Optimal shape design of coastal structures minimizing coastal erosion

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Abstract

Coastal erosion is and will be an increasing major environmental issue. We apply shape optimization techniques to the design of coastal structures such as breakwaters, groins and other innovative shapes. Actually, we compute the solution of a boundary value problem describing the water waves scattered by the structure and modify accordingly its shape, in order to minimize a pre-defined cost function taking into account the strength (energy) of the waterwaves. The optimization procedure relies on a new global semi-deterministic algorithm, able to pursue beyond local minima.

keywords. shape optimization, global recursive multi-layer optimization, coastal engineering, water waves, scattering, Helmholtz equation.

1 Hydrodynamics of water waves scattering.

We consider the case of a coastline provided with identical vertical emerged structures which we seek to control and optimize. These structures are fully reflective. In order to simplify the computation, we assume that the response of structures on a given incident wave is periodically reproduced along the coastline. So, we study the reflection on only one structure by using periodic lateral conditions (See figure 1). The depth in Ω is a constant equal to h. We assume an inviscid and irrotational flow with velocity $\mathbf{u} = (u, v, w)$ solution of

$$\mathbf{u} = -\nabla\Phi \tag{1.1}$$

where $\Phi = \Phi(\mathbf{x}, z, t)$ is a three dimensional velocity potential in the domain Ω and $\mathbf{x} = (x, y)$. Owing to boundary conditions (free surface and no-penetration at the bottom), the classical expression for the potential Φ is (

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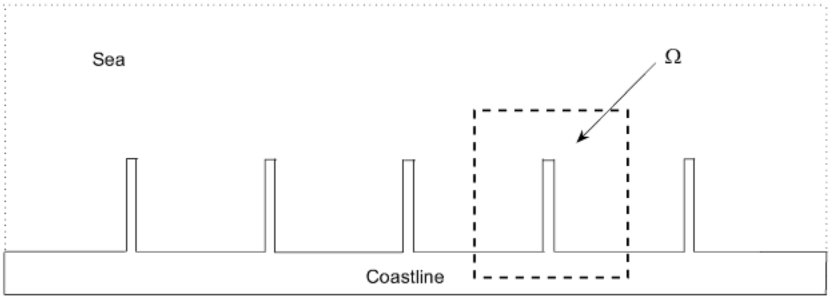

Figure 1: The coastline with five structures of defense

see [2])

$$\Phi(\mathbf{x}, z, t) = i\,\frac{g}{\sigma}\,\frac{\cosh k(h+z)}{\cosh kh}\,\eta(\mathbf{x}, t) \tag{1.2}$$

where free surface displacement $\eta(\mathbf{x}, t)$ is the free surface displacement. Water waves, assumed to be time harmonic, are described by the following expression :

$$\eta(\mathbf{x}, t) = \xi(\mathbf{x})e^{-i\sigma t}.$$

Thus, we are only interested in the space component $\xi(\mathbf{x})$. The incoming waves are assumed monochromatic.

$$\xi^i(\mathbf{x}) = ae^{i\mathbf{k}\cdot\mathbf{x}} \tag{1.3}$$

Moreover, we have the *dispersion equation.* $\sigma^2 = gk\tanh(kh)$ which for deep water waves (case $kh \gg \pi$) is reduced to $\sigma^2 = kg$ and $\xi(\mathbf{x})$ solves the Helmholtz equation.

$$\frac{\partial^2\xi}{\partial x^2} + \frac{\partial^2\xi}{\partial y^2} + k^2\xi(\mathbf{x}) = 0 \tag{1.4}$$

To compute the reflected free surface elevation, we decompose the total field as a sum $\xi = \xi^i + \xi^r$ where ξ^i is a given incident field and ξ^r the reflected field solution of a boundary value problem described below. The domain Ω is described in figure 1 where Γ_0 (shoreline and structures) is a fully reflective boundary, Γ_1 (off-shore)is an open boundary and $\Gamma_{2,(a,b)}$ (lateral sides) a periodic boundary. Thus ξ^r is solution of the following boundary value problem,

$$\begin{cases} \Delta\xi^r(x,y) + k^2\xi^r(x,y) = 0 & \text{on } \Omega \\ \dfrac{\partial\xi^r}{\partial\mathbf{n}} = -\dfrac{\partial\xi^i}{\partial\mathbf{n}} & \text{on } \Gamma_0 \\ \dfrac{\partial\xi^r}{\partial\mathbf{n}} - ik\xi^r = 0 & \text{on } \Gamma_1 \\ (\xi^i + \xi^r)_{|\Gamma_{2,a}} = (\xi^i + \xi^r)_{|\Gamma_{2,b}} & \end{cases} \tag{1.5}$$

This is a well-posed problem, see [1]. The numerical computation of the solution is performed with FEMLAB ® finite element solver on triangular grids.

2 Optimization.

A shape optimization problem consists in the minimization of a functional $J \in \mathbb{R}$, also called *cost function*. This function depends on $\mathbf{x}$, a design parameter defining the shape within the admissible set X, also called *control space* ([5]). We have a direct calculation loop for the functional: from a parameterization $\mathbf{x}$ we define a domain $\Omega(\mathbf{x})$ on which we compute the state equation solution $u(\Omega(\mathbf{x}))$ and the cost function $J(\mathbf{x})$:

$$J : \mathbf{x} \in X \mapsto \Omega(\mathbf{x}) \mapsto u(\Omega(\mathbf{x})) \mapsto J(\mathbf{x}, \Omega(\mathbf{x}), u(\Omega(\mathbf{x})))$$

The functional J can be penalized to include geometric and state constraints. To find numerically the minimum of J, it is necessary to use a minimization algorithm, e.g. steepest descent, genetic algorithms ... here we use a new *global* recursive algorithm ([3, 4]).

We present some examples of parameterizations for vertical emerged structure and cost function allowing to control the free surface elevation along the coastline. These choices are illustrative and other parameters and cost functions can be considered.

The incident waves will be either an unidirectional incident water wave of south-east direction (mono-directional case)or a set of incident wave direction (south-east,south,south-west) (multi-directional case). It is clear that the choice of J greatly influences the quality of optimization. The more physically relevant the cost function is, the better the results. We consider that the shape of the structure is efficient if it decreases the energy norm L^2 of water waves free surface in the admissible area Ω_{ad} representing the coastline between two successive structures. In addition, we ask for the solution to be as uniform as possible near the coastline by penalizing its standard deviation $\| \nabla\xi(\mathbf{x}) - \overline{\nabla\xi(\mathbf{x})} \|_{L^2(\Omega_{ad})}$ where $\overline{\nabla\xi(\mathbf{x})} = (\overline{\frac{\partial\xi}{\partial x}}, \overline{\frac{\partial\xi}{\partial y}})$ is a 2-dimensional average.

Thus we want to minimize the following cost function $J : X \to \mathbb{R}$, in the mono-directional case

$$J(\mathbf{x}) = \| \xi_{\theta_0}(\mathbf{x}) \|_{L^2(\Omega_{ad})} + \| \nabla\xi_{\theta_0}(\mathbf{x}) - \overline{\nabla\xi_{\theta_0}(\mathbf{x})} \|_{L^2(\Omega_{ad})} \tag{2.6}$$

in the multi-directional case

$$J(\mathbf{x}) = \sum_{\theta=-\frac{\pi}{4},-\frac{\pi}{2},-\frac{3\pi}{4}} \Big(\| \xi_\theta(\mathbf{x}) \|_{L^2(\Omega_{ad})} + \| \nabla\xi_\theta(\mathbf{x}) - \overline{\nabla\xi_\theta(\mathbf{x})} \|_{L^2(\Omega_{ad})} \Big) \tag{2.7}$$

where $\xi_\theta(\mathbf{x})$ is the total water wave resulting from the incident water wave of direction θ in the domain defined by the parameterization $\mathbf{x}$.

3 Some results

For the optimization, we consider a main wave period T of 2 seconds and an amplitude a of 2 meters. For the shape optimization of structures, we choose two different configurations. In a first case, we restrict ourselves with classical shapes (groins). In the second case, we remove all shape constraints and obtain innovating and non-intuitive shapes. The optimized case reduces the value of the cost function by 75% w.r.t. classical groins perpendicular to the shoreline. The shape obtained for the multi-directional case is all in all better than the one obtained in the mono-directional case. Surprisingly, the values of the cost function, for the SE direction, only differ by 4% whereas for incidental waves of southern (resp. SW) direction, they are 15% (resp. 60%) more effective (see fig. 2).

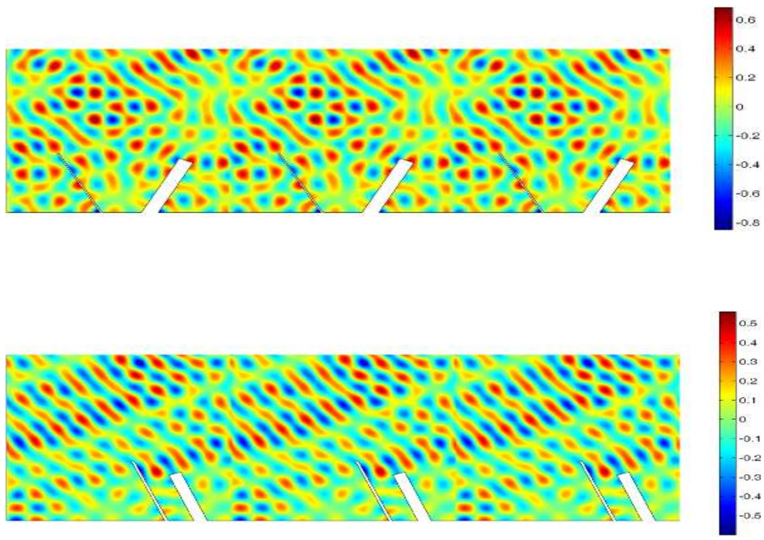

Figure 2: **Free surface elevation ξ resulting from a reflection** (**-Up**) on optimized structures for the mono-directional case, (**-Down**) on optimized structures for the multi-directional case.

Last, we let optimization be carried out without any constraints on the structures. So the optimized shapes obtained are not rectangular any more, and thus might be more difficult to realize in the real world.... The free surface elevation is shown on figure 3. For this configuration, the cost function decreases by more than 85% compared to case of rectangular structure perpendicular at the coastline (see fig. 3).

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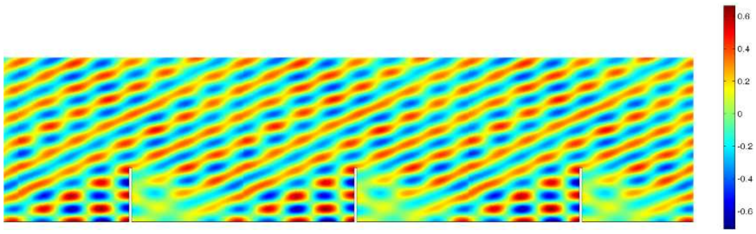

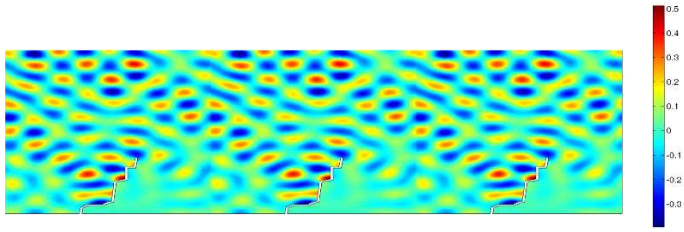

Figure 3: **Free surface elevation ξ resulting from a reflection (-Up)** on rectangular structures perpendicular at the coastline, (**-Down**) on optimized zig-zag structures.

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