Optimal Throughput-Delay Scaling in Wireless Networks – Part I: The Fluid Model

Abbas El Gamal, James Mammen, Balaji Prabhakar, and Devavrat Shah

Abstract

Gupta and Kumar (2000) introduced a random model to study throughput scaling in a wireless network with static nodes, and showed that the throughput per source-destination pair is $\Theta\left(1/\sqrt{n \log n}\right)$. Grossglauser and Tse (2001) showed that when nodes are mobile it is possible to have a constant throughput scaling per source-destination pair.

In most applications delay is also a key metric of network performance. It is expected that high throughput is achieved at the cost of high delay and that one can be improved at the cost of the other. The focus of this paper is on studying this trade-off for wireless networks in a general framework. Optimal throughput-delay scaling laws for static and mobile wireless networks are established. For static networks, it is shown that the optimal throughput-delay trade-off is given by $D(n) = \Theta(nT(n))$, where $T(n)$ and $D(n)$ are the throughput and delay scaling, respectively. For mobile networks, a simple proof of the throughput scaling of $\Theta(1)$ for the Grossglauser-Tse scheme is given and the associated delay scaling is shown to be $\Theta(n \log n)$. The optimal throughput-delay trade-off for mobile networks is also established. To capture physical movement in the real world, a random walk model for node mobility is assumed. It is shown that for throughput of $O\left(1/\sqrt{n \log n}\right)$, which can also be achieved in static networks, the throughput-delay trade-off is the same as in static networks, i.e., $D(n) = \Theta(nT(n))$. Surprisingly, for almost any throughput of a higher order, the delay is shown to be $\Theta(n \log n)$, which is the delay for throughput of $\Theta(1)$. Our result, thus, suggests that the use of mobility to increase throughput, even slightly, in real-world networks would necessitate an abrupt and very large increase in delay.

Index Terms

Queueing theory, random walks, scaling laws, throughput scaling, throughput-delay trade-off, wireless networks.

I. Introduction

A wireless network consists of a collection of nodes, each capable of transmitting to or receiving from other nodes. When a node transmits to another node, it creates interference for other nodes in its vicinity. When several nodes transmit simultaneously, a receiver can successfully receive the data sent by the desired transmitter only if the interference from the other nodes is sufficiently small. An important characteristic of wireless networks is that the topology of the nodes may not be known. For example, it may be a sensor network formed by a random configuration of nodes with wireless communication capability. The wireless nodes could also be mobile, in which case the topology could be continuously changing.

As the complexity of wireless networks increases, there is a need to develop better understanding of the fundamental trade-offs that govern their behavior. How much does interference limit throughput? How much does cooperation between the users help combat such interference? How does mobility affect network performance? Attempting to answer such questions by studying instances of wireless networks is not likely to lead to answers applicable to most of them.

In their seminal paper [11], Gupta and Kumar introduced a random network model for studying throughput scaling in a static wireless network, i.e., when the nodes do not move. They defined a random network to consist of n nodes distributed independently and uniformly on a unit disk. Each node has a randomly chosen destination node

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and can transmit at W bits per second provided that the interference is sufficiently small. Thus, each node can be simultaneously a *source,* S, a *destination,* D, and a *relay* for other source-destination (S-D) pairs. They showed that for almost all realizations of the random network, throughput scales as $\Theta(1/\sqrt{n \log n})$ per S-D pair [1]. Their result also showed that cooperation among users is essential to combating the adverse effects of interference. In [13], throughput scaling in static networks is studied and results similar to that of [11] are obtained using a different and simpler approach. In a related line of work (see e.g., [20], [14], [12]), the information-theoretic notion of network capacity is studied.

In [10], Grossglauser and Tse showed that by allowing the nodes to move, throughput scaling changes dramatically. Indeed, if node motion is independent across nodes and has a uniform stationary distribution, a constant throughput scaling ($\Theta(1)$) per S-D pair is feasible. Later, Diggavi, Grossglauser and Tse [5] showed that a constant throughput per S-D pair is feasible even with a more restricted mobility model.

In most networking applications delay is also a key performance metric along with throughput. Further, throughput that can be obtained from a network at the cost of increase in delay may not be useful. In this context the understanding of throughput-delay trade-off is key to achieving the quality of service required by the application. Indeed, a more useful description of the network capability would be in terms of its delay-constrained throughput. Most previous work has focused mainly on achieving the highest throughput with no consideration for delay or the trade-off between throughput and delay. In [2], a random network model with both mobile and static nodes is considered. The authors propose a routing algorithm that is almost optimal in terms of throughput and study its delay. In [18], throughput-delay trade-off in a mobile network is studied using an i.i.d. mobility model. In this model, each node is equally likely to be in any part of the network at each time instant.

In [7], we studied throughput and delay scaling for static and mobile networks and established optimal throughput-delay trade-offs. This paper provides a more complete treatment of our earlier results reported in [7] as well as some extensions. Further, we provide results on throughput-delay trade-off for mobile networks under a random walk model instead of the less realistic "hierarchical" Brownian motion model assumed in [7] [2]. In addition to establishing the optimal throughput-delay trade-off in wireless networks, the schemes we developed to achieve the optimal trade-off should help enhance our understanding of the fundamental interplay between such essential features of wireless networks as uncontrolled placement of nodes, common medium of communication leading to interference, and node mobility.

From previous throughput results in [11] and [10], one may make the following inferences about the trade-off between throughput and delay: (i) In a static random network a small transmission range is necessary to limit interference and hence to obtain a high throughput. This results in multi-hopping, and consequently leads to large delay. (ii) On the other hand, mobility allows nodes to approach each other closely. This not only allows the use of smaller transmission ranges, but more crucially, it allows the use of a single relay node, which boosts throughput to $\Theta(1)$. However, the delay is now dictated by the time it takes the relay to carry the packet to the neighborhood of the destination, which can be very large. The above observations point out three important features that influence the throughput and delay of any communication scheme in networks: (i) the number of hops, (ii) the transmission range, and (iii) the node mobility and velocity. We exploit these three features, to different degrees, to obtain the throughput-delay trade-off in an *optimal* way.

In the following section we provide needed definitions and summarize our main results. In Section III we establish the throughput-delay trade-off for static wireless networks. In Section IV we consider a mobile network model, present a scheme that achieves constant throughput, and determine its the corresponding delay. In Section V we established the trade-off for mobile networks.

II. MODELS, DEFINITIONS, AND MAIN RESULTS

We first present the static and mobile random network models along with the model for successful wireless transmission and then provide definitions of throughput and delay.

[1]Recall the following notation: (i) $f(n) = O(g(n))$ means that there exists a constant c and integer N such that $f(n) \leq cg(n)$ for $n > N$. (ii) $f(n) = o(g(n))$ means that $\lim_{n\to\infty} f(n)/g(n) = 0$. (iii) $f(n) = \Omega(g(n))$ means that $g(n) = O(f(n))$, (iv) $f(n) = \omega(g(n))$ means that $g(n) = o(f(n))$. (v) $f(n) = \Theta(g(n))$ means that $f(n) = O(g(n));\ g(n) = O(f(n))$.

[2]We note here that [7] incorrectly states the motion model as a Brownian motion model, whereas it is actually a hierarchical Brownian motion model.

Definition 1 (Random network model): The random network consists of n nodes distributed uniformly at random in a unit torus. The nodes are split into $n/2$ distinct source-destination (S-D) pairs at random. Time is slotted for packetized transmission. For simplicity, we assume that the time-slots are of unit length. In a *static network* nodes do not move. In the case of a *mobile network*, the mobility model, which we denote as the *random walk (RW) model*, is as follows. The unit torus is divided into n square cells of area $1/n$ each, resulting in a two-dimensional $\sqrt{n}\times\sqrt{n}$ discrete torus. The initial position of each node is equally likely to be any of the n possible cells independent of others. Each node independently performs a simple random walk on the two-dimensional $\sqrt{n}\times\sqrt{n}$ discrete torus. By a simple random walk, we mean the following: let a node be in cell $(i, j) \in \{0, \ldots, \sqrt{n}-1\}^2$ at time t, then, at time $t+1$ the node is equally likely to be in any of the four adjacent cells $\{(i-1, j), (i+1, j), (i, j-1), (i, j+1)\}$, where addition and subtraction are modulo $\sqrt{n}$.

Note that, implicitly this models a situation where each node moves $1/\sqrt{n}$ distance in unit time, that is, velocity scales as $1/\sqrt{n}$. Further, note that in the random walk model, nodes move independently according to a uniform stationary and ergodic distribution as in previous work [10]. The additional assumption of the random walk model is not required for throughput results and is used only in the analysis of delay.

At this point, we would like to argue that the random walk model is a good model for capturing real motion in the physical world due to its Markovian nature, so that the present position determines the distribution of the future position. It is sufficiently simple and well studied by probabilists, so as to allow analysis of complicated quantities such as queueing delay, which depends heavily on the motion model.

Definition 2 (Model for successful transmission): For successful transmission, we assume a model similar to the *Protocol model* as defined [11]. Under our *Relaxed Protocol model*, a transmission from node i to node j in a time-slot is successful if for any other node k that is transmitting simultaneously,

$$d(k, j) \geq (1 + \Delta)d(i, j) \quad \text{for} \quad \Delta > 0,$$

where $d(i, j)$ is the distance between nodes i and j. During a successful transmission, nodes send data at a constant rate of W bits per second.

To establish our results without being encumbered by issues related to scheduling packets in the network, we allow the packet size to be arbitrarily small. We refer to this as the *fluid model*. In this model, the data sent in a time-slot could correspond to multiple packets. Thus the time taken for a packet transmission may only be a small fraction of the time-slot itself. However, a packet received by a node in some time-slot cannot be transmitted by the node until the next time-slot. The scheduling problem is avoided using the fluid model by choosing the packet size to be small enough depending on the number of nodes in the network. In the second part of this paper [9], we extend our trade-off results for static networks to the case where packet size remains constant.

In the other commonly used model of successful transmission, namely the Physical model (e.g., [11], [10]), a transmission is successful if the signal to interference and noise ratio (SINR) is greater than some constant. It is well known [11] that if signal decays with distance r as $r^{-\delta}$ for $\delta > 1$, the Protocol model is equivalent to the Physical model, where each transmitter uses the same power. In the rest of the paper, we shall assume the Relaxed Protocol model.

Definition 3 (Scheme): A scheme Π for a random network is a sequence of communication policies, $\{\Pi_n\}$, where policy Π_n determines how communication takes place in a network of n nodes.

Definition 4 (Throughput of a scheme): Let $B_{\Pi_n}(i, t)$ be the number of bits of S-D pair i transferred in t time-slots under policy Π_n, for $1 \leq i \leq n/2$,. Note that this could be a random quantity for a given realization of the network. Scheme Π is said to have throughput $T_\Pi(n)$ if $\exists$ a sequence of events $A_\Pi(n)$ such that

$$A_\Pi(n) = \left\{ \min_{1\leq i\leq n/2} \liminf_{t\to\infty} \frac{1}{t} B_{\Pi_n}(i, t) \geq T_\Pi(n) \right\},$$

and $P\left(A_\Pi(n)\right) \to 1$ as $n\to\infty$.
We allow randomness in policies and as a result the set $A_\Pi(n)$ above is in the joint probability space including both the random network of size n and the policy. Randomness will be used in our schemes for mobile networks. We say that an event A_n occurs with high probability (*whp*) if $P\left(A_n\right) \to 1$ as $n\to\infty$.

Definition 5 (Delay of a scheme): The delay of a packet is the time it takes for the packet to reach its destination after it leaves the source. Let $D^i_{\Pi_n}(j)$ denote the delay of packet j of S-D pair i under policy Π_n, then the sample mean of delay (over packets that reach their destinations) for S-D pair i is

$$\bar{D}^i_{\Pi_n} = \limsup_{k\to\infty} \frac{1}{k}\sum_{j=1}^{k} D^i_{\Pi_n}(j).$$

The average delay over all S-D pairs for a particular realization of the random network is then

$$\bar{D}_{\Pi_n} = \frac{2}{n}\sum_{i=1}^{n/2} \bar{D}^i_{\Pi_n}.$$

The delay for a scheme Π is the expectation of the average delay over all S-D pairs, i.e.,

$$D_{\Pi}(n) = E[\bar{D}_{\Pi_n}] = \frac{2}{n}\sum_{i=1}^{n/2} E[\bar{D}^i_{\Pi_n}].$$

Definition 6 (Throughput-delay optimality): A pair $(T(n), D(n))$ is said to be throughput-delay (T-D) optimal if there exists a scheme Π with $T_{\Pi}(n) = \Theta(T(n))$ and $D_{\Pi}(n) = \Theta(D(n))$ and $\forall$ scheme Π' with $T_{\Pi'}(n) = \Omega(T(n))$, $D(\Pi')(n) = \Omega(D(n))$.

Definition 7 (Optimal throughput-delay trade-off): The optimal throughput-delay trade-off consists of all optimal T-D pairs.

Although we have introduced detailed notation in order to unambiguously define the above quantities, in the rest of this paper we shall avoid the use of subscripts to indicate the scheme and policy since the scheme or policy under consideration will be clear from the context. Also when describing a scheme Π, we shall just describe the policy Π_n for arbitrary n.

We would like to note that since the fluid model allows us to scale down the size of the packets, packet delay as defined here is not equivalent to the delay per bit. To measure delay per bit one would need to keep the packet size constant but then this would require scheduling in the network. The fluid model allows us to initiate a study of the network delay, without dealing with packet scheduling. The packet delay scaling as defined above would be the delay per bit scaling if the scheduling delay does not dominate the packet delay. This is the case in one of our schemes (Scheme 2 in Section IV), where we use packets of constant size. In subsequent work [9], we show that this is in fact the case for static networks as well.

In this paper, constants that do not depend on n are denoted as c_i. We would also like to note that in order to keep the discussion clear, we sometimes use looser bounds that result in the correct order even when tighter bounds in terms of constants can be readily obtained.

The main results of this paper are as follows.

Trade-off in the static random network: In Section III, we introduce a cellular scheme (Scheme 1) for static networks, which by varying cell size can trade-off throughput for delay. We then prove that the scheme is optimal leading to the following result.

Theorem 1: The optimal throughput-delay trade-off in a static random network is given by

$$T(n) = \Theta\left(\frac{D(n)}{n}\right), \tag{1}$$

when $T(n) = O\left(1/\sqrt{n \log n}\right)$.

The above result, which is illustrated in Figure 1, says that: (i) The highest throughput per node achievable in a static network is $\Theta\left(1/\sqrt{n\log n}\right)$, as Gupta and Kumar obtained. At this throughput the average delay $D(n) = \Theta\left(\sqrt{n/\log n}\right)$ (point Q in Figure 1). (ii) By increasing the cell size and hence the transmission radius, the average number of hops can be reduced. But because the interference is higher now, the throughput is lower. When throughput is $O\left(1/\sqrt{n\log n}\right)$, equation (1) shows the optimal delay-constrained throughput (segment PQ in Figure 1).

Mobile network at $\mathbf{T(n) = \Theta(1)}$: In [10], a two-hop scheme that achieves constant throughput scaling in mobile wireless networks was presented. It was expected that the delay would be high since mobility was utilized to communicate packets. Delay scaling, however, was not quantified. In Section IV, we introduce Scheme 2, which is essentially the same as the two-hop scheme in [10]. We show that this scheme achieves constant throughput scaling

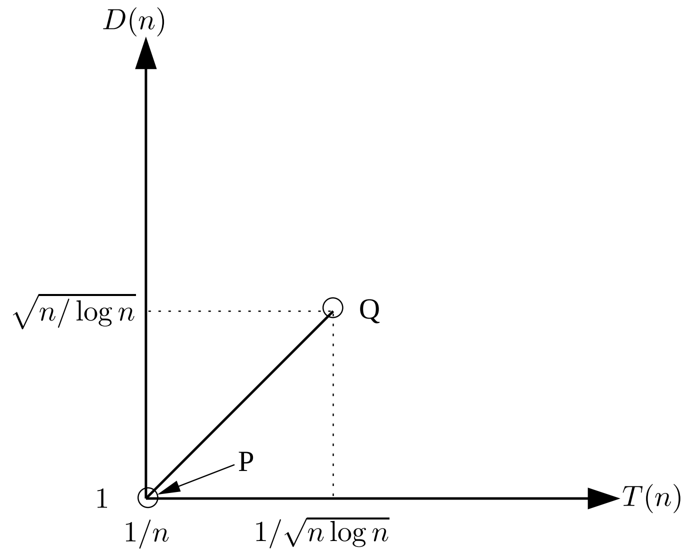

Fig. 1. Throughput-delay trade-off for the static random network. The scales of the axes are in terms of the orders in n.

for the mobile network and determine its associated constant. Using results and methods from random walks and queueing theory, we determine the exact order of delay for the random walk model of node mobility.

Theorem 2: Scheme 2 for the mobile random network has throughput $T(n) = \Theta(1)$ and its delay scales as

$$D(n) = \Theta\left(n \log n\right).$$

Point R in Figure 2 corresponds to the throughput-delay scaling provided by Scheme 2. As mentioned before, in contrast to other schemes in this paper that use a fluid model, the packet size remains fixed in Scheme 2. Note that any trade-off that can be achieved using constant-size packets can obviously be achieved using the fluid model since the constraint of requiring packets to have a constant size is removed.

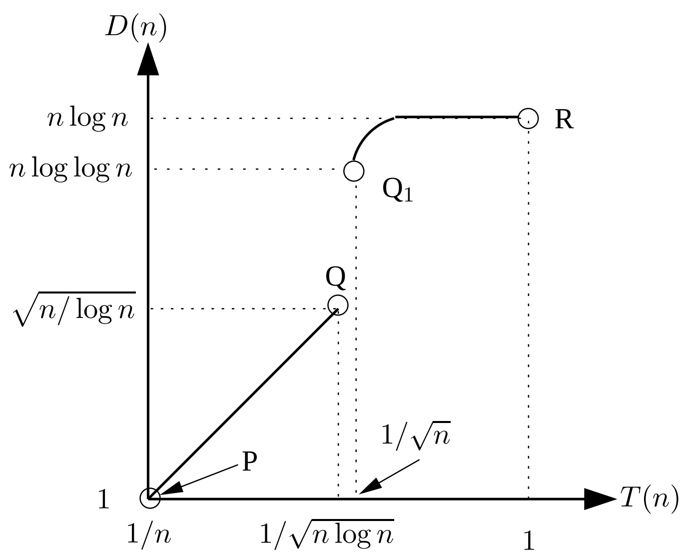

Fig. 2. Throughput-delay trade-off for the mobile random network. Again, the scales of the axes are in terms of the orders in n.

Trade-off in the mobile random network: In Section V we introduce Schemes 3(a) and 3(b) that achieve the optimal throughput-delay trade-off in mobile networks. To provide lower delay, Scheme 3(a) does not use mobility to relay packets. In fact, mobility makes this scheme significantly more complex than Scheme 1, even though the throughput-delay trade-off achieved is the same for both schemes. This is because packets need to chase the nodes to achieve low delay. However, mobility is essential for higher throughput and this is harnessed by Scheme 3(b) at the cost of higher delay. The throughput-delay results for mobile networks are as follows.

Theorem 3: The optimal throughput-delay trade-off for the mobile random network is as follows.

(a) For the range of $T(n) = O\left(1/\sqrt{n \log n}\right)$, similar to the case of static network,

$$T(n) = \Theta\left(\frac{D(n)}{n}\right).$$

(b) For $T(n) = \omega\left(1/\sqrt{n \log n}\right)$ and $T(n) = O(1/\log n)$ the optimal throughput-delay trade-off is characterized as,

$$T(n) = \Theta\left(\frac{1}{\sqrt{n\ a(n)\ \log n}}\right) \text{ and}$$

$$D(n) = \Theta\left(n \log\left(\frac{1}{a(n)}\right)\right),$$

where $a(n)$ is a parameter such that $a(n) = \Omega(\log n/n)$ and $a(n) \leq 1$.

Segment PQ in Figure 2 corresponds to Theorem 3(a) and segment Q_1R corresponds to Theorem 3(b). Note that the effect of mobility is to significantly increase the range of achievable throughput albeit at the expense of a very large delay.

Independently and around the same time, it has been shown in [15] using a Brownian motion model for node mobility that any throughput higher than $\Theta(1/\sqrt{n})$ comes at the expense of a very high delay. The same conclusion follows from the optimal throughput-delay trade-off for mobile networks obtained using the random walk mobility model in this paper.

The results for mobile networks provide several insights into the role of mobility in wireless networks.

- For throughput of $O\left(1/\sqrt{n \log n}\right)$, the trade-off in mobile networks is identical to that in static networks. This suggests that, although mobility can enhance the throughput of wireless networks, it does not alter the trade-off between throughput and delay for the range of throughputs achievable in static wireless networks. Further, the scheme achieving the above trade-off does not use the mobility of the nodes to communicate packets. This suggests that for low delay applications, mobility is in fact a hindrance and makes communication schemes significantly more complex.
- As soon as mobility is used to boost the throughput beyond $\Theta\left(1/\sqrt{n \log n}\right)$, the delay jumps up to $\Theta(n \log \log n)$. Thereafter, even though the throughput increases to $\Theta(1)$, the delay only increases to $\Theta(n \log n)$. In this sense there is almost no trade-off between throughput and delay for this range of high throughputs. This also means that if mobility is used to boost the throughput even slightly beyond that in static wireless networks then the delay shoots up to its highest value.

Apart from the above results, there are two other contributions of this paper.

1) Our work provides a unified framework for understanding the seemingly disparate throughput results of [11] and [10]. Further, the optimal throughput-delay trade-off for static and mobile wireless networks is also determined in the same framework.
2) In the course of establishing the optimal throughput-delay trade-off, we provide simpler proofs of previous throughput results. We note that our model is essentially the same as that in [11] and [10], with two minor differences — we assume that the network is located on a torus instead of a disk and we assume the Protocol model instead of the Physical model.

III. Throughput-Delay Trade-off for Static Networks

This section establishes the optimal throughput-delay trade-off in a static wireless network by providing a proof of Theorem 1. We first present Scheme 1 and compute the trade-off achievable using this scheme in Theorem 4. Theorem 5 provides a converse, which states that for a given delay scaling no scheme can provide a better throughput scaling than that of Scheme 1, thus establishing the optimality of Scheme 1 and also proving Theorem 1.

Our trade-off scheme is a multi-hop, time-division-multiplexed (TDM), cellular scheme with square cells of area $a(n)$ so that the unit torus consists of $1/a(n)$ cells as shown in Figure 3. In the following analysis, we ignore the edge effects due to $1/a(n)$ not being a perfect square. Before presenting the trade-off scheme, we present three lemmas about the geometry of the n nodes on the torus divided into square cells of area $a(n)$. Proofs of Lemmas 1 and 2 are in Appendix I.

Lemma 1: If $a(n) \geq 2\log n/n$, then each cell has at least one node *whp*.

We say that cell B *interferes* with another cell A if a transmission by a node in cell B can affect the success of a simultaneous transmission by a node in cell A.

Lemma 2: Under the Relaxed Protocol model, the number of cells that interfere with any given cell is bounded above by a constant c_1, independent of n.

A consequence of Lemma 2 is that there exists an interference-free schedule such that each cell becomes active regularly once in $1+c_1$ time-slots and it does not interfere with any other simultaneously transmitting cell.

We say that a cell is *active* in a time-slot if any of its nodes transmits in that time-slot. A consequence of Lemma 2 is that, there exists an interference-free schedule where each cell becomes active regularly, once in $1+c_1$ time-slots and no cell interferes with any other simultaneously transmitting cell.

Let the straight line connecting a source S to its destination D be called an S-D line.

Lemma 3: For $a(n) = \Omega(\log n/n)$, the number of S-D lines passing through each cell is $O\left(n\sqrt{a(n)}\right)$, *whp*.

Proof: Let H_i be the number of hops taken by a packet for S-D pair i, $1 \leq i \leq n/2$ in traveling from S to D along the S-D line by hops along adjacent cells of area $a(n)$. For each S-D pair, H_i depends on the distance L_i between S and D and also the orientation of the S-D line. Now $E[L_i]$ is a constant and hence since the hops are along cells having side-length $1/\sqrt{a(n)}$, it can be shown that

$$E[H_i] = \Theta\left(E[L_i]/\sqrt{a(n)}\right). \tag{2}$$

There are $m = 1/a(n)$ cells. Fix a cell j and define Y_i^j to be the indicator of the event that the S-D line of S-D pair i passes through cell j. That is,

$$Y_i^j = \begin{cases} 1 & \text{if any hop of S-D pair } i \text{ is in cell } j \\ 0 & \text{otherwise} \end{cases}$$

for $1 \leq i \leq n/2$ and $1 \leq j \leq m$. Summing up the total number of hops in the cell in two different ways we obtain

$$\sum_{i=1}^{n/2}\sum_{j=1}^{m} Y_i^j = \sum_{i=1}^{n/2} H_i.$$

Taking expectations on both sides and noting that all the $E[Y_i^j]$ are equal due to symmetry on the torus, we obtain

$$\frac{nm}{2}E[Y_i^j] = \frac{n}{2}E[H_i]. \tag{3}$$

From (2) and (3) it follows that $P\{Y_i^j = 1\} = \Theta\left(\sqrt{a(n)}\right)$. Now for a fixed cell j, the total number of S-D lines passing through it is given by $Y = \sum_{i=1}^{n/2} Y_i^j$. This is the sum of i.i.d. Bernoulli random variables since the position of each node is independent of that of the others and Y_i^j depends only on the positions of the source and destination nodes of S-D pair i. Moreover $E[Y] = \Theta\left(n\sqrt{a(n)}\right)$, which is $\Omega\left(\sqrt{n\log n}\right)$ since $a(n) = \Omega(\log n/n)$ and hence the Chernoff bound for the sum of i.i.d. Bernoulli random variables (e.g. see [17]) yields

$$P\{Y > (1+\delta)E[Y]\} \leq \exp(-E[Y]\delta^2/4).$$

Choosing $\delta = 2\sqrt{2\log n/E[Y]}$ results in

$$P\{Y > (1+\delta)E[Y]\} \leq 1/n^2.$$

Since $\delta = o(1)$, for $a(n) = \Omega(\log n/n)$ this means that $Y = O(E[Y])$ with probability $\geq 1 - 1/n^2$. Now using the union bound over $m = O(n/\log n)$ cells shows that the number of lines passing through each cell is $O(E[Y]) = O\left(n\sqrt{a(n)}\right)$ with probability $\geq 1 - 1/n$. ■

The above lemma shows that the number of S-D lines passing through each cell is $\leq c_2 n\sqrt{a(n)}$ *whp*, for an appropriate choice of the constant c_2.

Now we are ready to describe Scheme 1, which is parameterized by the cell area $a(n)$, where $a(n) = \Omega(\log n/n)$ and $a(n) \leq 1$.

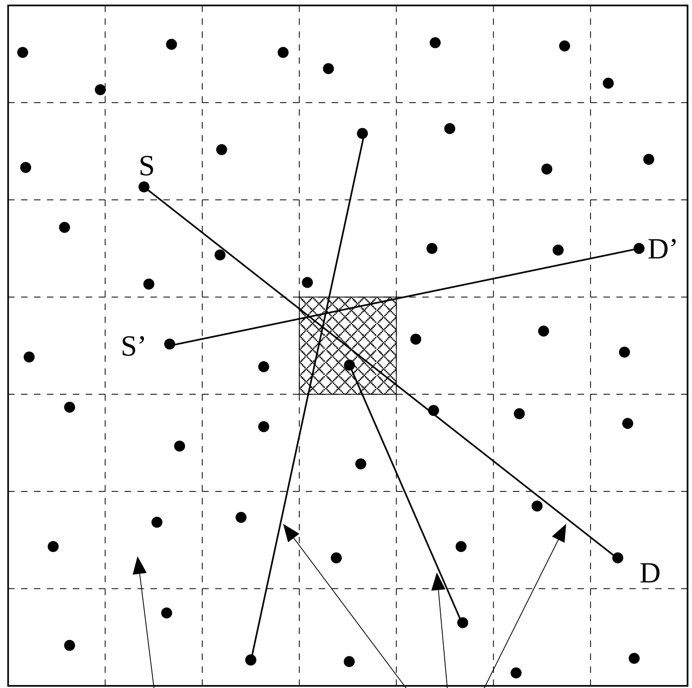

Fig. 3. The unit torus is divided into cells of size $a(n)$ for Scheme 1. The S-D lines passing through the shaded cell in the center are shown.

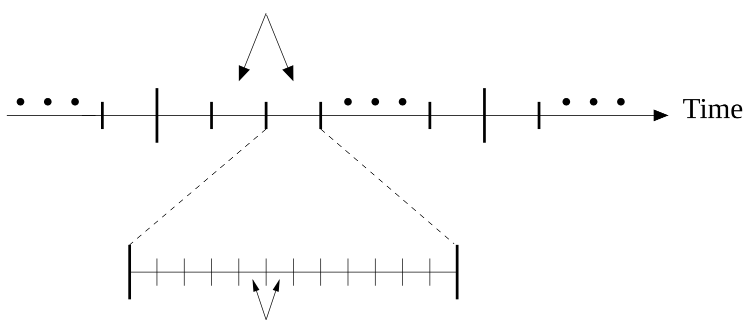

Fig. 4. The TDM transmission schedule of Scheme 1. Each cell becomes active once in $1 + c_1$ time-slots and each active time-slot is divided into several packet-slots.

Scheme 1: Static networks

1) Divide the unit torus using a square grid into square cells, each of area $a(n)$ (see Figure 3).
2) Verify whether the following conditions are satisfied for the given realization of the random network.
   - Condition 1: No cell is empty.
   - Condition 2: The number of S-D lines through each cell is at most $c_2 n\sqrt{a(n)}$.
3) If either of the above conditions is not satisfied then use a time-division policy, where each of the $n/2$ sources transmits directly to its destination in a round-robin fashion.
4) Otherwise, i.e., if both conditions are satisfied, use the following policy Σ_n:
   a) Each cell becomes active at a regular interval of $1+c_1$ time-slots (the constant c_1 comes from Lemma 2). Several cells which are sufficiently far apart become active simultaneously. Thus the scheme uses TDM

between nearby cells.

b) Let the straight line connecting a source S to its destination D be denoted as an S-D line. A source S transmits data to its destination D by hops along the adjacent cells lying on its S-D line as shown in Figure 3.

c) When a cell becomes active, it transmits a single packet for each of the S-D lines passing through it. This is again performed using a TDM scheme that slots each cell time-slot into *packet time-slots* as shown in Figure 4.

The point of trade-off at which Scheme 1 operates is determined by the parameter $a(n)$ and the dependence is made precise in the following theorem.

Theorem 4: For $a(n) \geq 2\log n/n$,

$$T(n) = \Theta\left(\frac{1}{n\sqrt{a(n)}}\right) \quad \text{and} \quad D(n) = \Theta\left(\frac{1}{\sqrt{a(n)}}\right),$$

i.e., the throughput-delay trade-off achieved by Scheme 1 is

$$T(n) = \Theta\left(\frac{D(n)}{n}\right) \quad \text{for} \quad T(n) = O\left(\frac{1}{\sqrt{n\log n}}\right).$$

Proof: If the time-division policy with direct transmission is used, then the throughput is $2W/n$ with a delay of 1. But since it happens with a vanishingly low probability, as shown by Lemmas 1 and 3, the throughput and delay for Scheme 1 are determined by that of policy Σ_n.

First we analyze the throughput of Scheme 1. When policy Σ_n is used, since Condition 1 is satisfied, each cell has at least one node. This guarantees that each source can send data to its destination by hops along adjacent cells on its S-D line. From Lemma 2, it follows that each cell gets to transmit a packet every $1 + c_1$ time-slots, or equivalently, the cell throughput is $\Theta(1)$. The total traffic through each cell is that due to all the S-D lines passing through the cell, which is $O\left(n\sqrt{a(n)}\right)$ since Condition 2 is also satisfied. This shows that

$$T(n) = \Theta\left(1/\left(n\sqrt{a(n)}\right)\right).$$

Next we compute the average packet delay $D(n)$. As defined earlier, packet delay is the sum of the amount of time spent in each hop. We first bound the average number of hops then use the fact that the time spent at each hop is constant, independent of n.

Since each hop covers a distance of $\Theta\left(\sqrt{a(n)}\right)$, the number of hops per packet for S-D pair i is $\Theta\left(d_i/\sqrt{a(n)}\right)$, where d_i is the length of S-D line i. Thus the number of hops taken by a packet averaged over all S-D pairs is $\Theta\left(\frac{1}{n}\sum_{i=1}^{n} d_i/\sqrt{a(n)}\right)$. Since for large n, the average distance between S-D pairs is $\frac{1}{n}\sum_{i=1}^{n} d_i = \Theta(1)$, the average number of hops is $\Theta\left(1/\sqrt{a(n)}\right)$.

Now note that by Lemma 2 each cell can be active once every constant number of cell time-slots and by Lemma 3 each S-D line passing through a cell can have its own packet time-slot within that cell's time-slot. Since we assumed that packet size scales in proportion to the throughput $T(n)$, each packet arriving at a node in the cell departs in the next active time-slot of the cell. Thus the delay is at most c_1 times the number of hops. From the above discussion, we conclude that the delay $D(n) = \Theta\left(1/\sqrt{a(n)}\right)$. This concludes the proof of Theorem 4. ■

Next we show that the throughput-delay trade-off provided by Scheme 1 is optimal for a static wireless network as far as the scaling is concerned.

Theorem 5: If any scheme has throughput, $T(n)$, and delay, $D(n)$, then $D(n) = \Omega(nT(n))$.

Proof: This proof uses techniques similar to those used in the proof of Theorem 2.1 in [11]. Consider a given fixed placement of $2n$ nodes in the unit torus. Let $\bar{L}$ be the sample mean of the lengths of the S-D lines for the given node placement and let the throughput of the scheme under consideration be λ. Consider a large enough time t, so that by definition, the total number of bits transported in the network is λnt. Let $h(b)$ be the number of hops taken by bit b, $1 \leq b \leq \lambda nt$ and let $r(b,h)$ denote the length of hop h of bit b. Therefore,

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} r(b,h) \geq \lambda nt\bar{L}. \tag{4}$$

Now, for two simultaneous transmissions from node i to node j and from node k to node l, consider

$$\begin{aligned} d(j,l) &\geq d(j,k) - d(l,k) \\ &\geq (1+\Delta)d(i,j) - d(l,k), \end{aligned} \tag{5}$$

and

$$\begin{aligned} d(j,l) &\geq d(l,i) - d(i,j) \\ &\geq (1+\Delta)d(l,k) - d(i,j). \end{aligned} \tag{6}$$

Combining (5) and (6), we obtain

$$d(j,l) \geq \frac{\Delta}{2}\left(d(i,j) + d(k,l)\right).$$

This result implies that if we place a disk around each receiver of radius $\Delta/2$ times the length of the hop, the disks must be disjoint for successful transmission under the Protocol model. Since a node transmits at W bits per second, each bit transmission time is $1/W$ seconds. During each bit transmission, the total area covered by the disks surrounding the receivers must be less than the total unit area. Summing over the Wt bits transmitted in time t and accounting for edge effects, we obtain

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} \frac{\pi}{4}\left(\frac{\Delta}{2}r(b,h)\right)^2 \leq Wt. \tag{7}$$

Let the total number of hops taken by all bits be $H = \sum_{b=1}^{\lambda nt} h(b)$. Then by convexity, it follows that

$$\left(\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)\right)^2 \leq \sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)^2. \tag{8}$$

Using (4), (7) and (8), we obtain

$$\left(\lambda nt\bar{L}\right)^2 \leq \left(\frac{16Wt}{\pi\Delta^2}\right) H. \tag{9}$$

Now defining $\bar{h}$ to be the sample mean of the number of hops over λnt bits, i.e., $\bar{h} = \frac{1}{\lambda nt}H$. Using this to rewrite (9) and the obvious fact that $\lambda \leq W$, we obtain

$$\lambda n \leq \min\left\{\frac{16W}{\pi\Delta^2\bar{L}^2}\bar{h},\ nW\right\}. \tag{10}$$

By the law of large numbers, $\bar{L} = \Theta(1)$ *whp*. Moreover the rate of convergence is exponential in n. Let A_n be the set such that $\bar{L} = \Theta(1)$ and let $I(A_n)$ be the indicator of set A. Then from (10), we have

$$\begin{aligned} nE[\lambda] &\leq E\left[\frac{16W}{\pi\Delta^2\bar{L}^2}\bar{h}I(A_n)\right] + E\left[nWI(A_n^c)\right] \\ &\leq c_5 E[\bar{h}] + o(1), \end{aligned} \tag{11}$$

where the last term is $o(1)$ since $P(A_n^c)$ converges to 0 exponentially.

By definition, if a scheme has throughput $T(n)$ then there exists a set B_n on which $\lambda \geq T(n)$ and $P(B_n)$ converges to 1. Therefore we have

$$\begin{aligned} E[\lambda] &= E[\lambda I(B_n)] + E[\lambda I(B_n^c)] \\ &\geq T(n)(1 - o(1)). \end{aligned} \tag{12}$$

From (11) and (12), it follows that $nT(n)(1-o(1)) \leq c_5 E[\bar{h}] + o(1)$, which is the same as $E[\bar{h}] = \Omega(nT(n))$. Now each packet spends at least one time-slot at each hop and hence the delay of each packet is at least as much as the number of hops it takes. As a result, if $D(n)$ is the delay of the scheme under consideration then by definition, $D(n) \geq E[\bar{h}]$. Thus we have shown that for any scheme, $D(n) = \Omega(nT(n))$. ■

IV. Mobile network with $T(n) = \Theta(1)$

In this section we consider a random mobile network with the random walk mobility model, which is similar to the model introduced by Grossglauser and Tse in [10]. They showed that under the Physical model $T(n) = \Theta(1)$ is achievable. Recall that we assume the n nodes form $n/2$ distinct S-D pairs in a torus of unit area. The motion of each node is independent, ergodic and uniform on the $\sqrt{n} \times \sqrt{n}$ discrete torus. Thus, at a given time, a node is equally likely to be in any part of the torus independent of the location of any other node.

We first present a scheme (which is similar to the one in [10]) and show that it achieves constant throughput and then analyze its delay in Subsection IV-A. The analysis of delay for this scheme will also help in characterizing the throughput-delay trade-off in mobile wireless networks in Section V.

Scheme 2: $T(n) = \Theta(1)$ in mobile networks

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1) Divide the unit torus into n square cells, each of area $1/n$.
2) Each cell becomes active once in every $1 + c_1$ time-slots (as a consequence of Lemma 2).
3) In an active cell, the transmission is always between two nodes within the same cell.
4) Each time-slot is divided into two sub-slots A and B. The following is done in each active cell.
   a) In sub-slot A, if one or more source nodes are present and the cell contains two or more nodes, pick one source node at random. With probability $0 < p_1 < 1$, the randomly chosen node transmits its packet to another randomly chosen node in the same cell, which acts as a relay. This node could also happen to be the destination. And with probability $1 - p_1$ it does nothing.
   b) In sub-slot B, if the cell contains one or more destination nodes and two or more nodes in all, pick one destination node at random. Another randomly chosen node in the same cell acts as a relay and transmits to this destination a packet that is destined for it if it has one. This node could also happen to be the source. Otherwise nothing happens.

We now show that this scheme achieves a constant throughput scaling using a simpler proof than the one in [10]. Our proof is based on showing that the total network throughput is $\Theta(n)$. The symmetry of the scheme and the use of at most one relay ensure that this total network throughput is equally divided among the $n/2$ S-D pairs resulting in $T(n) = \Theta(1)$.

Theorem 6: The throughput in a mobile random network using Scheme 2 is $T(n) = \Theta(1)$.

Proof: Consider the transmission of packets from sources in sub-slot A over a period of T time-slots. Due to division into sub-slots the packet size is $W/2$ bits. Let $A(i,t)$ be the number of packets transmitted from source i in time-slot t and let $A(t) = \sum_{i=1}^{n/2} A(i,t)$ be the total number of packets transmitted in time-slot t. The number of bits transmitted by source i in time-slot t is just $WA(i,t)/2$.

Next we determine $E[A(t)]$. Let $C(t)$ be the number of cells which contain at least one of the $n/2$ source nodes and two or more nodes in all. Then from the description of Scheme 2 it follows that $E[A(t)] = p_1 E[C(t)]/(1+c_1)$. Now let I_i be the indicator for the event that cell i contains at least one source node and two or more nodes in all. Let E_1 be the event that cell i contains exactly one source node and E_2 be the event that cell i contains two or more source nodes. Similarly let F_1 be the event that cell i contains one or more destination nodes. Also let

$$p_k = \binom{n/2}{k}\left(\frac{1}{n}\right)^k \left(1 - \frac{1}{n}\right)^{n/2-k} \rightarrow \frac{e^{-1/2}}{2^k k!}.$$

That is, p_k is equal to the probability that k nodes are in cell i. Then for $1 \le i \le n$,

$$\begin{aligned} E[I_i] &= P\left((E_1 \cap F_1) \cup E_2\right) \\ &= P(E_1)P(F_1) + P(E_2) \\ &= p_1(1-p_0) + (1 - p_0 - p_1) \\ &\rightarrow \frac{1}{2}e^{-1/2}\left(1 - e^{-1/2}\right) + 1 - e^{-1/2} - \frac{1}{2}e^{-1/2} \\ &= c_3 > 0, \end{aligned} \tag{13}$$

as $n \to \infty$. Therefore,

$$\begin{aligned} E[A(t)] &= \frac{p_1}{1+c_1} E[C(t)] \\ &= \frac{p_1}{1+c_1} E\left[\sum_{i=1}^{n} I_i\right] \\ &= \frac{np_1}{1+c_1} E[I_1] \\ &\to c_4 n, \end{aligned} \tag{14}$$

where $c_4 = c_3 p_1/(1+c_1) > 0$.

Now the mobile random network is an irreducible finite-state Markov chain and $A(t)$ is a bounded non-negative function of the state of this Markov chain at time t. Therefore by the ergodicity of such a Markov chain,

$$\lim_{T\to\infty} \frac{1}{t} \sum_{t=1}^{T} A(t) = E[A(t)] \to c_4 n.$$

Thus the total rate at which packets are transmitted from sources is $\Theta(n)$. From the symmetry of the nodes and the randomness of the scheme it follows that each of the $n/2$ sources transmits at rate of $\Theta(1)$. These packets either reach the destination or are queued at the relay nodes, in which case they are transmitted to their final destinations in some of the B sub-slots. By choosing $0 < p_1 < 1$ in Scheme 2, we have ensured that the arrival rate to each queue is less than the rate at which the queue can be serviced. This ensures the stability of the queues as a result of which the throughput per S-D pair is just the rate at which each source transmits data. And we have shown that this is $\Theta(1)$ thus proving that Scheme 2 yields $T(n) = \Theta(1)$. ■

The above proof also shows that the constant throughput per S-D pair that can be achieved is close to $c_4 W/2$ bits per second for large enough n. Now $c_3 = 0.2095$ and reasonably small values of Δ result in $1 + c_1 = 16$, which results in $c_4 = 0.13p_1$. Thus under Scheme 2, for large enough n, the throughput between each source-destination pair is about 0.65% of the maximum possible value of W bits per second.

A. Analysis of Delay

The nodes perform independent random walks. Hence only $\Theta(1/n)$ of the packets belonging to any S-D pair reach their destination in a single hop (which happens when both S and D are in the same cell in sub-slot A). Thus, most of the packets reach their destination via a relay node, where the delay has two components: (i) *hop-delay*, which is constant and independent of n, and (ii) *mobile-delay*, which is the time a packet spends at the relay while it is moving until it is delivered to its destination. Next, we analyze the mobile-delay.

Relay-queue: From the description of Scheme 2, for each S-D pair, each of the remaining $n-2$ nodes acts as a relay. Each node maintains a separate queue for each of the S-D pairs as illustrated in Figure 5. Thus the mobile-delay is the expected delay at such a relay-queue. By symmetry, all such queues at all relay nodes are identical. Consider one such relay-queue, i.e., fix an S-D pair and a relay node R. To compute the expected delay of this relay-queue, we need to study the characteristics of its arrival and departure processes.

A packet arrives at the relay-queue when (i) R is in the same cell as S, (ii) the cell becomes active, (iii) S and R are chosen as a transmit-receive pair, and (iv) S transmits a packet (which happens with probability p_1). Similarly, a departure can occur from the queue when (i) R is in the same active cell as D, (ii) the cell becomes active, and (iii) R and D are chosen as a transmit-receive pair. Such a time-slot is called a *potential departure instant* and the sequence of inter-potential-departure times is called the *potential-departure process*. A packet actually departs, only if, in addition to the above, R also has a packet for D, i.e., the relay-queue is not empty. In the analysis below, we ignore the effect of the the cell becoming active once in $1 + c_1$ time-slots since the actual delay is $1 + c_1$ times the delay computed in this manner by ignoring it.

We say that two nodes *meet* if they are in the same cell. The joint position of two nodes due to independent random walks can also be viewed as a difference random walk relative to the position of one node. Then the inter-meeting times are just the inter-visit times of state $(0, 0)$ for the difference random walk on a $\sqrt{n} \times \sqrt{n}$ discrete torus. Hence the inter-meeting times of nodes S and R, $\tau_0, \tau_1, \ldots$, form an i.i.d. sequence. Let τ be a

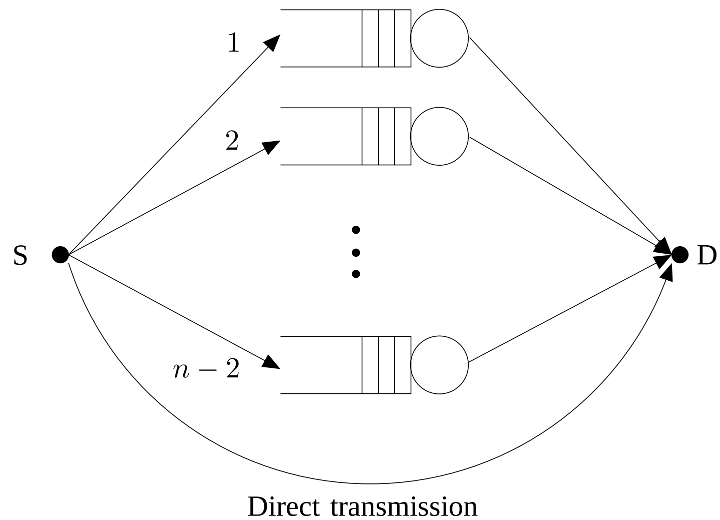

Fig. 5. For any S-D pair, the remaining $n-2$ nodes act as relays. Each node maintains a separate queue for each of the $n-2$ S-D pairs.

random variable with their common distribution. The moments of τ that will be required later are given in the following lemma, which is proved in Appendix II.

Lemma 4: The first and second moments of τ are given by

$$E[\tau] = n, \quad E[\tau^2] = \Theta(n^2 \log n).$$

In what follows, we obtain upper and lower bounds of the same order on the delay of the relay-queue, thus pinning down the exact order of delay scaling. To obtain an upper bound, we progressively define queues that are simpler to analyze. We first upper bound the delay of the relay-queue by that of another queue, $\mathcal{Q}_1$, which has i.i.d. inter-arrival times. The delay of $\mathcal{Q}_1$ is then upper bounded by that of $\mathcal{Q}_2$, which, in addition, has i.i.d. inter-potential-departure times. The delay of $\mathcal{Q}_2$ is then upper bounded by the sum of delays through two queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, in tandem. Queue, $\mathcal{Q}_3$, is a GI/GI/1 queue, so that Kingman's upper bound can be used. Queue, $\mathcal{Q}_4$, is not a GI/GI/1 queue, but we are still able to bound the delay using the moments of the inter-arrival and inter-potential-departure times. For both these queues, the required moments can be expressed in terms of the moments of the inter-meeting time of two nodes, which we obtain using random walk analysis. We now proceed to the details.

Upper bound: As mentioned above, the inter-meeting times of nodes S and R form an i.i.d. sequence. The inter-arrival times, however, are not independent because an arrival occurs with probability p_1 only if S and R are chosen as a transmit-receive pair. The probability that S and R are chosen as a transmit-receive pair depends on the number of other nodes in the same cell. If S and R are not chosen, in spite of being in the same cell, the likelihood of there being many more nodes in the same cell increases. Due to the random walk model of the node mobility, if there is a crowding of nodes in some part of the network, it remains crowded for some time in the future. Hence due to the Markovian nature of node mobility, inter-arrival times in the relay-queue are not independent.

Consider a queue, $\mathcal{Q}_1$, in which there is an arrival with probability p_1 whenever S and R meet, irrespective of whether S and R are chosen as a transmit-receive pair. The inter-arrival times of this queue are then stochastically dominated by the inter-arrival times of the relay-queue. Let the potential-departure process of $\mathcal{Q}_1$ be the same as that of the relay-queue. Then the delay of $\mathcal{Q}_1$ provides an upper bound on the delay of the relay queue. It is easy to see that the sequence of inter-arrival times of $\mathcal{Q}_1$ is i.i.d. and that the common distribution is that of the sum of G independent copies of τ, where $G \sim \text{Geom}(p_1)$. Recall that τ is the inter-meeting time of S and R.

Let the potential-departure process of the relay-queue (and also $\mathcal{Q}_1$) be denoted by $\{S_i\}$. Next we will study this process in order to replace it with another coupled process with less frequent potential departures. Recall that the potential departure time-slots are the ones in which a packet can be emptied from the queue if the queue is not empty. These are the times when R and D are in the same cell and are chosen as a transmit-receive pair. Let $\alpha_0, \alpha_1, \ldots$ be the time-slots in which R and D are in the same cell and let $E_i, i = 0, 1, \ldots$, be the indicator of the event that α_i is also a potential service instant. That is, E_i is an indicator for the event that R and D are chosen as a transmit-receive pair at time α_i.

Let α^i denote $\{\alpha_0, \dots, \alpha_i\}$ and E^i denote $\{E_0, \dots, E_i\}$. Then $P(E_i = 1|\alpha^i, E^{i-1}), i = 0, 1, \dots$ is the probability that $E_i = 1$ given $\alpha_0, \dots, \alpha_i$ and $E_0, \dots, E_{i-1}$. This is the probability that R and D are chosen as a transmit-receive pair given that they are in the same cell in time-slot α_i and given the entire past consisting of $\alpha_0, \dots, \alpha_i$ and $E_0, \dots, E_{i-1}$. From the description of Scheme 2 it is clear that the probability of the event $\{E_i = 1\}$ depends on the number of other nodes in the cell containing R and D. This, in turn, depends on the entire past of the processes $\{E_i\}$ and $\{\alpha_i\}$ due to the Markovian nature of node mobility.

Thus the potential-departure process is generated by the processes $\{\alpha_i\}$ and $\{E_i\}$. This is because time-slot α_i, when R and D are in the same cell for the ith time, is chosen as a potential departure instant with probability $P(E_i = 1|\alpha^i, E^{i-1})$.

Due to the dependence on the past of $\{E_i\}$, the inter-potential-departure times are also dependent. We will next show that this dependence is not too much, in the sense that irrespective of the past, the probability of the event $\{E_i = 1\}$ is greater than a positive constant that does not depend on n. The following lemma is proved in Appendix II.

Lemma 5: There exists a constant, c_6, (independent of n) such that for all large enough n,

$$P\left(E_i = 1|\alpha^i, E^{i-1}\right) \geq c_6 > 0.$$

Now let $\mathcal{Q}_2$ be a queue such that each time-slot in which R and D meet is chosen to be a potential departure instant with probability c_6. Then by Lemma 5, the inter-potential-departure times for this queue would be stochastically dominated by those for $\mathcal{Q}_1$. If $\mathcal{Q}_2$ has the same arrival process as $\mathcal{Q}_1$ then the delay of $\mathcal{Q}_2$ is an upper bound on that of $\mathcal{Q}_1$.

As before, the sequence of inter-potential-departure times of $\mathcal{Q}_2$ is i.i.d. and the common distribution is that of the sum of G independent copies of τ, where $G \sim \text{Geom}(c_6)$. As a result, we have upper bounded the delay of the relay-queue by the delay of $\mathcal{Q}_2$, which has i.i.d. inter-arrival times and i.i.d. inter-potential-departure times. To obtain an upper bound on the delay, we only use the first two moments of the inter-arrival time and the inter-potential-departure time. Since both of these are sums of a Geometric number of independent inter-meeting times, it is easy to check that their moments are of the same order as that of the inter-meeting time. As a result, the constants p_1 and c_6 do not affect the delay scaling. Further, $\mathcal{Q}_2$ is stable as long as the arrival rate is less than the service rate, i.e., $p_1 < c_6$. Since we are interested in determining the delay scaling, for simplicity, we assume that in $\mathcal{Q}_2$, an arrival occurs whenever S and R meet with probability 0.5 and a potential departure occurs whenever R and D meet.

Now we bound the delay of $\mathcal{Q}_2$ by the sum of the delays through two queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, in tandem. Both $\mathcal{Q}_3$ and $\mathcal{Q}_4$ will be shown to have delay of $O(n \log n)$, which implies that the delay of $\mathcal{Q}_2$ is $O(n \log n)$. Queues $\mathcal{Q}_3$ and $\mathcal{Q}_4$ are constructed as follows. The arrival process of $\mathcal{Q}_3$ is the same as that of $\mathcal{Q}_2$. The potential-departure process of $\mathcal{Q}_3$ is an i.i.d. Bernoulli process with parameter $2/3n$ (or potential departure rate $\frac{2}{3n}$). An arrival occurs at $\mathcal{Q}_4$ whenever there is a potential-departure at $\mathcal{Q}_3$. If $\mathcal{Q}_3$ is non-empty, then the arrival to $\mathcal{Q}_4$ is the head-of-line packet transferred from $\mathcal{Q}_3$ to $\mathcal{Q}_4$ or else a dummy packet is fed to $\mathcal{Q}_4$. Thus the arrival process at $\mathcal{Q}_4$ is the same as the potential-service process at $\mathcal{Q}_3$. By construction, the delay of a packet through this tandem of queues, $\mathcal{Q}_3$ and $\mathcal{Q}_4$, upper bounds the delay experienced by a packet through $\mathcal{Q}_2$. Now from Lemmas 6 and 7 stated below and proved in Appendix II, both $\mathcal{Q}_3$ and $\mathcal{Q}_4$ have an expected delay of $O(n \log n)$.

Lemma 6: The expected delay of a packet through $\mathcal{Q}_3$ is $O(n \log n)$.

Lemma 7: The expected delay of a packet through $\mathcal{Q}_4$ is $O(n \log n)$.

Hence the expected delay of the packets of each S-D pair relayed through each relay R is $O(n \log n)$. The delay of a scheme is the expectation of the packet delay averaged over all S-D pairs and all relay nodes. Hence it follows that the delay of Scheme 2 is $O(n \log n)$.

Lower bound: We now establish a lower bound on the delay of Scheme 2. Consider a packet arrival at the relay node when it is in cell (i, j). Let the destination be in cell (k, l), which is equally likely to be any one of the n cells since the destination performs an independent random walk. Using the difference random walk, the delay is at least equal to the time required for the random walk to reach state (k, l) starting from state (i, j). Hence the expected value of the delay can be lower bounded as

$$
\begin{aligned}
E[D] &\geq \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \pi(i,j)\pi(k,l)E_{(i,j)}T_{(k,l)} \\
&= \Theta(n \log n), \qquad (15)
\end{aligned}
$$

where (15) is from p. 11 in Chapter 5 of [1].

Combining this lower bound with the earlier upper bound leads to the following theorem.

Theorem 7: The delay of Scheme 2 is

$$D(n) = \Theta\left(n \log n\right).$$

V. Throughput-delay trade-off in mobile networks

In this section we establish the optimal throughput-delay trade-off for mobile random networks under the random walk (RW) model for node mobility. To achieve this trade-off, we introduce Scheme 3. The scheme is divided into two parts based on the range of throughput handled by it. Scheme 3(a) is for $T(n) = O(1/\sqrt{n \log n})$, while Scheme 3(b) is for $T(n) = \omega(1/\sqrt{n \log n})$. These ranges are dealt with separately since the schemes achieving the optimal trade-off in these ranges are fundamentally different. In the low throughput range, the optimal scheme cannot use the mobility of nodes to communicate packets. In fact, Scheme 3(a) is significantly more complex than Scheme 1. Even though it provides the same trade-off, it needs to overcome difficulties created by node mobility. On the other hand, in the high throughput range, it is essential to use the mobility of nodes to communicate packets.

Both these schemes divide the network into square cells of area $a(n)$, which is a parameter that determines the point of trade-off. We would like to note that these cells are a part of the scheme and are unrelated to the n cells used in the definition of the random walk mobility model.

A. *Trade-off for low throughput*

Scheme 3(a) described below requires the packet size to scale down as $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, and similar to Scheme 1, it is a cellular TDMA scheme. Due to the mobility of the nodes, the packets need to chase their destination nodes, which makes the scheme and its analysis significantly more complex. For the sake of our proof technique, the scheme drops a packet that is unsuccessful in chasing down its destination for long. A more precise description of the scheme follows.

Scheme 3(a): Mobile networks at low throughput

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1) Divide the unit torus into square cells, each of area $a(n)$ (see Figure 3).
2) A cellular TDMA transmission scheme is used, in which, each cell becomes active at regularly scheduled cell time-slots (see Figure 4). From Lemma 2, each cell gets a chance to be active once every $1 + c_1$ cell time-slots.
3) A packet is sent from its source S to its destination D by chasing the destination for at most $k(n) = \Theta(\log \log n)$ stages as follows:
   a) Consider a packet that is generated at S, when S is in cell C^0. Let D be in cell C^1 at that time.
   b) Set $k = 1$.
   c) In stage k, the packet is sent from cell C^{k-1} to cell C^k via hops along adjacent cells on the line joining centers of cells C^{k-1} and C^k.
   d) When the packet reaches cell C^k, assume that the destination D is in the cell C^{k+1}. If $C^{k+1} = C^k$ then the packet is delivered to D. Otherwise, set $k = k + 1$.
   e) If $k < k(n)$, repeat (c)-(d), else drop the packet.
4) In every active time-slot of a cell, each of the source nodes residing in the cell at that time generates a new packet for its respective destination. These new packets are transmitted in the same time-slot.
5) Packet size scales as $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ and hence in an active time-slot a cell can transmit $\Theta\left(n\sqrt{a(n)}\right)$ packets. If at any time instant a cell has more packets than it can transmit then the excess packets are dropped.
6) Packets transmitted to a cell not containing any node are dropped.

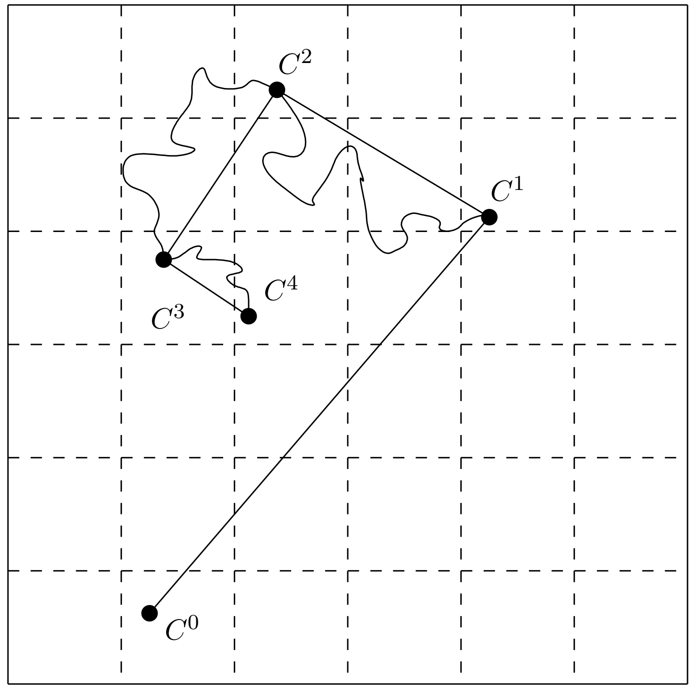

Fig. 6. In scheme 3(a), the unit torus is divided into square cells of area $a(n)$. In stage 1 of chasing, a packet starts from cell C^0 containing its source, and moves by hops along adjacent cells towards cell C^1 which contains its destination node at that time. By the time it reaches cell C^1, its destination has moved to cell C^2. So in stage 2 of chasing, the packet hops from cell C^2 to cell C^3. And this continues for at most $k(n)$ stages or until the packet reaches the cell containing its destination.

In the above scheme packets are dropped if one of the following three scenarios occurs.

(i) Empty cells: Due to mobility of nodes, it is possible that a cell may be empty in some time-slot. In part b) of step 3) above, a packet may be lost if it is transmitted to any empty cell.

(ii) Overloading of cells: In the case of Scheme 1 we could provide a guarantee *whp* on the maximum number of S-D lines passing through each cell. In this case, due to mobility, the number of S-D lines passing through a cell may exceed its capacity of $\Theta\left(n\sqrt{a(n)}\right)$ packets. If this occurs, the excess packets are dropped as mentioned in step 5) above.

(iii) Unsuccessful chasing: A packet that does not reach its destination after $k(n)$ stages of chasing is also dropped.

The following theorem shows that the fraction of packets dropped is negligible, i.e., goes to 0 as $n \to \infty$. We assume that error correction is employed to combat this packet loss, however this requires only a constant fraction of the total throughput and hence does not affect the throughput scaling. Thus, in spite of node mobility, Scheme 3(a) achieves the same throughput-delay trade-off as Scheme 1 for static networks.

Theorem 8: Scheme 3(a) achieves the throughput-delay trade-off given by:

$$T(n) = \Theta\left(\frac{D(n)}{n}\right), \quad \text{for} \quad T(n) = O\left(\frac{1}{\sqrt{n \log n}}\right).$$

We first analyze the throughput of Scheme 3(a). The delay of a packet, as per the description of the scheme, is $(1 + c_1)$ times the number of hops taken by a packet. In the process of determining the throughput, we shall determine the average number of hops per packet during the $k(n)$ stages. This will allow us to determine the average packet delay.

As explained in the description of Scheme 3(a) above, each source generates a packet of size $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ each time the cell it belongs to become active. Each cell becomes active once every $1 + c_1$ time-slots, according to Lemma 2. Hence, independent random walk of nodes under the RW model implies that each node is in an active cell for a constant fraction $1/(1+c_1)$ of the time *w.p. 1* due to ergodicity. That is, each source node generates traffic at rate $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ under Scheme 3(a). Thus, to show that each S-D pair achieves throughput $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, it is sufficient to show that the fraction of the packets dropped under Scheme 3(a) goes to 0 as $n \to \infty$.

To show that the fraction of dropped packets goes to 0 as $n \to \infty$, we need to bound the total traffic generated by all $k(n)$ stages. The total traffic due to all $k(n)$ stages is the number of packets that a cell is required to transmit in a time-slot. We analyze this in the following three lemmas by utilizing arguments similar to that in the proof of Lemma 3. See Appendix III for proofs of Lemmas 8 and 9. For simplicity of analysis, we assume that each cell becomes active every time-slot instead of $1 + c_1$ time-slots. This simplification does not change the results in the order notation.

Lemma 8: The number of packets of stage 1 passing through each cell in a time-slot is $O\left(n\sqrt{a(n)}\right)$ with probability at least $1 - 1/n^3$.

Lemma 9: For $k \geq 2$, the number of packets of stage k passing through each cell in a time-slot is $O\left(\left(\frac{n^2}{m}\right)^{2^{-k}}\right)$ with probability at least $1 - 1/n^3$.

Lemma 10: The number of packets passing through each cell in a time-slot is $O\left(n\sqrt{a(n)}\right)$ with probability at least $1 - 1/n^{2.9}$.

Proof: There are $k(n) = O(\log\log n)$ stages. Using the union bound over $k(n)$ stages and the bounds given by Lemma 8 and Lemma 9, we can show that the total number of packets (due to all $k(n)$ stages), $P(n)$, passing through each cell is

$$P(n) \;\; = \;\; O(n\sqrt{a(n)}) + \sum_{k=2}^{k(n)} O\left(\left(\frac{n^2}{m}\right)^{2^{-k}}\right), \tag{16}$$

with probability at least $1 - k(n)/n^3 \geq 1 - 1/n^{2.9}$.

Next, we evaluate the summation on the right hand side of (16). Consider the largest index k such that

$$\left(\frac{n^2}{m}\right)^{2^{-k+1}} \geq 2\left(\frac{n^2}{m}\right)^{2^{-k}}. \tag{17}$$

This gives

$$\left(\frac{n^2}{m}\right)^{2^{-k}} \geq 2. \tag{18}$$

That is,

$$k \leq \frac{\log\log n^2/m - \log\log 2}{\log 2}.$$

Note that the parameter $k(n)$ in Scheme 3(a) is chosen to satisfy this condition. Moreover for $k < k(n)$, the ratio of consecutive terms,

$$\frac{\left(\frac{n^2}{m}\right)^{2^{-k+1}}}{\left(\frac{n^2}{m}\right)^{2^{-k}}} = \left(\frac{n^2}{m}\right)^{2^{-k}} \geq 2,$$

due to (18). As a consequence,

$$\begin{aligned}
\sum_{k=2}^{k(n)} \left(\frac{n^2}{m}\right)^{2^{-k}} &\leq \sum_{k=2}^{k(n)} \left(\frac{n^2}{m}\right)^{2^{-2}} 2^{-k+2} \\
&= \frac{\sqrt{n}}{m^{1/4}} \sum_{k=0}^{k(n)-2} 2^{-k} \\
&\leq \frac{2\sqrt{n}}{m^{1/4}}.
\end{aligned} \tag{19}$$

Replacing (19) in (16), we obtain that, for large enough n, with probability at least $1 - 1/n^{2.9}$,

$$P(n) \;\; = \;\; O(n\sqrt{a(n)}) + O\left(\frac{\sqrt{n}}{m^{1/4}}\right). \tag{20}$$

Now, $a(n) = \Omega(\log n/n)$. Hence, $n\sqrt{a(n)} = \Omega\left(\sqrt{n \log n}\right)$. Since $m = \Omega(1)$, this will imply that the right hand side of (20) is $O(n\sqrt{a(n)})$. ∎

Remark: The packet size is of order $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$ and the associated constant in this Θ notation is chosen such that the total data due to all the packets can be supported in one time-slot.

Proof of Theorem 8: We show that the fraction of packets dropped due to (i) overloading of cells, (ii) unsuccessful chasing, and (iii) empty cells, goes to 0 as $n \to \infty$. This will immediately imply that the throughput of Scheme 3(a) is as claimed in Theorem 8.

Packets dropped due to overloading of cells: Since the number of hops in each stage is $O\left(1/\sqrt{a(n)}\right)$ and there are at most $\Theta(\log \log n)$ stages, the total number of packets in the network is $O\left(n \log \log n/\sqrt{a(n)}\right)$. Thus the process describing the number of packets in each of the $1/a(n)$ cells is a finite-state Markov chain that is induced by the underlying Markov chain due to the random walk of nodes on the $\sqrt{n} \times \sqrt{n}$ discrete torus as specified by the RW model. This is aperiodic and hence an ergodic Markov chain. Hence by choosing packets of size $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, the fraction of time when the the number of packets through any cell exceeds its capacity is $p_n \leq 1/n^{2.9}$. Now consider a long time duration t in which $nT(n)t$ bits corresponding to nt packets are sent from the sources to their destinations. Of these, at most $O\left(n \log \log n p_n t/\sqrt{a(n)}\right)$ is dropped due to overloading in cells since the total number of packets in the network is $O\left(n \log \log n/\sqrt{a(n)}\right)$. Hence if $p_n = o\left(\log \log n \sqrt{a(n)}\right)$, the fraction of packets dropped approaches 0 as $n \to \infty$, which is indeed the case here.

Packets dropped due to unsuccessful chasing: We need to determine the fraction of packets dropped due to the destination moving away from its initial position in the last stage, i.e., stage $k(n)$. Recall that the choice of $k(n)$ is such that it is the largest index with property (17). Hence

$$\left(\frac{n^2}{m}\right)^{2^{-k(n)}} \geq 2\left(\frac{n^2}{m}\right)^{2^{-k(n)-1}},$$

which in turn yields

$$m^{2^{-k(n)}} \leq 4. \tag{21}$$

Using (21) in (67), we obtain

$$l_{k(n)} = O\left(\frac{m}{n}\right).$$

Now using the fact that $m = O\left(n/\log n\right)$, it follows that $l_{k(n)} \to 0$ as $n \to \infty$. On the other hand, for our motion model where the nodes move according to a two-dimensional random walk on the $\sqrt{n} \times \sqrt{n}$ discrete torus and cell size of $\Omega(\log n/n)$, the average time taken by a node to move out of a cell is $\Omega(\log n)$. Hence the probability that a packet is dropped due to unsuccessful chasing tends to 0 as $n \to \infty$.

Packets dropped due to empty cells: Finally in any time-slot, the packets that need to be relayed through cells that do not contain any nodes are lost. Now consider a fixed time-slot. Each node is equally likely to be in any of the cells, thus by Lemma 1, the probability that any cell is empty is $\leq 1/n^2$. Again, using ergodicity and the bound on the number of packets transmitted per time-slot, which is $O\left(n\sqrt{a(n)}\right)$, the fraction of packets dropped due to cells being empty goes to 0 as $n \to \infty$.

Thus, the net fraction of packets dropped due to (i) overloading in cells, (ii) empty cells and (iii) unsuccessful chasing goes to 0 as $n \to \infty$. In other words, almost all the packets that are generated in Scheme 3(a) reach their destination successfully. As noted before, the number of packets generated by each source per unit time is $\Theta(1)$, and since each packet is of size $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$, the net throughput per S-D pair is $\Theta\left(1/\left(n\sqrt{a(n)}\right)\right)$. This completes the proof of the achievability of throughput as claimed in Theorem 8.

Average Delay: Next we compute the average delay of packets. Under Scheme 3(a), the average delay of a packet in stage k is l_k as it makes l_k hops on average in the k^{th} stage. Hence from (67), we obtain that the average delay,

$D(n)$, is

$$\begin{aligned} D(n) &= \sum_{k=1}^{k(n)} l_k \\ &\leq \sqrt{c_5 m} + \sum_{k=2}^{k(n)} \frac{c_5 m}{n} \left(\frac{n^2}{m}\right)^{2^{-k}}. \end{aligned} \tag{22}$$

Using (19) in (22), for large enough n, we obtain

$$\begin{aligned} D(n) &\leq \sqrt{c_5 m} + \frac{2c_5 m\sqrt{n}}{nm^{1/4}} \\ &\leq c_5\sqrt{m} + 2c_5\sqrt{m}\sqrt{\frac{\sqrt{m}}{n}} \\ &= O\left(\sqrt{m}\right), \end{aligned} \tag{23}$$

where (23) holds because $m = O(n/\log n) = o(n)$. Thus as claimed the average packet delay $D(n) = O\left(\sqrt{m}\right)$. This completes the proof of Theorem 8.

■

B. *Trade-off for high throughput*

To obtain a throughput higher than $\Theta\left(1/\sqrt{n\log n}\right)$, we need to use mobility, and to keep the delay as low as possible, we need to use multiple hops cleverly. This naturally leads to a scheme that combines Scheme 3(a) (which provides low throughput with low delay) and Scheme 2 (which provides high throughput with high delay). A first approach would be to *time-share* between Scheme 3(a) and Scheme 2. It is easy to show that such a scheme can achieve any throughput in the range from $\Theta\left(1/\sqrt{n\log n}\right)$ to $\Theta(1)$, but the average delay remains fixed at $\Theta(n\log n)$. By using Scheme 3(b), which is a careful combination of Scheme 2 and Scheme 3(a), the throughput-delay trade-off can be slightly improved. And it turns out that the performance of Scheme 3(b) is optimal.

Scheme 3(b) uses the chasing technique of Scheme 3(a) by using hopping along adjacent cells of size $\Theta(\log n/n)$ as the underlying packet transport mechanism. However this chasing is not done all the way from each source to its destination. Instead, the mobility of an intermediate mobile relay node is employed as in Scheme 2 to increase the throughput. The chasing technique of Scheme 3(a) is used only to send packets from a source to a mobile relay node or from a mobile relay node to a destination when they are sufficiently close. Nodes get sufficiently close to each other due to their mobility and the amount of closeness is captured by the parameter $b(n)$ of the scheme, which determines the trade-off point.

In the description of the scheme and the proof we refer to two ways of measuring distance between two nodes.

1) The *step-distance* or distance in terms of steps between two nodes with positions (i_1, j_1) and (i_2, j_2) on the $\sqrt{n}\times\sqrt{n}$ discrete torus is $|i_2 - i_1| + |j_2 - j_1|$, where the subtraction is modulo $\sqrt{n}$. Thus this distance is simply the Manhattan distance on the underlying discrete torus.
2) The *hop-distance* or distance in terms of hops, which is the number of hops a packet would take along adjacent cells of the straight line joining the nodes to reach from one node to the other. The cells in this case are determined by the scheme and are always of area of $\Omega(\log n/n)$ in Scheme 3(b).

Scheme 3(b): Mobile networks at high throughput

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1) Divide the unit torus into square cells each of area $a(n) = \Theta(\log n/n)$ as in Figure 7. The scheme uses a parameter $b(n)$ that determines the point of trade-off. Let $l(n) = c_6\sqrt{nb(n)}$ and $c_0(n) = c_7\sqrt{nb(n)}$.
2) A cellular TDMA transmission scheme is used, in which, each cell becomes active at regularly scheduled cell time-slots. By Lemma 2, these active time-slots are at most $1 + c_1$ time-slots apart.
3) Each node in the network maintains a separate FIFO queue for each of the $n/2$ S-D pairs in the network.
4) Each time-slot is divided into two equal subslots, A and B. The network operates in two phases – SR (source to relay) phase in the A subslots and RD (relay to destination) phase in the B subslots.

5) SR phase: (Source-to-Relay in the A subslots)
   a) For the SR phase, each source node maintains a counter and a state variable for every other node. Let S_{ij}^{SR} be the value of the state variable and C_{ij}^{SR} be the value of the counter at source node i for some mobile node j. The state variable S_{ij}^{SR} is binary valued and is used to determine whether node i should send packets to node j or not. Each counter is initially at count 0 and is operated as follows:
      i) If $C_{ij}^{\mathrm{SR}} = 0$ and the step-distance between nodes i and j is greater than $l(n)$ then set $C_{ij}^{\mathrm{SR}} = -1$.
      ii) If $C_{ij}^{\mathrm{SR}} = -1$ and the step-distance between nodes i and j is no greater than $l(n)$ then set $C_{ij}^{\mathrm{SR}} = c_0(n)$ and with probability $p_0, 0 < p_0 < 1$ set $S_{ij}^{\mathrm{SR}} = 1$ otherwise reset to $S_{ij}^{\mathrm{SR}} = 0$.
      iii) In each A subslot of the SR phase, the counter decrements by one until it reaches 0.
   b) In the SR phase when a cell becomes active, every source node i in the cell sends a packet intended for its destination to every other node j in the network for which $C_{ij}^{\mathrm{SR}} > 0$ and $S_{ij}^{\mathrm{SR}} = 1$. These nodes act as relay nodes for this source node. These packets reach these relay nodes using the transport mechanism of Scheme 3(a) during the A subslots of the SR phase.
6) RD phase: (Relay-to-Destination in the B subslots)
   a) For the RD phase, each mobile node i maintains a counter for every destination node j denoted by C_{ij}^{RD}. They are initially set to 0 and operated in the same way as the counters for the SR phase as follows: That is, each C_{ij}^{RD} is set to 0,-1 or $c_0(n)$ based on the step-distance between nodes i and j and the previous value of the counter. Each counter is initially at count 0 and is operated as follows:
      i) If $C_{ij}^{\mathrm{RD}} = 0$ and the step-distance between nodes i and j is greater than $l(n)$, set $C_{ij}^{\mathrm{RD}} = -1$.
      ii) If $C_{ij}^{\mathrm{RD}} = -1$ and the step-distance between nodes i and j is no greater than $l(n)$, set $C_{ij}^{\mathrm{RD}} = c_0(n)$.
      iii) In each B subslot of the SR phase, the counter decrements by one until it reaches 0.
   b) In the RD phase when a cell becomes active, every node i in the cell sends a packet to every other destination node j in the network, for which $C_{ij}^{\mathrm{RD}} > 0$, if it has a packet intended for that destination node. That is, if the FIFO queue corresponding to a destination is not empty, then a packet for that destination is emptied out of the queue. These packets reach their respective destinations using the transport mechanism of Scheme 3(a) during the B subslots constituting the RD phase.
7) Packet size scales as $\Theta\left(1/\sqrt{n^3 b(n)^3 \log n}\right)$ and hence in an active time-slot, a cell can transmit $\Theta\left(\sqrt{n^3 b(n)^3 \log n}\right)$ packets. If in any time-slot, a cell has more packets than it can transmit then the excess packets are dropped.
8) Packets transmitted to a cell not containing any node are dropped.

As shown in Figure 7, in Scheme 3(b), a source node S sends a packet intended for its destination to a mobile relay node R, which is no farther than $l(n)$ hops initially. It continues to do so for $c_0(n)$ time slots during which it is improbable that S and R get too far (i.e. farther than $\Theta(l(n))$ step-distance) from each other due to the random walk mobility model. These packets are sent using the chasing strategy of Scheme 3(a) by hops along adjacent cells of size $\Theta(\log n/n)$. This mobile relay R, in turn, sends the packet to the destination D when R and D get sufficiently close. Sending with probability $p_0 < 1$ ensures that arrival rate to each relay node is less than the service rate so that the queues at the mobile relay nodes are stable.

Theorem 9: Scheme 3(b) provides the following throughput and delay.

(i)

$$T(n) = \Theta\left(\frac{1}{\sqrt{nb(n)\log n}}\right),$$

(ii)

$$D(n) = O\left(n \log\left(\frac{1}{b(n)}\right)\right),$$

where the parameter satisfies $b(n) = \Omega(\log n/n)$ and $b(n) = O(1)$. That is, the achievable throughput-delay trade-off for Scheme 3(b) is

$$T(n) = \Theta\left(\frac{D(n)}{n}\right).$$

We first analyze the throughput of this scheme and then its delay. The throughput analysis requires extensions of the techniques used in the proof of Theorem 8.

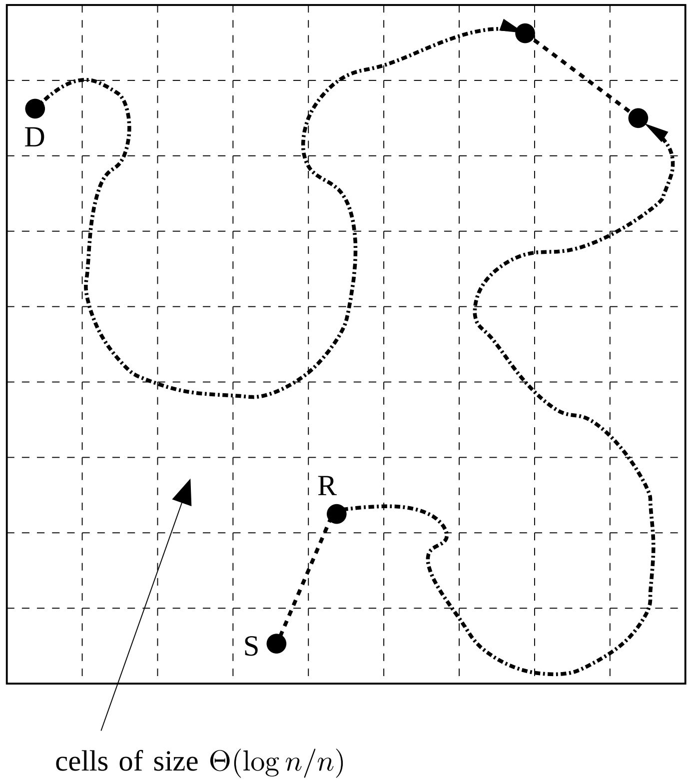


Fig. 7. Scheme 3(b) is a cellular TDM scheme like Scheme 3(a). All other mobile nodes act as relays for each S-D pair as in Scheme 2. Sources send packets to relays and relays send packets to destinations only when they are sufficiently close using the chasing technique of Scheme 3(a).

Proof of Theorem 9(i): The throughput analysis involves showing that (i) if no packets are dropped then the throughput is as claimed, and that (ii) the fraction of packets dropped by the underlying packet transport mechanism of Scheme 3(a) is negligible.

First note that the traffic in the SR and RD phases are similar and hence it is sufficient to analyze just the SR phase. Let $C_{ij}(t)$ denote the value of the counter at source node i for some mobile node j at time t. Let $X_{ij}(t)$ be the indicator of the event that $C_{ij}(t)$ is positive, i.e., $X_{ij}(t) = 1$ if $C_{ij}(t) > 0$ and zero otherwise. For simplicity, assume that each cell becomes active every time-slot (instead of once every $1 + c_1$ time slots) and whenever $X_{ij}(t) = 1$, source i transmits a new packet to a relay j (instead of also considering the state variable $S_{ij}(t)$ which is positive with probability p_0). It is easy to see that these assumptions do not affect the results in the order notation.

Now let $\{\tilde{T}_k\}$ be the intervals between consecutive transitions of $X_{ij}(t)$ from 0 to 1 (see Figure 8). Then $\{\tilde{T}_k\}$ is an i.i.d. sequence due to the independent random walks of the nodes. Let $\tilde{T}$ be a random variable with the common distribution of these random variables. As shown in Lemma 11

$$E[\tilde{T}] = \Theta\left(\sqrt{n/b(n)}\right).$$

The random walk of nodes i and j on the discrete torus is a stationary and ergodic process and hence the process $X_{ij}(t)$ derived from it is also stationary and ergodic. Hence *w.p. 1*,

$$\lim_{s\to\infty} \sum_{t=1}^{s} X_{ij}(t)/s = E[X_{ij}(t)].$$

The quantity above is the rate at which packets are sent from source node i to relay node j in t time-slots.

Now each time $X_{ij}(t)$ makes a transition from 0 to 1, it remains at 1 for $c_0(n)$ time-slots. Hence using the random variable version of the elementary renewal theorem (see [19]), it is easy to see that *w.p. 1*,

$$\begin{aligned}
\lim_{s\to\infty}\sum_{t=1}^{s} X_{ij}(t)/s &= c_0(n)/E[\tilde{T}] \\
&= \Theta\left(\sqrt{nb(n)}\frac{1}{\sqrt{n/b(n)}}\right) \\
&= \Theta(b(n)). \qquad (24)
\end{aligned}$$

Hence we have

$$P\{X_{ij}(t)=1\} = E[X_{ij}(t)] = \Theta(b(n)). \qquad (25)$$

Each source node uses the other $n-2$ nodes as relays. By considering all these $n-2$ relays and the direct path, it follows from (24) that if no packets are dropped then the throughput for each S-D pair in terms of packets is

$$\lim_{s\to\infty}\sum_{t=1}^{s}\sum_{j\neq i} X_{ij}(t)/s = \Theta(nb(n)).$$

Now since the packet size is $\Theta\left(1/\left(n^{3/2}b(n)^{3/2}\sqrt{\log n}\right)\right)$ it is immediate that if no packets are dropped then the throughput per S-D pair is $\Theta\left(1/\sqrt{nb(n)\log n}\right)$ as claimed. The rest of the proof shows that the fraction of packets dropped goes to zero. In Scheme 3(b), as in Scheme 3(a), packets are dropped due to (i) overloading in cells, (ii) transmission to empty cells, and (iii) unsuccessful chasing. Scheme 3(b) uses cells of size $\Theta(\log n/n)$ and hence as shown in the proof of Scheme 3(a), the fraction of packets being dropped due to (ii) and (iii) goes to 0. To establish (i), it is sufficient to show that the number of packets passing through each cell is $O\left(\sqrt{n^3 b(n)^3\log n}\right)$ *whp*, which we do next.

Next let $D_{ij}(t)$ be the distance in steps from node i to node j. If $X_{ij}(t)$ makes a transition from 0 to 1 at time τ then we know that $D_{ij}(\tau) = l(n)$. After this $X_{ij}(t)$ stays at 1 for $c_0(n)$ time-slots. Since nodes i and j are moving according to independent random walks, the distance between them increases by at most two steps in each time-slot. Therefore, $D_{ij}(t) \le l(n) + c_0(n) = O(l(n))$ for $\tau \le t \le \tau + c_0(n)$. Thus we have

$$D_{ij}(t) = O(l(n)) \;\text{ given }\; X_{ij}(t) = 1. \qquad (26)$$

Now let $L_{ij}(t)$ be the distance in hops between nodes i and j if $X_{ij}(t) = 1$ and 0 otherwise. Then $L_{ij}(t) = O(D_{ij}/\sqrt{\log n}))$ since the cells are of size $\log n/n$ and so

$$\begin{aligned}
&E[L_{ij}(t)] \\
&= E[L_{ij}(t)|X_{ij}(t)=1]P\{X_{ij}(t)=1\} \\
&= E[L_{ij}(t)|X_{ij}(t)=1]\Theta(b(n)) \qquad (27) \\
&= O(E[D_{ij}(t)|X_{ij}(t)=1]b(n)/\sqrt{\log n}) \qquad (28) \\
&= O(l(n)b(n)/\sqrt{\log n}) \\
&= O(b(n)^{3/2}\sqrt{n/\log n}), \qquad (29)
\end{aligned}$$

where (27) is due to (25) and (28) follows from (26).

For exactly the same reasons as those pointed out while analyzing the traffic of Scheme 3(a), the traffic in stage 1 of Scheme 3(b) dominates the traffic of all other stages. Hence we will only analyze traffic of stage 1 here. Now let $Y_{ij}^c(t)$ be the number of packets in stage 1 of source i to node j passing through cell c at time t. As shown during the analysis of traffic in stage 1 of Scheme 3(a), we have $Y_{ij}^c(t) \in \{0,1,2\}$.

Now a packet sent out from source i at time $t-k$ passes through some cell in the network at time t if and only if $L_{ij}(t-k) > k$. There are $m = n/\log n$ cells in the network and summing over all these cells we obtain

$$\sum_{c=1}^{m} Y_{ij}^c(t) = \sum_{k=0}^{\infty} I\{L_{ij}(t-k) > k\},$$

where $I(A)$ is the indicator function of the event A. Taking expectations on both sides, using the symmetry of the cells and 29, this gives

$$\begin{aligned} mE[Y_{ij}^c(t)] &= \sum_{k=0}^{\infty} P\{L_{ij}(t-k) > k\} \\ &= \sum_{k=0}^{\infty} P\{L_{ij}(t) \geq k\} \\ &= E[L_{ij}(t)] \\ &= O(b(n)^{3/2}\sqrt{n/\log n}). \end{aligned}$$

Hence we have

$$E[Y_{ij}^c(t)] = O\left(b(n)^{3/2}\sqrt{\log n/n}\right). \tag{30}$$

The total number of packets passing through cell c in time-slot t is given by $Y = \sum_i \sum_{j\neq i} Y_{ij}^c(t)$. The total number of distinct source-relay pairs is $\Theta(n^2)$. For each such pair, by symmetry and (30), $E[Y_{ij}^c(t)] = O\left(b(n)^{3/2}\sqrt{\log n/n}\right)$. Hence the expected number of packets passing through cell c in time-slot t is

$$E[Y] = \Theta\left(n^2 E[Y_{SR}^C(t)]\right) = O\left(\sqrt{n^3 b(n)^3 \log n}\right). \tag{31}$$

Thus it is sufficient to show that $Y = \Theta\left(E[Y^C(t)]\right)$ *whp* in order to establish that the number of packets through each cell is $O\left(\sqrt{n^3 b(n)^3 \log n}\right)$ *whp*. Next, we establish this.

Note that terms of the form $Y_{ii}^c(t)$ are always 0 and terms of the form $Y_{ij}^c(t)$ and $Y_{ik}^c(t)$ for $j \neq k$ are independent. So are terms of the form $Y_{ij}^c(t)$ and $Y_{kj}^c(t)$ and terms where all indices are different. Another important property which we use crucially is that if nodes j and k are not in cell c and $Y_{ij}^c = Y_{ik}^c = 1$ then $Y_{jk}^c(t)$ is necessarily 0. This happens because $Y_{ij}^c(t) = Y_{ik}^c(t) = 1$ implies that nodes j and k lie on the same side of cell c under consideration and hence the line connecting j and k does not pass through cell c. The same holds when $Y_{ji}^c = Y_{ki}^c = 1$. This is not true however if either of the nodes j and k lies in cell c. However this can be handled by dealing with the first and last hops separately which is easy since it depends only on the distribution of nodes themselves. Hence we ignore this aspect for the sake of simplicity.

Now fix a cell c and a time-slot t and define $Z_{ij} = 1$ if $Y_{ij}^c(t) > 0$ and 0 otherwise. Then $Y_{ij}(t) \leq 2Z_{ij}$ and hence

$$\begin{aligned} Y &\leq 2\sum_{i=1}^{n/2}\sum_{j=1}^{n} Z_{ij} \\ &= \sum_{i=1}^{n/2}\sum_{j=i+1}^{n} Z_{ij} + \sum_{i=2}^{n/2}\sum_{j=1}^{i-1} Z_{ij}. \end{aligned}$$

In what follows, we will show that the first sum above is of the same order as its expected value *whp*. The same technique would show that the same is true for the second sum also *whp* and hence that the total traffic Y is of the same order as its expected value *whp*.

So consider the $M = \Theta(n^2)$ terms in the first sum, i.e., $\{Z_{ij}, 1 \leq i \leq n/2, i < j \leq n\}$. These are identically distributed Bernoulli random variables with $p_n = P\{Z_{ij} = 1\} = O\left(b(n)^{3/2}\sqrt{\log n/n}\right)$. The dependence properties of $\{Y_{ij}(t)\}$ carry over to $\{Z_{ij}\}$ so that, for example, $Z_{ij} = Z_{ik} = 1$ implies that $Z_{jk} = Z_{kj} = 0$. For simplicity we rewrite $\{Z_{ij}, 1 \leq i \leq n/2, i < j \leq n\}$ as $\{Z_i, 1 \leq i \leq M\}$. Now let $\{\tilde{Z}_i, 1 \leq i \leq M\}$ be i.i.d. Ber(p_n) random variables and let $Z = \sum_{i=1}^{M} Z_i$ and $\tilde{Z} = \sum_{i=1}^{M} \tilde{Z}_i$. Now

$$\begin{aligned} E[Z^k] &= \sum_{(i_1,\ldots,i_k)\in\{1,\ldots,M\}^k} E\left[Z_{i_1},\ldots,Z_{i_k}\right] \\ &= \sum_{(i_1,\ldots,i_k)\in\{1,\ldots,M\}^k} P\{Z_{i_1} = \ldots = Z_{i_k} = 1\}. \end{aligned}$$

Similarly, we can write $E[\tilde{Z}^k]$ as the sum of terms of the form $P\{\tilde{Z}_{i_1} = \ldots = \tilde{Z}_{i_k} = 1\}$. Consider a term of the form $P\{Z_{i_1} = \ldots = Z_{i_k} = 1\}$. If all the Z_is contained in this term are independent then this is the same as the corresponding term of $E[\tilde{Z}^k]$. When there is a dependence, the term becomes zero due to the particular nature of dependence as mentioned above, due to which for some distinct indices i, j, k, $Z_i = Z_j = 1$ implies $Z_k = 0$. However, for $E[\tilde{Z}^k]$ the corresponding term is still non-zero. Hence $E[Z^k] \leq E[\tilde{Z}^k]$ for all $k \geq 0$. As a result, for any $t > 0$,

$$E[\exp(tZ)] \leq E[\exp(t\tilde{Z})]. \tag{32}$$

So we can write for $t > 0$,

$$\begin{aligned} P\{Z > (1+\delta)E[Z]\} &\leq \frac{E[\exp(tZ)]}{\exp((1+\delta)tZ)} \\ &\leq \frac{E[\exp(t\tilde{Z})]}{\exp((1+\delta)t\tilde{Z})} \\ &\leq \exp(-E[\tilde{Z}]\delta^2/2), \end{aligned}$$

using the Chernoff bounding technique for the sum of i.i.d Bernoulli random variables. Now $E[\tilde{Z}] = O(Mp_n) = O\left(n^{3/2}b(n)^{3/2}\sqrt{\log n}\right)$. Choosing an appropriate δ we have $Z = O(E[Z])$ *whp*. Hence the total number of packets through a cell is $O\left(n^{3/2}b(n)^{3/2}\sqrt{\log n}\right)$ *whp*. Using the union bound over the $n/\log n$ cells establishes the claim about the throughput of Scheme 3(b). ■

Next we analyze the delay for Scheme 3(b). The delay of a packet is determined by the queueing delay at a relay node. We first upper bound this queueing delay by that of a GI/GI/1 queue and then use Kingman's upper bound for the GI/GI/1 queue. To use this upper bound, we require to compute the first two moments of the inter-arrival times and the service times. For Scheme 3(b), these moments are related to the corresponding moments of the hitting times of subsets of the torus.

Proof of Theorem 9(ii): Two types of relaying are used in Scheme 3(b). First, there is relaying by hops along adjacent cells of size of $\Theta(\log n/n)$ when a packet is sent from a source to a mobile relay node which we call relaying by hops. Second, the mobile relay node carries the packet until it gets near the destination for further hop-relaying. Thus the total delay experienced by a packet in moving from its source S to its destination D via a mobile relay R involves two types of delay - (i) the hop-delay $D_h(n)$, which is the delay in relaying by hops from S to R and then from R to D, and (ii) the queueing delay $D_q(n)$ in the queue at R for that S-D pair. Only packets that reach from S to R directly (i.e., without an intermediary mobile relay node) are not subject to queuing delay, which form a negligible fraction.

First we determine $D_h(n)$, the delay due to relaying by hops. Consider a packet that is relayed by hops from its source S to a mobile relay node R or its destination. The counters in Scheme 3(b) ensure that this process starts only when S and R are initially within a step-distance of $l(n)$ hops. Thereafter due to the random walk model for mobility of nodes, the average distance between S and R monotonically increases with time and at time $c_0(n)$ the average distance in terms of number of steps is less than $l(n) + \Theta(\sqrt{c_0(n)}) = \Theta(l(n))$ since $l(n) = \Theta(c_0(n))$. Hence from the analysis of Scheme 3(a) it can be seen that the delay due to relaying by hops along cells of size $\Theta(\log n/n)$ is

$$D_h(n) = \Theta\left(l(n)/\sqrt{\log n}\right) = \Theta\left(\sqrt{nb(n)/\log n}\right). \tag{33}$$

Now we proceed to determine $D_q(n)$, the queueing delay. Since the underlying packet transport mechanism is that of Scheme 3(a), packets are dropped in the network. However packets are dropped during the relaying by hops and hence does not affect the queuing delay at the mobile relay nodes.

For any S-D pair the delay at each mobile relay node is the same and by symmetry each S-D pair has the same delay. Hence we only need to compute the queueing delay for any one such queue. So fix an S-D pair and a mobile relay node R and let the queue at this relay node be called $\mathcal{Q}_1$. First consider the arrival process to this queue which is depicted in Figure 8. The solid line in in the figure is non-zero when S and R when S and R are at a distance no more than $l(n)$ hops. The counter is set to $c_0(n)$ the first time this happens and the dashed line is non-zero when the counter is positive. Packets are sent from S to R during this period when the counter is positive if the corresponding state variable is 1. Note that some of the arriving packets may be dropped due to the way

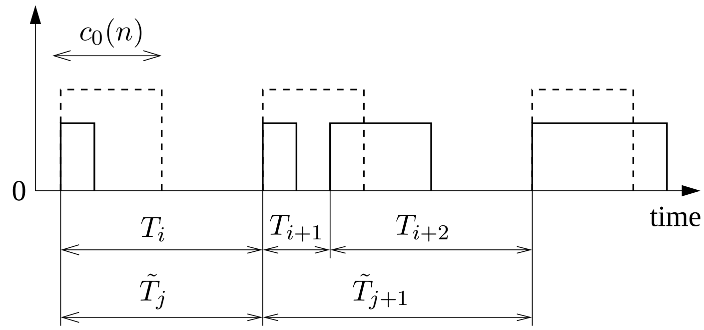


Fig. 8. The solid line is non-zero when S and R are within a distance of $l(n)$ hops of each other. The dashed line is non-zero when the counter at S for R is positive. Packets are sent from S to R when the dashed line is non-zero if the state variable is 1 as explained in the description of Scheme 3(b).

Scheme 3(a) operates. However by considering a queue $\mathcal{Q}_2$ in which these packet drops are ignored we obtain an upper bound on the queueing delay in $\mathcal{Q}_1$.

Packets arrive at R with a delay of $D_h(n)$ due to hopping. However these packets arrive in order (although some of the packets may arrive together) because of the way Scheme 3(a) works. Now the step-distance between S and R when any packet departs is at most $l(n) + c_0(n)$ and so $D_h(n) \leq (c_6 + c_7)\sqrt{nb(n)/\log n} = D_h^{max}$ (say). So if we consider a queue in which each packet arrives exactly D_h^{max} time-slots after its departure from S, then the sum of the average delay of this queue and D_h^{max} is an upper bound to the delay in $\mathcal{Q}_2$. Now as will be shown later (and as claimed in the Theorem), it turns out the the queueing delay is of an order strictly greater than that of D_h^{max} hence it does not matter in the order of the average delay of $\mathcal{Q}_2$. Moreover the arrival and service processes are jointly stationary since they are based on the motion of nodes S, R and D. Hence in order to determine an upper bound of the same order as the actual delay we can instead consider a queue, say $\mathcal{Q}_3$, in which arrivals occur when the counter is positive and the state variable is 1.

Let $\tilde{T}$ be a random variable denoting the time interval between two instants when the counter is set to $c_0(n)$ consecutively. Then $\{\tilde{T}_j\}$ in Figure 8 is a sequence of i.i.d. random variables each with the same distribution as $\tilde{T}$. Also let T be a random variable denoting the time interval between two slots when the distance between S and R decreases to $l(n)$ from greater than $l(n)$ consecutively. Then $\{T_i\}$ in Figure 8 is a sequence of i.i.d. random variables each with the same distribution as T.

The departure process is similar and departures occur when the corresponding counter is positive if the queue is not already empty. The delay for this queue is the same as that for $\mathcal{Q}_4$, a queue in which a single arrival occurs at the start of each period of length $c_0(n)$ of arrivals in $\mathcal{Q}_3$ and a departure occurs at the beginning of each period of $c_0(n)$ departures in $\mathcal{Q}_3$. The inter-arrival times in $\mathcal{Q}_4$ form an i.i.d. process and the distribution of the inter-arrival time is the same as the sum of K independent copies of $\tilde{T}$ where K is an independent Geometric random variable with parameter p_0. Hence the first two moments of the inter-arrival time in $\mathcal{Q}_4$ are the same as those of $\tilde{T}$ which is the time interval between two instants when the counter is set to $c_0(n)$ consecutively. Similarly the inter-service times are also i.i.d. and hence as done in the delay analysis of Scheme 2, we can upper bound the delay of $\mathcal{Q}_4$ by that of a GI/GI/1 queue $\mathcal{Q}_5$ whose service time distribution is the same as the inter-service time distribution of $\mathcal{Q}_4$. Further using Kingman's upper bound for the GI/GI/1 queue as in the delay analysis of Scheme 2 and the fact that the moments of the inter-arrival time and service time in $\mathcal{Q}_5$ are of the same order, we can write

$$D_q(n) = O\left(E[\tilde{T}^2]/E[\tilde{T}]\right). \tag{34}$$

Substituting the moments of $\tilde{T}$ from Lemma 11 which is presented after this proof, we obtain

$$D_q(n) = O(n\log(1/b(n))). \tag{35}$$

As a result the delay for Scheme 3(b) scales as

$$D(n) = D_h(n) + D_q(n) = O(n\log(1/b(n))).$$

■

Remark: At the choice of $b(n) = \Theta(1)$ the performance of the scheme is vastly different for $b(n) = 1$ and $b(n) \neq 1$. This discontinuity of trade-off is the consequence of using mobility since $b(n) = 1$ means that the mode of operation is that of Scheme 3(a) where mobility is not used to move the packets toward their destinations. The delay jumps up immediately as mobility is used with $b(n) \neq 1$.

Lemma 11: For $\tilde{T}$, as defined earlier,

$$E[\tilde{T}] = \Theta\left(\sqrt{\frac{n}{b(n)}}\right), \quad \text{and}$$

$$E[\tilde{T}^2] = O\left(\log\left(\frac{1}{b(n)}\right)\sqrt{\frac{n^3}{b(n)}}\right).$$

Proof: Consider the counter at a source node for some other node. Recall that $\tilde{T}$ is a random variable denoting the time interval between two consecutive transitions of the counter from -1 to $c_0(n)$. For simplicity, assume that each cell becomes active every time-slot (instead of once in $1 + c_1$ time slots). Then by definition, $\tilde{T} \geq c_0(n)$. Let T be a random variable denoting the time interval between two slots when the distance between S and R decreases to $l(n)$ from greater than $l(n)$ consecutively. Then $\{T_i\}$ in Figure 8 is a sequence of i.i.d. random variables each with the same distribution as T.

Consider the random variable $K = \inf\{k : \; T_1 + \cdots + T_k > c_0(n)\}$, where T_i are i.i.d. with the distribution of T. Then, by definition

$$\tilde{T} \;=\; \sum_{k=1}^{K} T_k. \tag{36}$$

From definition of K, $\sum_{k=1}^{K-1} T_k \leq c_0(n)$. Hence, from (36)

$$\tilde{T} > T_K \;\; \text{and} \;\; \tilde{T} \leq c_0(n) + T_K. \tag{37}$$

This implies that,

$$E[T] \;\leq\; E[\tilde{T}] \;\leq\; c_0(n) + E[T], \tag{38}$$

and

$$E[\tilde{T}^2] \;\leq\; c_0^2(n) + c_0(n)E[T] + E[T^2]. \tag{39}$$

Next we compute $E[T], E[T^2]$ to determine $E[\tilde{T}], E[\tilde{T}^2]$. In the random walk motion model, each node moves according to a simple random walk on the discrete $\sqrt{n} \times \sqrt{n}$ torus. Let $X(t) \in \{(i,j) : 0 \leq i,j \leq \sqrt{n} - 1\}$ be such a random walk on the $\sqrt{n} \times \sqrt{n}$ torus. Since T is determined by the independent random walks of S and R on the torus, equivalently we can study it using a difference random walk of a single node as was done in the analysis of delay for Scheme 2.

Now let A be the set of cells of the torus which are at a distance no greater than $l(n)$ from $(0,0)$, i.e.,

$$A = \{(i,j) : d((i,j),(0,0)) \leq l(n)\}.$$

And let ∂A be the set of cells of the torus which are exactly at distance $l(n)$ from $(0,0)$, i.e.,

$$\partial A = \{(i,j) : d((i,j),(0,0)) = l(n)\}.$$

Then T as defined above is the time taken by a node performing a difference random walk to perform another transition from A^c to A starting from such a transition. Since we are interested only in the exact order of the moments, we can consider the simple random walk instead of the difference random walk on the discrete torus. Hence to determine $E[T]$ and $E[T^2]$ we can redefine T as follows.

Let π denote the stationary distribution for the random walk on the torus which we know is uniform and let for a set B let $\pi(B) = \sum_{v \in B} \pi(v)$ be the probability of the set B under π. By the probability distribution π_B, we mean $\pi_B(v) = \pi(v)/\pi(B)$ if $v \in B$ and zero otherwise.

Now instead of the random walk $\{X(t)\}$, consider a Markov chain given by $Z(t) = (X(t-1), X(t))$. The state space of $\{Z(t)\}$ is clearly the set of directed edges of the torus. Let ∂A^+ be the set of edges of the torus directed from A to A^c and let ∂A^- be the set of edges of the torus directed form A^c to A. For this new Markov chain $\{Z_t\}$, T is the first return time to the set ∂A^-, i.e.,

$$T = \inf\{t \geq 1 : Z(t) \in \partial A^-\},$$

starting from $Z(0) \sim \pi_{\partial A^-}$.

Before proceeding to compute the first two moments of T, note that there are $4n$ states in the state space corresponding to the 4 directed edges emanating from each vertex of the torus. Moreover the stationary distribution of $\{Z(t)\}$ is also uniform. This can be easily verified (e.g. see the proof of Lemma 6.5 in [17] or the proof of Lemma 7 in Chapter 3 of [1]). Also note that the number of states in A is $4l(n) = 4c_6\sqrt{nb(n)}$. Now using Kac's formula, we obtain

$$\begin{aligned} E[T] = &= E_{\pi_{\partial A^+}} T \\ &= 1/\pi(\partial A^+) \qquad (40) \\ &= 4n/c_8\sqrt{nb(n)} \\ &= c_8\sqrt{n/b(n)}. \qquad (41) \end{aligned}$$

Now using equation (21) in Chapter 2 of [1], we obtain

$$\begin{aligned} E[T^2] &= E_{\pi_{\partial A^+}}[T^2] \\ &= \left(2E_\pi T + 1\right)/\pi(\partial A^+) \\ &= \left(2E_\pi T + 1\right)E[T], \qquad (42) \end{aligned}$$

where the last equality follows from (40).

Let $\mathcal{E}$ be the set of all directed edges of the torus and let $\mathcal{E}_A$ be the set of directed edges that are between vertices in A. Consider the following two possible cases, based on the starting position of $Z(0)$, to compute $E_\pi[T]$.

Case 1. Suppose $Z(0) = e \in \mathcal{E} - (\mathcal{E}_A \cup \partial A^-) \stackrel{\triangle}{=} \mathcal{E}_1$. Under this situation for $Z(\cdot)$ to visit any edge in ∂A^-, the original random walk needs to enter A starting from a node in A^c chosen with the uniform distribution restricted to A^c. Thus it is the same as T_A, the hitting time of set A of vertices, starting with initial distribution $\pi(A^c)$. Using Lemma 12 following this proof, it follows that

$$E_{\pi_{A^c}} T_A = O(n \log(1/b(n))).$$

Thus,

$$E_{\pi_{\mathcal{E}_1}}[T] = O(n \log(1/b(n))). \qquad (43)$$

Case 2. Suppose $Z(0) = e \in \mathcal{E}_A \cup \partial A^- \stackrel{\triangle}{=} \mathcal{E}_2$. Under this starting condition, the original random walk starts inside the set A. Hence visiting edge of ∂A^- requires that the original random walk first get out of the set A, and then visit ∂A^- given that $Z(\cdot)$ is in $\mathcal{E} - (\mathcal{E}_A \cup \partial A^-)(= \mathcal{E}_1)$. From basic first passage time results for one-dimensional random walks, it is easy to see that the expected time to get out of set A starting from any position inside A is $O\left(l(n)^2\right)$. Using this and *Case 1*, we obtain

$$\begin{aligned} E_{\pi_{\mathcal{E}_2}}[T] &\leq E_{\pi_{\mathcal{E}_1}}[T] + O(l(n)^2) \\ &= O\left(n \log(1/b(n)\right) + O\left(l(n)^2\right) \\ &= O\left(n \log(1/b(n)\right). \qquad (44) \end{aligned}$$

From Case 1 and Case 2, using (43) and (44), we obtain

$$E_\pi T = O(n \log(1/b(n))). \qquad (45)$$

Finally combining (42) and (45),

$$E[T^2] = O\left(\log(1/b(n))\sqrt{\frac{n^3}{b(n)}}\right). \qquad (46)$$

■

The following lemma used in the above proof is proved in Appendix III.

Lemma 12: Consider the subset $A = \{(i,j) : 0 \leq i,j < \sqrt{m}\}$ on a two-dimensional $\sqrt{n} \times \sqrt{n}$ discrete torus. That is, A is a square set of $\sqrt{m} \times \sqrt{m}$ cells of the discrete torus. Let T_A be the hitting time of A then

$$E_{\pi_{A^c}} T_A = O\left(n \log\left(\frac{n}{m}\right)\right).$$

C. Optimality of Scheme 3

In this section, we establish the optimality of Scheme 3 under the random walk (RW) model. Consider any communication scheme operating under the RW model. Then the distance traveled by a packet between its source and destination is the sum of the distances traveled by hops and the total distance traveled by the mobile relays that are used by the packet under this scheme. Let $\bar{l}(n)$ be the sample mean of the distance traveled by hops (i.e., by wireless transmission) and let $\bar{r}(n)$ be the sample mean of the distance traveled by a packet per hop. In the following lemma, proved in Appendix III, we obtain a bound on throughput scaling as a function of $\bar{l}(n)$ and $\bar{r}(n)$ using a technique similar to the one used in Theorem 5. We then show that to achieve this optimal throughput, the minimum delay incurred is of the same order as the delay of Scheme 3, which will establish the optimality of Scheme 3.

Lemma 13: Consider any scheme such that the sample mean of the distance traveled by hops is $\bar{l}(n)$ and the sample mean of the distance traveled per hop is $\bar{r}(n)$. Then its achievable throughput,

$$T(n) = O\left(\frac{1}{n\bar{l}(n)\bar{r}(n)}\right). \tag{47}$$

The above lemma is proved in Appendix III. Next we state a result regarding the mean hitting time of a subset of cells for a random walk on the $\sqrt{n} \times \sqrt{n}$ discrete torus. It is a consequence of Lemma 2.1 in [4] and the strong approximation (of random walk by Brownian motion) results as used in the proof of Theorem 1.1 in [4].

Lemma 14: Let T_r be the time to hit a set of cells of the discrete torus contained in a disk of radius $r < R/2$ around a point x starting from the boundary of a disk of radius R around x. Then for a symmetric random walk on $\sqrt{n} \times \sqrt{n}$ discrete torus,

$$E[T_r] = \Theta(n \log r^{-1}).$$

Now we are ready to prove the optimality of Schemes 3(a) and 3(b) using Lemma 13 and Lemma 14.

Optimality of Scheme 3(a): Consider any communication scheme that uses cellular transmission and possibly utilizes node mobility to achieve the optimal throughput-delay trade-off for $T(n) = O(1/\sqrt{n \log n})$. Let $\bar{l}(n)$ be the sample mean of the distance traveled by hops under this scheme. Let $\bar{r}(n)$ be the sample mean of the hop distance, then by Lemma 13, $T(n) = O\left(1/\left(n\bar{l}(n)\bar{r}(n)\right)\right)$. For this scheme, the average delay due to hops alone, i.e., the time taken to travel by hops is $\Theta\left(\bar{l}(n)/\bar{r}(n)\right)$.

Now, if $\bar{l}(n) = \Theta(1)$, $T(n) = O\left(1/\left(n\bar{r}(n)\right)\right)$ and $D(n) = \Omega(1/\bar{r}(n))$. That is, $T(n) = O\left(D(n)/n\right)$. Hence from Theorem 8, among all schemes that have $\bar{l}(n) = \Theta(1)$, Scheme 3(a) is optimal. As a consequence, if Scheme 3(a) is not optimal then it must be that $\bar{l}(n) = o(1)$ for the optimal scheme.

If $\bar{l}(n) = o(1)$, then at least a constant fraction of the packets must travel a distance of $\Theta(1)$ using the node mobility. From Lemma 14, it follows that for such packets, the delay is $\Omega(n)$. But then, $D(n) = \Omega(n) = \omega(nT(n))$ since $T(n) = o(1)$. Hence, Scheme 3(a) is optimal among all schemes with $\bar{l}(n) = o(1)$.

This shows that Scheme 3(a) provides the optimal throughput-delay trade-off as far as the scaling is concerned.

Optimality of Scheme 3(b): Consider an optimal communication scheme that uses cellular transmission, possibly along with the mobility of nodes, to achieve the optimal throughput-delay trade-off for $T(n) = \omega(1/\sqrt{n \log n})$. Let $\bar{l}(n)$ and $\bar{r}(n)$ be as defined above. By Lemma 13, $T(n) = \omega(1/\sqrt{n \log n})$ requires that $\bar{l}(n) = o(1)$. But from the preceding discussion, when $\bar{l} = o(1)$, the mobile-delay (the time spent at a mobile relay node) dominates the hop-delay (the time spent in performing hops). Thus when throughput is $\omega(1/\sqrt{n \log n})$, to maximize the throughput for a given delay any optimal scheme must have the smallest possible $\bar{r}(n)$, which is $\Theta(\sqrt{\log n / n})$. Therefore, any optimal scheme for this range of high throughput has $T(n) = \Theta\left(1/\left(\bar{l}(n)\sqrt{n \log n}\right)\right)$.

Consider a throughput-delay optimal scheme. For any such scheme, fixing a throughput $T(n)$, fixes $\bar{l}(n)$, which is the average distance traveled by hops. The goal of an optimal scheme then, is to travel this distance by hops in a manner so as to minimize the average time for a packet to reach its destination.

Consider the transmission of a packet p starting from its source S and moving toward its destination D, initially at a distance d from S. Recall that a packet travels a distance $\bar{l} = \bar{l}(n)$ through hops and the rest through the motion of the nodes relaying it. Define t_p to be the time it takes the packet p, after leaving its source S, to reach its destination D. We ignore the time required for hops as the mobile delay dominates the total delay. Let $E[t_p]$ be the expectation of t_p for a given $\bar{l}$ and d. Note that the expectation is over the distribution induced by the random walks of the nodes.

We claim the following.

Lemma 15: For any $\bar{l}$ and d, a scheme that minimizes $E[t_p]$ must perform all the hops the first time the packet is at a distance less than or equal to $\bar{l}$ from its destination D.

Proof: For $\bar{l} \geq d$, the Lemma clearly holds. Next consider the case where $\bar{l} < d$. To prove the Lemma for this case, consider the following two schemes. Scheme A uses the entire hop distance $\bar{l}$ when the packet reaches within a distance $\bar{l}$ of D for the first time, which is consistent with the claim of the lemma. Scheme B uses a hop of length ϵ when the packet is at a distance $\tilde{d}$ ($d > \tilde{d} > \bar{l}$) from D, and uses the remaining hop distance $\bar{l} - \epsilon$ at the end, as in Scheme A.

We want to show that, on average, a packet takes longer to reach D in Scheme B than in Scheme A. For simplicity, we assume that D is fixed. This does not affect generality as all nodes perform independent symmetric random walks.

Consider the path of a packet originating at distance d from D. Until the packet reaches within a distance $\tilde{d}$ of D, its path is the same in both schemes. As illustrated in Figure V-C, under Scheme B, at point X, which is at a distance $\tilde{d}$ from D, the packet travels a distance ϵ by hops toward D to reach Y. Under Scheme A, the packet remains at point X. At this instant, the remaining time for the packet to reach D under Scheme A, t_A, is the time taken to reach a ball $B(D, \bar{l})$ starting from X, and under Scheme B, it is the time t_B taken to reach $B(D, \bar{l} - \epsilon)$, starting from Y. We now show that on average $t_{\mathrm{A}} < t_{\mathrm{B}}$. Consider a point D' on the line X–D at distance ϵ from D (as depicted in Figure V-C). Since all nodes perform independent symmetric random walks, the probability that a path starting from X reaches $B(D, \bar{l})$ is the same as the probability that any path starting from Y reaches $B(D', \bar{l})$. Note that, by construction, $B(D, \bar{l} - \epsilon) \subset B(D', \bar{l})$. Hence the time for a packet at Y to reach $B(D', \bar{l})$ is stochastically dominated by the time needed to reach $B(D, \bar{l} - \epsilon)$. This proves that the time taken by Scheme A is strictly smaller than the time taken by Scheme B on average.

Using the above argument inductively for all hops establishes the Lemma. ■

The above Lemma shows that a throughput-delay optimal scheme must utilize all the hops at the end. Since in Scheme 3(b), half the hops are performed at the end, it follows that its throughput-delay trade-off is of the same order as that of an optimal scheme. The argument used above is for the case when the scheme allows only one copy of each packet in the network at any time. Clearly the scaling is unaffected by the use of $\Theta(1)$ copies of each packet.

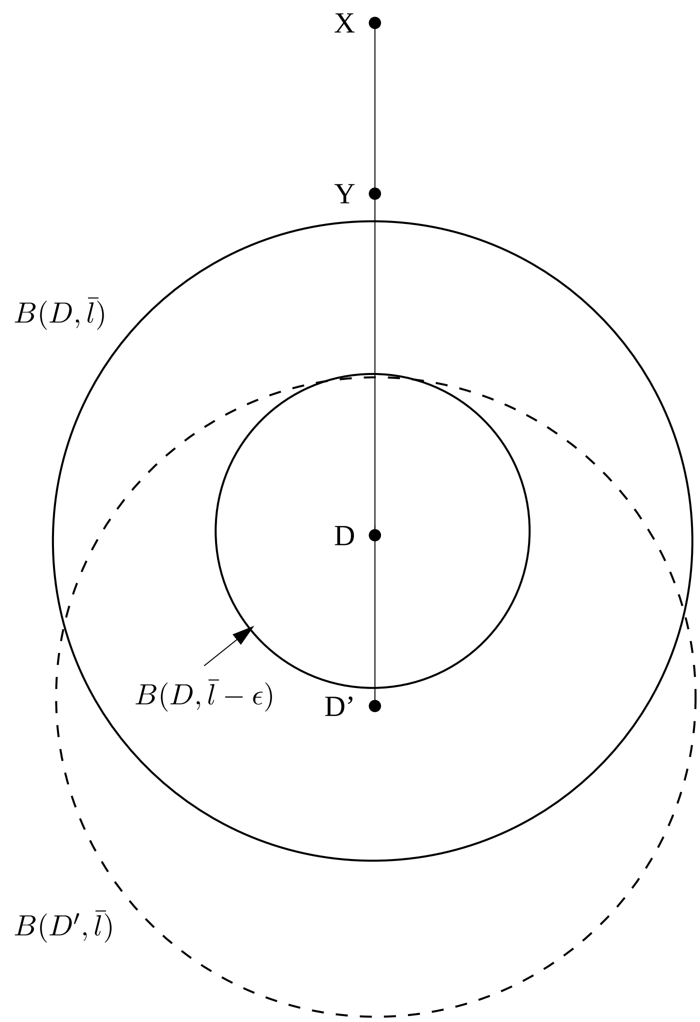
The optimality of Schemes 3(a) and 3(b) establishes the following theorem.

Theorem 10: Among all schemes such that the number of copies of any packet in the network at any time is $\Theta(1)$, Scheme 3 obtains the optimal throughput-delay trade-off for mobile networks.

VI. Conclusion

Optimal throughput scaling with the number of nodes in static and mobile wireless networks was established in [11] and [10] using a random network framework. Delay scaling, however, was not as well addressed in earlier work. This paper established the optimal trade-off between throughput and average packet delay in static and mobile wireless networks using a similar random network framework to that in [11] and [10].

For static networks, we showed that the optimal throughput-delay trade-off is given by $D(n) = \Theta(nT(n))$. For mobile networks, we presented a scheme similar to the Grossglauser-Tse scheme in [10] and showed that the delay corresponding to $\Theta(1)$ throughput scales as $\Theta(n \log n)$ when the nodes move according to independent random walks. Further, we described and analyzed a scheme that achieves the optimal throughput-delay trade-off for mobile networks by varying the number of hops, the transmission range, and the degree to which node mobility is used. For the low throughput range achieved in static networks, we found that the trade-off for mobile networks is the



same as that for static networks. For higher throughputs, there is almost no trade-off between throughput and delay – the same maximum delay is incurred regardless of the throughput. Our results utilized a unified framework for static and mobile networks and our random network model resulted in simpler proofs of previous throughput results. The framework and proofs should provide a better understanding of the essential characteristics of large wireless networks.

Several questions remain to be tackled for a better theoretical understanding of the data communication aspect of random wireless networks. With the exception of Scheme 2, the paper assumed a fluid model in which the packet size is allowed to be arbitrarily small. A priori, it is not clear what the trade-off will be with the additional constraint that packet size remains constant. In the second part of this paper [9], we show that for static networks the trade-off remains unchanged even with packets of constant size. We believe that the same should hold for mobile networks, but such a result remains to be established.

The Protocol model used in this paper and the Physical model have been used by many researchers. These models, however, do not take transmission energy into account. Modeling energy consumption and determining the trade-offs between throughput, delay and energy is an important issue with initial steps already taken by several researchers (e.g., [3], [8], [6]).

In this paper we assumed a random walk model for node mobility. One expects that the scaling results would be the same for a larger class of Markovian motion models. Determining the class of such motion models would be another future challenge.

Appendix I

Proof of Lemma 1: Let A_i be the event that cell i is not empty and let $m = 1/a(n)$ be the number of cells. Then

$$P(A_i) = 1 - (1 - 1/m)^n \to 1 - e^{-n/m}.$$

With $m \leq n/2 \log n$, it follows that $P(A_i) \geq 1 - 1/n^2$ and hence an application of the union bound completes the proof. ■

Proof of Lemma 2: Consider a node in a cell transmitting to another node within the same cell or in one of its 8 neighboring cells. Since each cell has area $a(n)$, the distance between the transmitting and receiving nodes cannot be more than $r = \sqrt{8a(n)}$. Under the Relaxed Protocol model, data is successfully received if no node within distance $\bar{r} = (1 + \Delta)r$ of the receiver transmits at the same time. Therefore, the number of interfering cells, c_1, is at most

$$c_1 \leq 2\frac{\bar{r}^2}{a(n)} = 16(1 + \Delta)^2,$$

which, for a constant Δ, is a constant, independent of n (and $a(n)$). ■

Appendix II

Proof of Lemma 5: Let the cells on the torus be numbered 1 to n. Let $X^i(t)$ be the cell in which node i resides at time t for $i = 1, \ldots, n$. Without loss of generality let node 1 be the relay node and node 2 be the destination node. Now $X(t) = (X^3(t), \ldots, X^n(t))$ is a Markov chain formed by the independent random walks of the $n-2$ nodes other than the relay and the destination on the two-dimensional torus. The corresponding equilibrium distribution is independent and uniform, i.e., each node is equally likely to be in any of the n cells. This implies that at time α_0, $X^1(\alpha_0) = X^2(\alpha_0) = U_0$ where U_0 is uniformly distributed on $\{1, \ldots, n\}$. And nodes 3 to n are also distributed independently and uniformly over the n cells. Due to symmetry on the torus, by arguing inductively, it can be seen that for $k = 0, 1, \ldots, X^1(\alpha_k) = X^2(\alpha_k) = U_k \sim \text{Uniform}\{1, \ldots, n\}$.

For typographical ease, let $F = \{E_i = 1\}$. Then for a network of n nodes, we have

$$\begin{aligned}
&P\left(E_i = 1|\alpha^i, E^{i-1}\right) \\
&= \sum_{X(\alpha_i)} P\left(F|X\left(\alpha_i\right), \alpha^i, E^{i-1}\right) P\left(X\left(\alpha_i\right)|\alpha^i, E^{i-1}\right) \\
&= \sum_{X(\alpha_i)} P\left(F|X\left(\alpha_i\right)\right) P\left(X\left(\alpha_i\right)|\alpha^i, E^{i-1}\right) && (48) \\
&\geq \min_{X(\alpha_i)} P(F|X(\alpha_i)) \sum_{X(\alpha_i)} P(X(\alpha_i)|\alpha^i, E^{i-1}) \\
&= \min_{X(\alpha_i)} P\left(F|X\left(\alpha_i\right)\right), && (49)
\end{aligned}$$

where (48) is true since E_i is independent of everything else given $X(\alpha_i)$ since it determines the configuration of occupancies of all the n cells by the other $n-2$ nodes. Next we show that the configuration that minimizes $P(F|X(\alpha_i))$ is the one in which $n-2$ cells contain one node each.

In time-slot α_i, let F_k be the event that R and D meet in cell k and let s_k, d_k denote the cell occupancies, i.e., number of source and destination nodes respectively in cell k (not including R and D) for $1 \leq k \leq n$. Obviously, s_k, d_k are determined by $X(\alpha_i)$. Recall that R and D are equally likely to meet in any of the n cells and hence

$$\begin{aligned}
P(F|X(\alpha_i)) &= \sum_{k=1}^{n} P(E_i = 1|X(\alpha_i), F_k)P(F_k) \\
&= \frac{1}{n}\sum_{k=1}^{n} \frac{1}{(1 + d_k)(1 + s_k + d_k)}. && (50)
\end{aligned}$$

Now clearly for $l \geq 0$ and $k \geq 1$

$$\frac{1}{(k+1)(k+2)(k+l+1)} < \frac{1}{2},$$

and for $l \geq 1$ and $k \geq 0$

$$\frac{1}{(k+1)(k+l+1)(k+l+2)} < \frac{1}{2}.$$

With simple algebraic manipulations, these can be rewritten as

$$\frac{1}{(k+l+1)(k+1)} + \frac{1}{2} < \frac{1}{(k+2)(k+l+1)} + 1, \tag{51}$$

$$\frac{1}{(k+l+1)(k+1)} + \frac{1}{6} < \frac{1}{(k+1)(k+l+2)} + 1. \tag{52}$$

Consider two cells with occupancies $s_1 = l, d_1 = k+1$ and $s_2 = 0, d_2 = 0$. Then their contribution to the sum in (50) is given by the right hand side of (51). Now by moving one of the nodes from cell 1 into cell 2 we obtain $s_1 = l, d_1 = k$ and $s_2 = 0, d_2 = 1$. For this case, the left hand side of (51) gives the contribution to the sum in (50). Thus the last inequality says that if a cell contains more than 1 destination nodes and there is another empty cell then $P(F|X(\alpha_i))$ can be reduced by moving one of the destination nodes to an empty cell. Similarly (52) says that if a cell contains more than 1 source nodes and there is another empty cell then $P(F|X(\alpha_i))$ can be reduced by moving one of the source nodes to an empty cell. But in our case with n cells and $n-2$ nodes, empty cells always exist. Hence starting from any initial cell occupancies, we progressively obtain that $P(F|X(\alpha_i))$ is minimized when each of the $n-2$ nodes occupies a different cell.

Next we compute the term in (49), which is the probability of R and D being chosen as a transmit-receive pair for the worst case of cell occupancies mentioned above. Using (50), we obtain

$$\begin{aligned} \min_{X(\alpha_i)} P\left(F|X\left(\alpha_i\right)\right) &= \frac{1}{n}\left(\frac{(n-2)}{2}\left[\frac{1}{4}+\frac{1}{2}\right]+2\right) \\ &= \frac{3}{8} - \frac{5}{4n} \\ &\to \frac{3}{8}. \end{aligned} \tag{53}$$

This proves the lemma for any choice of $c_6 < 3/8$. ■

Remark: It is interesting to obtain an upper bound on $P(E_i = 1|\alpha^i, E^{i-1})$ in the same manner. It is easy to see that the configuration that maximizes this probability is the one in which all $n-2$ nodes are in the same cell. Hence using (50) and recalling that $F = \{E_i = 1\}$, we obtain

$$P(E_i = 1|\alpha^i, E^{i-1}) \leq \frac{1}{n}\left((n-1) + \frac{4}{n^2}\right) = 1 - \frac{1}{n} + \frac{4}{n^3} \to 1.$$

This shows that $3/8 - 5/4n \leq P(E_i = 1|\alpha^i, E^{i-1}) \leq 1 - 1/n + 4/n^3$, indicating the extent of dependence in the process.

Proof of Lemma 4: We need to compute the first and second moments of τ, which is the inter-meeting time of two nodes, i and j, moving according to independent random walks on a $\sqrt{n} \times \sqrt{n}$ discrete torus. Let the position of node i at time t be $X^i(t) = (X_1^i(t), X_2^i(t))$, where $X_k^i(t) \in \{0, \ldots, \sqrt{n}-1\}$ for $k \in \{1, 2\}$.

Now consider the difference random walk between nodes i and j, defined by $X^{ij}(t) = (X_1^{ij}(t), X_2^{ij}(t))$, where $X_k^{ij}(t) = X_k^i(t) - X_k^j(t) \mod \sqrt{n}$, for $k = 1, 2$. The meeting time of two nodes $i,\ j$ is identified by the event $\{X^{ij}(t) = (0,0)\}$. Thus the inter-meeting time is the stopping time

$$T^{ij} = \inf\{t \geq 1 : X^{ij}(t) = (0,0), X^{ij}(0) = (0,0)\}.$$

This is in fact the first return time to state $(0,0)$. Since we are only interested in the scaling orders of the first two moments, we instead consider the first return time to state $(0,0)$ for a simple random walk $X(t)$ on a $\sqrt{n} \times \sqrt{n}$ discrete torus. The first return time to state $(0,0)$ is given by

$$T = \inf\{t \geq 1 : X(t) = (0,0),\ X(0) = (0,0)\}.$$

First note that, $X(t)$ is a Markov chain with a uniform equilibrium distribution π, i.e. $\pi(i,j) = 1/n$ for $0 \leq i, j \leq \sqrt{n} - 1$. For any finite-state ergodic Markov chain, the expectation of the first return time to any state is the reciprocal of the equilibrium probability of the Markov chain being in that state. In particular, the average first return time to state $(0,0)$ is n, i.e. $E[T]=n$. By the same argument, $E[\tau]=n$.

Next, we wish to compute $E[T^2]$, which is of the same order as $E[\tau^2]$. First consider the quantity

$$T_0 = \inf\{t \geq 0 : X(t) = (0,0)\},$$

which is the hitting time of state $(0,0)$. Let $E_{(i,j)}T_{(k,l)}$ denote the expected time to first hit state (k,l) starting from state (i,j). Then

$$\begin{aligned}
E_\pi[T_0] &= \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)} T_{(0,0)} \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \pi(i,j) E_{(i,j)} T_{(k,l)} \qquad (54)\\
&= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \frac{1}{n} \pi(i,j) E_{(i,j)} T_{(k,l)} \\
&= \sum_{i,j=0}^{\sqrt{n}-1} \sum_{k,l=0}^{\sqrt{n}-1} \pi(i,j)\pi(k,l) E_{(i,j)} T_{(k,l)} \\
&= \Theta(n \log n), \qquad (55)
\end{aligned}$$

where (54) holds because $\sum_{ij} E_{(i,j)}T_{(0,0)} = \sum_{ij} E_{(i,j)}T_{(k,l)}$ for any $0 \leq k,l \leq \sqrt{n}-1$ due to symmetry of states corresponding to cells on the torus. The validity of (55) is from page 11 of Chapter 5 in [1].

Using Kac's formula (see Corollary 24 in Chapter 2 of [1]) and (55), we obtain

$$\begin{aligned}
E[T^2] &= \frac{2E_\pi[T_0]+1}{\pi(0,0)} \\
&= 2\Theta\left(n^2 \log n\right) + n.
\end{aligned}$$

Therefore, we obtain $E[\tau]=n$ and $E[\tau^2]=\Theta(n^2\log n)$. ■

Proof of Lemma 6: An arrival occurs to $\mathcal{Q}_3$ when S and R meet with probability 0.5. Let $\{X_i\}$ be the sequence of inter-arrival times to this queue. Then, X_i are i.i.d. with $E[X_1] = 2E[\tau] = 2n$ and $E[X_1^2] = \Theta(E[\tau^2]) = \Theta(n^2 \log n)$ from Lemma 4. The potential-departure process is an i.i.d. Bernoulli process with parameter $1/1.5n$. Let $\{Y_i\}$ be the sequence of service times then Y_i is a Geometric random variable with mean $1.5n$. Hence $E[Y_1]=1.5n$ and $E[Y_1^2]=\Theta(n^2)$. Let D_3 denote the delay of a packet through $\mathcal{Q}_3$. The service process is independent of the arrival process and hence $\mathcal{Q}_3$ is a GI/GI/1 FCFS queue. Then, by Kingman's upper bound [19] on the expected delay for a GI/GI/1 – FCFS queue, the expected delay of $\mathcal{Q}_3$ is upper bounded as

$$\begin{aligned}
E[D_3] &= O\left(\frac{E[X_1^2]+E[Y_1^2]}{E[X_1]}\right) = O\left(\frac{n^2\log n + n^2}{n}\right) \\
&= O\left(n \log n\right).
\end{aligned}$$

■

Proof of Lemma 7: Consider the service process of $\mathcal{Q}_4$, which is 1 at a potential departure instant and 0 otherwise. This is a stationary, ergodic process since the inter-potential-departure times are i.i.d. with mean n. The Bernoulli arrival process to $\mathcal{Q}_4$ is independent of the service process with mean inter-arrival time $1.5n$. Since the arrival and service processes form a jointly stationary and ergodic process with mean service time strictly less than mean inter-arrival time, the queue has a stationary, ergodic distribution with finite expectation as shown by [16]. Thus $\mathcal{Q}_4$ is stable.

Let $\tilde{Q}_t$ be the number of packets in the queue in time-slot t and let Q_i be the number of packets in the queue at potential departure instant i. Thus the process $\{Q_i\}$ is obtained by sampling $\{\tilde{Q}_t\}$ at potential departure instants. Let A_{i+1} be the number of arrivals between potential departure instants i and $i+1$. Then the evolution of Q_i is given by

$$Q_{i+1} = Q_i - \mathbf{1}_{\{Q_i>0\}} + A_{i+1}. \qquad (56)$$

Comparing the evolution of the process $\{Q_i\}$ with that of $\{\tilde{Q}_t\}$ shows that $\{Q_i\}$ also has a stationary, ergodic distribution. Recall that τ is the inter-meeting time of R and D. Then since the arrival process is Bernoulli and the inter-potential departure times are i.i.d. with common distribution that of τ, it is clear the $\{A_i\}$ is a stationary process. Let $\tilde{Q}$, Q and A be random variables with the common stationary marginals of $\{\tilde{Q}_t\}$, $\{Q_i\}$ and $\{A_i\}$ respectively. Then taking expectation in (56) under the stationary distribution, we obtain

$$P(Q > 0) = E[A]. \tag{57}$$

The arrival process is i.i.d. Bernoulli and hence conditioned on τ, the distribution of A is Binomial $(\tau, 2/3n)$. Since $E[\tau] = n$ from Lemma 4, we obtain

$$E[A] \;=\; E[E[A|\tau]] \;=\; E\left[\frac{\tau}{1.5n}\right] \;=\; \frac{2}{3}. \tag{58}$$

Squaring (56), taking expectation, using the independence of Q_i and A_{i+1} and then rearranging terms, we obtain

$$2(1 - E[A])E[Q] = P(Q > 0) + E[A^2] - 2E[A]P(Q > 0).$$

Using (57) and (58) in the above, we obtain

$$\begin{aligned} E[Q] &= \frac{E[A] + E[A^2] - 2E[A]^2}{1(1 - E[A])} \\ &= \frac{3}{2}\left(E[A^2] - \frac{2}{9}\right). \end{aligned} \tag{59}$$

Recall that conditioned on τ the distribution of A is Binomial $(\tau, 2/3n)$ and hence

$$\begin{aligned} E[A^2] &= E[E[A^2|\tau]] \\ &= \frac{2E[\tau]}{3n} + \frac{4}{9n^2}\left(E[\tau^2] - E[\tau]\right) \\ &= \left(\frac{2}{3} - \frac{4}{9n}\right) + \frac{4}{9n^2}\Theta(n^2 \log n) \\ &= \Theta(\log n), \end{aligned} \tag{60}$$

where we used Lemma 4. As a result it follows from (59) that

$$E[Q] = \Theta(\log n). \tag{61}$$

Next, we will bound $E[\tilde{Q}]$ using $E[Q]$. To this end, consider a time-slot t and let the number of potential departures before time-slot t be $I(t)$. Thus time-slot t is flanked by potential departures $I(t)$ and $I(t)+1$. Then $\tilde{Q}_t \le Q_{I(t)} + A_{I(t)+1}$. Also using the fact that $\{\tilde{Q}_t\}$ is ergodic, with probability 1, we have

$$\begin{aligned} E[\tilde{Q}] &= \lim_{T\to\infty} \frac{1}{T}\sum_{k=1}^{T} \tilde{Q}_k \\ &\le \lim_{T\to\infty} \frac{1}{T}\sum_{j=1}^{I(T)+1} \left(Q_j \tau_{j+1} + A_{j+1}\tau_{j+1}\right) \\ &= \lim_{T\to\infty} \frac{I(T)+1}{T}\frac{1}{I(T)+1} \\ &\qquad \sum_{j=1}^{I(T)+1} \left(Q_j \tau_{j+1} + A_{j+1}\tau_{j+1}\right) \\ &= \frac{1}{E[\tau]}\left(E[Q_1\tau_2] + E[A_1\tau_1]\right) && (62) \\ &= \frac{1}{n}\left(E[Q]E[\tau] + \frac{2}{3n}E[\tau^2]\right) && (63) \\ &= O(\log n). && (64) \end{aligned}$$

We used the fact that $I(T)/T \to 1/E[\tau]$ by the elementary renewal theorem [19] in (62) and the independence of Q_j and τ_{j+1} in (63). Let D_4 denote the delay of a packet through $\mathcal{Q}_4$. Using Little's formula, since the arrival rate is $2/3n$, we conclude that

$$E[D_4] = \frac{3n}{2} E[\tilde{Q}] = \frac{3n}{2} O(\log n) = O(n \log n).$$

■

APPENDIX III

Proof of Lemma 8: Consider a fixed cell, say cell 1, out of $m = 1/a(n)$ cells. First we determine the traffic due to stage 1 in time-slot 0 which is the number of packets that cell 1 is required to transmit in time-slot 0.

Let Y_i be the number of packets of the S-D pair i at cell 1 in time-slot 0. We claim that no more than two packets of S-D pair i can be passing through cell 1 in time-slot 0. This is because k packets of S-D pair i can pass through cell 1 at the same time if and only if for k consecutive time-slots, the source node S moves closer to cell 1 by 1 hop along the line joining S and D. Now, cells are of size at least $\Theta(\log n/n)$, whereas according to the RW model, the nodes can only move $\Theta\left(1/\sqrt{n}\right)$ distance in unit time. Hence this can hold for at most $k = 2$. Thus, $Y_i \in \{0, 1, 2\}$. Next, we compute $E[Y_i]$.

Let $L_i(s)$ be the length (in terms of number of hops) of the straight line joining the center of cells that contain S and D at time s of S-D pair i. Now, a packet generated at time $-s, s \geq 0$ can be part of stage 1 at time 0, only if $L_i(-s) > s$. This is true because otherwise packet generated at time $-s$ is already transmitted for s times till time 0 and hence it is either in stage 2 or has reached the destination. Using this, we obtain

$$\sum_{j=1}^{m} Y_i(j) \leq \sum_{s=0}^{\infty} I(\{L_i(-s) > s\}),$$

where $I(A)$ is the indicator function of the event A and $Y_i(j)$ is the number of packets of stage 1 of the i^{th} S-D pair at cell j at time 0. By symmetry, $E[Y_i(j)] = E[Y_i(1)] = E[Y_i]$. Then,

$$\begin{aligned} mE[Y_i] &\leq \sum_{s=0}^{\infty} P\{L_i(-s) > s\} \\ &\stackrel{(a)}{=} \sum_{s=0}^{\infty} P\{L_i(0) > s\} \\ &= E[L_i(0)] \\ &\stackrel{(b)}{=} \Theta\left(1/\sqrt{a(n)}\right). \end{aligned}$$

Above, (a) holds because of the stationarity of $L_i(\cdot)$ under the RW model. And (b) is true because under the RW model the average distance between an S-D pair is $\Theta(1/\sqrt{a(n)})$ hops since the actual physical distance is $\Theta(1)$.

From above, $E[Y_i] = \Theta\left(\sqrt{a(n)}\right)$. Now let $Z_i = I(\{Y_i \neq 0\})$. Then by Markov's inequality,

$$P(\{Z_i = 1\}) = P(\{Y_i \geq 1\}) \leq E[Y_i].$$

The total number of packets passing through cell 1 at time 0 is $\sum_{i=1}^{n/2} Y_i \leq 2\sum_{i=1}^{n/2} Z_i$. Since under the RW model, all $n/2$ S-D pairs move independently, $\{Z_i\}$ are independent and due to symmetry they are distributed identically. Thus we can define $\{\tilde{Z}_i\}$ to be i.i.d. Bernoulli random variables with parameter $p_n = \Theta\left(\sqrt{a(n)}\right)$ so that Z_i is stochastically dominated by $\tilde{Z}_i$. As a result, the total number of packets is stochastically dominated by $2\sum_{i=1}^{n/2} \tilde{Z}_i$. By an application of the Chernoff bound, $\sum_{i=1}^{n/2} \tilde{Z}_i \leq np_n$ with probability at least $1 - 1/n^4$ for large enough n. Thus, cell 1 has $O(np_n) = O\left(n\sqrt{a(n)}\right)$ packets passing through it in time-slot 0 with probability at least $1 - 1/n^4$. Due to symmetry, the same is true for all m cells. Hence, by the union bound, each of the m cells has $O\left(n\sqrt{a(n)}\right)$ packets of stage 1 in time-slot 0 with probability at least $1 - m/n^4 \geq 1 - 1/n^3$. ■

Proof of Lemma 9: Let $H(C^{k-1}, C^k)$ denote the number of hops made by a packet in stage k for $k = 1, \ldots, k(n) - 1$. For a given packet this is the hop distance between C^{k-1} and C^k (as explained in the description of Scheme 3(a)). Let $l_k = E[H(C^{k-1}, C^k)]$. Since, each source node S is at a physical distance of $\Theta(1)$ from its destination on average, $l_1 = \Theta(1/\sqrt{a(n)})$. It is clear that the traffic of stage k depends on l_k, which in turn depends on l_{k-1}. Hence, we first determine the relation between l_k and l_{k-1} for $k \geq 2$ in order to evaluate l_k.

By definition, $H(C^{k-1}, C^k)$ is the displacement of the destination node of the packet while the packet was being transported in stage k from cell C^{k-2} to cell C^{k-1}. Under Scheme 3(a), if a packet is not dropped, it reaches C^{k-1} from C^{k-2} in time $H(C^{k-2}, C^{k-1})$ since each hop takes a constant amount of time. The average distance moved by destination node in this time is $\Theta\left(\sqrt{H(C^{k-2}, C^{k-1})/n}\right)$ under the RW model. That is, the destination node is displaced from C^{k-1} by $\Theta\left(\sqrt{H(C^{k-2}, C^{k-1})/na(n)}\right)$ hops on average. Putting this together, we obtain the following.

$$\begin{aligned} l_k &= E[H(C^{k-1}, C^k)] \\ &= E\left[E\left[H(C^{k-1}, C^k)|H(C^{k-2}, C^{k-1})\right]\right] \\ &= E\left[\Theta\left(\sqrt{\frac{H(C^{k-2}, C^{k-1})}{na(n)}}\right)\right] \\ &= \Theta\left(\frac{E\left[\sqrt{H(C^{k-2}, C^{k-1})}\right]}{\sqrt{na(n)}}\right) \\ &= O\left(\frac{\sqrt{E[H(C^{k-2}, C^{k-1})]}}{\sqrt{na(n)}}\right), \end{aligned} \tag{65}$$

where (65) follows from Jensen's inequality. Equation (65) gives us the following recursion. For $k \geq 2$,

$$l_k = O\left(\sqrt{\frac{ml_{k-1}}{n}}\right). \tag{66}$$

Now, as noted earlier, $l_1 = O\left(1/\sqrt{a(n)}\right) = O\left(\sqrt{m}\right)$. For ease of presentation, let $c_5 \geq 1$ be constant such that for large enough n, $l_1 \leq \sqrt{c_5 m}$ and $l_k \leq \sqrt{c_5 m l_{k-1}/n}$. Putting all this together, we obtain

$$\begin{aligned} l_k &\leq \sqrt{\frac{c_5 m l_{k-1}}{n}} \\ &\leq \left(\frac{c_5 m}{n}\right)^{\sum_{j=1}^{k-1} 2^{-j}} \quad l_1^{2^{-k+1}} \\ &\leq \left(\frac{c_5 m}{n}\right)^{1-2^{-k+1}} \quad (c_5 m)^{2^{-k}} \\ &\leq \frac{c_5 m}{n} \left(\frac{n^2}{m}\right)^{2^{-k}}. \end{aligned} \tag{67}$$

Now we are ready to analyze the traffic load on cells due to stage k, $k \geq 2$. The analysis of stage 1 showed that at most 2 packets of the same S-D pair can pass through a cell in the same time-slot. This happens only if the source moves in a particular way so that in adjacent time-slots the distance to the cell being considered reduces by one hop. Similarly by considering the motion of both the source and the destination it can be shown that in stage 2 the number of packets of the same S-D pair that can pass through a cell is at most 3. For stage $k \geq 3$, since a packet originates from a fixed cell, rather than from a mobile node like S, it follows that the number of packets of the same S-D pair passing through a cell in the same time-slot is no more than 3.

Consider stage k. As in stage 1, let Y_i be the number of packets for S-D pair i at cell 1 at time 0. From the above discussion, $Y_i \in \{0, 1, 2, 3\}$. Let $L_i(s)$ denote the length in hops, $H(C^{k-1}, C^k)$, for a packet of the S-D pair i entering stage k in time-slot s. Due to the symmetry of the cells under the RW model (using arguments similar to that used in analysis of stage 1) and (67), we obtain

$$\begin{aligned} mE[Y_i] &\leq \sum_{s=0}^{\infty} P(\{L_i(-s) > s\}) \\ &= \sum_{s=0}^{\infty} P(\{L_i(0) > s\} \\ &= E[L_i(0)] \\ &= O\left(\frac{m}{n}\left(\frac{n^2}{m}\right)^{2^{-i}}\right). \end{aligned}$$

Thus,

$$E[Y_i] = O\left(\frac{1}{n}\left(\frac{n^2}{m}\right)^{2^{-i}}\right). \tag{68}$$

Again using the same method as in the analysis of stage 1, it can be shown that the total number of stage k packets passing through each cell is $O\left(\left(\frac{n^2}{m}\right)^{2^{-k}}\right)$ with probability at least $1-1/n^3$. ■

Proof of Lemma 12: Let $X(t) = (X_1(t), X_2(t))$ be a two-dimensional simple RW on the $\sqrt{n} \times \sqrt{n}$ discrete torus. Let $B = \{1, \sqrt{m}, \ldots, (\sqrt{n/m}-1)\sqrt{m}\}$, i.e., B is the set of elements of $\{0, \ldots, \sqrt{n}-1\}$ which are multiples of $\sqrt{m}$. Let $Y_1(t) \in B$ be the discrete-time process such that $Y_1(t)$ is the last (including the current) state visited by $X_1(t)$ in B. Thus the process $Y_1(t)$ is a coarser version of $X_1(t)$ that changes state only when $X_1(t)$ moves $\sqrt{m}$ steps away on the one-dimensional discrete torus. Similarly, define $Y_2(t)$ to be the last state visited by $X_2(t)$ in B and let $Y(t) = (Y_1(t), Y_2(t))$.

Now let $Z(t)$ be the process obtained by sampling $Y(t)/\sqrt{m}$ whenever its value changes. Thus $Z(t) \in \{0, \ldots, \sqrt{n/m}\}$ proceeds in steps and ignores the random amount of time that $Y(t)$ spends in each step. Now $Z(t)$ is a random walk on the discrete $\sqrt{n/m} \times \sqrt{n/m}$ torus, such that the next state is one of the eight possible neighbors. Let T_0^Z be the hitting time of state $(0,0)$ for the random walk $Z(t)$. This is the number of steps taken by $Z(t)$ to hit $(0,0)$. Since we are only interested in the order, we can use the corresponding moment for the simple random walk instead. Thus we obtain

$$E_\pi T_0^Z = O\left(\frac{n}{m}\log\left(\frac{n}{m}\right)\right).$$

Now $Z(T_0^Z) = (0,0)$ implies that for some time T, $X(T) \in A$. In fact, T is the random time required for T_0^Z steps of $Z(t)$. Hence $E[T]$ is an upper bound on $E[T_A]$. Let $\tilde{T}(i)$ be the time required for step i of $Z(t)$, i.e., the time for $Y(t)$ to change state for the i^{th} time. The $\tilde{T}(i)$ are clearly i.i.d. Moreover, T_0^Z is independent of $\tilde{T}(i)$ and

$$T = \sum_{i=1}^{T_0^Z} \tilde{T}(i).$$

Let $\tilde{T}_1$ and $\tilde{T}_2$ be the random times required for $Y_1(t)$ and $Y_2(t)$ respectively, to change their states. Then $E[\tilde{T}_1] = E[\tilde{T}_2] = m$, since this is the time required for a simple random walk on the integers to exit from $\{-\sqrt{m}+1, \ldots, \sqrt{m}-1\}$. Moreover, $\tilde{T}(i)$ is dominated by $\tilde{T}_1$ since $Y(t)$ changes state if either of $Y_1(t)$ or $Y_2(t)$ change. Hence $E[\tilde{T}(i)] \leq E[\tilde{T}_1] = m$.

Now in using T to upper bound T_A, we need to take care of the possibility that $X(t)$ may not start from a state that is an element of $B \times B$ and hence $Y(t)$ is undefined until both $X_1(t)$ and $X_2(t)$ hit some element of B. But this time is at most equal to $\max T_1, T_2 \leq T_1 + T_2$. Hence allowing for the initial expected time for $X(t)$ to reach one of the elements of B, which is at most $2m$, we obtain

$$\begin{aligned}\frac{n-|A|}{n}E_{\pi_{A^c}}T_A &= E_\pi T_A \\ &\leq E_\pi T + 2m \\ &\leq E_\pi \sum_{i=1}^{T_0^Z} \tilde{T}(i) + 2m \\ &= E\left[T_0^Z\right] E\left[\tilde{T}(1)\right] + 2m \\ &= O\left(n \log\left(\frac{n}{m}\right)\right).\end{aligned}$$

■

Proof of Lemma 13: If the throughput of the network is $T(n)$, the number of bits transmitted in a large enough time t is at least $ntT(n)/2$. We will ignore this factor of $1/2$ as it does not affect the scaling. Now consider a bit b, where $1 \leq b \leq ntT(n)$. Let $h(b)$ denote the number of hops taken by bit b and let $r(b,h)$ denote the length of hop h of bit b. Then

$$\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} r(b,h) \geq ntT(n)\bar{l}(n). \tag{69}$$

Using the same reasoning as the one which led to (7), we obtain

$$\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{\pi}{4}\left(\frac{\Delta}{2} r(b,h)\right)^2 \leq Wt. \tag{70}$$

Let the total number of hops taken by all bits be

$$H = \sum_{b=1}^{ntT(n)} h(b).$$

Then by convexity, we obtain

$$\left(\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)\right)^2 \leq \sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)^2. \tag{71}$$

Combining (70) and (71) we obtain,

$$\left(\sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)\right)^2 \leq \frac{16Wt}{\pi\Delta^2 H} = c_3 \frac{t}{H}, \tag{72}$$

where c_3 is a constant that does not depend on n. Substituting from (69) into (72) and rearranging we obtain

$$\frac{ntT(n)\bar{l}(n)}{H}\bar{r}(n) \leq c_3 \frac{t}{H}, \tag{73}$$

where

$$\bar{r}(n) = \sum_{b=1}^{ntT(n)} \sum_{h=1}^{h(b)} \frac{1}{H} r(b,h)$$

is the sample mean of hop-lengths over H hops as defined earlier. Rearranging we obtain

$$T(n) \leq \frac{c_3}{n\bar{l}(n)\bar{r}(n)}. \tag{74}$$

This completes the proof of Lemma 13. ■

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REFERENCES

[1] D. Aldous and J. Fill, "Reversible Markov Chains and Random Walks on Graph," Monograph in preparation, available at http://www.stat.berkeley.edu/users/aldous/RWG/book.html.
[2] N. Bansal and Z. Liu, "Capacity, Mobility and Delay in Wireless Ad hoc Networks," *Proc. IEEE INFOCOM*, 2003.
[3] A. F. Dana and B. Hassibi, "On the Power Efficiency of Sensory and Ad-hoc Wireless Networks," *Proc. IEEE ISIT*, pp. 412, 2003.
[4] A. Dembo, Y. Peres, L. Rosen, and O. Zeitouni, "Cover Times for Brownian Motion and Random Walks in Two Dimensions," *Annals of Mathematics*, 160(2), pp. 433-464, 2004.
[5] S. N. Diggavi, M. Grossglauser and D. Tse, "Even One-Dimensional Mobility Increases Ad Hoc Wireless Capacity," *Proc. IEEE ISIT*, Laussane, Switzerland, July 2002.
[6] A. El Gamal and J. Mammen, "Optimal Hopping in Ad Hoc Wireless Networks," to appear, *Proc. IEEE INFOCOM*, 2006.
[7] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Throughput-Delay Trade-off in Wireless Networks," *Proc. IEEE INFOCOM*, Hong Kong, 2004.
[8] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Optimal Throughput-Delay Trade-off in Energy Constrained Wireless Networks," *Proc. IEEE ISIT*, pp. 439, Chicago, 2004.
[9] A. El Gamal, J. Mammen, B. Prabhakar, and D. Shah, "Optimal Throughput-Delay Scaling in Wireless Networks – Part II: Constant-size Packets," submitted, available at http://www.stanford.edu/˜jmammen/papers/it-TDpkt.pdf.
[10] M. Grossglauser and D. Tse, "Mobility Increases the Capacity of Ad-hoc Wireless Networks," *Proc. IEEE INFOCOM*, Anchorage, Alaska, pp. 1360-1369, 2001.
[11] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Trans. Inform. Theory*, 46(2), pp. 388-404, March 2000.
[12] A. Jovicic, P. Viswanath, and S. R. Kulkarni," Upper Bounds to Transport Capacity of Wireless Networks," *IEEE Trans. Inform. Theory*, 50(11), pp. 2555-2565, November 2004.
[13] S. R. Kulkarni and P. Viswanath, "A Deterministic Approach to Throughput Scaling in Wireless Networks," *IEEE Trans. Inform. Theory*, 50(6), pp. 1041-1049, June 2004.
[14] O. Leveque and E. Telatar, "Information theoretic upper bounds on the capacity of large extended ad hoc wireless networks," *IEEE Trans. Inform. Theory*, 51 (3), pp. 858-865, March 2005.
[15] X. Lin, G. Sharma, R. R. Mazumdar, and N. B. Shroff, "Degenerate Delay-Capacity Trade-offs in Ad Hoc Networks with Brownian Mobility," submitted, available at http://min.ecn.purdue.edu/˜linx/paper/it05.pdf.
[16] R. Loynes, "Stability of Queue with non-independent arrival and service times," *Proc. Camb. Philos. Soc.*, 58:497-520, 1962.
[17] R. Motwani and P. Raghavan, "Randomized algorithms," *Cambridge Univ. Press*, 1995.
[18] M. J. Neely and E. Modiano, "Capacity and Delay Tradeoffs for Ad-Hoc Mobile Networks," *IEEE Trans. Inform. Theory*, June 2005.
[19] R. W. Wolff, "Stochastic Modeling and the Theory of Queues," *Prentice-Hall*, 1989.
[20] L. Xie and P. R. Kumar, "A Network Information Theory for Wireless Communication: Scaling Laws and Optimal Operation," *IEEE Trans. Inform. Theory*, 50(5), pp. 748-767, May 2004.

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