheduling algorithm can be used as a simulator of proposed changes.

## 1.2 Train Schedule Reliability

When trains are scheduled on a rail corridor the objective is to achieve a given level of customer service whilst minimising overall operating costs. Customer service in this context is made up of several attributes which include overall journey time of trains and

the degree to which those journey times are achieved on a daily basis. In the context of freight movements, the benefits of improved reliability need to be estimated on a train by train basis. Each train is usually loaded with freight from a range of customers and origin-destination flows. The elasticity of demand with respect to transit time reliability will differ for each customer, commodity, and origin-destination combination.

The overall timetable reliability is a measure of the likely performance of the timetable as a whole, in terms of schedule adherence. The reliability of arrivals is a critical performance measure for all rail markets. The ability of rail systems to compete effectively relies to a large extent on consistent transit time reliability (Fowkes et al., 1991; Industry Commission, 1991; Bureau of Industry Economics, 1993).

If the train operations are conducted under single line conditions, the transit time reliability is a function of a range of factors. The degree of 'slackness' built into the schedule; the number and position of train conflicts; priorities for each train; terminal congestion; number and nature of scheduled stops; and train speeds are all influencing variables. Analytically based models designed to quantify the amount of delay risk associated with each track segment, train and the schedule as a whole are given by Higgins et al. (1995a).

There are several ways in which investment in track related infrastructure can reduce delays and hence improve transit time reliability. Four main investment strategies may have significant impact on the probabilities associated with train delays, namely:

(a) Investment in major track strengthening to increase maximum allowable speeds. The higher speeds have the potential to reduce conflict related delays and improve train recoverability;

(b) Investment designed to alter track alignments, both vertical and horizontal, thus increasing average train speeds;

(c) Investment in additions to the number and length of sidings where trains can cross and pass each other on single line track. Conflict related delays are directly affected by the number, length and location of sidings;

(d) Investment in advanced train control and communication systems to allow trains to proceed at shorter headways, and with less stops required for safe train operations.

The benefits of some of the above investment usually extend beyond transit reliability gains. For example, in the case of track rehabilitation and upgrading, those benefits may include reductions in overall transit times; reduction in accident risks; lower track maintenance costs; increased train productivity from higher maximum allowable axle-loads; reductions in rolling stock maintenance costs due to improved vehicle-track interaction; and improved locomotive productivity through the use of more modern equipment.

In order to obtain maximum benefits it is usually necessary to combine a number of investment strategies into a coherent and complementary package of capital expenditure projects. For example, the gains in reliability from track upgrading projects can be

augmented by investment in terminal infrastructure to allow faster loading/unloading of trains, and by locomotive investment to enable higher train speeds.

## **2 Past Work**

Work on an optimum solution to the train scheduling problem started in the early seventies by Szpigel (1973) who developed a linear programming model to determine the best overtaking and crossing positions given that the departure times and upper velocities of the trains are known. A Branch and Bound method is used to resolve the conflicts and lower bound to the remaining delay is generated by relaxing the remaining conflicts. Minimising the sum of the travel times was the objective and only small problems were tested. Petersen et al (1986) considered a similar dispatch algorithm which calculates the crosses, segment transit times and determines which train takes the sidings in order to minimise the total travel times.

Kraft (1987) takes a different approach by developing a dispatching rule giving the optimal time advantage for a particular train based on train priority, track running times and the delay penalties of each train. This rule was used to resolve crossing conflicts in the Branch and Bound procedure.

Kraay et al (1991) proposes a model which paces trains in order to conserve fuel and, at the same time, keep the lateness of trains at a minimum. The fuel consumption is a function of friction and gradient of track, speed and mass of the train, and air friction. Two heuristics were proposed which are able to find solutions to realistic size problems. Jovanovic et al (1991) uses a similar constraint framework as part of a decision support model called 'SCAN' which is based upon combinatorial optimisation and simulation. Mills et al (1991) formulated a discrete network type model by discretising the departure and arrival time variables. The discretisation allows the use of the shortest path algorithm to update the journey of each train. A procedure recursively updates the path of each train until a feasible schedule is found. The solution procedure is an approximation.

Mees (1991) models the single line rail as a network structure where each segment is an arc (a siding is considered as an extra arc), separated by nodes (considered as track intersections or stations). The network is time-space with a fixed schedule time span and headways are obtained by allowing only one train per arc at a time segment. A solution procedure similar to Mills et al (1991) is used to find a feasible solution.

Due to the difficulty of finding an optimal solution to large problems the trend has been towards finding an approximate solutions. The objective in this paper is to present a lower bound that will allow the branch and bound procedure to find the optimal solution to realistic size problems in reasonable time.

## **3 Model Formulation**

This section deals with the definition and derivation of the model. The resulting formulation is a non-linear mixed integer program for which the integer part is solved using an intelligent branch and bound procedure.

### 3.1 Variables and Model Assumptions

The set of trains is given by  $I=\{1,2,\dots,m,m+1,\dots,N\}$  for which inbound trains are from 1 to  $m$  and outbound are from  $m+1$  to  $N$ . The variables used in the model are listed and described in this section.

Let:  $P = \{P_1, P_2\}$

where:

$P_1 =$  set of single line tracks,  $P_2 =$  set of double line tracks

The integer decision variables for determining which train traverses a section first (also determines the position of conflict resolution) are given by:

$A_{ijp} = \begin{cases} 1 & \text{if inbound train } i \leq m \text{ traverses track segment } p \in P_1 \text{ before inbound train } j \leq m \\ 0 & \text{otherwise} \end{cases}$

$B_{ijp} = \begin{cases} 1 & \text{if inbound train } i \leq m \text{ traverses track segment } p \in P_1 \text{ before outbound train } j > m \\ 0 & \text{otherwise} \end{cases}$

$C_{ijp} = \begin{cases} 1 & \text{if outbound train } i > m \text{ traverses track segment } p \in P_1 \text{ before outbound train } j > m \\ 0 & \text{otherwise} \end{cases}$

The arrival and departure time decision variables are as follows:

$X^i_{aq} =$  arrival time of train  $i \in I$  at station  $q \in Q$

$X^i_{dq} =$  departure time of train  $i \in I$  from station  $q \in Q$

$X^i_{oi} =$  departure time of train  $i \in I$  from its origin station

$X^i_{Di} =$  arrival time of train  $i \in I$  at its destination station

The input parameters are defined as follows:

$h_p =$  minimum headway between two trains on segment  $p \in P_1$

$d_p =$  length of segment  $p \in P$

$Y^i_{oi} =$  earliest departure time of train  $i \in I$  from origin station

$Y^i_{Di} =$  planned arrival time of train  $i \in I$  at destination station

$\underline{v}_p^i =$  minimum allowable velocity of train  $i \in I$  on segment  $p \in P$

$\bar{v}_p^i =$  maximum achievable average velocity of train  $i \in I$  on segment  $p \in P$

$W_i =$  priority of train  $i \in I$  (highest for passenger trains)

$S^i_q =$  scheduled stop time for train  $i \in I$  at station  $q \in Q$ .

An illustration of the ordering of a single track used for the model in this paper is given in Figure 2 where the set of sidings is represented by  $Q=\{1,2,\dots,NS\}$  and here, track  $(p-2) \in P_2$ .

The following assumptions are made with regard to the model in this section:

- The track is divided into segments which are separated by sidings.
- Crossing and overtaking can occur at any siding or double line track segments.
- Trains can follow each other on a track segment with a minimum headway. If only one train is permitted on some track segments then the headway on these track segments is increased to the track length.

- For double track sections, it is assumed one lane will be allocated for inbound trains and one lane will be allocated for outbound trains. Usually, signal points will be set up this way.
- Scheduled stops are permitted at any intermediate siding for any train

The model will require various information to make use for the input to the model. The specific information is as follows:

- An unresolved train plan to make available the number of overtake and cross interferences for each train.
- The priorities of each train. These are determined by several factors such as the type of train, customer contract agreements and train load.
- The upper and lower velocity limits for each train (which are dependent on the physical characteristics of the track segment and the train).
- The times of any scheduled train stops. These stops may include loading/unloading, refuelling and crew changes.

### 3.2 Objective Function and Constraints

The objective function used in the model takes the following form:

$$\text{Min } \sum_i W_i^* (\text{delay of train } i \in I \text{ at destination}) + \text{Train Operating Costs} \quad (1)$$

For the purposes of the solution procedure (namely Branch and Bound), the delay of train  $i \in I$  is comprised of two parts. These are the current delay of train  $i \in I$  at any point in time and a lower bound estimate of remaining overtake and crossing delay from this point. The model is subject to various constraints to ensure safe operation, enforce speed restrictions and permit stops. The following and overtake constraints for outbound trains  $i, j \in I$  are as follows:

$$M^* C_{ijp} + X^i_{aq+1} \geq X^j_{aq+1} + h_p \quad \forall p \in P_1 \text{ and } i, j > m \quad (2)$$

$$M^* C_{ijp} + X^i_{dq} \geq X^j_{dq} + h_p \quad \forall p \in P_1 \text{ and } i, j > m \quad (3)$$

and for inbound trains  $i, j \in I$ :

$$M^* A_{ijp} + X^i_{aq} \geq X^j_{aq} + h_p \quad \forall p \in P_1 \text{ and } i, j \leq m \quad (4)$$

$$M^* A_{ijp} + X^i_{dq+1} \geq X^j_{dq+1} + h_p \quad \forall p \in P_1 \text{ and } i, j \leq m \quad (5)$$

Equation (2) implies that if train  $j \in I$  goes first, then train  $i \in I$  must depart station  $q \in Q$  after train  $j \in I$  plus the minimum headway, and arrive at station  $(q+1) \in Q$  after train  $j \in I$  plus the headway. Equation (3) is similar except train  $i \in I$  goes first. Equations (4) and (5) are the same as equations (2) and (3), but for inbound trains. The constraints for the case when two trains approach each other are:

$$\begin{aligned}
h_p + X^j_{aq+1} &\leq X^i_{dq+1} + M * B_{ijp} \\
h_p + X^i_{aq} &\leq X^j_{dq} + M * (1 - B_{ijp})
\end{aligned}
\quad \bigcup_{p \in P_1, (i \leq m, j > m)} \quad (6)$$

Equation (6) implies that if outbound train  $j \in I$  goes first, inbound train  $i \in I$  must depart station  $q \in Q$  after train  $j \in I$  arrives plus a safety headway. Constant  $M$  is chosen large enough so that both equations in each crossing and overtake constraint are satisfied. Given the upper and lower velocities for each train on each segment, the upper and lower limits for traversal time of train  $i \in I$  on segment  $p \in P_1$  are given by:

$$\begin{aligned}
\frac{d_p}{\bar{v}_p^i} &\leq X^i_{aq+1} - X^i_{dq} \leq \frac{d_p}{\underline{v}_p^i} \quad i > m, p \in P \\
\frac{d_p}{\bar{v}_p^i} &\leq X^i_{aq} - X^i_{dq+1} \leq \frac{d_p}{\underline{v}_p^i} \quad i \leq m, p \in P
\end{aligned} \quad (7)$$

To stop trains from departing before their scheduled times and trains departing intermediate stations before they arrive, the following constraints are included.

$$\begin{aligned}
X^i_{oi} &\geq Y^i_{oi} \\
X^i_{aq} + S^i_q &\leq X^i_{dq}
\end{aligned}
\quad \bigcup_{i \in I, q \in Q} \quad (8)$$

The objective is to minimise equation (1) subject to constraints given by equations (2) - (8). The solution procedure to solve this model can be found in Appendix A. The algorithm for determining the lower bound is described in Appendix B.

## 4 Model Applications

### 4.1 Model Performance Analysis

The solution procedure to solve the model of section 3.2 along with the lower bound estimates (LB1) have been implemented in FORTRAN on a 80486 PC. The model was tested on train schedules varying from 9 trains to 30 trains and was compared to a Branch and Bound Procedure with a lower bound calculated by relaxing the remaining conflict constraints (LB2). The exact objective function used is a combination of tardiness and fuel consumption (Mills et al (1991)).

The results were also compared to a Tabu Search (TS) heuristic solution (Glover 1993). The TS was allowed to run until a certain number of iterations elapses when no improvement in solution value is found. The neighbourhood was defined as the movement of a conflict to a neighbouring siding and only a sample of the neighbourhood was searched at each iteration. A full description of the use of the TS heuristic can be found in Higgins et al (1995b).

Different examples were solved for each problem size by varying the earliest departure time of each train, upper velocities and due arrival times. As shown in Table 1, the examples within each problem size contain different numbers of conflicts. The latter are a representation of the complexity of each problem. Results for the synthetic problems are summarised in Table 1. The results are given in terms of the number of times the schedule



had to be updated due to the addition or change of a resolved conflict. The TS results shown are the average values of 5 trials.

As shown in Table 1, the new lower bound presented in this paper (LB1) produced significant reductions in the number of calculations for all problems. The optimum solution to most of the smaller problems was found in the minimum possible time. Results compared favourably to the Tabu Search heuristic in terms of number of calculations and average travel time per train.

However, the lower bound (LB1) took longer to calculate than LB2. The significance of this towards the overall CPU time depends on the way the schedule is updated when a resolved conflict is changed or added. For the procedure used in this paper (described in Appendix B) the CPU comparison between columns LB1 and LB2 would be valid if the values for LB1 in Table 1 were multiplied by 2.5. If a network flow algorithm or a software package such as GAMS/MINOS is used to update the schedule the significance of the calculation of LB1 towards the overall CPU time would be very low. The unresolved and resolved schedules for the 30 train problem is illustrated in Figures 3 and 4 respectively.

## **4.2 Option Testing**

The algorithm was tested on an actual section of railroad. The rail corridor contains 14 sidings and 31 trains are scheduled on the busiest day of the week. The four types of trains scheduled are: fast freight, heavy freight, electric passenger and locomotive hauled passenger trains. Each train class has a different maximum achievable velocity. For the purposes of this paper it is assumed all sidings are sufficiently long to accommodate any train size. The algorithm only required 46 seconds to determine the optimum solution to the problem. The optimum schedule is demonstrated in Figure 5. The efficiency of the algorithm will allow the rapid updating of an optimum solution whenever an unexpected delay occurs.

The solution procedure presented here was used to demonstrate the effects of two different types of operating changes, namely: increases in demand through additional trains and reduction in the number of sidings available for conflict resolution. As shown in Figure 5, the line is currently operating with significant spare capacity. The effects of train delays (in terms of average increase travel time per train in the optimal schedule), when sidings are removed is demonstrated in Figure 6. To prevent any possible bias, the siding positions in the original problem are taken to be equally spaced. When a siding is removed the remaining sidings are spaced equally. It is only when the number of sidings are reduced below nine that the increase in average transit time becomes significant. If the original number of sidings was reduced to nine, the schedule would operate with an increase in average transit time of less than 2 percent.

However, this analysis does not take into account the ability of a given set of sidings to cater for unexplained events such as track and rolling-stock related delays on the line. Some sidings are required to provide insurance against delay risks, rather than to act as conflict resolution locations under 'ideal' conditions.

The second application consisted of scheduling additional trains at already congested periods of the existing schedule. Three new fast freight trains were added at a time, one for each peak period. The average effects of adding these extra trains to the original 31 train, 14 siding problem is demonstrated in Figure 7. The average transit time increases almost linearly with the number of trains in the schedule. The addition of 18 trains increases the average transit time by 8.4 percent. The schedule with 49 trains contained nearly 100 train conflicts and was solved within 10 minutes. The effects of adding extra trains and removing sidings simultaneously is demonstrated in Table 2. The numbers in the table represent the increase in average travel time as a proportion of the base problem. The increase in average travel time tends to be linear for the increase in number of trains and exponential for the reduction in sidings.

## 5 Conclusions

This paper has presented an on-line model for the scheduling of trains on a single line track. The solution procedure used a lower bound estimate of the remaining overtake and crossing delay to reduce the search space in the branch and bound tree. The lower bound estimate was based on the calculation of the least cost path of each train, while assuming the path of a train is independent of previous accumulated conflict delay. The calculation of the lower bound is shown to be of low order polynomial time. This has allowed the optimal solution to realistic size problems to be found in a reasonable time.

As well as an online scheduling tool, the model presented here can be used for long-range planning of railroad operations. In Australia, there are two main infrastructure planning issues which are currently under investigation, namely: the upgrading of main line track to allow higher speeds and heavier axle loads; and the need to extend sidings to allow for longer trains. The scheduling optimisation model can be used to evaluate both these investment strategies. The impact on the schedule of extending some sidings and not others can be assessed by using the model to simulate the effect of the proposed changes on future schedules. The removal of sidings has a cost in terms of flexibility and feasibility of schedules. As part of the same research project, the authors have developed a model to optimise the location of sidings for a given set of train departure times and mean train speeds (Higgins et al 1994b).

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## Appendix A Solution Procedure

### A1 General

The solution procedure described in this section is based on Branch and Bound and uses the depth first search for the resolution of conflicts. Each node in the tree involves the solution of a non-linear program which can be solved using a specialised package program such as GAMS/ MINOS. An algorithm for determining the lower bound can be found in Appendix B. The procedure is as follows:

1. Solve the objective function (1) subject to the velocity, departure time and scheduled stop constraints (7,8) ignoring the cross and overtake constraints (2)-(6). This will give an unresolved train plan.
2. From the train graph in step 1. pick out the first conflict in time. Identify the two trains  $i, j \in I$  involved and the segment  $p \in P$  which this conflict occurs. There will be two alternatives to resolve this conflict, train  $i \in I$  is delayed, or train  $j \in I$  is delayed. For each of the alternatives, update the train journey by solving the sub problem with the conflict between these two trains. ie. solve objective function (1) st (7,8) and the appropriate overtake or crossing constraint from equation (2)-(6). For each of these two alternatives determine the lowerbound estimate of the remaining crossing and overtake delay. Add the delay cost of the lower bound estimate of remaining delay to the overall cost.
3. Pick the alternative (resolution) with the lowest cost. If the cost is greater than the current upperbound then go to step 6(a) (the rest of the branch is pruned).
- 3(a). From this resolution (conflict) node, pick the next conflict in the train graph. Identify the trains involved and the segment which this conflict occurs.
4. For each of the two alternatives, update the train journey by solving the subproblem with objective function (1) subject to the constraints (7,8) and a subset of the fixed integer constraints (2)-(6) when considering the current conflict, and previous conflicts (ie. conflicts on the same branch and higher in the branch and bound tree). Calculate the lower bound estimate of remaining conflict delay for each of these alternatives and add the delay cost of the lower bound estimate of remaining delay to the overall cost.
5. If there are no more conflicts then go to step 6 otherwise go to step 3.
6. Mark the best solution in this last alternative pair of resolutions. If the cost of this solution is less than the current upper bound then let the upper bound equal the cost of this new solution.
- 6(a). Trace up the Branch and Bound tree, one level at a time until a level with an unbranched node and a cost lower than the upper bound is found. Pick the node with the smallest cost in this level. If the top level in the branch and bound tree is reached when tracing up, then the procedure terminates and the current solution is the best solution otherwise go to step 3(a).

### A2 Solving the Non-Linear Subproblem

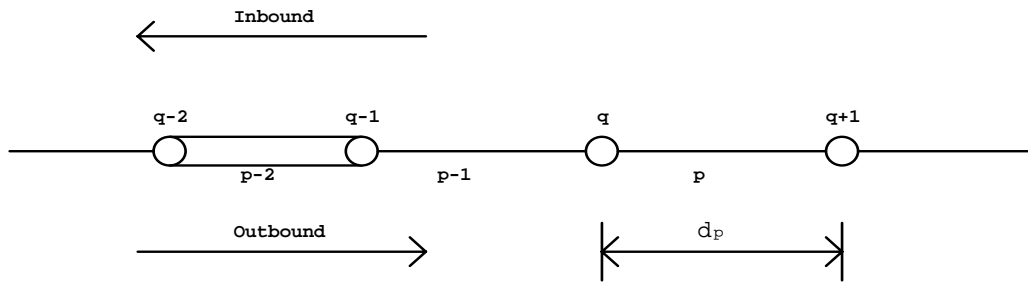
The non-linear subproblems (fixed integer variables) are solved using an approximate iterative procedure which builds up a schedule given a set of resolved conflicts. The objective function (equation 1) in the model is assumed to have the components tardiness and fuel costs of each train. It is assumed the cost of tardiness has a higher priority than

fuel costs. A train will not travel slow to arrive at the destination station late unless the train's priority is set to a minimum. Situations occur when a train is not able to make a connection on time or the due arrival time of the train is extremely late. In such a case, the train will be delayed in every conflict and the train is paced between conflicts.

When constructing the schedule using the iterative procedure, it is assumed a train will travel at the maximum achievable average velocity on any track segment, unless the train is to be delayed in a conflict at the next siding. In this case the train will be paced to arrive at the siding at the last moment. If a train has slack time, it is paced to arrive at the destination station on time from its last conflict. Full details of the algorithm can be found in Higgins et al (1994a).

For solving the non-linear subproblem, the approximate procedure gave near identical results to GAMS/MINOS 5.2 for all size problems tried when the objective function of minimum tardiness and fuel consumption is used. For a problem of 32 trains and 14 sidings, GAMS/MINOS 5.2 took 8 minutes (on a 80486DX PC) to solve one instance and required 8 megabytes of RAM. The approximate iterative procedure was able to solve nearly a thousand instances of this size problem in one minute with near identical results.

**Figure 1:** Australia interstate rail network



**Figure 2:** Sample of a network showing the single and double track segments.



No. of Trains (No. of Sidings)	No. of Conflicts	Branch & Bound (LB1)	Branch & Bound (LB2)	Tabu Search (TS)
9 (6)	11	24#	97	620
	12	24#	211	411
	12	24#	282	543
	13	28#	452	436
11 (8)	11	42	433	789
	14	30#	1052	402
	15	38#	1129	421
	19	56	1026	1140
15 (8)	13	34#	402	816
	14	36#	332	407
	14	36#	1066	1151
	21	74	1190	783
20 (8)	19	36#	1054	995
	19	96	1190	860
	25	56#	1268	812
	27	122	1440	405
25 (12)	23	46#	1338	982
	24	120	1270	384
	34	72	1872	1062
	40	120	2304	1108
30 (12)	50	194	2265	1020
	53	250	2820	1268

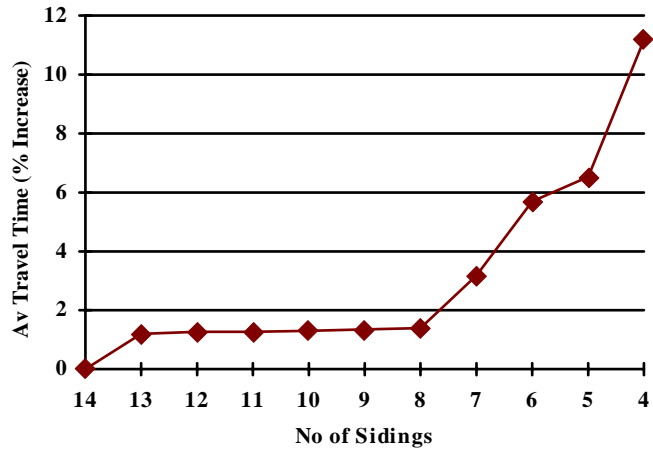
# Optimum solution found in minimum possible time

**Table 1:** Comparison between solution techniques: Number of schedule updates

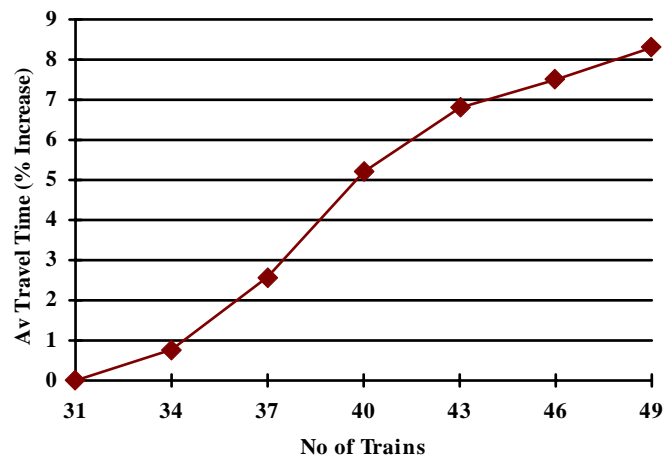
**Figure 3:** Unresolved train schedule consisting of 30 trains

**Figure 4:** Optimal train schedule

**Figure 5:** Real life problem consisting of 31 trains and 14 sidings



**Figure 6:** Effect of decreasing number of sidings



**Figure 7:** Effect of increasing demand

No. of Sidings	No. of Trains						
	31	34	37	40	43	46	49
14	1.000	1.007	1.029	1.052	1.068	1.075	1.083
13	1.012	1.015	1.038	1.060	1.071	1.086	1.087
12	1.012	1.017	1.042	1.062	1.074	1.097	1.097
11	1.012	1.019	1.043	1.072	1.082	1.114	1.114
10	1.013	1.024	1.044	1.071	1.082	1.114	1.114
9	1.013	1.038	1.051	1.072	1.088	1.115	1.118
8	1.014	1.051	1.062	1.078	1.097	1.120	1.123
7	1.032	1.054	1.068	1.097	1.122	1.150	1.158
6	1.057	1.092	1.128	1.146	1.155	1.168	1.180
5	1.065	1.096	1.126	1.155	1.170	1.200	1.210
4	1.112	1.154	1.178	1.230	1.258	1.269	1.279

**Table 2:** Two way effect of adding sidings and trains

**Av Travel Time (% Increase)**

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**Figure 1:** Australia interstate rail network

**Figure 2:** Sample of a network showing the single and double track segments.

**Figure 3:** Unresolved train schedule consisting of 30 trains

**Figure 4:** Optimal train schedule

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**Figure 6:** Effect of decreasing number of sidings

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**Table 1:** Comparison between solution techniques: Number of schedule updates

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**Figure B1:** Demonstration of delay times

**Figure B2:** Train graphs to demonstrate best path