

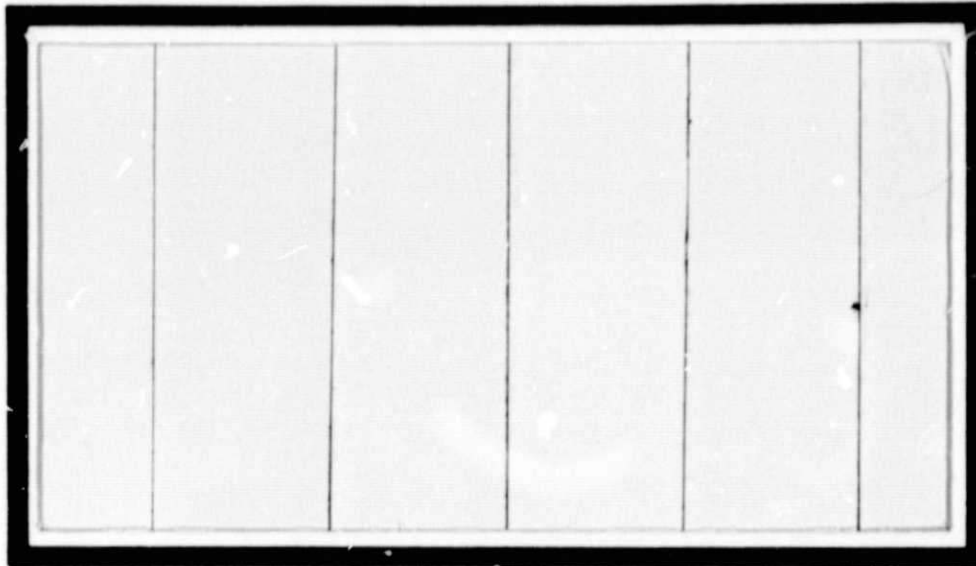
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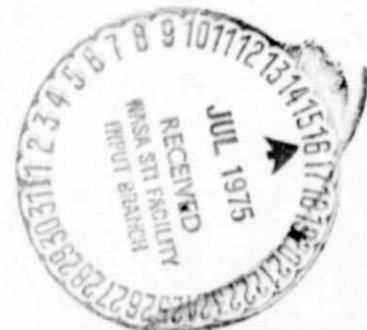
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DEPARTMENT OF MATHEMATICS

TEXAS A&M UNIVERSITY

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OPTIMAL SELECTION OF PASSES

by

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Report #2

OPTIMAL SELECTION OF PASSES

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1. Introduction

This report presents preliminary numerical results obtained from the application of a linear feature selection technique to the determination of combinations of passes which best discriminate between a given set of crops in a given area of interest. The results obtained are not purported to hold in a general situation, but only for the given set of crops and the given (but unknown) levels of several factors (such as soil type, fertilizer practice, etc.) holding in the area of interest. Nevertheless, some insight into optimal choices of passes can be gained through the application of the technique to data sets with different levels of those factors which are known to affect the spectral signatures. The technique could also be useful in determining those factors which significantly affect the choice of passes used for discrimination between a given set of crops.

For a given set of r registered ERTS passes, the input data consists of a $4r$ -dimensional mean vector and a $4r \times 4r$ covariance matrix for each of the m classes (assumed to be multivariate normal). For each of the combinations of r passes taken k at a time ($1 \leq k \leq r$) the portions of the mean vectors and covariance matrices pertaining to the k passes i_1, i_2, \dots, i_k are used to compute a $4k$ -dimensional vector $B(i_1, \dots, i_k)$ of norm one whose linear combination of the channels in the passes i_1, i_2, \dots, i_k minimizes the probability of misclassification, $g(B)$, for the transformed (one-dimensional) densities of the m classes. The

feature selection technique and associated computer program are described in [1].

In Section 2 we present numerical results obtained when using 6 registered passes of ERTS data from Pill County North.

2. Numerical Results--Hill County North

Input data for the runs described in this section consisted of 24-dimensional mean vectors and 24 x 24 covariance matrices computed from training data using six registered passes of ERTS data from Hill County North. The six registered ERTS passes (identified below) were, for purposes of this note, assigned the following pass numbers and channel numbers. The associated biological phases for Winter Wheat are also given.

| Pass Number | Channel Numbers | Date of Pass | Scene Number | Biological Phase (Winter Wheat) |
|-------------|-----------------|---------------|--------------|---------------------------------|
| 1 | 1-4 | Aug. 20, 1973 | 1393-17392 | Harvest |
| 2 | 5-8 | Aug. 3, 1973 | 1376-17452 | Yellow/Harvest |
| 3 | 9-12 | July 16, 1973 | 1358-17453 | Heading/Yellow |
| 4 | 13-16 | June 27, 1973 | 1339-17400 | Heading |
| 5 | 17-20 | June 11, 1973 | 1332-17460 | Heading |
| 6 | 21-24 | May 23, 1973 | 1304-17461 | Jointing (Greening) |

(Note that the passes are numbered in reverse chronological order)

The crops under consideration and their designated class numbers were:

| Class | Crop |
|-------|--------------|
| 1 | Barley |
| 2 | Stubble |
| 3 | Grass |
| 4 | Winter Wheat |
| 5 | Spring Wheat |
| 6 | Fallow |

For each of the sixty-three combinations of 6 passes taken k at a time ($1 \leq k \leq 6$), the appropriate signatures were used to compute a vector B of norm one (and appropriate dimension) whose linear combination of the channels minimizes the one-dimensional probability of misclassification. The resulting values, $g(B)$, of the one-dimensional probability of misclassification determined by the optimal B vectors for the various combinations of passes appear in Tables 1-5 below. The value for $g(B)$ using all channels within all six passes was .2699. The graphs of the six transformed density functions determined by the optimal B vector for the optimal one, two, three, four, five and six passes appear in Figures 1-6, respectively.

Additional runs were made using all six passes where Spring Wheat and Winter Wheat were alternately deleted and the remaining five crops (renumbered in same order) considered. The results from these runs appear in Figures 7 and 8, respectively.

The confusion matrices associated with Figures 1-8 appear in Tables 6 and 7.

| Pass | 1 | 2 | 3 | 4 | 5 | 6 |
|------|-------|-------|-------|-------|-------|-------|
| g(B) | .5226 | .4811 | .4326 | .6600 | .4488 | .4832 |

Table 1. Values of g(B) for $\binom{6}{1}$

| Passes | 1,2 | 1,3 | 1,4 | 1,5 | 1,6 |
|--------|-------|-------|-------|-------|-------|
| g(B) | .4700 | .4045 | .4873 | .4276 | .4703 |
| Passes | 2,3 | 2,4 | 2,5 | 2,6 | 3,4 |
| g(B) | .4342 | .4730 | .3873 | .3807 | .4256 |
| Passes | 3,5 | 3,6 | 4,5 | 4,6 | 5,6 |
| g(B) | .3551 | .4332 | .4453 | .4756 | .3962 |

Table 2. Values of g(B) for $\binom{6}{2}$

| Passes | 1,2,3 | 1,2,4 | 1,2,5 | 1,2,6 | 1,3,4 |
|--------|-------|-------|-------|-------|-------|
| g(B) | .3980 | .4349 | .3762 | .3305 | .3964 |

| Passes | 1,3,5 | 1,3,6 | 1,4,5 | 1,4,6 | 1,5,6 |
|--------|-------|-------|-------|-------|-------|
| g(B) | .3453 | .3618 | .4239 | .4432 | .2910 |

| Passes | 2,3,4 | 2,3,5 | 2,3,6 | 2,4,5 | 2,4,6 |
|--------|-------|-------|-------|-------|-------|
| g(B) | .4164 | .3266 | .3789 | .3844 | .3762 |

| Passes | 2,5,6 | 3,4,5 | 3,4,6 | 3,5,6 | 4,5,6 |
|--------|-------|-------|-------|-------|-------|
| g(B) | .2951 | .3485 | .4259 | .3048 | .3104 |

Table 3. Values of g(B) for $\binom{6}{3}$

| Passes | 1,2,3,4 | 1,2,3,5 | 1,2,3,6 | 1,2,4,5 | 1,2,4,6 |
|--------|---------|---------|---------|---------|---------|
| g(B) | .3917 | .3215 | .3226 | .3732 | .3247 |

| Passes | 1,2,5,6 | 1,3,4,5 | 1,3,4,6 | 1,3,5,6 | 1,4,5,6 |
|--------|---------|---------|---------|---------|---------|
| g(B) | .2764 | .3377 | .3521 | .2873 | .2873 |

| Passes | 2,3,4,5 | 2,3,4,6 | 2,3,5,6 | 2,4,5,6 | 3,4,5,6 |
|--------|---------|---------|---------|---------|---------|
| g(B) | .3229 | .3738 | .2925 | .2909 | .3005 |

Table 4. Values of $g(B)$ for $\binom{6}{4}$

| Passes | 1,2,3,4,5 | 1,2,3,4,6 | 1,2,3,5,6 | 1,2,4,5,6 | 1,3,4,5,6 | 2,3,4,5,6 |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|
| g(B) | .3171 | .3140 | .2733 | .2718 | .2842 | .2886 |

Table 5. Values of $g(B)$ for $\binom{6}{5}$

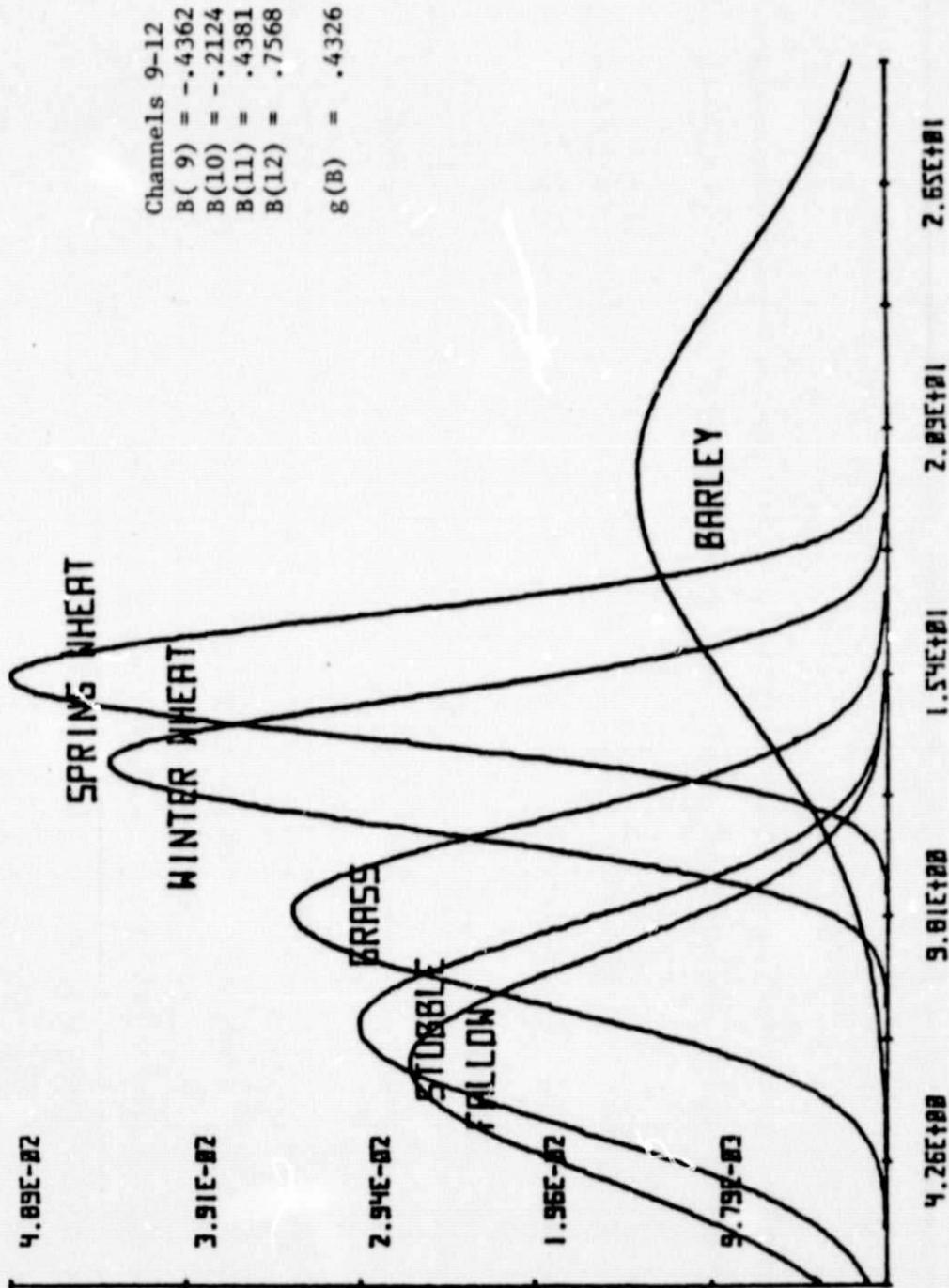


Fig. 1. Optical B, g(5), Transformed densities: Best one pass-3.

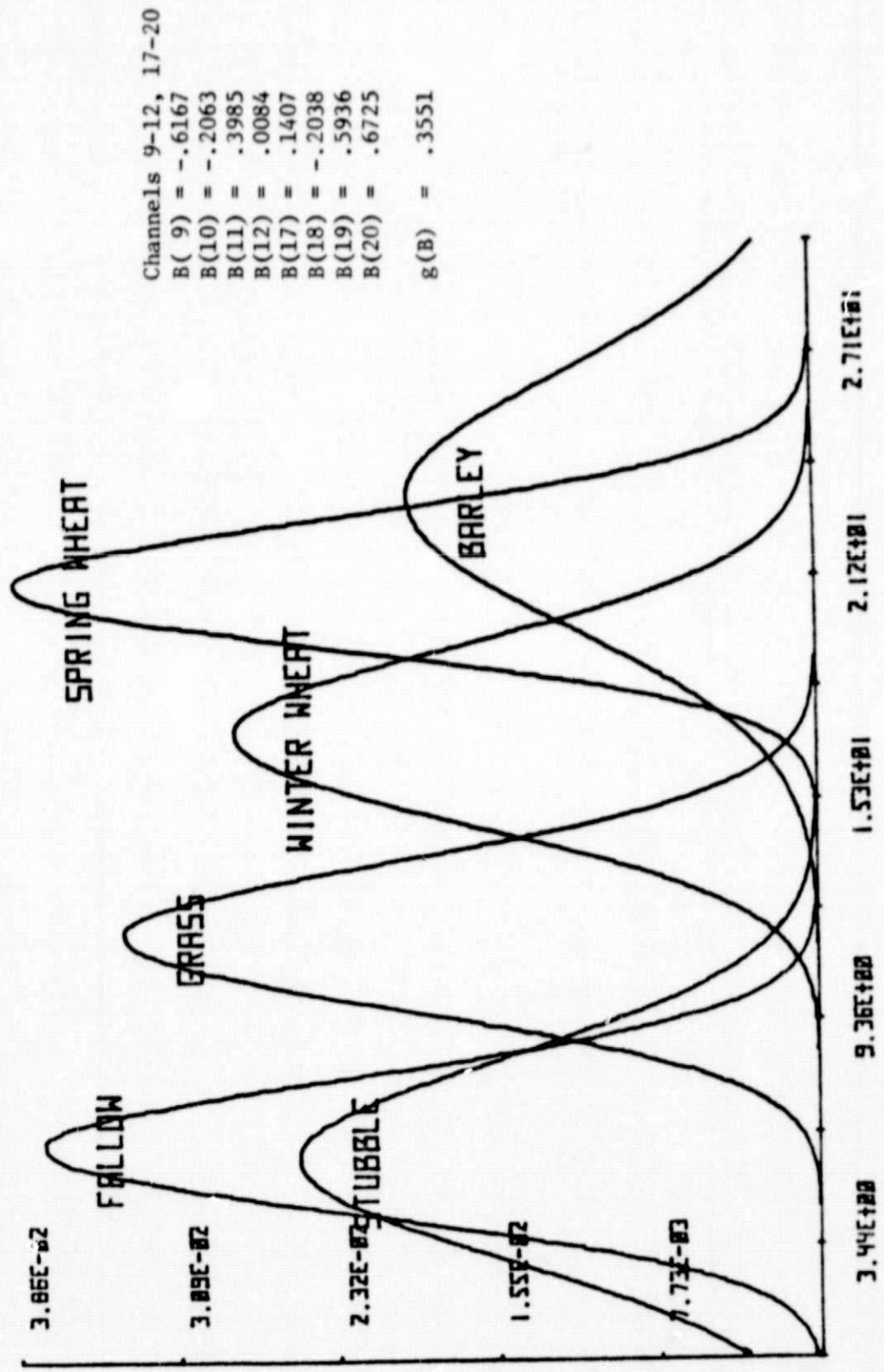


Fig. 2. Optimal B, g(B), Transformed densities: Best two passes-3,5.

Channels 1-4, 17-24
 B(1) = .0016
 B(2) = .1567
 B(3) = .0805
 B(4) = .0767
 B(17) = -.0779
 B(18) = -.2536
 B(19) = .0968
 B(20) = .3538
 B(21) = .0655
 B(22) = -.2743
 B(23) = -.0438
 B(24) = .8227
 g(B) = .2910

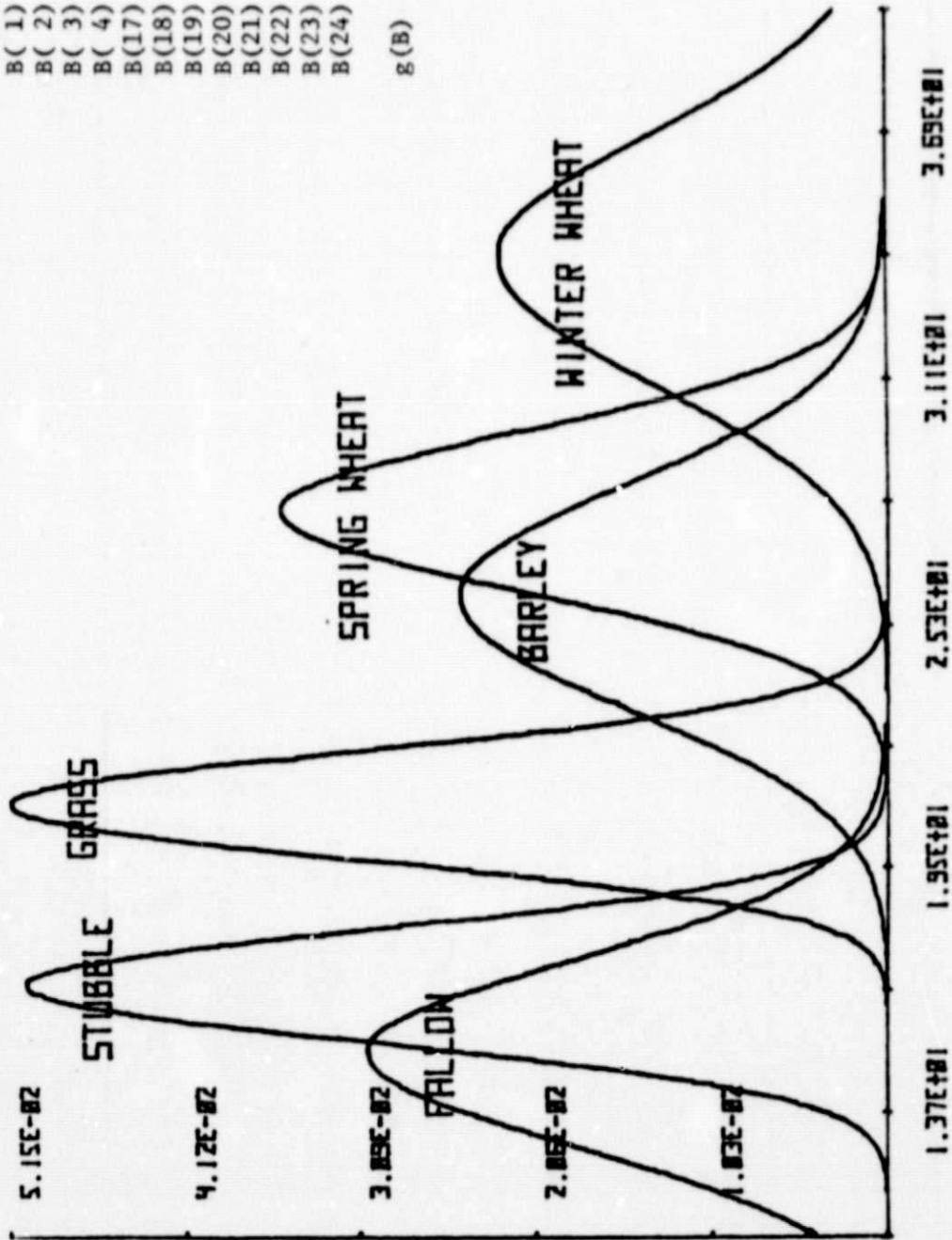


Fig. 3. Optimal B, g(B), Transformed densities: Best three passes-1,5,6.

Channels 1-8, 17-24

B(1) = .0038
 B(2) = .0908
 B(3) = .0875
 B(4) = .1604
 B(5) = .0092
 B(6) = .0578
 B(7) = .0508
 B(8) = .0034
 B(17) = -.0789
 B(18) = -.2124
 B(19) = .0946
 B(20) = .3456
 B(21) = .0695
 B(22) = -.3161
 B(23) = -.0417
 B(24) = .8162

g(3) = .2764

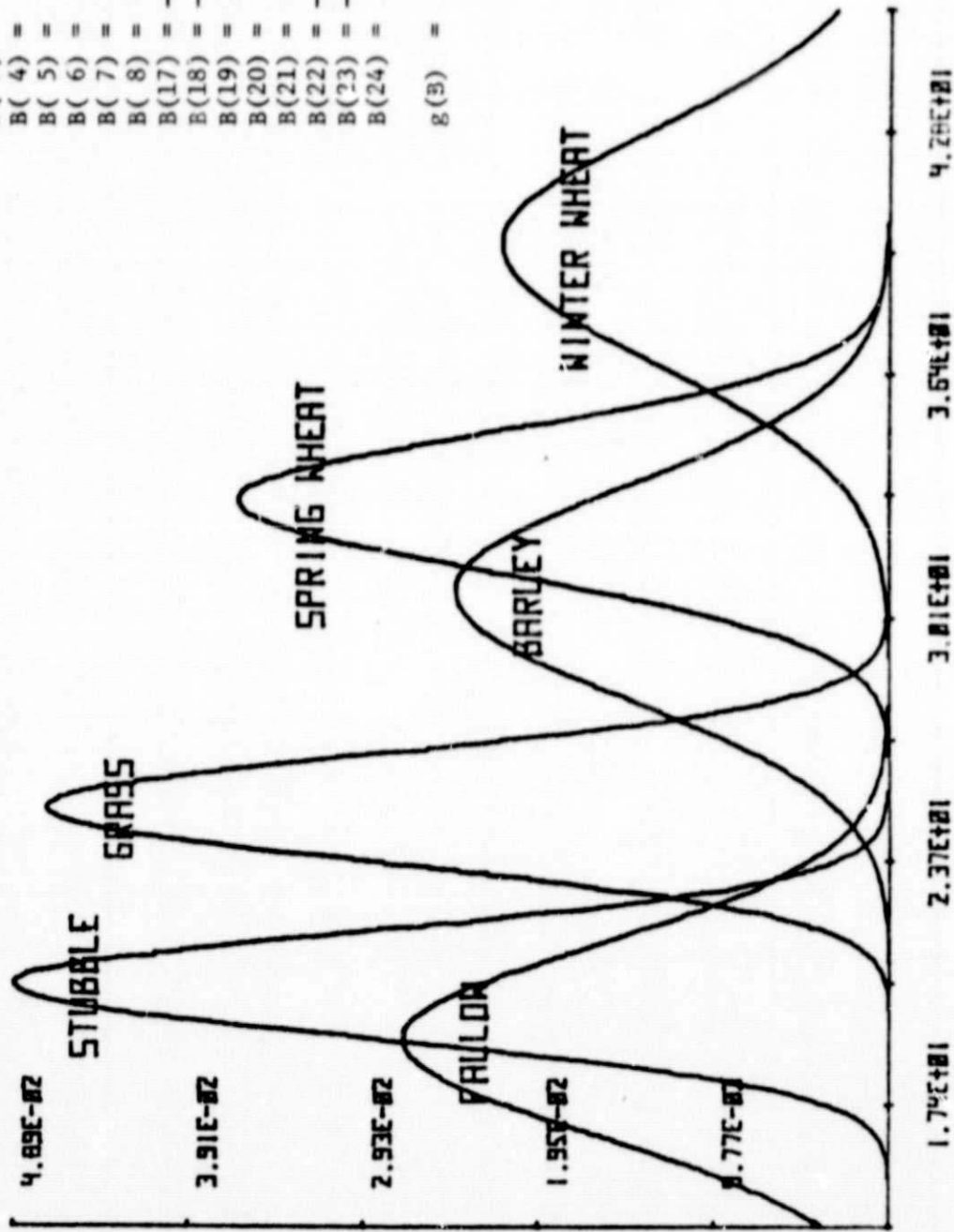


Fig. 4. Optimal B, g(B), Transformed densities: Best four passes-1,2,5,6.

Channels 1-8, 13-24

B(1) = -.0244
 B(2) = -.0678
 B(3) = -.0980
 B(4) = -.1546
 B(5) = -.0045
 B(6) = -.0468
 E(7) = -.0589
 B(8) = -.0181
 B(13) = .2297
 B(14) = -.1478
 B(15) = -.0488
 B(16) = .0296
 B(17) = .0830
 B(18) = .2098
 B(19) = -.1269
 B(20) = -.2987
 B(21) = -.0444
 B(22) = .3041
 B(23) = .0175
 B(24) = -.7914
 g(B) = .2718

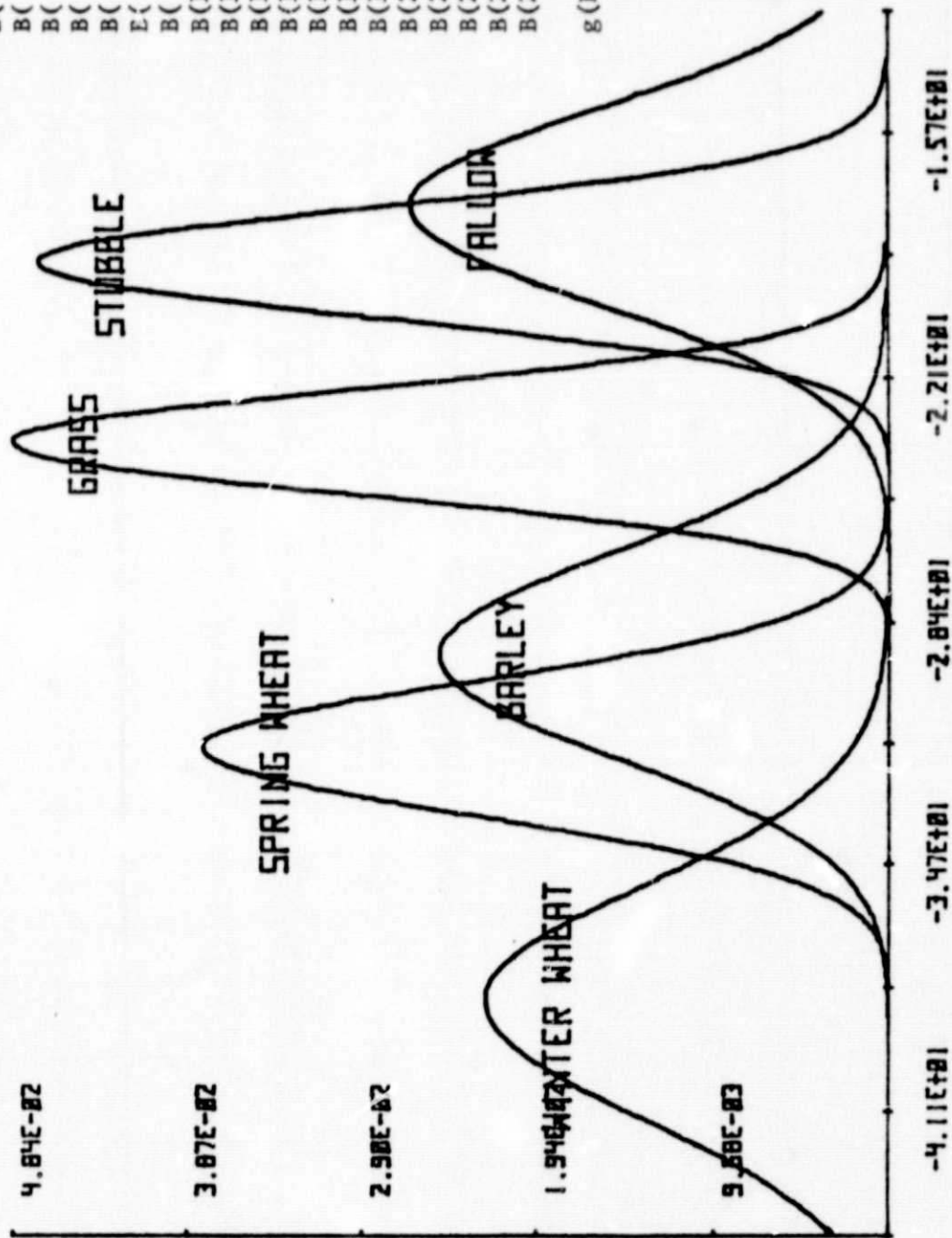


Fig. 5. Optimal B, g(B), Transformed densities: Best five passes-1,2,4,5,6.

Channels 1-24
 B(1) = -.0069
 B(2) = -.0759
 B(3) = -.1168
 B(4) = -.1916
 B(5) = .0542
 B(6) = -.0424
 B(7) = -.0729
 B(8) = -.0960
 B(9) = -.0960
 B(10) = -.0217
 B(11) = .0359
 B(12) = .0660
 B(13) = .2060
 B(14) = -.1399
 B(15) = -.0319
 B(16) = .0081
 B(17) = .1042
 B(18) = .2067
 B(19) = -.1302
 B(20) = -.3086
 B(21) = -.0255
 B(22) = .3128
 B(23) = .0148
 B(24) = -.7626
 g(B) = .2699

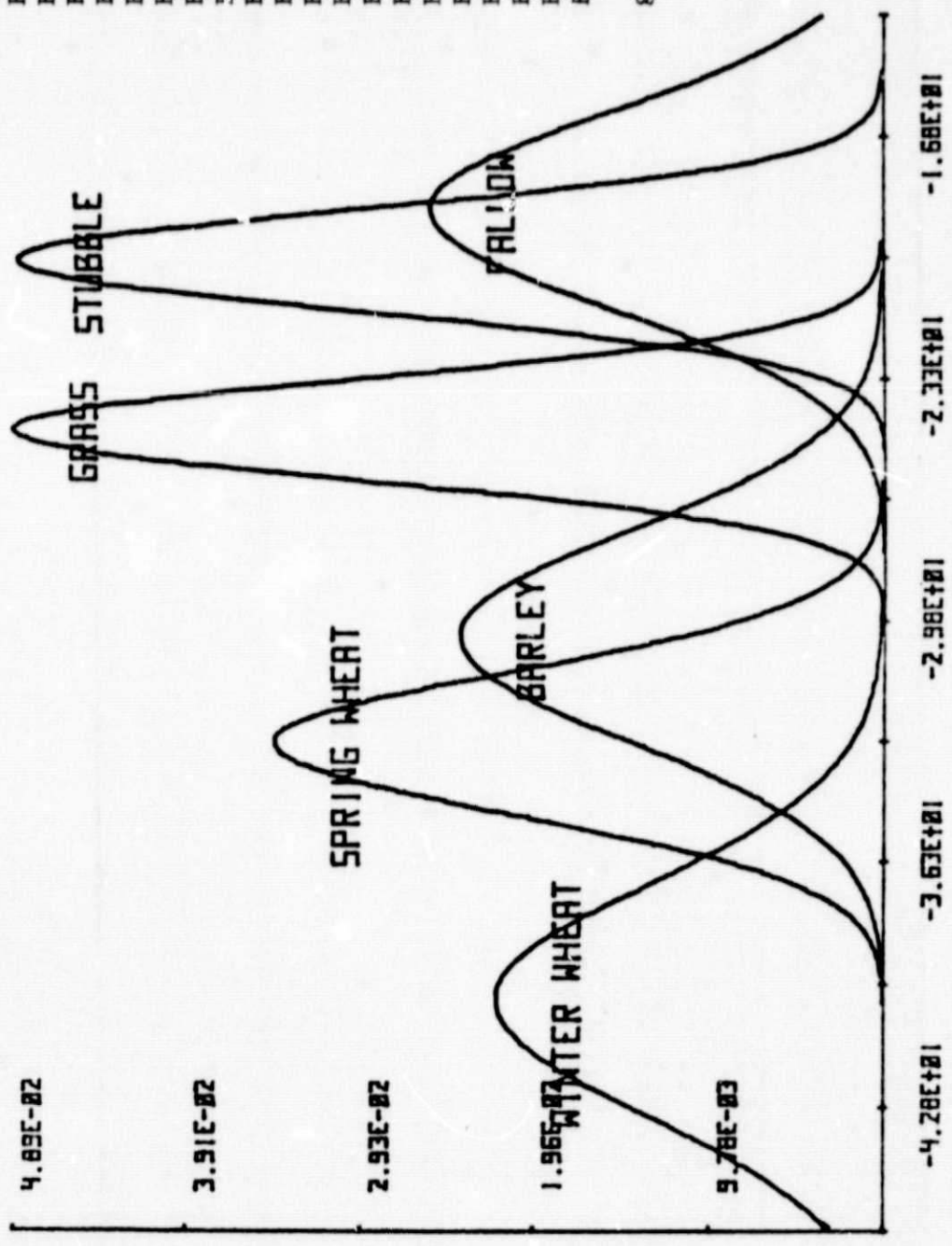


Fig. 6. Optimal B, g(B), Transformed densities: All passes-1,2,3,4,5,6.

Channels 1-24
 B(1) = .1185
 B(2) = .0259
 B(3) = -.0827
 B(4) = -.3607
 B(5) = -.0676
 B(6) = .1548
 B(7) = -.1239
 B(8) = -.3585
 B(9) = .3096
 B(10) = -.1510
 B(11) = .2829
 B(12) = .2331
 B(13) = .1013
 B(14) = -.1326
 B(15) = -.0466
 B(16) = -.0326
 B(17) = -.1502
 B(18) = .2032
 B(19) = -.3334
 B(20) = -.3820
 B(21) = .0890
 B(22) = .0627
 B(23) = -.0558
 B(24) = .2361
 g(B) = .2298

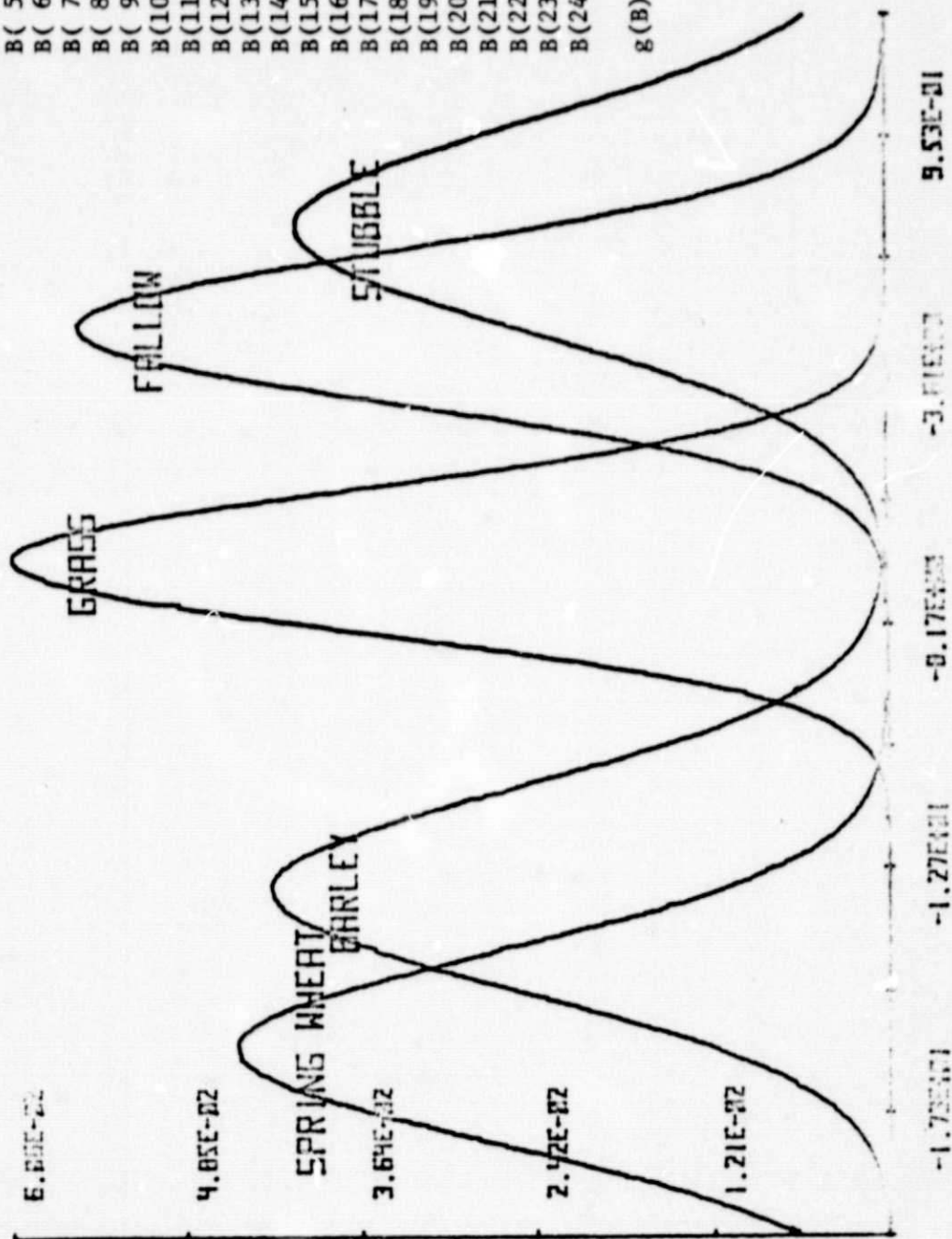


Fig. 7. Five classes (no Winter Wheat): Optimal B, g(B), Transformed densities: All passes.

Channels 1-24
 B(1) = .1783
 B(2) = -.1129
 B(3) = -.1789
 B(4) = -.4214
 B(5) = -.1422
 B(6) = .0319
 B(7) = -.3098
 B(8) = -.1218
 B(9) = .2948
 B(10) = -.0689
 B(11) = .1912
 B(12) = .2480
 B(13) = -.0114
 B(14) = .0239
 B(15) = -.0611
 B(16) = -.0349
 B(17) = -.1142
 B(18) = .2285
 B(19) = -.1552
 B(20) = -.3886
 B(21) = .1882
 B(22) = .3325
 B(23) = -.0322
 B(24) = .1762
 g(B) = .1842

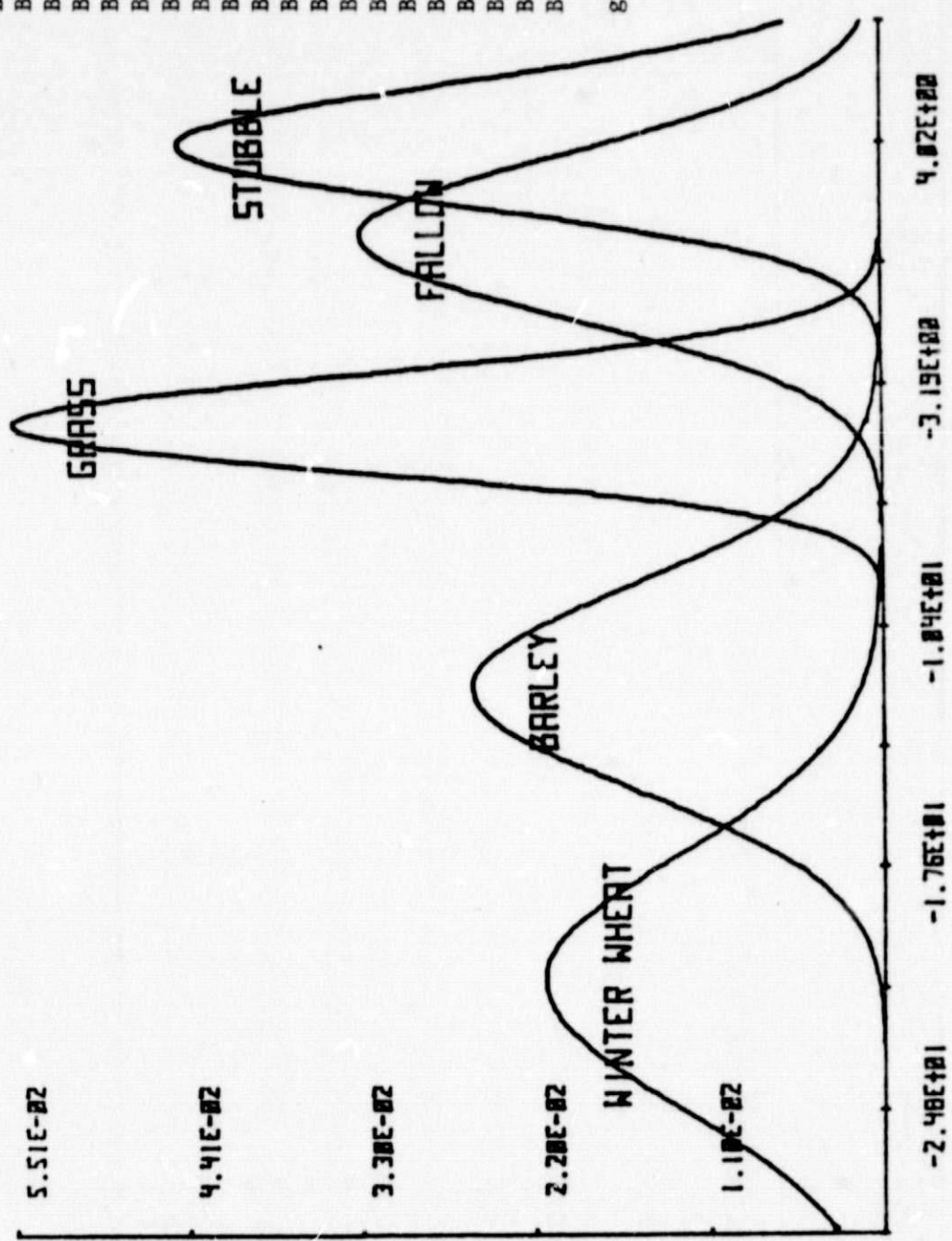


Fig. 8. Five classes (no Spring Wheat): Optimal B, g(B), Transformed densities: All passes.

| | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|------------------|
| .668 | .300 | .000 | .003 | .050 | .000 | .525 | .000 | .000 | .005 | .121 | .000 |
| .007 | .373 | .203 | .001 | .000 | .316 | .000 | .307 | .019 | .000 | .000 | .16 ^c |
| .037 | .274 | .562 | .131 | .003 | .187 | .004 | .116 | .825 | .122 | .000 | .059 |
| .084 | .028 | .183 | .593 | .216 | .019 | .102 | .001 | .096 | .680 | .122 | .000 |
| .201 | .001 | .015 | .272 | .731 | .001 | .369 | .000 | .000 | .192 | .757 | .000 |
| .003 | .324 | .036 | .000 | .000 | .478 | .000 | .576 | .060 | .000 | .000 | .776 |

Best one pass - 3

| | | | | | | | | | | | |
|------|------|------|------|------|------|------|------|------|------|------|------|
| .423 | .000 | .053 | .005 | .193 | .000 | .475 | .000 | .049 | .003 | .175 | .000 |
| .004 | .794 | .049 | .000 | .000 | .410 | .002 | .813 | .050 | .000 | .000 | .441 |
| .137 | .050 | .897 | .000 | .005 | .043 | .115 | .048 | .901 | .000 | .001 | .065 |
| .037 | .000 | .000 | .864 | .074 | .000 | .028 | .000 | .000 | .890 | .055 | .000 |
| .399 | .000 | .000 | .131 | .729 | .000 | .380 | .000 | .000 | .106 | .769 | .000 |
| .000 | .156 | .000 | .000 | .000 | .547 | .000 | .139 | .000 | .000 | .000 | .494 |

Best two passes - 3, 5

Best three passes - 1, 5, 6

Best four passes - 1, 2, 5, 6

Table 6. Confusion matrices.

| | | | | | |
|------|------|------|------|------|------|
| .528 | .000 | .047 | .002 | .176 | .001 |
| .002 | .825 | .048 | .000 | .000 | .451 |
| .117 | .048 | .905 | .000 | .001 | .083 |
| .017 | .000 | .000 | .894 | .059 | .000 |
| .335 | .000 | .000 | .104 | .764 | .000 |
| .000 | .127 | .000 | .000 | .000 | .465 |

All passes - 1, 2, 3, 4, 5, 6

| | | | | | |
|------|------|------|------|------|------|
| .493 | .000 | .047 | .003 | .168 | .000 |
| .002 | .810 | .047 | .000 | .000 | .446 |
| .112 | .049 | .906 | .000 | .001 | .066 |
| .025 | .000 | .000 | .894 | .052 | .000 |
| .368 | .000 | .000 | .103 | .779 | .000 |
| .000 | .141 | .000 | .000 | .000 | .487 |

Best five passes - 1, 2, 4, 5, 6

| | | | | |
|------|------|------|------|------|
| .863 | .000 | .021 | .111 | .000 |
| .000 | .817 | .000 | .000 | .322 |
| .052 | .001 | .929 | .000 | .096 |
| .085 | .000 | .000 | .889 | .000 |
| .000 | .183 | .049 | .000 | .581 |

Five classes (no Spring Wheat)

| | | | | |
|------|------|------|------|------|
| .749 | .000 | .021 | .199 | .000 |
| .000 | .629 | .000 | .000 | .195 |
| .032 | .018 | .926 | .000 | .059 |
| .219 | .000 | .000 | .801 | .000 |
| .000 | .353 | .053 | .000 | .746 |

Five classes (no Winter Wheat)

Table 7. Confusion matrices

3. Remarks

The preliminary numerical results in the previous section indicate that fewer optimally chosen passes could be used while maintaining most of the discriminatory power inherently available in the original data. Again, we note that the results give insight only for the crops considered and levels of factors reflected by the data used. However, by identifying the various factors (and levels thereof) affecting the spectral signatures, and by formulating a regression model one could use the feature selection technique to determine the regression coefficients for predicting optimal passes for a given set of crops.

Another use of the feature selection technique as applied to multiple-pass registered data is the generation of enhanced grey scale displays by using a single linear combination of all channels of all designated passes as opposed to a single channel within a single pass.

References

1. L. F. Guseman, Jr. and Bruce P. Marion, LFSPMC: Linear Feature Selection Prog. @ Using the Probability of Misclassification, Report #3, NAS-9-13894-1S, Texas A&M University, Department of Mathematics, January, 1975.