Optimal Selection of Channel Sensing Order in Cognitive Radio

Hai Jiang, *Member, IEEE*, Lifeng Lai, *Member, IEEE*, Rongfei Fan, and H. Vincent Poor, *Fellow, IEEE*

***Abstract*—This paper investigates the optimal sensing order problem in multi-channel cognitive medium access control with opportunistic transmissions. The scenario in which the availability probability of each channel is known is considered first. In this case, when the potential channels are identical (except for the availability probabilities) and independent, it is shown that, although the intuitive sensing order (i.e., descending order of the channel availability probabilities) is optimal when adaptive modulation is not used, it does not lead to optimality in general with adaptive modulation. Thus, a dynamic programming approach to the search for an optimal sensing order with adaptive modulation is presented. For some special cases, it is proved that a simple optimal sensing order does exist. More complex scenarios are then considered, e.g., in which the availability probability of each channel is unknown. Optimal strategies are developed to address the challenges created by this additional uncertainty. Finally, a scheme is developed to address the issue of sensing errors.**

***Index Terms*—Cognitive radio, medium access control, dynamic programming.**

I. Introduction

Wireless networks have experienced rapid growth during the past two decades. This has caused a spectrum scarcity problem, since much of the prime wireless spectrum has been licensed for specific applications. However, according to recent measurements of wireless spectrum usage, the licensed spectrum is actually severely under-utilized. Thus, the idea of dynamically accessing the spectrum has attracted considerable attention, which allows unlicensed users (referred to as *secondary users*) to access the spectrum at a particular time and location when and where licensed users (referred to as *primary users*) are not active. Recently cognitive radio has demonstrated the potential to enable dynamic spectrum access, mainly because of its ability to adapt to dynamic spectral environments [1].

In such systems, in order to avoid interference to the primary network, it is necessary for secondary users to determine (usually via spectrum sensing) whether there exist primary activities in the spectrum before their secondary transmissions. In this research, we consider the scenario in which a secondary user has a number of potential wireless channels, and the secondary user can sense one channel at a time. Since the secondary user cannot sense two or more channels simultaneously, one task is to determine which channel to observe at a given time so as to fully utilize the spectral opportunities. Similar problems of multi-channel medium access control have attracted considerable recent attention. In [2], an analytical framework is provided based on the theory of partially observable Markov decision processes. The availability of each channel is assumed to follow a Markov chain, whose transition matrix is known to secondary users. In [3], [4], the dynamic channel selection problem is formulated as a multi-armed bandit problem. In [5], [6], an optimal channel probing and transmission policy is derived, with an assumption that *recall* (i.e., using one of the previously sensed channels) and *guess* (i.e., using a channel that has not been sensed yet) are allowed.

In this paper, we investigate the multi-channel cognitive medium access control problem, in which multiple potential channels (i.e., frequency bands) are available. Unlike [2]–[4], we consider a cognitive radio network with *opportunistic transmissions* explained as follows. The secondary user will not only sense a channel to decide whether it is free, but will also estimate the channel coefficient to decide the transmission rate. If a channel is sensed to be free, but the channel quality between the secondary transceiver pair is not satisfactory, the secondary user may still skip this channel and keep sensing other channels. The opportunistic transmission creates a new degree of freedom for the secondary user. The goal of the secondary user is to find a free and good channel as quickly as possible. In this case, the channel sensing problem can be formulated as an optimal stopping rule problem [7] if the channel sensing order is determined in advance. Our aim is to find an optimal channel sensing order such that the user achieves the maximal gain. Single-channel and multi-channel medium access control with opportunistic transmissions have been studied in [8] and [9], respectively. The major difference between [8], [9] and our work is that in [8] and [9], the single and multiple channels are assumed to be always available, and hence, different sensing orders do not make any difference in the achieved gain. In our research, we examine instead the optimal sensing order problem for the cognitive radio setup in which the channels are not always available (due to activities of the primary users) and the channel availability probabilities are different from one another. We show that the intuitive and plausible strategy that the secondary user senses the channels with the larger availability probabilities first does not generally lead to an optimal solution. We then

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H. Jiang and R. Fan are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4 (e-mail: hai.jiang@ece.ualberta.ca, rongfei@ualberta.ca).

L. Lai and H. V. Poor are with the Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA (e-mail: {llai, poor}@princeton.edu).

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propose a dynamic programming approach to obtain an optimal solution. We also examine several scenarios with certain structures and find simple optimal rules for these scenarios. Further, we investigate more complex scenarios (e.g., with unknown channel availability probabilities), and the scenario with channel sensing errors as well.

The rest of this paper is organized as follows. The network model is described in Section II. The optimal sensing order problem in a generic case is investigated in Section III. Some special cases with resulting simple sensing orders are studied in Section IV, while more complex scenarios are discussed in Section V. The impact of sensing errors is discussed and addressed in Section VI, followed by concluding remarks and further discussions in Section VII.

II. Network Model

Consider a secondary user and a number, N, of potential channels having IDs from 1 to N. The secondary user is operated in a time-slotted fashion, where the length of each time slot is T. The secondary user also has a sensing order $(s_1, s_2, ..., s_N)$, which is a permutation of the set $\{1, 2, ..., N\}$. In a given time slot, the secondary user senses the channels sequentially according to the sensing order, until it stops at a channel based on a specific criterion (e.g., the channel is sensed to be free and it has acceptable channel quality), and transmits its information in that channel during the remainder of that time slot. It is assumed that accurate channel sensing is achieved, and there is no sensing error. The impact of channel sensing errors is to be discussed in Section VI. Fig. 1 shows the channel sensing and information transmission procedure at the secondary user in a time slot, where the user stops at channel s_k, and τ denotes the time needed for sensing a channel and estimating the channel gain, which is assumed to be the same in all the channels.

In each time slot, channel i ($1 \le i \le N$) is free (i.e., no primary user activity) with probability $\theta_i (\in (0, 1))$, which is the availability probability of that channel. With little loss of generality, we assume that no two channels have the same availability probability. For each channel, the busy/free status in each slot is assumed to be independent of the status in other slots, and also to be independent of the status in other channels as well. We consider fading channels. For each channel, the signal-to-noise ratio (SNR) is fixed within a time slot, and changes randomly at the beginning of the next time slot. The channel SNR γ is assumed to be independent and identically distributed across time slots and across different channels, with a common probability density function (pdf) denoted by $h_{SNR}(\gamma)$. Opportunistic transmissions are used. If the secondary user decides to transmit in a free channel i, the achievable transmission rate is $f(SNR_i)$ where SNR_i is the SNR of the secondary user in channel i, and $f(\cdot)$ is a non-descending function mapping SNR to the transmission rate. With this model, the channel sensing problem becomes an optimal stopping problem [7], [9] as described below.

If channel s_i is sensed busy, then the secondary user proceeds to sense channel s_{i+1}. When channel s_i is sensed free, the secondary user will transmit at channel s_i as long as the achieved transmission rate at channel s_i is greater than

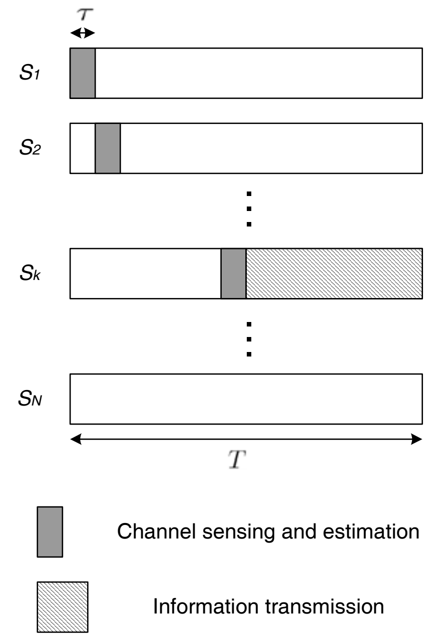

Fig. 1. The channel sensing and information transmission procedure at a secondary user in a time slot.

the expected rate if the user proceeds to the next channel (i.e., channel s_{i+1}). Hence, the *reward* (throughput) at channel s_i is given by

$$u_i = \begin{cases} c_i f(SNR_{s_i}), & \text{if } i = N \text{ (stop at channel } s_i) \\ c_i f(SNR_{s_i}), & \text{if } i \ne N \text{ and } c_i f(SNR_{s_i}) > U_{i+1} \\ & \quad \text{(stop at channel } s_i) \\ U_{i+1}, & \text{otherwise (proceed to channel } s_{i+1}) \end{cases} \tag{1}$$

where c_i is the *effectiveness* of transmitting at channel s_i, given by $c_i = 1 - \frac{i \cdot \tau}{T}$ (from Fig. 1), and U_{i+1} ($i \le N-1$) is the expected reward at channel s_{i+1} if the user proceeds to that channel, given by

$$U_{i+1} = \begin{cases} \theta_{s_{i+1}} \mathbb{E}[u_{i+1}] + (1 - \theta_{s_{i+1}}) U_{i+2}, & \text{if } i < N-1 \\ \theta_{s_{i+1}} \mathbb{E}[u_{i+1}], & \text{if } i = N-1 \end{cases} \tag{2}$$

where $\mathbb{E}[\cdot]$ denotes expectation. The sequence $\{U_1, U_2, ..., U_N\}$ can be obtained recursively from U_N to U_1 using (1) and (2) [9]. No recall is allowed, i.e., the secondary user cannot return to transmit at a previously sensed channel. This requirement is reasonable for the following reason. In a secondary network, it is possible that a number of secondary users exist. If a secondary user returns to access a previously free channel, a collision may happen since the channel may be accessed by another secondary user at that time.

The expected reward U_1 represents the expected transmission rate of the secondary user with the N channels, and is dependent on the sensing order. The goal of this paper is to find an optimal sensing order and the corresponding stopping rule.

III. Optimal Selection of Sensing Order

A. Is the Intuitive Sensing Order Optimal?

The goal of channel sensing is to find an acceptable wireless channel (i.e., a channel that is free and has satisfactory channel gain) as quickly as possible. In a general stopping rule problem, it is shown that an optimal sensing order cannot be determined by a function of the moments of the random variables to be observed [10]. However, observation overhead (e.g., the τ in Fig. 1) is not considered in this finding. And additionally, in the general case discussed, each random variable can have any arbitrary distribution. Thus the preceding finding may be too "general" for our special case, in which all the channels (viewed as random variables to be observed) are statistically identical (except for the availability probabilities) and independent. For our case, it is not unreasonable to expect that the user should check the channels according to descending order of θ_i, referred to as *Intuitive Sensing Order*. In other words, the first moment of the random variables is used to determine the sensing order. Indeed, the Intuitive Sensing Order is optimal when adaptive modulation is not used, as shown in the following lemma. Without adaptive modulation, we assume that whenever the user transmits, the rate is a constant R [1].

Lemma 1: If adaptive modulation is not used, the user should stop at the first free channel, and the Intuitive Sensing Order is optimal in terms of expected reward U_1.

Proof: When adaptive modulation is not used, we have

$$f(SNR_i) \equiv R, \quad \forall i. \tag{3}$$

At any channel s_i, if it is sensed to be free, the transmission rate if the user stops here is $c_iR > c_{i+1}R \geq U_{i+1}$, i.e., greater than the expected transmission rate if the user proceeds to channel s_{i+1}. So from (1), the user should stop at the first available channel.

We prove the optimality of the Intuitive Sensing Order by contradiction. Suppose an optimal sensing order is $(s_1, s_2, ..., s_N)$, and there exists $k < N$ such that $\theta_{s_k} < \theta_{s_{k+1}}$. Then from (2), we have

$$\begin{aligned} & U_k \\ = \; & \theta_{s_k} c_k R + (1-\theta_{s_k})\big(\theta_{s_{k+1}} c_{k+1} R + (1-\theta_{s_{k+1}})U_{k+2}\big) \\ = \; & (\theta_{s_k} c_k + \theta_{s_{k+1}} c_{k+1})R - \theta_{s_k}\theta_{s_{k+1}} c_{k+1} R \\ & +(1-\theta_{s_k})(1-\theta_{s_{k+1}})U_{k+2} \end{aligned} \tag{4}$$

where we define $U_{N+1} = 0$ (for the case $k = N-1$).

We get a new sensing order by switching the order of channels s_k and s_{k+1}, and keeping all the other channels the same. We have

$$\begin{aligned} U_k^{\text{new}} &= (\theta_{s_{k+1}} c_k + \theta_{s_k} c_{k+1})R - \theta_{s_k}\theta_{s_{k+1}} c_{k+1} R \\ & \quad +(1-\theta_{s_k})(1-\theta_{s_{k+1}})U_{k+2}. \end{aligned} \tag{5}$$

[1] When adaptive modulation is not used, it is possible that a transmission may fail due to poor channel quality. So it may be desirable to let the user skip a free channel when the channel gain is below a threshold λ. This case is actually equivalent to the case in which 1) the availability probability of channel $i \in \{1, ..., N\}$ is $\theta'_i = \theta_i \cdot \text{Prob}\{\text{channel gain} \geq \lambda\}$, and 2) whenever the user transmits, the rate is R.

Since $c_k > c_{k+1}$ and $\theta_{s_k} < \theta_{s_{k+1}}$, it can be easily seen that $U_k < U_k^{\text{new}}$. Further we have

$$\begin{aligned} U_{k-1} &= \theta_{s_{k-1}} c_{k-1} R + (1-\theta_{s_{k-1}})U_k \\ &< \theta_{s_{k-1}} c_{k-1} R + (1-\theta_{s_{k-1}})U_k^{\text{new}} = U_{k-1}^{\text{new}}. \end{aligned} \tag{6}$$

Similarly, we have $U_1 < U_1^{\text{new}}$. This contradicts the assumption that the sensing order $(s_1, s_2, ..., s_N)$ is optimal. ■

On the other hand, when adaptive modulation is adopted, the Intuitive Sensing Order may no longer be optimal. We demonstrate this by examples as follows.

Let Γ denote the mean channel gain of each channel. Consider the case of Rayleigh fading, in which the pdf of the channel gain is

$$h_{SNR}(\gamma) = \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}}, \quad \gamma > 0. \tag{7}$$

Assume that the transmission rate achievable in channel i (when channel i is free) is $\log(1 + SNR_i)$. The secondary user senses the channels according to the order $(s_1, s_2, ..., s_N)$. Then we have

$$U_N = \theta_{s_N} \int_0^\infty c_N \log(1+\gamma) \cdot \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}} d\gamma = \theta_{s_N} c_N e^{1/\Gamma} \psi\left(\frac{1}{\Gamma}\right) \tag{8}$$

where

$$\psi(x) = \int_x^\infty \frac{e^{-t}}{t} dt. \tag{9}$$

For $1 \leq i < N$, we have

$$\begin{aligned} U_i &= \theta_{s_i} \mathbb{E}[u_i] + (1-\theta_{s_i})U_{i+1} \\ &= \theta_{s_i} \Bigg[\int_{c_i \log(1+\gamma) > U_{i+1}} c_i \log(1+\gamma) \cdot \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}} d\gamma \\ & \quad + \int_{c_i \log(1+\gamma) \leq U_{i+1}} U_{i+1} \cdot \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}} d\gamma \Bigg] \\ & \quad +(1-\theta_{s_i})U_{i+1} \\ &= \theta_{s_i} c_i \int_{e^{\frac{U_{i+1}}{c_i}}-1}^{\infty} \log(1+\gamma) \cdot \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}} d\gamma \\ & \quad + \theta_{s_i} U_{i+1} \int_0^{e^{\frac{U_{i+1}}{c_i}}-1} \frac{1}{\Gamma} e^{-\frac{\gamma}{\Gamma}} d\gamma + (1-\theta_{s_i})U_{i+1} \\ &= \theta_{s_i} c_i e^{1/\Gamma} \psi\left(\frac{e^{\frac{U_{i+1}}{c_i}}}{\Gamma}\right) + U_{i+1}. \end{aligned} \tag{10}$$

Let the expected reward for a sensing order $(s_1, s_2, ..., s_N)$ be denoted by

$$G(\theta_{s_1}, \theta_{s_2}, ..., \theta_{s_N}) = U_1 \tag{11}$$

where $G(x_1, x_2, ..., x_k)$ means the expected reward for the case in which i) there are k potential channels with availability probabilities $x_1, x_2, ...,$ and x_k, respectively, and ii) the sensing order is to sense first the channel with availability probability x_1, then the channel with x_2, ..., and finally the channel with x_k.

Lemma 2: There exist θ_1 and θ_2 such that $\theta_1 > \theta_2$, but $G(\theta_1, \theta_2) < G(\theta_2, \theta_1)$.

Proof: Let $\tau/T = 0.01$, $\Gamma = 10$, $\theta_1 = 0.9$, and $\theta_2 = 0.5$. Then $G(\theta_1, \theta_2) = 1.95 < G(\theta_2, \theta_1) = 2.02$. ■

Lemma 2 indicates that the Intuitive Sensing Order is not optimal in general.

Lemma 3: There exist θ_1, θ_2 and θ_3 so that $G(\theta_1, \theta_2) > G(\theta_2, \theta_1)$ and $G(\theta_2, \theta_3) > G(\theta_3, \theta_2)$, but $G(\theta_1, \theta_2, \theta_3) < G(\theta_2, \theta_1, \theta_3)$.

Proof: Let $\tau/T = 0.01$, $\Gamma = 10$, $\theta_1 = 0.2$, $\theta_2 = 0.6$, and $\theta_3 = 0.9$. Then $G(\theta_1, \theta_2) = 1.3672 > G(\theta_2, \theta_1) = 1.3600$; $G(\theta_2, \theta_3) = 2.0741 > G(\theta_3, \theta_2) = 2.0060$; and $G(\theta_1, \theta_2, \theta_3) = 2.1215 < G(\theta_2, \theta_1, \theta_3) = 2.1257$. ■

Lemma 3 indicates that concatenation of optimal sensing orders does not lead to an optimal order in general.

B. A Dynamic Programming Solution for Optimal Sensing Order Search

Since the Intuitive Sensing Order is not optimal in general, it may seem necessary to use brute force search to find an optimal order. The computational complexity of brute force search is $O(N \cdot N!)$ if the complexity in calculating the expected reward at a specific channel is $O(1)$. In this situation, a low-complexity method is desired, and this is our target. Before describing our low-complexity method, we present the following lemma.

Lemma 4: For a set of N channels with availability probabilities θ_1, θ_2, ..., and θ_N, if $(s_1, s_2, ..., s_N)$ is an optimal sensing order, then for any $k \in \{1, 2, ..., N\}$, $U_k^{(s_1, s_2, ..., s_N)}$ is not less than $U_k^{(s_1, s_2, ... s_{k-1}, P(s_k, ..., s_N))}$, where $P(s_k, ..., s_N)$ denotes any permutation of $(s_k, ..., s_N)$, and $U_k^{(x_1, x_2, ..., x_N)}$ is given by (2) when the sensing order is $(x_1, x_2, ..., x_N)$.

Proof: We use proof by contradiction. Suppose there exist $k \in \{1, 2, ..., N\}$ and $P(s_k, ..., s_N)$ such that $U_k^{(s_1, s_2, ..., s_N)} < U_k^{(s_1, s_2, ... s_{k-1}, P(s_k, ..., s_N))}$. Then we have (12) on the top of the next page. Using similar steps, finally we obtain

$$U_1^{(s_1, s_2, ..., s_N)} < U_1^{(s_1, s_2, ..., s_{k-1}, P(s_k, ..., s_N))} \quad (13)$$

which contradicts the condition that $(s_1, s_2, ..., s_N)$ is an optimal sensing order. This completes the proof. ■

Based on Lemma 4, we can formulate a dynamic programming solution for the optimal sensing order problem of the secondary user, as follows.

Stage 1 – Calculate the maximal expected reward[2] value associated with the final channel in the sensing order. In this stage, a state is represented by the set of sensed channels $(i_1, i_2, ..., i_{N-1})$. So there exist $\binom{N}{N-1}$ states. For state $(i_1, i_2, ..., i_{N-1})$, the maximal reward is

$$\Omega^1_{\max}(i_1, i_2, ..., i_{N-1}) = \theta_{l \in \mathcal{N} \backslash \{i_1, i_2, ..., i_{N-1}\}} \int_0^\infty c_N f(\gamma) h_{SNR}(\gamma) d\gamma \quad (14)$$

where the superscript '1' means the stage number and $\mathcal{N}$ denotes the set of all N potential channels. Since the computational complexity for the calculation at a channel is $O(1)$, the computational complexity at this stage is $O\Big(\binom{N}{N-1}\Big)$.

Stage k $(1 < k \leq N)$ – Calculate the maximal reward value associated with the kth from the last channel in the sensing order, based on results in stage $k-1$. At this stage, $N-k$ channels, denoted by $(i_1, i_2, ..., i_{N-k})$, have been sensed. Thus there are $\binom{N}{N-k}$ states. Each state $(i_1, i_2, ..., i_{N-k})$ has k possible transitions to stage $k-1$. The lth $(1 \leq l \leq k)$ transition leads to state $(i_1, ..., i_{N-k}, j_l)$ at stage $k-1$, which means channel $j_l \in \mathcal{N} \backslash \{i_1, i_2, ..., i_{N-k}\}$ is first sensed subsequently. The reward associated with the lth transition is denoted as $F\big(j_l, \Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l)\big)$ given by

$$\begin{aligned} &F\big(j_l, \Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l)\big) \\ &= \theta_{j_l} \Big[\int_{f^{-1}\left(\frac{\Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l)}{c_{N-k+1}}\right)}^{\infty} c_{N-k+1} f(\gamma) h_{SNR}(\gamma) d\gamma \\ &\quad + \Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l) \int_0^{f^{-1}\left(\frac{\Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l)}{c_{N-k+1}}\right)} \\ &\quad h_{SNR}(\gamma) d\gamma \Big] + (1 - \theta_{j_l}) \Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l). \end{aligned} \quad (15)$$

So the maximal reward of state $(i_1, i_2, ..., i_{N-k})$ is

$$\Omega^k_{\max}(i_1, i_2, ..., i_{N-k}) = \max_{1 \leq l \leq k} F\big(j_l, \Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l)\big). \quad (16)$$

At each state, it is also recorded which transition leads to the maximal reward. The computational complexity of each state is $O(k)$, with $O(1)$ for each transition. So the total computational complexity at stage k is $O\Big(k\binom{N}{N-k}\Big)$.

After the maximal reward value is obtained at stage N (i.e., the stage when no channel has been sensed yet), an optimal sensing order can be traced back according to the recorded optimal transition at each state.

The computational complexity of the preceding dynamic programming solution is

$$O\left(\sum_{k=1}^{N} k \binom{N}{N-k}\right) = O(N \cdot 2^{N-1}). \quad (17)$$

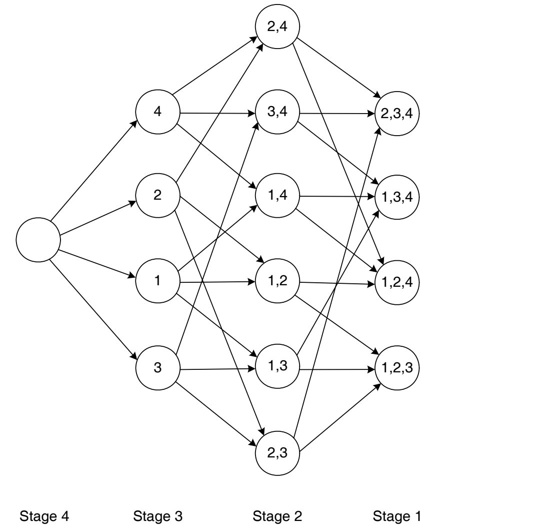

Fig. 2. The dynamic programming procedure with $N = 4$.

Fig. 2 illustrates the dynamic programming procedure for the case $N = 4$. Table I shows a comparison of the computational complexity of brute force search and our dynamic programming solution when N varies from 2 to 20. We can see that our solution can significantly lower the computational overhead, especially when N is large.

C. Numerical Results

For a secondary user with three potential channels, Fig. 3 shows the expected reward U_1 with the optimal sensing order

[2]For presentation simplicity, we omit the qualifier "expected" in the sequel.

$$
\begin{aligned}
& U_{k-1}^{(s_1,s_2,\ldots,s_N)} \\
= \;& \theta_{s_{k-1}}\left[\int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_N)}}{c_{k-1}}\right)}^{\infty} c_{k-1}f(\gamma)h_{SNR}(\gamma)d\gamma + U_k^{(s_1,s_2,\ldots,s_N)}\int_0^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_N)}}{c_{k-1}}\right)} h_{SNR}(\gamma)d\gamma\right] \\
& +(1-\theta_{s_{k-1}})U_k^{(s_1,s_2,\ldots,s_N)} \\
= \;& \theta_{s_{k-1}}\left[\int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}^{\infty} c_{k-1}f(\gamma)h_{SNR}(\gamma)d\gamma + \int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_N)}}{c_{k-1}}\right)}^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)} c_{k-1}f(\gamma)h_{SNR}(\gamma)d\gamma\right. \\
& \left.+U_k^{(s_1,s_2,\ldots,s_N)}\int_0^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_N)}}{c_{k-1}}\right)} h_{SNR}(\gamma)d\gamma\right] + (1-\theta_{s_{k-1}})U_k^{(s_1,s_2,\ldots,s_N)} \\
\leq \;& \theta_{s_{k-1}}\left[\int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}^{\infty} c_{k-1}f(\gamma)h_{SNR}(\gamma)d\gamma + U_k^{(s_1,s_2,\ldots s_{k-1},P(s_k,\ldots,s_N))}\int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_N)}}{c_{k-1}}\right)}^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}\right. \\
& \left. h_{SNR}(\gamma)d\gamma + U_k^{(s_1,s_2,\ldots,s_N)}\int_0^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_N)}}{c_{k-1}}\right)} h_{SNR}(\gamma)d\gamma\right] + (1-\theta_{s_{k-1}})U_k^{(s_1,s_2,\ldots,s_N)} \\
\leq \;& \theta_{s_{k-1}}\left[\int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}^{\infty} c_{k-1}f(\gamma)h_{SNR}(\gamma)d\gamma + U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}\int_0^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}\right. \\
& \left. h_{SNR}(\gamma)d\gamma\right] + (1-\theta_{s_{k-1}})U_k^{(s_1,s_2,\ldots,s_N)} \\
< \;& \theta_{s_{k-1}}\left[\int_{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}^{\infty} c_{k-1}f(\gamma)h_{SNR}(\gamma)d\gamma + U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}\int_0^{f^{-1}\left(\frac{U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))}}{c_{k-1}}\right)}\right. \\
& \left. h_{SNR}(\gamma)d\gamma\right] + (1-\theta_{s_{k-1}})U_k^{(s_1,s_2,\ldots,s_{k-1},P(s_k,\ldots,s_N))} \\
= \;& U_{k-1}^{(s_1,s_2,\ldots s_{k-1},P(s_k,\ldots,s_N))}.
\end{aligned}
\tag{12}
$$

TABLE I
THE COMPUTATIONAL COMPLEXITY OF BRUTE FORCE SEARCH AND DYNAMIC PROGRAMMING.

N	2	4	6	8	10	15	20
Brute force search complexity $O(\cdot)$	4	96	4320	3.2×10^5	3.6×10^7	2.0×10^{13}	4.9×10^{19}
Dynamic programming complexity $O(\cdot)$	4	32	192	1024	5120	2.5×10^5	1.0×10^7

when θ_1 and θ_2 are fixed, while θ_3 varies from 0.1 to 0.9. A Rayleigh fading channel model is used, in which the pdf of the channel gain is $h_{SNR}(\gamma) = \frac{1}{\Gamma}e^{-\frac{\gamma}{\Gamma}}, \gamma > 0$. And the transmission rate achievable in channel i (when channel i is free) is $\log(1+SNR_i)$. In Fig. 3, the legend shows the values of θ_1 and θ_2. It can be seen that, when θ_1 and θ_2 are large (e.g., 0.9 and 0.8, respectively), increasing θ_3 does not bring about a significant increase in U_1. This is because when a channel has a large availability probability, the chance that the secondary user stops at this channel is also large. So the impact of other channels is not significant. On the other hand, when θ_1 and θ_2 are small (e.g., 0.2 and 0.1, respectively), increasing θ_3 leads to a significant change in U_1, as shown in Fig. 3.

IV. SIMPLE OPTIMAL SENSING ORDERS IN SPECIAL CASES

In some special cases, a simple optimal sensing order exists, with much lower computational complexity (than both dynamic programming and brute force search), as we show in this section.

In a real system, it may not be practical to have continuous variable values for the transmission rate in adaptive modulation (e.g., to achieve $\log(1+SNR_i)$ exactly). Thus adaptive modulation with finite states is more practical. To implement this, for each channel, $M-1$ thresholds divide the SNR range $(0,\infty)$ into M regions, with region i corresponding to an achievable rate R_i ($R_1 \geq R_2 \geq \ldots \geq R_M$). When a channel is free, the probability of achievable rate R_i is denoted as p_i, which can be obtained from the channel gain distribution. We

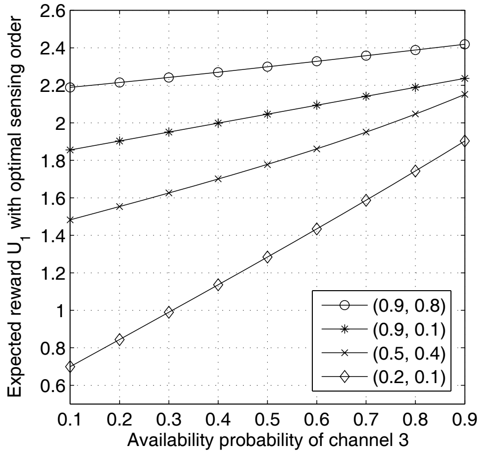

Fig. 3. The expected reward U_1 with optimal sensing order when θ_3 varies in a three-potential-channel case.

have $\sum_{j=1}^{M} p_j = 1$. Let $\overline{R} = \sum_{j=1}^{M} p_j R_j$.

Lemma 5: If $\theta_i \geq \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}}$, $\forall i$, then at each sensed channel except the last one, the secondary user should stop at the channel (i.e., transmit its information) only if the channel is free and the achievable transmission rate in the channel is R_1.

Proof: Consider a sensing order $(s_1, s_2, ..., s_N)$. If channel s_k $(1 \leq k < N)$ is sensed free, then the expected reward if the user proceeds to channel s_{k+1} is

$$\begin{aligned} U_{k+1} &\geq \theta_{s_{k+1}} c_{k+1} \overline{R} \geq \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}} \cdot c_{k+1} \overline{R} \\ &\geq \frac{c_k}{c_{k+1}} \frac{R_2}{\overline{R}} \cdot c_{k+1} \overline{R} = c_k R_2. \end{aligned} \quad (18)$$

On the other hand, if the secondary user proceeds to channel s_{k+1}, then its transmission effectiveness is not larger than c_{k+1}, as the transmission effectiveness is c_{k+1}, c_{k+2}, ..., and c_N if the secondary user stops at channel s_{k+1}, s_{k+2}, ..., and s_N, respectively. Since the maximum achievable rate is R_1, we have

$$U_{k+1} \leq c_{k+1} R_1 < c_k R_1. \quad (19)$$

From (18) and (19), the expected reward of proceeding to channel s_{k+1} is smaller than $c_k R_1$ and larger than (or equal to[3]) $c_k R_2$. Therefore, at channel s_k, if the channel is free and the achievable transmission rate is R_1, the secondary user transmits in channel s_k; otherwise (i.e., either channel s_k is busy, or the channel is free but the achievable transmission rate is among $\{R_2, R_3, ..., R_M\}$), the secondary user proceeds to channel s_{k+1}. This completes the proof. ■

From Lemma 5, we have the following simple optimal sensing order case.

Lemma 6: If $\theta_i \geq \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}}$, $\forall i$, then in any optimal sensing order denoted by $(s_1, s_2, ..., s_N)$, 1) the first $N-1$ channels should be in descending order of their channel availability probabilities, i.e., $\theta_{s_1} > \theta_{s_2} > ... > \theta_{s_{N-1}}$; and 2) $\theta_{s_{N-1}} > \theta_{s_N}$ if $c_{N-1} p_1 R_1 > c_N \overline{R}$, and $\theta_{s_{N-1}} < \theta_{s_N}$ if $c_{N-1} p_1 R_1 < c_N \overline{R}$.

[3]The equality happens only when 1) $k = N-1$, and 2) $\theta_{s_N} = \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}}$.

Proof: We first prove claim 1) using proof by contradiction. Suppose there exists $k \leq N-2$, such that $\theta_{s_k} < \theta_{s_{k+1}}$. From Lemma 5, the expected reward at channel s_k is

$$\begin{aligned} U_k &= \theta_{s_k} p_1 c_k R_1 + (1 - \theta_{s_k} p_1) U_{k+1} \\ &= \theta_{s_k} p_1 c_k R_1 + (1 - \theta_{s_k} p_1) \\ &\quad \cdot \Big(\theta_{s_{k+1}} p_1 c_{k+1} R_1 + (1 - \theta_{s_{k+1}} p_1) U_{k+2} \Big) \\ &= \theta_{s_k} p_1 c_k R_1 + \theta_{s_{k+1}} p_1 c_{k+1} R_1 - \theta_{s_k} \theta_{s_{k+1}} p_1^2 c_{k+1} R_1 \\ &\quad + (1 - \theta_{s_k} p_1)(1 - \theta_{s_{k+1}} p_1) U_{k+2}. \end{aligned}$$

If we switch the positions of channels s_k and s_{k+1} in the sensing order, a new order can be obtained. For the new order we have

$$\begin{aligned} U_k^{\text{new}} &= \theta_{s_{k+1}} p_1 c_k R_1 + \theta_{s_k} p_1 c_{k+1} R_1 - \theta_{s_k} \theta_{s_{k+1}} p_1^2 c_{k+1} R_1 \\ &\quad + (1 - \theta_{s_k} p_1)(1 - \theta_{s_{k+1}} p_1) U_{k+2}. \end{aligned} \quad (20)$$

Then we have

$$U_k^{\text{new}} - U_k = (\theta_{s_{k+1}} - \theta_{s_k})(c_k - c_{k+1}) p_1 R_1 > 0. \quad (21)$$

Using a similar derivation to that used in the proof of Lemma 4, we obtain

$$U_1^{\text{new}} > U_1. \quad (22)$$

This contradicts the condition that the sensing order $(s_1, s_2, ..., s_N)$ is optimal.

Next we prove claim 2). For the optimal sensing order $(s_1, s_2, ..., s_N)$, from Lemma 5 we have

$$\begin{aligned} U_{N-1} &= \theta_{s_{N-1}} p_1 c_{N-1} R_1 + (1 - \theta_{s_{N-1}} p_1) U_N \\ &= \theta_{s_{N-1}} p_1 c_{N-1} R_1 + (1 - \theta_{s_{N-1}} p_1) \theta_{s_N} C_N \overline{R} \\ &= \theta_{s_{N-1}} p_1 c_{N-1} R_1 + \theta_{s_N} C_N \overline{R} - \theta_{s_{N-1}} \theta_{s_N} p_1 C_N \overline{R}. \end{aligned}$$

If we switch the positions of channel s_{N-1} and s_N in the sensing order, we obtain a new sensing order, and we have

$$U_{N-1}^{\text{new}} = \theta_{s_N} p_1 c_{N-1} R_1 + \theta_{s_{N-1}} C_N \overline{R} - \theta_{s_{N-1}} \theta_{s_N} p_1 C_N \overline{R} \quad (23)$$

and

$$U_{N-1} - U_{N-1}^{\text{new}} = (\theta_{s_{N-1}} - \theta_{s_N})(c_{N-1} p_1 R_1 - C_N \overline{R}). \quad (24)$$

From Lemma 4, it can be seen that $U_{N-1} \geq U_{N-1}^{\text{new}}$. Thus we have $\theta_{s_{N-1}} > \theta_{s_N}$ if $c_{N-1} p_1 R_1 > C_N \overline{R}$; and $\theta_{s_{N-1}} < \theta_{s_N}$ if $c_{N-1} p_1 R_1 < C_N \overline{R}$. If $c_{N-1} p_1 R_1 = C_N \overline{R}$, the new sensing order is also optimal. ■

From Lemma 6, we can have the following observations.

- If $\theta_i \geq \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}}$, $\forall i$, and $c_{N-1} p_1 R_1 > c_N \overline{R}$, then the Intuitive Sensing Order is optimal.
- If $\theta_i \geq \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}}$, $\forall i$, and $c_{N-1} p_1 R_1 = c_N \overline{R}$, then in the optimal sensing order $(s_1, s_2, ..., s_N)$, we have $\theta_{s_1} > \theta_{s_2} > ... > \theta_{s_{N-1}}$. To obtain an optimal sensing order, we need to calculate the expected rewards U_1's in N possible sensing orders, each with the last channel being channel 1, 2,..., and N, respectively, and the first $N-1$ channels being in descending order of their channel availability probabilities. The computational complexity for each sensing order is $O(N)$. So the computational complexity of the optimal sensing order search is $O(N^2)$.
- If $\theta_i \geq \frac{c_{N-1}}{c_N} \frac{R_2}{\overline{R}}$, $\forall i$, and $c_{N-1} p_1 R_1 < c_N \overline{R}$, then in the optimal sensing order $(s_1, s_2, ..., s_N)$, we have

$\theta_{s_1} > \theta_{s_2} > ... > \theta_{s_{N-1}}$, and channel s_{N-1} should be the one with the minimum availability probability. Accordingly the computational complexity of the optimal sensing order search is $O\big(N(N-1)\big)$.

Lemma 7: If there exists $m \in \{1, 2, ..., M\}$, such that $R_{m+1} \leq U_k/c_{k-1} < R_m$[4] for $\forall k \in \{2, 3, ..., N\}$ in any sensing order, then the first $N-1$ channels in any optimal sensing order should be in descending order of their channel availability probabilities.

Proof: We use proof by contradiction. Consider an optimal sensing order $(s_1, s_2, ..., s_N)$. Suppose there exists $k \leq N-2$, such that $\theta_{s_k} < \theta_{s_{k+1}}$. Since $R_{m+1} \leq U_{k+2}/c_{k+1} < R_m$, we have

$$U_{k+1} = \theta_{s_{k+1}} \sum_{j=1}^{m} p_j c_{k+1} R_j + (1 - \theta_{s_{k+1}} \sum_{j=1}^{m} p_j) U_{k+2}. \quad (25)$$

And further we have

$$\begin{aligned} U_k &= \theta_{s_k} \sum_{j=1}^{m} p_j c_k R_j + (1 - \theta_{s_k} \sum_{j=1}^{m} p_j) U_{k+1} \\ &= \theta_{s_k} c_k \sum_{j=1}^{m} p_j R_j + \big(1 - \theta_{s_k} \sum_{j=1}^{m} p_j\big) \Big(\theta_{s_{k+1}} c_{k+1} \\ &\quad \cdot \sum_{j=1}^{m} p_j R_j + \big(1 - \theta_{s_{k+1}} \sum_{j=1}^{m} p_j\big) U_{k+2} \Big) \\ &= \big(\theta_{s_k} c_k + \theta_{s_{k+1}} c_{k+1}\big) \sum_{j=1}^{m} p_j R_j \\ &\quad - \theta_{s_k} \theta_{s_{k+1}} c_{k+1} \sum_{j=1}^{m} p_j \sum_{j=1}^{m} p_j R_j \\ &\quad + (1 - \theta_{s_k} \sum_{j=1}^{m} p_j)(1 - \theta_{s_{k+1}} \sum_{j=1}^{m} p_j) U_{k+2}. \end{aligned} \quad (26)$$

Now we switch the positions of channels s_k and s_{k+1} in the sensing order, and obtain a new sensing order. The expected reward at the kth sensing of the new order is

$$\begin{aligned} U_k^{\text{new}} &= \big(\theta_{s_{k+1}} c_k + \theta_{s_k} c_{k+1}\big) \sum_{j=1}^{m} p_j R_j \\ &\quad - \theta_{s_k} \theta_{s_{k+1}} c_{k+1} \sum_{j=1}^{m} p_j \sum_{j=1}^{m} p_j R_j \\ &\quad + (1 - \theta_{s_k} \sum_{j=1}^{m} p_j)(1 - \theta_{s_{k+1}} \sum_{j=1}^{m} p_j) U_{k+2}. \end{aligned} \quad (27)$$

And we have

$$U_k^{\text{new}} - U_k = (\theta_{s_{k+1}} - \theta_{s_k})(c_k - c_{k+1}) \sum_{j=1}^{m} p_j R_j > 0. \quad (28)$$

Using a similar derivation to that used in the proof of Lemma 4, we further have

$$U_1^{\text{new}} > U_1. \quad (29)$$

This contradicts the condition that the sensing order $(s_1, s_2, ..., s_N)$ is optimal. ■

[4]We set R_{M+1} to 0.

Directly applying Lemma 7 leads to the following lemma.

Lemma 8: For N potential channels, if $\overline{R} \sum_{i=1}^{N} \theta_i \leq R_M$, then the first $N-1$ channels in any optimal sensing order should be in descending order of their channel availability probabilities.

Proof: For any sensing order $(s_1, s_2, ..., s_N)$, we have

$$\begin{aligned} U_N &= \theta_{s_N} c_N \overline{R} \\ U_k &= \theta_{s_k} \sum_{1 \leq j \leq M;\ c_k R_j > U_{k+1}} p_j c_k R_j \\ &\quad + \Big(1 - \theta_{s_k} \sum_{1 \leq j \leq M;\ c_k R_j > U_{k+1}} p_j\Big) U_{k+1} \\ &< \theta_{s_k} \sum_{1 \leq j \leq M} p_j c_k R_j + U_{k+1} \\ &= \theta_{s_k} c_k \overline{R} + U_{k+1}, \qquad for \quad 1 \leq k \leq N-1. \end{aligned}$$

It can be seen that $U_k \leq \sum_{i=k}^{N} \theta_{s_i} c_i \overline{R}$, for $\forall k \in \{1, ..., N\}$, and further we have

$$U_k \leq \sum_{i=k}^{N} \theta_{s_i} c_i \overline{R} < c_k \overline{R} \sum_{i=1}^{N} \theta_{s_i} < c_{k-1} \overline{R} \sum_{i=1}^{N} \theta_i \leq c_{k-1} R_M$$

which leads to $U_k/c_{k-1} < R_M$, for $\forall k \in \{1, 2, .., N\}$. Here we have $c_0 = 1$. From Lemma 7, the first $N-1$ channels in any optimal sensing order should be in descending order of their channel availability probabilities. This completes the proof. ■

Lemma 8 indicates that when the summation of the N channel availability probabilities is small enough, a simple optimal sensing order may exist. On the other hand, it is possible that $R_M = 0$. In this case we have the following lemma.

Lemma 9: For the case with $R_M = 0$ and $R_{M-1} > 0$, if $\overline{R} \sum_{i=1}^{N} \theta_i \leq R_{M-1}$, then the first $N-1$ channels in any optimal sensing order should be in descending order of their channel availability probabilities.

Proof: Similar to the proof of Lemma 8, we have $U_k/c_{k-1} < R_{M-1}$, for $\forall k \in \{1, 2, .., N\}$. On the other hand, $U_k/c_{k-1} \geq R_M = 0$. From Lemma 7, the first $N-1$ channels in any optimal sensing order should be in descending order of their channel availability probabilities. ■

V. More Complex Scenarios

In the preceding sections, the availability probability of each channel is fixed, and is known *a priori*. In addition, the duration of a time slot is assumed to be sufficient to sense up to N channels sequentially. In the sequel, more complex scenarios will be discussed.

A. Non-fixed and Unknown $\boldsymbol{\theta}$

We consider the scenario in which the channel availability probabilities, θ_i's, are not fixed for the long term. Rather $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_N)$ is fixed in a single time slot but randomly changes at the beginning of the next time slot according to joint pdf $g(\boldsymbol{\theta})$. At the beginning of each time slot, the secondary user does not know the exact values of the θ_i's. After each sensing, the cognitive user can obtain posterior pdf $g^{\text{new}}(\boldsymbol{\theta})$ using Bayes' formula as detailed in the sequel. The secondary user thus has finer information about the value of $\boldsymbol{\theta}$ and can then use this updated information in making future decisions.

1) Without Adaptive Modulation: We know from Section III that, when adaptive modulation is not adopted, we should sense the channels according to descending order of the θ_i's if these are fixed and known to the secondary user. Similarly, with unknown $\boldsymbol{\theta}$, one may guess that the secondary user should sense the channels according to descending order of $\bar{\theta}_i = \int \theta_i g(\boldsymbol{\theta}) d\boldsymbol{\theta}$. However, this may not lead to optimal performance. Generally, the sensing of each channel at each slot will have two effects: short-term gain and long-term gain. The short-term gain refers to the opportunity for packet transmission in the current slot (if the channel is sensed free), while the long-term gain refers to the ability to update the state information about the channel (e.g., about $g(\boldsymbol{\theta})$), which can benefit later decisions about which channels to sense in subsequent slots. It is obvious that selecting the largest $\bar{\theta}_i$ to sense based on available information maximizes only the short-term gain. An optimal solution should thus balance between short-term gain and long term gain. The optimal balance can be achieved via dynamic programming as follows. As adaptive modulation is not used, clearly the user should transmit at the first sensed-free channel, with constant rate R.

Stage 1 – If the user proceeds to sense the final channel in the sensing order, then a state is represented by the set of sensed channels $(i_1, i_2, ..., i_{N-1})$. So there exist $\binom{N}{N-1}$ states. Since the user has proceeded to the final channel, this means that the previous $N-1$ sensed channels are all busy. Thus the pdf of $\boldsymbol{\theta}$ at state $(i_1, i_2, ..., i_{N-1})$ can be updated via Bayes' formula as follows:

$$g^{\text{new}}_{(i_1,i_2,...,i_{N-1})}(\boldsymbol{\theta}) = \frac{\left(\Pi_{j\in\{i_1,i_2,...,i_{N-1}\}}(1-\theta_j)\right)g(\boldsymbol{\theta})}{\int \left(\Pi_{j\in\{i_1,i_2,...,i_{N-1}\}}(1-\theta_j)\right)g(\boldsymbol{\theta})d\boldsymbol{\theta}}.$$

Thus, the maximal reward at this state is

$$\Omega^1_{\max}(i_1, i_2, ..., i_{N-1}) = c_N R \int \theta_{j\in\mathcal{N}\backslash\{i_1,i_2,...,i_{N-1}\}} g^{\text{new}}_{(i_1,i_2,...,i_{N-1})}(\boldsymbol{\theta})d\boldsymbol{\theta}. \quad (30)$$

Stage k $(1 < k \leq N)$ – We assume that the first $N-k$ channels in the sensing order have been sensed to be busy. A state is then represented by the set of sensed channels $(i_1, i_2, ..., i_{N-k})$, and so there are $\binom{N}{N-k}$ states at this stage. At state $(i_1, i_2, ..., i_{N-k})$, the pdf of $\boldsymbol{\theta}$ is updated as

$$g^{\text{new}}_{(i_1,...,i_{N-k})}(\boldsymbol{\theta}) = \frac{\left(\Pi_{j\in\{i_1,i_2,...,i_{N-k}\}}(1-\theta_j)\right)g(\boldsymbol{\theta})}{\int \left(\Pi_{j\in\{i_1,i_2,...,i_{N-k}\}}(1-\theta_j)\right)g(\boldsymbol{\theta})d\boldsymbol{\theta}}.$$

Each state $(i_1, i_2, ..., i_{N-k})$ has k possible transitions to stage $k-1$. The lth $(1 \leq l \leq k)$ transition leads to state $(i_1, ..., i_{N-k}, j_l)$ at stage $k-1$, which means channel $j_l \in \mathcal{N}\backslash\{i_1, i_2, ..., i_{N-k}\}$ is first sensed subsequently. So the maximal reward at state $(i_1, i_2, ..., i_{N-k})$ is given by

$$\Omega^k_{\max}(i_1, ..., i_{N-k}) = \max_{1\leq l\leq k}\left\{c_{N-k+1}R\int \theta_{j_l} g^{\text{new}}_{(i_1,...,i_{N-k})}(\boldsymbol{\theta})d\boldsymbol{\theta} + \left(1-\int \theta_{j_l} g^{\text{new}}_{(i_1,...,i_{N-k})}(\boldsymbol{\theta})d\boldsymbol{\theta}\right)\cdot \Omega^{k-1}_{\max}(i_1, ..., i_{N-k}, j_l)\right\}.$$

At each state, we also record which transition leads to the maximal reward. And after the maximal reward is obtained at stage N (i.e., the stage at which no channel has been sensed yet), an optimal sensing order can be traced back according to the recorded optimal transition at each state.

2) With Adaptive Modulation: When adaptive modulation is adopted, the user may skip a channel if the channel is available but of poor quality. A dynamic programming approach similar to that in Section V-A1 can be used to find an optimal balance between short-term and long-term gains, but with higher complexity. In the following, only the differences between this and the non-adaptive modulation case are discussed.

Stage 1 – Assume that there is one channel remaining unsensed. With the same subset of $N-1$ sensed channels, different sensing results of the $N-1$ channels will lead to different updated version of $g(\boldsymbol{\theta})$. Therefore, we have $\binom{N}{N-1}2^{N-1}$ states, where $\binom{N}{N-1}$ is the number of subsets of the $N-1$ sensed channels, and the factor 2^{N-1} is due to the fact that each sensed channel may be either free or busy. A state is denoted by $((i_1, z_{i_1}), (i_2, z_{i_2}), ..., (i_{N-1}, z_{i_{N-1}}))$, where $i_1, i_2, ..., i_{N-1}$ are the sensed channels, and z_{i_l} is the sensing result ('1' for free and '0' for busy) of channel i_l. The state has maximal reward

$$\Omega^1_{\max}\left((i_1, z_{i_1}), ..., (i_{N-1}, z_{i_{N-1}})\right) = \int \theta_{m\in\mathcal{N}\backslash\{i_1,...,i_{N-1}\}} g^{\text{new}}_{((i_1,z_{i_1}),(i_2,z_{i_2}),...,(i_{N-1},z_{i_{N-1}}))}(\boldsymbol{\theta})d\boldsymbol{\theta} \cdot \int_0^\infty c_N f(\gamma) h_{SNR}(\gamma)d\gamma \quad (31)$$

where the updated pdf of $\boldsymbol{\theta}$ is

$$g^{\text{new}}_{((i_1,z_{i_1}),(i_2,z_{i_2}),...,(i_{N-1},z_{i_{N-1}}))}(\boldsymbol{\theta}) = \frac{\left(\Pi_{j\in\{i_1,i_2,...,i_{N-1}\}}\omega_j\right)g(\boldsymbol{\theta})}{\int \left(\Pi_{j\in\{i_1,i_2,...,i_{N-1}\}}\omega_j\right)g(\boldsymbol{\theta})d\boldsymbol{\theta}} \quad (32)$$

with

$$\omega_j = \begin{cases} \theta_j, & \text{if } z_j = 1 \\ 1-\theta_j, & \text{if } z_j = 0. \end{cases} \quad (33)$$

Stage k $(1 < k \leq N)$ – Assume there are k channels remaining unsensed, in which case there are $\binom{N}{N-k}2^{N-k}$ possible states. For state $\left((i_1, z_{i_1}), (i_2, z_{i_2}), ..., (i_{N-k}, z_{i_{N-k}})\right)$, if channel j_l $(1 \leq l \leq k,\ j_l \in \mathcal{N}\backslash\{i_1, ..., i_{N-k}\})$ is selected to sense, it leads to state $\left((i_1, z_{i_1}), ..., (i_{N-k}, z_{i_{N-k}}), (j_l, 1)\right)$ in stage $k-1$ if the channel is sensed free, and to state $\left((i_1, z_{i_1}), ..., (i_{N-k}, z_{i_{N-k}}), (j_l, 0)\right)$ in stage $k-1$ if the channel is sensed busy. So the maximal reward at state $\left((i_1, z_{i_1}), (i_2, z_{i_2}), ..., (i_{N-k}, z_{i_{N-k}})\right)$ is given in (34) on the top of the next page. In (34), the updated pdf of $\boldsymbol{\theta}$ is obtained similarly to (32).

B. Limited Sensing Capacity in Each Time Slot

When the number of potential channels, N, is large, it is likely that the user cannot sequentially sense all possible N channels in a time slot, i.e., we will have $N\tau > T$. Let B denote the maximum number of channels that can be sensed sequentially in a time slot, i.e., $B = \lfloor \frac{T}{\tau} \rfloor$ ($\lfloor\cdot\rfloor$ denoting the floor function). Adaptive modulation is adopted.

$$\begin{aligned}&\Omega^k_{\max}\big((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}})\big)\\=&\max_{1\le l\le k, j_l\in\mathcal{N}\setminus\{i_1,...,i_{N-k}\}}\Bigg\{\int\theta_{j_l}g^{\text{new}}_{((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}}))}(\boldsymbol{\theta})d\boldsymbol{\theta}\\&\cdot\Bigg[\int_{f^{-1}\left(\frac{\Omega^{k-1}_{\max}((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}}),(j_l,1))}{c_{N-k+1}}\right)}^{\infty}c_{N-k+1}f(\gamma)h_{SNR}(\gamma)d\gamma\\&+\Omega^{k-1}_{\max}\big((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}}),(j_l,1)\big)\int_0^{f^{-1}\left(\frac{\Omega^{k-1}_{\max}((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}}),(j_l,1))}{c_{N-k+1}}\right)}h_{SNR}(\gamma)d\gamma\Bigg]\\&+\left(1-\int\theta_{j_l}g^{\text{new}}_{((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}}))}(\boldsymbol{\theta})d\boldsymbol{\theta}\right)\Omega^{k-1}_{\max}\big((i_1,z_{i_1}),...,(i_{N-k},z_{i_{N-k}}),(j_l,0)\big)\Bigg\}.\end{aligned}\quad(34)$$

1) With Fixed and Known $\boldsymbol{\theta}$: When the θ_i's are fixed and known to the secondary user, we have the following lemma.

Lemma 10: Any optimal sensing order is a permutation of the channels with the B maximal values of θ_i.

Proof: We use proof by contradiction. Suppose an optimal sensing set and order is $(s_1,s_2,...,s_B)$, and there exists a channel s_l $(1\le l\le B)$ that is not among the channels with the B maximal values of θ_i. Among the channels with the B maximal values of θ_i, we select a channel that is not in $(s_1,...,s_{l-1},s_{l+1},...,s_B)$ and denote it by j. Thus $\theta_j>\theta_{s_l}$. It can be easily seen that

$$U_l^{(s_1,...,s_{l-1},j,s_{l+1},...,s_B)}>U_l^{(s_1,...,s_{l-1},s_l,s_{l+1},...,s_B)}.\quad(35)$$

Using a similar method to that in the proof of Lemma 4, we have

$$U_1^{(s_1,...,s_{l-1},j,s_{l+1},...,s_B)}>U_1^{(s_1,...,s_{l-1},s_l,s_{l+1},...,s_B)}.\quad(36)$$

This contradicts the condition that $(s_1,s_2,...,s_B)$ is an optimal sensing order. ■

Based on Lemma 10, the user needs to select only the channels with the B maximal values of θ_i, and uses the method in Sections III-B (for the general case) or IV (for simple cases) to find an optimal sensing order.

2) With Non-fixed and Unknown $\boldsymbol{\theta}$: When the exact value of $\boldsymbol{\theta}$ is not fixed and unknown *a priori*, and the user has only the initial information of $g(\boldsymbol{\theta})$, an optimal sensing order may not be selected from the channels with the B maximal values of $\bar{\theta}_i$. Instead, a dynamic programming approach similar to that in Section V-A2 can be used. The difference lies in that: 1) there are B stages, and 2) the number of states in stage k $(1\le k\le B)$ is $\binom{N}{B-k}2^{B-k}$. The corresponding dynamic programming procedure is a straightforward modification of those above, and thus is omitted here.

VI. Impact of Channel Sensing Errors and Solution

In the preceding sections, we have assumed that there are no channel sensing errors. However, in a practical cognitive radio network, channel sensing error is inevitable. In this section, we investigate the impact of channel sensing errors and further propose a solution to address this problem. In what follows, it is assumed that the availability probability of each channel is fixed, and is known *a priori*. Adaptive modulation is adopted.

In cognitive radio, the secondary user needs to sense whether the primary user is present. In this detection problem, a channel sensing error is represented by either a *false alarm* (i.e., the primary users are idle, but the secondary user senses the channel as busy) or a *missed detection* (i.e., the primary users are active, but the secondary user senses the channel as free). In the former case, the spectrum opportunity is wasted, while in the latter case, a collision with the primary receivers may happen. When a secondary user senses a channel, let p_f and p_m denote the probability of false alarm and missed detection, respectively, which are assumed to be known *a priori*. For simplicity of exposition, suppose $(1,2,...,N)$ is an optimal sensing order for the N potential channels. For each channel $i\in\{1,2,...,N\}$, let θ_i denote the channel availability probability, and $\hat{\theta}_i$ the perceived availability probability of channel i. Then we have

$$\hat{\theta}_i=\theta_i(1-p_f)+(1-\theta_i)p_m.\quad(37)$$

So if the secondary user proceeds to channel i, the probability of stopping at channel i is

$$\hat{\theta}_i\int_{f^{-1}\left(\frac{\hat{U}_{i+1}}{c_i}\right)}^{\infty}h_{SNR}(\gamma)d\gamma$$

where $\hat{U}_{i+1}$ is the expected reward at channel $i+1$ if the user proceeds to that channel, and can be obtained recursively from $\hat{U}_N$ to $\hat{U}_1$ (similar to the method to obtain U_i in Section II). So for channel i, if there is primary activity in a time slot, the probability of a collision at primary receivers is

$$\begin{aligned}p_{c,i}=&\left(\prod_{k=1}^{i-1}\left(1-\hat{\theta}_k\int_{f^{-1}\left(\frac{\hat{U}_{k+1}}{c_k}\right)}^{\infty}h_{SNR}(\gamma)d\gamma\right)\right)\\&\cdot p_m\cdot\int_{f^{-1}\left(\frac{\hat{U}_{i+1}}{c_i}\right)}^{\infty}h_{SNR}(\gamma)d\gamma.\end{aligned}\quad(38)$$

On the right-hand side of (38), the three factors are the probability that the secondary user proceeds to channel i, the probability of missed detection in channel i, and the probability that the channel gain of the secondary user at channel i is satisfactory, respectively.

Let $p^*\in(0,1]$ denote the tolerable probability of collisions with primary users in each channel. If not all the $p_{c,i}$ values are bounded by p^*, actions need to be taken. In this research, a *trusting probability* denoted by $\alpha_i \in (0,1]$ is introduced for channel i. If the physical layer at the secondary user indicates a busy channel after sensing, the result is trusted, while if a free channel is indicated, the result is trusted with probability α_i. So we have

$$\hat{\theta}_i = \alpha_i\big(\theta_i(1-p_f) + (1-\theta_i)p_m\big), \quad (39)$$

and

$$p_{c,i} = \left(\prod_{k=1}^{i-1}\left(1-\hat{\theta}_k \int_{f^{-1}\left(\frac{\hat{U}_{k+1}}{c_k}\right)}^{\infty} h_{SNR}(\gamma)d\gamma\right)\right)\cdot p_m \cdot\alpha_i \cdot \int_{f^{-1}\left(\frac{\hat{U}_{i+1}}{c_i}\right)}^{\infty} h_{SNR}(\gamma)d\gamma. \quad (40)$$

Accordingly, we propose the following procedure to bound the probability of collisions with primary receivers in each channel.

- Step 1: Set $\alpha_i = 1,\ \forall i \in \{1, ..., N\}$;
- Step 2: Determine $\hat{\theta}_i$ and $\hat{U}_i$, $\forall i$, and calculate $p_{c,i}$, $\forall i$;
- Step 3: Find the channel with the maximal value of $p_{c,i}$, and denote this channel index as i';
- Step 4: If $p_{c,i'} \leq p^*$, terminate; otherwise, proceed to Step 5;
- Step 5: Define a scale $\rho = \frac{p^*(1-\delta)}{p_{c,i'}}$ (in which $\delta \ll 1$ is a very small positive value), scale $\alpha_{i'}$ by a factor ρ (i.e., $\rho\alpha_{i'} \to \alpha_{i'}$), and proceed to Step 2.

It can be seen that the termination condition of the preceding procedure is $p_{c,i} \leq p^*,\ \forall i$.

Lemma 11: The preceding procedure terminates in a finite number of updates. Here an update refers to scaling of an α_i once.

Proof: If $p_m \leq p^*$, from (40) it can be seen that $p_{c,i} \leq p^*,\ \forall i$. No update is needed. So in the following we consider only the case with $p_m > p^*$.

In Step 5, before $\alpha_{i'}$ is scaled by ρ, we have $p_{c,i'} > p^*$. So

$$\rho = \frac{p^*(1-\delta)}{p_{c,i'}} < 1-\delta. \quad (41)$$

For any channel $j \in \{1, ..., N\}$, if it has experienced a number, l_j, of updates, its trusting probability will be $\alpha_j < 1\cdot(1-\delta)^{l_j}$. So we have

$$\begin{aligned} p_{c,j} &= \left(\prod_{k=1}^{j-1}\left(1-\hat{\theta}_k \int_{f^{-1}\left(\frac{\hat{U}_{k+1}}{c_k}\right)}^{\infty} h_{SNR}(\gamma)d\gamma\right)\right)\cdot p_m \\ &\quad \cdot\alpha_j \cdot \int_{f^{-1}\left(\frac{\hat{U}_{j+1}}{c_j}\right)}^{\infty} h_{SNR}(\gamma)d\gamma \\ &\leq p_m\cdot\alpha_j \\ &< p_m\cdot(1-\delta)^{l_j}. \end{aligned} \quad (42)$$

It can be seen that, if $l_j \geq \left\lceil \frac{\log p^* - \log p_m}{\log(1-\delta)} \right\rceil$ ($\lceil\cdot\rceil$ denoting the ceiling function), we have

$$\begin{aligned} p_{c,j} < p_m\cdot(1-\delta)^{l_j} &\leq p_m\cdot(1-\delta)^{\left\lceil \frac{\log p^* - \log p_m}{\log(1-\delta)} \right\rceil} \\ &\leq p_m\cdot(1-\delta)^{\frac{\log p^* - \log p_m}{\log(1-\delta)}} = p^*. \end{aligned}$$

As aforementioned, when $p_{c,j} < p^*$, α_j will not be scaled further. So it can be concluded that for each channel, its trusting probability will be scaled for at most $\left\lceil \frac{\log p^* - \log p_m}{\log(1-\delta)} \right\rceil$ times. And for the N channels, the number of updates will be bounded by $N\left\lceil \frac{\log p^* - \log p_m}{\log(1-\delta)} \right\rceil$. This completes the proof. ■

A similar solution can be derived if the channels do not have the same tolerable collision probability with the primary receivers, e.g., p_i^* for channel i. The difference lies in that: i) in Step 3, the channel with the maximal $p_{c,i}/p_i^*$ is selected; ii) in Step 4, the termination condition is $p_{c,i'}/p_{i'}^* \leq 1$; and iii) in Step 5, $\rho = \frac{p_{i'}^*(1-\delta)}{p_{c,i'}}$.

VII. Conclusions and Further Discussions

For cognitive radio networks with multiple potential channels and opportunistic transmissions, we have shown that the Intuitive Sensing Order may not be optimal when adaptive modulation is used. We have also provided dynamic programming solutions for determining optimal sensing orders. In several special cases, we have also devised simple and optimal sensing orders. The optimal sensing policies described in this paper will provide useful insights into the design of multi-channel medium access control protocols in cognitive radio networks.

In this research, we have assumed the busy/free status of each channel is independent from one slot to another. So the derived optimal sensing order can be used in each slot. On the other hand, when the channel busy/free status among different time slots is correlated (e.g., follows an on-off Markov model as in [2]), the sensing of each channel at each slot will have both short-term gain (i.e., immediate transmission opportunity) and long term gain (exploration of channel statistics, which benefits sensing of subsequent channels). So an optimal solution should strike a balance between them. Therefore, the channel sensing order may be different from one time slot to another. Generally a dynamic programming approach can be used to find optimal sensing orders in a fixed number of slots, which maximizes the total throughput in these slots. As the computational complexity is high, an interesting future research topic is to find sub-optimal solutions with lower computational complexity and comparable performance.

When there are multiple secondary users, it may not be optimal that all the secondary users use the same sensing order. For example, assume adaptive modulation is not used. When all the secondary users use the same sensing order, they will all select the first free channel in the order. Even if a good contention resolution is in place, only one channel will be used at any slot. In this case, a game theoretic approach might be helpful to find effective strategies of sensing orders at the secondary users.

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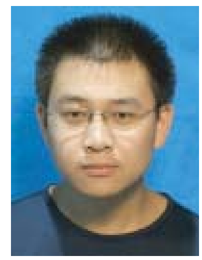
Hai Jiang (S'04-M'07) received the B.S. degree in 1995 and the M.S. degree in 1998, both in electronics engineering, from Peking University, China and the Ph.D. degree (with an Outstanding Achievement in Graduate Studies Award) in 2006 in electrical engineering from the University of Waterloo, Canada. He was a Postdoctoral Fellow at the Department of Electrical Engineering, Princeton University from September 2006 to June 2007. Since July 2007, he has been an Assistant Professor at the Department of Electrical and Computer Engineering, University of Alberta, Canada. His research interests include radio resource management, cellular/WLAN interworking, and cross-layer design for wireless multimedia communications. He is an Associate Editor for IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.

Lifeng Lai (M'07) received the B.E. and M. E. degrees in Information Science and Electrical Engineering from Zhejiang University, Hangzhou, China in 2001 and 2004 respectively, and the PhD degree in Electrical and Computer Engineering from the Ohio State University in Columbus, OH, in 2007. From March 2007, he has been working at Princeton University, first as a department guest then as a postdoctoral research associate. He was a Distinguished University Fellow of the Ohio State University from 2004 to 2007. His current research interest includes network information theory, wireless networks security and sequential analysis of wireless networks.



Rongfei Fan received the B.E. degree in 2007 in electrical engineering from Harbin Institute of Technology, China. He is currently a graduate student at the Department of Electrical and Computer Engineering, University of Alberta, Canada. His research interests include cognitive radio, ultra-wideband, and localization in wireless sensor networks.

H. Vincent Poor (S'72-M'77-SM'82-F'87) received the Ph.D. degree in EECS from Princeton University in 1977. From 1977 until 1990, he was on the faculty of the University of Illinois at Urbana-Champaign. Since 1990 he has been on the faculty at Princeton, where he is the Michael Henry Strater University Professor of Electrical Engineering and Dean of the School of Engineering and Applied Science. Dr. Poor's research interests are in the areas of stochastic analysis, statistical signal processing and their applications in wireless networks and related fields. Among his publications in these areas is the recent book *MIMO Wireless Communications* (Cambridge University Press, 2007).

Dr. Poor is a member of the National Academy of Engineering, a Fellow of the American Academy of Arts and Sciences, and a former Guggenheim Fellow. He is also a Fellow of the Institute of Mathematical Statistics, the Optical Society of America, and other organizations. In 1990, he served as President of the IEEE Information Theory Society, and in 2004-07 he served as the Editor-in-Chief of the IEEE TRANSACTIONS ON INFORMATION THEORY. Recent recognition of his work includes the 2005 IEEE Education Medal and the 2007 IEEE Marconi Prize Paper Award.