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R. Viswanathan

*Southern Illinois University Carbondale, viswa@engr.siu.edu*

S. C. A. Thomopoulos

*Southern Illinois University Carbondale*

R. Tumuluri

*Southern Illinois University Carbondale*

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# Optimal Serial Distributed Decision Fusion

R. VISWANATHAN, Member, IEEE

S.C.A. THOMOPOULOS, Member, IEEE

R. TUMULURI

Southern Illinois University at Carbondale

**The problem of distributed detection involving  $N$  sensors is considered. The configuration of sensors is serial in the sense that the  $(j-1)$ th sensor passes its decision to the  $j$ th sensor and that the  $j$ th sensor decides using the decision it receives and its own observation. When each sensor employs the Neyman-Pearson test, the probability of detection is maximized for a given probability of false alarm, at the  $N$ th stage. With two sensors, the serial scheme has a performance better than or equal to the parallel fusion scheme analyzed in the literature. Numerical examples illustrate the global optimization by the selection of operating thresholds at the sensors.**

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Authors' current addresses: R. Viswanathan and S.C.A. Thomopoulos, Department of Electrical Engineering, College of Engineering and Technology, Southern Illinois University at Carbondale, Carbondale, IL 62901-6603; R. Tumuluri, Department of Electrical Engineering, Purdue University, West Lafayette, IN.

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## I. INTRODUCTION

The theory of distributed detection is receiving a lot of attention in the literature [1-10]. Typically, a number of sensors process the data they receive and decide in favor of one of the hypotheses about the origin of the data. In a two-class decision problem, the hypotheses would be signal present ( $H_1$ ) or the signal absent ( $H_0$ ). These decisions are then sent to a fusion center where a final decision regarding the presence of the signal is made. This scheme, which can be termed parallel decision making, is shown in Fig. 1. In order to maximize the probability of detection at the fusion center for a fixed probability of false alarm, the tests used at the fusion center and at the sensors must be Neyman-Pearson (N-P) [3, 8]. The above result is based on the assumption that the data at the sensors conditioned on the hypothesis are statistically independent. If the conditional independence is removed, the threshold of the N-P tests become data dependent and does not yield any easy solution for optimization [16].

We consider a serial distributed decision scheme (Fig. 2), (in [4] this is called a tandem network). Though the serial fusion is very sensitive to link failures, its performance analysis is of interest. In [4], the tandem network was analyzed with Baye's cost as the optimality criterion. Though analytical equations are given, no performance analysis for typical channels or comparison of performance with respect to the parallel fusion was provided. Here we aim to fill this gap.

In Section II we derive the relevant equations describing the operation of the serial scheme based on the knowledge that all the sensors employ the N-P test. In Section III we show that the global optimality is guaranteed when each stage employs the N-P test. Section IV examines the conditions under which the performance of the serial scheme is definitely not inferior to the parallel scheme. Some numerical examples are also presented to illustrate the performance.

## II. DEVELOPMENT OF KEY EQUATIONS

Consider the serial configuration of distributed sensors shown in Fig. 2. Denote the sensor decisions as  $u_1, u_2, \dots, u_N$ . The  $j$ th sensor receives the decision  $u_{j-1}$  and its own observation  $Z_j$  to make its decision  $u_j$ . The decision  $u_N$  at the  $N$ th sensor is the fused decision about the hypotheses. We assume that the data at the sensors, conditioned on each hypothesis, are statistically independent. This implies that  $Z_j$  and  $u_{j-1}$  are also conditionally independent. As mentioned earlier, the  $j$ th sensor employs an N-P test using the data  $(Z_j, u_{j-1})$ . The optimality of this assumption is explored in the next section.

Denoting the distributions of  $Z_j$  as  $p(Z_j|H_1)$  and  $p(Z_j|H_0)$ , the likelihood ratio becomes

$$\frac{L(Z_j, u_{j-1}|H_1)}{L(Z_j, u_{j-1}|H_0)}$$

$$= \frac{p(Z_j|H_1)[P_{D,j-1}\delta(u_{j-1}-1) + (1-P_{D,j-1})\delta(u_{j-1})]}{p(Z_j|H_0)[P_{F,j-1}\delta(u_{j-1}-1) + (1-P_{F,j-1})\delta(u_{j-1})]} \quad (1)$$

where

$$P_{D,j-1} = \Pr(u_{j-1} = 1 | H_1)$$

$$P_{F,j-1} = \Pr(u_{j-1} = 1 | H_0)$$

$u_{j-1} = k$  implies that the  $(j-1)$ th sensor decides  $H_k$ ,  $k = 0, 1$ , and  $\delta(x)$  is the Kronecker delta function defined as  $\delta(x) = \begin{cases} 1 & x = 0 \\ 0 & x \neq 0 \end{cases}$  and  $L(\cdot)$  is the likelihood

function [14].

Therefore, the test at the  $j$ th sensor is given by

$$\frac{p(Z_j|H_1)}{p(Z_j|H_0)} \frac{P_{D,j-1}}{P_{F,j-1}} \underset{H_0}{\overset{H_1}{\geq}} t, \quad \text{if } u_{j-1} = 1$$

$$\frac{p(Z_j|H_1)}{p(Z_j|H_0)} \frac{1 - P_{D,j-1}}{1 - P_{F,j-1}} \underset{H_0}{\overset{H_1}{\geq}} t, \quad \text{if } u_{j-1} = 0 \quad (2)$$

where  $t$  a threshold to be determined.

Equivalently,

$$\Lambda(Z_j) \underset{H_0}{\overset{H_1}{\geq}} \begin{cases} t_{j,1}, & \text{if } u_{j-1} = 1 \\ t_{j,0}, & \text{if } u_{j-1} = 0 \end{cases} \quad (3)$$

where

$$\Lambda(Z_j) = \frac{p(Z_j|H_1)}{p(Z_j|H_0)}$$

and

$$\frac{t_{j,1}}{t_{j,0}} = \frac{P_{F,j-1}}{P_{D,j-1}} \frac{1 - P_{D,j-1}}{1 - P_{F,j-1}}$$

Many times it is convenient to use the log likelihood ratio,  $\ln \Lambda(Z_j) = \Lambda^*(Z_j)$ . Hence,

$$\Lambda^*(Z_j) \underset{H_0}{\overset{H_1}{\geq}} \begin{cases} t_{j,1}^*, & \text{if } u_{j-1} = 1 \\ t_{j,0}^*, & \text{if } u_{j-1} = 0 \end{cases} \quad (4)$$

and

$$t_{j,1}^* = t_{j,0}^* + \ln \left( \frac{P_{F,j-1}}{1 - P_{F,j-1}} \frac{1 - P_{D,j-1}}{P_{D,j-1}} \right), \quad j = 2, \dots, N.$$

For the first stage,  $t_{1,1}^* = t_{1,0}^*$ .

#### A. False Alarm and Detection Probabilities

At the  $j$ th stage, the false alarm probability is given by

$$P_{F,j} = \Pr(\Lambda^*(Z_j) > t_{j,0}^* | H_0, u_{j-1} = 0) \Pr(u_{j-1} = 0 | H_0) + \Pr(\Lambda^*(Z_j) > t_{j,1}^* | H_0, u_{j-1} = 1) \times \Pr(u_{j-1} = 1 | H_0). \quad (5)$$

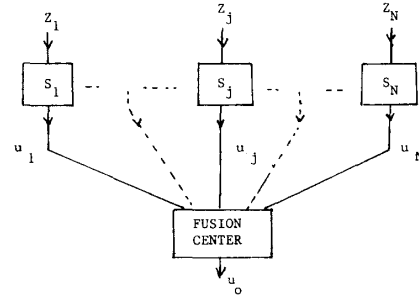


Fig. 1. Parallel decision fusion.

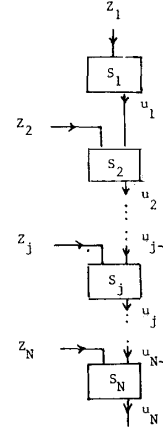


Fig. 2. Serial decision fusion.

Let

$$a_j = \Pr(\Lambda^*(Z_j) > t_{j,0}^* | H_0)$$

$$b_j = \Pr(\Lambda^*(Z_j) > t_{j,1}^* | H_0)$$

$$c_j = \Pr(\Lambda^*(Z_j) > t_{j,0}^* | H_1)$$

$$d_j = \Pr(\Lambda^*(Z_j) > t_{j,1}^* | H_1). \quad (6)$$

Using (5), (6), and the conditional independence assumption, we have

$$P_{F,j} = a_j(1 - P_{F,j-1}) + b_j P_{F,j-1}. \quad (7)$$

Similarly,

$$P_{D,j} = c_j(1 - P_{D,j-1}) + d_j P_{D,j-1}. \quad (8)$$

Knowing the distribution of the observations  $Z_j$  and using (4), (6)–(8), it is possible to compute the  $P_{D,j}$ s recursively provided the  $P_{F,j}$ s are specified. If the  $P_{F,j}$ s are kept the same, the serial configuration exhibits some nice properties [5]. However, for a given  $P_{F,N}$  at the  $N$ th stage, this procedure does not guarantee a maximum  $P_{D,N}$ . In order to globally optimize the performance, that is to maximize  $P_{D,N}$  for a given  $P_{F,N}$ , we need a multidimensional search with respect to the variables  $P_{F,j}$ s,  $j = 1, 2, \dots, (N-1)$ . The results obtained using the numerical search procedure are presented in Section IV.

In Fig. 3 a functionally equivalent form of the serial decision fusion is shown. Each sensor, except the first one, sends two decisions  $u_{j,0}$  and  $u_{j,1}$  depending on whether the previous sensor decides a 0 or a 1, respectively. These decisions are arrived by using (3). The fusion center uses the decision from the first sensor and sequentially picks the appropriate decisions from the sensors to arrive at the final decision  $u_0$  which is either  $u_{N,0}$  or  $u_{N,1}$ . Performance-wise, the configuration in Fig. 3 is equivalent to the serial scheme. The equivalent configuration does not have the time delay problem associated with the serial configuration. However, both are highly sensitive to link failures.

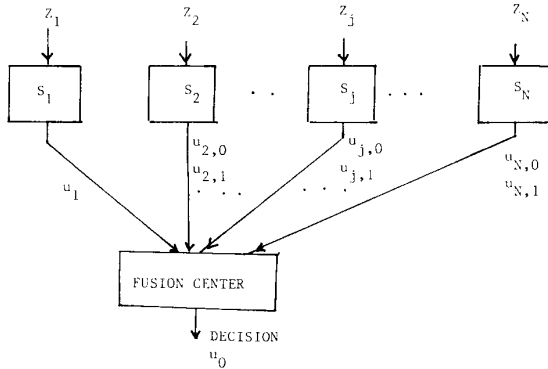


Fig. 3. Functionally equivalent configuration of serial network.

### III. GLOBAL OPTIMALITY

The global optimization problem is to find the tests at each stage of the serial configuration such that the probability of detection  $P_{D,N}$  is maximized for a given  $P_{F,N}$ . Here, we show that the global optimality is achieved when each sensor employs the N-P test.

**THEOREM 1.** *Given that the observations at each stage in a serial distributed detection environment with  $N$  sensors are independent identically distributed (IID), the probability of detection is maximized for a given probability of false alarm, at the  $N$ th stage, when each stage employs the N-P test.*

**PROOF.** Consider the last two stages. At the  $N$ th stage, the N-P test using the data  $(Z_N, u_{N-1})$  maximizes  $P_{D,N}$  for a fixed  $P_{F,N}$  [11, 13]. Let

$$L^* \equiv \ln \frac{p(Z_N, u_{N-1}|H_1)}{p(Z_N, u_{N-1}|H_0)}$$

$$\Lambda^*(Z_N) = \ln \frac{p(Z_N|H_1)}{p(Z_N|H_0)}. \quad (9)$$

Call  $\Lambda^*(Z_N)$ ,  $P_{F,N-1}$ , and  $P_{D,N-1}$  as  $\Lambda^*$ ,  $P_F$  and  $P_D$ , respectively, for simplicity. Then,

$$\Pr(L^* < \lambda | H_1) = P_D \Pr\left(\Lambda^* + \ln\left(\frac{P_D}{P_F}\right) < \lambda | H_1\right) + (1 - P_D) \Pr\left(\Lambda^* + \ln\left(\frac{1 - P_D}{1 - P_F}\right) < \lambda | H_1\right). \quad (10)$$

Denote the cumulative distributions and the density functions of  $\Lambda^*$  under  $H_1$  and  $H_0$  as  $F_1^*(\cdot)$ ,  $f_1^*(\cdot)$  and  $F_0^*(\cdot)$ ,  $f_0^*(\cdot)$ , respectively. Since the left-hand side of (10) is one minus the probability of detection, we have

$$1 - P_{D,N} = P_D F_1^*\left(\lambda - \ln\left(\frac{P_D}{P_F}\right)\right) + (1 - P_D) F_1^*\left(\lambda - \ln\left(\frac{1 - P_D}{1 - P_F}\right)\right). \quad (11)$$

Similarly,

$$1 - P_{F,N} = P_F F_0^*\left(\lambda - \ln\left(\frac{P_D}{P_F}\right)\right) + (1 - P_F) F_0^*\left(\lambda - \ln\left(\frac{1 - P_D}{1 - P_F}\right)\right). \quad (12)$$

We require for a fixed  $P_{F,N}$  and for any arbitrary but fixed  $P_F$  at the  $(N-1)$ th stage, the  $P_{D,N}$  to be a monotonic increasing function of the  $P_D$  at the  $(N-1)$ th stage. Observe that if the  $P_D$  of the  $(N-1)$ th stage is changed, then the threshold  $\lambda$  at the  $N$ th stage changes in order that  $P_{F,N}$  remains fixed. Taking the derivative of (12) w.r.t.  $P_D$  and equating the result to zero, we obtain

$$\frac{d\lambda}{dP_D} = \frac{\frac{P_F}{P_D} f_0^*(x_1) - \frac{1 - P_F}{1 - P_D} f_0^*(x_2)}{P_F f_0^*(x_1) + (1 - P_F) f_0^*(x_2)} \quad (13)$$

where

$$x_1 = \lambda - \ln(P_D/P_F)$$

$$x_2 = \lambda - \ln\left(\frac{1 - P_D}{1 - P_F}\right).$$

Similarly,

$$\frac{d(1 - P_{D,N})}{dP_D} = F_1^*(x_1) - F_1^*(x_2) + \left[ P_D f_1^*(x_1) \left(\frac{d\lambda}{dP_D} - \frac{1}{P_D}\right) + (1 - P_D) f_1^*(x_2) \left(\frac{d\lambda}{dP_D} + \frac{1}{1 - P_D}\right) \right]. \quad (14)$$

A reasonable requirement is  $P_D > P_F$ . This implies that  $F_1^*(x_1) - F_1^*(x_2)$  is less than zero. Hence, a sufficient

condition for  $\frac{dP_{D,N}}{dP_D} > 0$  is that the term in the brackets in (14) be less than or equal to zero. After some simplification, using (13), we obtain the following sufficiency condition:

$$\frac{\frac{f_1^*(x_2)}{f_0^*(x_2)}}{\frac{f_1^*(x_1)}{f_0^*(x_1)}} \leq e^{x_2 - x_1} \quad (15)$$

However, from the result that the likelihood ratio of the likelihood ratio is the likelihood ratio itself [11, pp. 46], it follows that (15) is satisfied with equality.

#### IV. PERFORMANCE ANALYSIS

##### A. Numerical Results

By using the algorithm developed in Section II, we can obtain the best  $P_{D,N}$  for a given  $P_{F,N}$  by using a search procedure on the variables,  $P_{F,i}$ ,  $i = 1, \dots,$

$(N - 1)$ . We have recursively used the one-dimensional optimization routine FMIN [15] for this purpose. The algorithm also requires the zero of a function in order to obtain the thresholds at each stage (7). The ZEROIN routine in [15] is used to solve for the zeros. The convergence to the optimum value is obtained in the case of 2 sensors and 3 sensors. For performance comparison, we also considered the following parallel fusion schemes: two sensors, identical thresholds at the sensors, AND, OR rules, and three sensors, identical thresholds at the sensors, AND, OR, majority logic rules. In the three-sensor case we also consider two other rules, termed *F1* and *F2*. *F1* corresponds to the Boolean function  $u_0 = u_1 + u_2u_3$  and *F2* corresponds to  $u_0 = u_1(u_2 + u_3)$ . For *F1* and *F2*, sensors numbered 2 and 3 operate at the same thresholds. In all the cases the observations at the sensors are taken to be IID. Two channel models, namely the constant signal detection in additive white Gaussian noise (AWGN) and the detection of a slowly fluctuating Rayleigh target [3, 12] are considered.

Figs. 4–6 show the performance of two sensors in AWGN channel and Figs. 7–9 show the performance

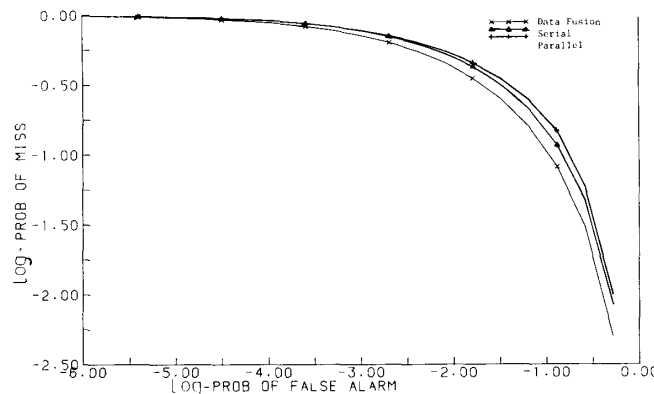


Fig. 4. Performance of serial scheme with two sensors: constant signal in Gaussian noise and signal-to-noise ratio of 5 dB.

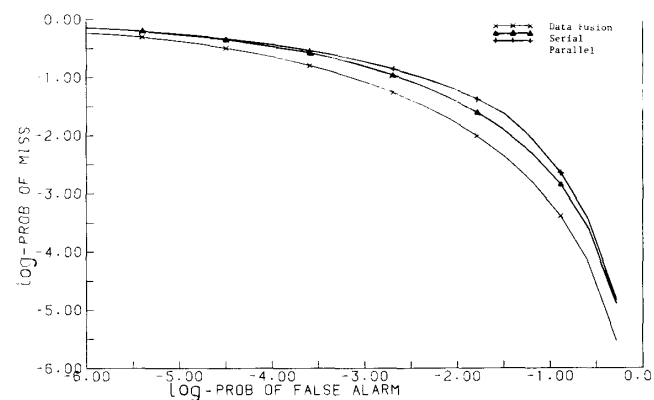


Fig. 5. Performance of serial scheme with two sensors: constant signal in Gaussian noise and signal-to-noise ratio of 10 dB.

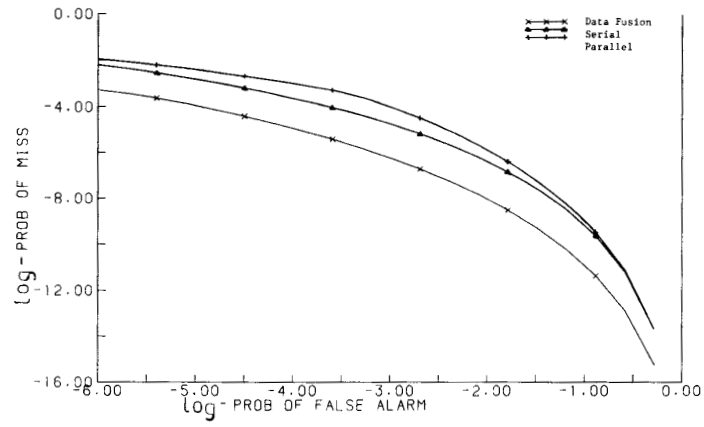


Fig. 6. Performance of serial scheme with two sensors: constant signal in Gaussian noise and signal-to-noise ratio of 15 dB.

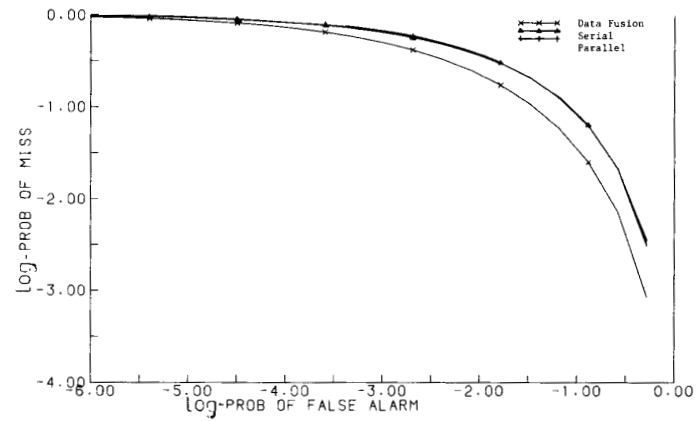


Fig. 7. Performance of serial scheme with three sensors: constant signal in Gaussian noise and signal-to-noise ratio of 5 dB.

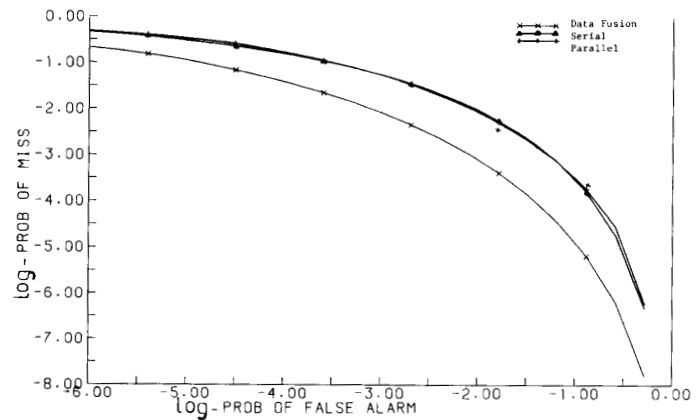


Fig. 8. Performance of serial scheme with three sensors: constant signal in Gaussian noise and signal-to-noise ratio of 10 dB.

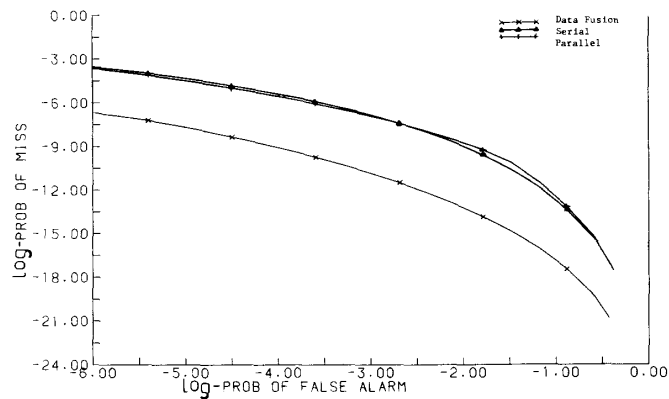


Fig. 9. Performance of serial scheme with three sensors: constant signal in Gaussian noise and signal-to-noise ratio of 15 dB.

with three sensors. The curve named parallel is the best of the several parallel decision rules mentioned above and the data fusion corresponds to the centralized detection scheme which uses data available at all the sensors. With two sensors, the serial performs better than the parallel, especially at larger signal-to-noise ratios. With three sensors, the performance of the two schemes are nearly the same. Also, either of them is poor compared with the data fusion. This is due to the loss associated with the distributed detection. In Rayleigh target detection with two or three sensors, the OR rule is better than the rest of the parallel fusion rules. Moreover, the numerical computation shows that the serial is equivalent to OR for this channel. Theoretically establishing the equivalence has not been possible. In the sense that the serial is only as good as the OR rule, one can term the Rayleigh channel as conservative (Theorem 2 in the next subsection implies that the serial should be at least as good as the OR rule). Figs. 10–15 show the performances of different schemes for the Rayleigh target

detection. In Figs. 13–15, the performances of  $F1$  and  $F2$  are equivalent and hence the corresponding graphs coincide with each other.

### B. Comparison with Parallel Scheme

An optimal parallel fusion is the parallel scheme of Fig. 1 which gives the largest possible probability of detection for a given probability of false alarm at the fusion. Only a monotone increasing switching function, called the positive unate function [17], qualifies as a candidate for the optimal fusion switching function. This can be easily proved from the requirement that the optimal scheme employs likelihood ratio test at the fusion. One property of monotone increasing function is that function, when expressed as a sum of products does not contain any complemented variables. A switching function which can be expressed as a sequence of two input and one output functions is a positive unate function and hence qualifies as a candidate for the optimal parallel

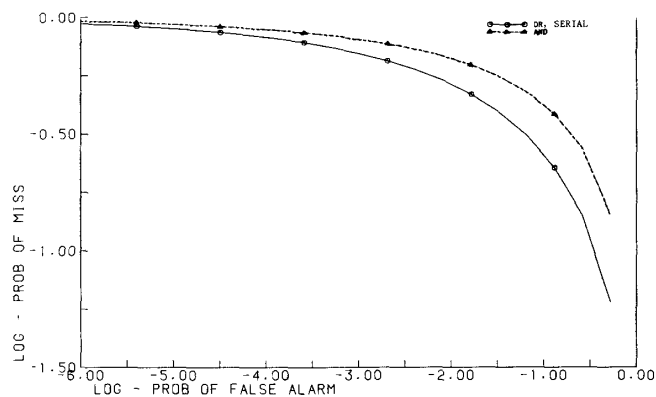


Fig. 10. Performance of serial and parallel schemes for Rayleigh target detection with two sensors: energy-to-noise density ratio of 5 dB.

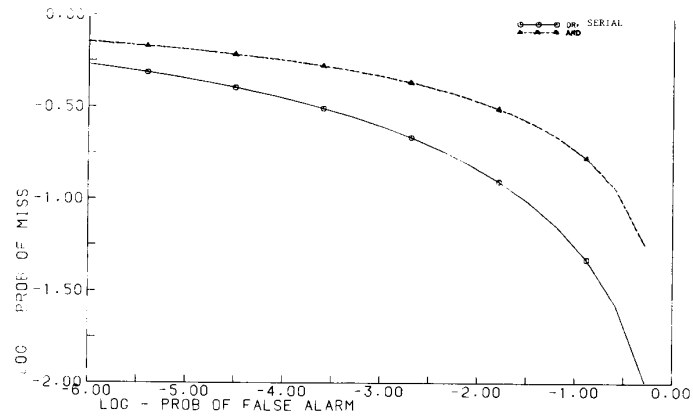


Fig. 11. Performance of serial and parallel schemes for Rayleigh target detection with two sensors: energy-to-noise density ratio of 10 dB.

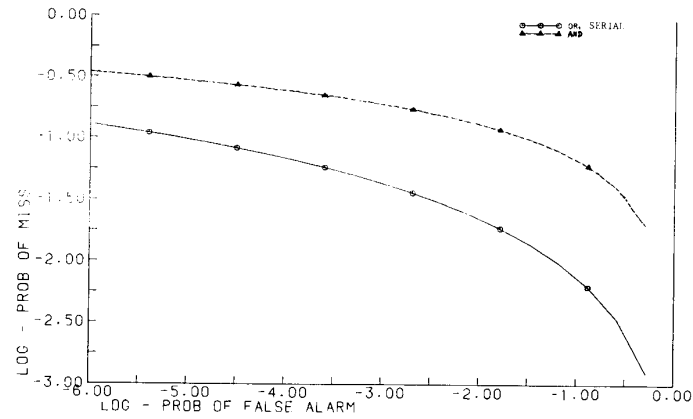


Fig. 12. Performance of serial and parallel schemes for Rayleigh target detection with two sensors: energy-to-noise density ratio of 15 dB.

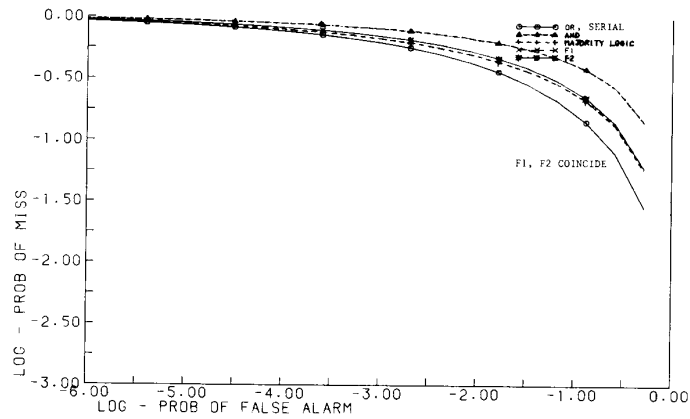


Fig. 13. Performance of serial and parallel schemes for Rayleigh target detection with three sensors: energy-to-noise density ratios of 5 dB.



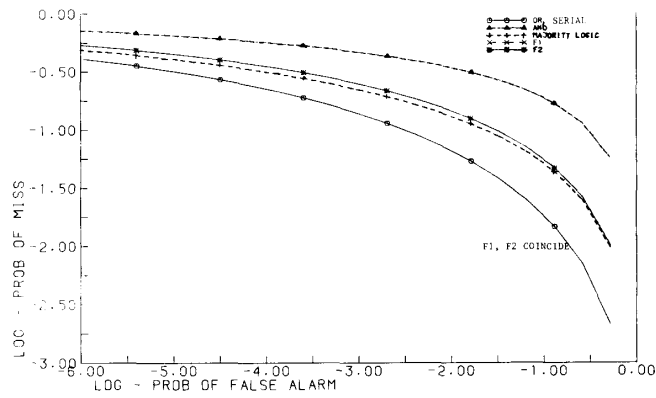


Fig. 14. Performance of serial and parallel schemes for Rayleigh target detection with three sensors: energy-to-noise density ratio of 10 dB.

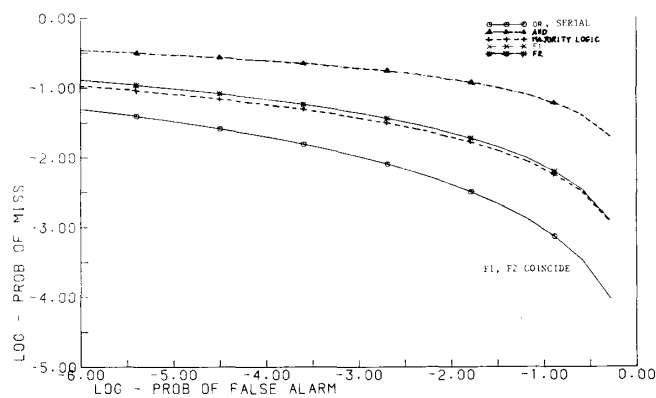


Fig. 15. Performance of serial and parallel schemes for Rayleigh target detecting with three sensors: energy-to-noise density ratio of 15 dB.

fusion function. An example of one such switching function of three variables is shown in Fig. 16. Fig. 16 also shows the serial scheme with three sensors.

Theorem 2 (given below) establishes a sufficient condition for the performance of the optimal serial scheme to be not inferior to the performance of the optimal parallel scheme.

**THEOREM 2.** *If the switching function corresponding to the optimal parallel fusion can be realized in terms of a sequence of two variable functions with single output, then the optimal serial scheme is better than or equal to the optimal parallel scheme.*

**PROOF.** Consider the conservative situation in which the decision variable  $u_1$  in Fig. 16(a) and (b) are identical and each stage of the serial scheme operates at the corresponding false alarms of the parallel scheme (in the Appendix we show that it is possible to achieve such an operation). The  $u_2$  in Fig. 16(b) is a function of  $u_1$  and the observation  $Z_2$ . Since the mapping of  $(u_1, u_2)$  to  $\hat{u}_2$  in

the parallel is contained in the mapping of  $(u_1, Z_2)$  to  $\hat{u}_2$  in the serial, the detection power  $P_{D,2}^*$  attained at  $P_{F,2}$  in the serial is greater than or equal to  $P_{D,2}$ . Similarly,  $u_0$  in the parallel is a function of  $u_2$  and  $u_3$  only whereas in the serial it is a function of  $\hat{u}_2$  and the observation  $Z_3$ . It is observed that the  $\hat{u}_2$  of the serial has the same false alarm  $P_{F,2}$  of the parallel but has a greater than or equal power. For the serial case, the proof of Theorem 1 shows that the detection probability of any stage operating at certain false alarm is a monotone nondecreasing function of the detection probability of the previous stage operating at some false alarm. It then follows that  $P_{D,0}^*$  is greater than or equal to  $P_{D,0}$ . By induction the proof is complete for any  $N$ . Conservatively it is assumed that the false alarm at each stage of the serial is identical to the one in the parallel scheme. If the serial scheme false alarms are optimized then definitely  $P_{D,0}^*$  cannot be less than  $P_{D,0}$ .

From Theorem 2, we observe that for the case of two sensors, the optimal serial is better than or equal to the optimal parallel scheme. With three sensors, it is better

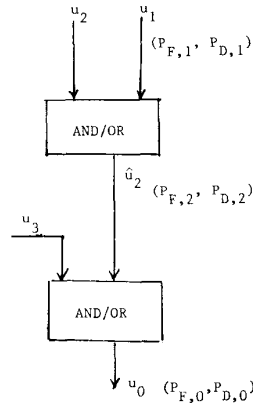


Fig. 16(a). Example of two input and one output parallel fusion function with three sensors.

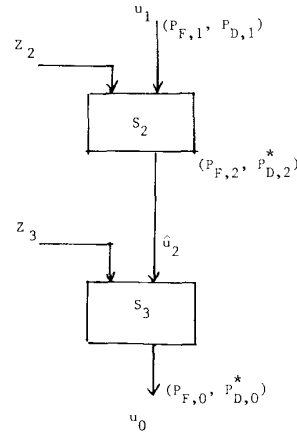


Fig. 16(b). Serial scheme with three sensors.

than or equal unless the optimal parallel is a majority decision logic. In such a case, only an actual performance assessment determines which is better. As mentioned earlier, in the case of Rayleigh channel with two or three sensors, the numerical results show that the optimal serial is just equivalent to OR. In this sense the Rayleigh channel can be termed conservative. Also, in Figs. 7-9, over the range of false alarms where the parallel outperforms the serial, the best of the parallel is the majority decision rule. In the range where serial is better, the best of the parallel belongs to the class of Theorem 2.

## V. CONCLUSION

A serial distributed network of  $N$  sensors detecting the presence or absence of a signal is analyzed. When the sensor observations conditioned on the hypothesis are statistically independent, the sensors employ  $N$ -P test for maximizing the detection probability for a given false alarm probability at the  $N$ th stage (Theorem 1). For certain noise distributions, the parallel structure requiring its fusion scheme to belong to a certain class of switching functions, is not superior to the serial scheme (Theorem 2). As a drawback, any serial network is vulnerable to link failures. Some numerical examples illustrate the performance of the optimal serial decision scheme.

In the case of Rayleigh target detection with two and three sensors, the performances of the serial and the OR fusion rule are equal. For AWGN channel and two sensors, the serial performs better than the parallel. However, with three sensors the performance is essentially the same. It is not known whether there exists any channel, practical or hypothetical, such that the serial is better than the parallel for a distributed network with three or more sensors. Considering the complexity of the serial scheme and the results from this limited study, the choice seems to favor the parallel fusion for the distributed detection problem.

## APPENDIX

It is shown here that any false alarm is realizable at any stage of a serial configuration. Let us denote for simplicity  $P_{F,j-1}$ ,  $P_{F,j}$ ,  $P_{D,j-1}$ ,  $t_{j,1}$ ,  $t_{j,0}$ ,  $a_j$ , and  $b_j$  by  $\alpha$ ,  $\alpha_0$ ,  $\beta$ ,  $t_1$ ,  $t_0$ ,  $a$ , and  $b$ , respectively. Therefore, using (2) and (3), and (7)

$$\alpha_0 = (1 - \alpha)a + \alpha b$$

$$t_0 = t \frac{1 - \alpha}{1 - \beta}$$

$$t_1 = t \frac{\alpha}{\beta}. \quad (A1)$$

The likelihood ratio  $\Lambda$  (from (3)) and hence  $a$  and  $b$  are continuous functions of  $t$ . Hence, for a fixed  $\alpha$ ,  $\alpha_0$  is a continuous function of  $t$ . Let the support of the distribution of  $\Lambda$  be between  $t_1$  and  $t_h$  ( $t_1 \geq 0$  and  $t_h \leq \infty$ ). As  $t_0$  approaches  $t_1$ ,  $a$ ,  $b$ , and  $\alpha_0$  approach 1 and as  $t_1$  approaches  $t_h$ ,  $a$ ,  $b$ , and  $\alpha_0$  approach 0. Therefore, any  $\alpha_0$  in  $(0, 1)$  can be obtained.

Please note that the method employed here is suggested by one of the reviewers.

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**Ramanarayanan Viswanathan** (S'81—M'83) received the B.E. (Hons.) degree in electronics and communication engineering from the University of Madras, India, in 1975, the M.E. degree with distinction in electrical communication engineering from the Indian Institute of Science, Bangalore, in 1977, and the Ph.D. degree in electrical engineering from Southern Methodist University, Dallas, Tex. in 1983.

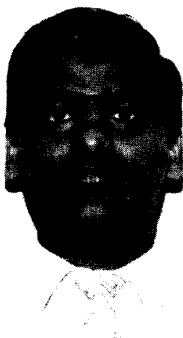
From 1977 until 1980 he worked as a Deputy Engineer in the Digital Communication Department of the Research and Development Division of Bharat Electronics Limited, Bangalore, India. Since 1983 he has been an Assistant Professor in the Electrical Engineering Department at Southern Illinois University, Carbondale, Ill. His research interests include detection and estimation theory, statistical theory of communication and spread spectrum systems.



**Stelios C.A. Thomopoulos** (S'80—M'83) was born in Athens, Greece, on June 15, 1955. He received his Diploma of Electrical & Mechanical Engineering from the National Technical University of Athens in 1978, and both his M.S. and Ph.D. in electrical engineering from the State University of New York at Buffalo in 1981 and 1983, respectively.

Since 1983, he has been an Assistant Professor in the Department of Electrical Engineering at Southern Illinois University, Carbondale, Ill. His areas of interest include data networks, distributed processing, robotics, artificial intelligence, and neural networks.

Dr. Thomopoulos is a member and Licensed Engineer of the Technical Chamber of Greece.



**Ramakrishna J. Tumuluri** received his B.E. in electrical engineering from Gujarat University, India, in 1984 and his M.S. degree in electrical engineering from Southern Illinois University, Carbondale, Ill. in 1987. He is currently enrolled in the Ph.D. program in electrical engineering at Purdue University, West Lafayette, Ind.

His research interests include detection and estimation theory, computer networking, and artificial intelligence.

He is a member of Phi-Kappa-Phi and is a gold medal winner of Gujarat University, India.