Optimal Trade-Offs in Distribution Protection Design

Farajollah Soudi and Kevin Tomsovic

***Abstract*—The number, type and location of the protective devices on a distribution feeder have a direct effect on the system reliability. In earlier work, a technique was developed to design a protective system in order to minimize the *SAIFI* index. This paper extends earlier results by using a goal programming approach to achieve compromises among various engineering objectives. The design goals are: a) to minimize the *SAIFI* and *ASIFI* indices by identifying types and locations of protective devices and b) to achieve a reasonable trade-off between a decrease in the *SAIFI* index and an increase in *MAIFI* index by identifying where a fuse saving scheme should be applied. Numerical examples highlight the approach.**

***Index Terms*—Distribution reliability, distribution systems, fuzzy optimization, goal programming, integer programming, linear programming, protection design.**

I. INTRODUCTION

DISTRIBUTION reliability can be analyzed based on either customer or load based indices. Utilities use calculation of these reliability indices to prioritize capital and maintenance expenditures, evaluate the system performance or provide a basis to establish service continuity criteria. Recent surveys indicate that the most common indices used in the industry are customer based indices, including System Average Interruption Frequency Index (*SAIFI*) and Momentary Average Interruption Frequency Index (*MAIFI*). In addition, utilities may use the load based indices of, Average System Interruption Frequency Index (*ASIFI*) and Average System Interruption duration Index (*ASIDI*) [1], [2].

In the recent years, due to the increase in electronic loads, customers have become less tolerant of momentary faults. This has resulted in renewed interest in the *MAIFI* index. For many years, utilities have allowed increased momentary outages in order to achieve a minor improvement in the number of the permanent outages by applying a fuse saving scheme. A fuse saving scheme protects fuses from momentary faults on their load side through proper response of the source side line recloser. Such a scheme will increase the number of momentary outages for all customers on the load side of the recloser; however, permanent outages will decrease for customers on the load side of the fuse and remain unchanged for others. Finally, recloser tripping can influence voltage quality but is not considered here.

The selection of a reliability index for a study usually depends on the type of the customers on a given distribution circuit. A circuit with primarily residential customers would probably focus on the *SAIFI* index, while for a industrial or commercial *ASIFI* index may be more appropriate. If a circuit has a mixture of customers types, then both indices may need to be considered.

In general, the number, type and location of the protective devices on a distribution feeder have a direct effect on the reliability. In [3], a proposed technique identifies type of the protective devices at the predetermined locations on a distribution feeder based on the objective of minimizing reliability indices, such as *SAIFI*, *ASIFI*, or by minimizing cost while achieving desired performance levels. In [4], a technique was proposed to identify the number, type and location of the protective devices in order to minimize the *SAIFI* index. This paper extends [4] by using a goal programming approach to optimize the effectiveness of protective devices. The goals are: a) to minimize the *SAIFI* and *ASIFI* indices by identifying types and locations of protective devices and b) to achieve a reasonable trade-off between a decrease in the *SAIFI* index and an increase in the *MAIFI* index by identifying where a fuse saving scheme should be applied.

II. PROPOSED ALGORITHM

A. Goal Programming

It is often not possible to encompass the objectives of an optimization problem within a single overriding function. A traditional approach to multiple objective problems is to assign weights to the individual objectives. The difficulty in determining weights *a priori* usually renders this approach ineffective. Another approach is to select one objective as a primary and assign acceptable minimum and maximum values to the remaining objectives. These secondary objectives are then treated as constraints. The drawback of this approach is that, if careful consideration is not given while selecting the initial acceptable values, a feasible solution may not exist. Furthermore, Pareto optimality is not guaranteed.

In goal programming, all objectives are treated as constraints after assigning each a specific numerical goal level [5]. The goal constraints are conditions which are desired but not required. Positive and negative deviational variables are introduced and the new objective is to minimize the sum of these deviations. In addition, some goals might be considered more important than others or possibly a deviation in one direction might be more significant than deviations in the opposite direction. These differences can be taken into account by assigning different weighting factors.

In general, a goal programming looks for a solution where either all goals are achieved or nearly so. The objective is to minimize

$$z = \sum_{i=1}^{m} (w_i^+ y_i^+ + w_i^- y_i^-) \tag{1}$$

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with the goal constraints as

$$\sum_{j=1}^{n} c_{ij}x_j + y_i^- - y_i^+ = g_i \qquad \forall i$$

$$y_i^+,\, y_i^- \geq 0 \qquad \forall i,\, j \tag{2}$$

where

m	is the number of the objectives,
n	is the number of decision variables
x_j, c_{ij}	are cost coefficients,
y_i	are deviations from a goal
g_i, and w_i	are weighting factors.

Weights are selected to ensure appropriate tradeoffs between objectives but require an appropriate scaling of the deviational variables. One approach is to normalize the deviations based on the original cost coefficients. For example, weighting by the Euclidean norm (2) becomes

$$\sum_{j=1}^{n} c_{ij}x_j + \eta(y_i^- - y_i^+) = g_i \qquad \forall i$$

$$y_i^+,\, y_i^- \geq 0 \qquad \forall i,\, j \tag{2a}$$

where

$$\eta = \|c_{ij}\| = \left[\sum_{j=1}^{n} c_{ij}^2\right]^{1/2}.$$

Another formulation is discussed in the following section.

B. Fuzzy Goal Programming

The general development of the fuzzy mathematical programming problem can be found in [6], [7]. The structure is similar to the goal programming approach; however, the methodology for formulating the objectives as constraints is carefully defined. For the purpose of this problem, the fuzzy decision problem requires:

- Formulation of the objectives (and possibly fuzzy constraints) as a membership function which represents the degree each objective is satisfied on a scale of [0, 1].
- Definition of the overall satisfaction with a decision as dependent on an appropriate aggregate function of all objectives. The function typically selects the minimum (maximum) among the objectives but other operators from the triangular norm (conorm) class [8] may be appropriate. For example, the minimum operator would ensure that attention is focused on the least satisfied objective while in contrast, the product operator rewards large improvements in specific objectives.
- Establishing the goal g_i for each objective by performing a single objective optimization. This determines the best performance possible for each objective. The worst case for each objective can be found from these intermediate solutions.

The primary objectives in the proposed formulation are minimization of various reliability indices so the desired goals can be written as $g_i^{\min}$. Since these goals are lower bounds then negative deviations are deleted from the equations and only positive deviations are added and penalized. With $g_i^{\max}$ and $g_i^{\min}$ determined by solutions of the individual objective functions, reformulation of the goal programming to eliminate y_i^- yields

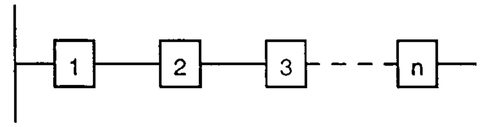

Fig. 1. A simple main or lateral feeder with n sections.

$$\sum_{j=1}^{n} c_{ij}x_j - y_i^+ = g_i^{\min} \qquad \forall i$$

$$y_i^+ \geq 0 \qquad \forall i,\, j. \tag{3}$$

Weights are now found by normalizing over the range between the maximum and minimum of the goals so that

$$w_i^+ = (g_i^{\max} - g_i^{\min})^{-1} \qquad \forall i \tag{4}$$

and the objective function is

$$z = \max_i\, w_i^+ y_i^+. \tag{5}$$

A similar development can be found for the maximization problem.

C. Formulation

In [4], a binary programming optimization is utilized to identify type and location of the protective devices on a distribution feeder. The proposed technique identifies type and location of the specific number of the protective devices on a distribution feeder in order to minimize the *SAIFI* index. The distribution feeder is assumed to be radial in construction.

In that proposed formulation, a distribution feeder is divided into four categories: a main feeder, lateral one, lateral two or lateral three. A lateral one category is short and will not be fused. The effect of this lateral on reliability can be included in the feeder section from which it branches. A lateral two will only be fused and its effect on the *SAIFI* index is constant. All other laterals are category three. Thus, only the main feeder and category three laterals are explicit in the optimization.

Consider Fig. 1 as a main or a category three lateral with n_1 possible locations for installing protective devices. If α is the number of category three laterals, then there will be $\sum_{i=1}^{\alpha+1} \eta_i$ possible locations; however, there are only m single and three phase protective devices available for installation.

1) Customer Based Index vs. Load Based Index: The *SAIFI* index for a distribution feeder is defined as

$$\text{SAIFI} = \frac{\sum \lambda_i N_i}{N_T} \tag{6}$$

where N_i is the number of customers in section i, N_T is the total number of customers on the feeder and λ_i is the net failure rate for section i (the sum of all individual failure rates between the substation and the section). The numerator of (6) is written here as

$$\sum \lambda_i N_i = \sum_{q=1}^{\alpha+\beta+1} A_q \tag{7}$$

where α is the number of category three laterals, β is the number of category two laterals, the first term is the contributions from the main feeder and category three laterals and the second term is the contribution of the category two laterals. The expressions for these terms can be found in the appendix. Since the contribution of the category two lateral is constant then minimizing

$$z = \sum_{q=1}^{\alpha+1} A_q \tag{8}$$

is equivalent to minimizing the *SAIFI* index.

The constraints for this problem include those on coordination, number of devices, economic cost, and various other design parameters. In addition, there are constraints due to the reduction of the integer programming problem to zero-one linear programming problem. These are detailed in reference [4].

When load based indices are used, the formulation will be similar to *SAIFI* index except the number of the customer will be replaced with the connected load. For example, the *ASIFI* index for a distribution feeder is defined as

$$\text{ASIFI} = \frac{\sum \lambda_i L_i}{L_T} \tag{9}$$

where L_i is the connected load in section i and L_T is the total connected land on the feeder. Again, the numerator of (9) is written as

$$\sum \lambda_i L_i = \sum_{q=1}^{\alpha+\beta+1} A'_q \tag{10}$$

and minimizing the *ASIFI* index is equivalent to

$$z = \sum_{q=1}^{\alpha+1} A'_q. \tag{11}$$

As was stated earlier, depending on the type of the customers on a distribution circuit, a utility may wish to find an optimal solution, which considers both *SAIFI* and *ASIFI* indices. For the proposed approach, the first step is to establish a specific numerical goal for both the *SAIFI* and *ASIFI* indices. The goal on these indices can be found by solving two separate optimization problems, which minimize the *SAIFI* and *ASIFI* indices. The objectives are then treated as constraints as indicated in Section II. In order to have a feasible solution, the numerical goals for these two constraints must be greater than or equal to the optimal solution obtained from the above problems. The goal constraint far the *SAIFI* index is

$$\frac{\sum_{q=1}^{\alpha+1} A_q}{N_T} - y_1^+ = g_1^{\min} \tag{12}$$

and for the *ASIFI* index

$$\frac{\sum_{q=1}^{\alpha+1} A'_q}{L_T} - y_2^+ = g_2^{\min}. \tag{13}$$

Now, the objective function is to minimize

$$z = w_1^+ y_1^+ + w_2^+ y_2^+ \tag{14}$$

or if using fuzzy goal programming to minimize

$$z = \max\left[\frac{y_1^+}{g_1^{\max} - g_1^{\min}}, \frac{y_2^+}{g_2^{\max} - g_2^{\min}}\right]. \tag{15}$$

2) Permanent vs. Momentary Outages: The majority of faults on a distribution circuit are momentary. The effect of momentary faults on customers will depend on the type of the protective device. Customers within the reach of an automatic protective device will have a momentary outage with the outage duration dependent on the recloser timing. Customers on a fused lateral may experience a permanent outage unless a fuse saving scheme is applied. With a fuse saving scheme, all temporary faults are cleared by an automatic device but permanent faults are cleared by the fuse. Thus, the cost of applying a fuse saving scheme is that the number of momentary outages will increase.

The formulation in this work assumes the location of all protective devices is determined by minimizing the permanent outage indices. This approach is based on reasoning that momentary faults will occur in a relatively fixed proportion to permanent faults throughout a given feeder. The designer can decide where to apply a fuse saving scheme to achieve an acceptable trade-off between permanent and temporary faults. In the following development, an overscore notation is introduced in order to represent the net failure rate for a particular protection zone. To begin, note that the minimum *MAIFI* index is when no fusing saving scheme is applied so

$$\text{MAIFI}^{\min} = \frac{\sum_{i=1}^{n} \hat{\gamma}_i \hat{N}_i}{N_T} \tag{16}$$

where

- n is the number of the breaker and line reclosers,
- $\hat{N}_i$ is the number of the customers downstream from section i, and
- $\hat{\gamma}_i$ is the net temporary failure rate for the protection zone of a protective device i.

The number of momentary outages reaches the maximum value and the number of permanent outages reaches the minimum value if a fuse saving scheme is applied for all of the automatic devices. Thus, the maximum increase, from the system with no fuse saving, in *MAIFI* is

$$\Delta\text{MAIFI}^{\max} = \frac{\sum_{i=1}^{n} \hat{N}_i \sum_{q\in I_1} (\hat{\gamma}_q + \hat{\lambda}_q) - \sum_{q\in I_1} \hat{N}_q \hat{\lambda}_q}{N_T} \tag{17}$$

where I_1 is the set of fused laterals within the fuse saving zone for recloser or breaker i, $\hat{\lambda}_q$ and $\hat{\gamma}_q$ are net permanent and temporary failure rates, respectively, for the fused lateral q, $\hat{N}_q$ is the number of the customers downstream from the fused lateral q. The maximum decrease, from the system with no fuse saving and resulting in the minimum value for *SAIFI*, will be

$$\Delta\text{SAIFI}^{\max} = \frac{\sum_{q\in I_1} \hat{N}_q \hat{\gamma}_q}{N_T}. \tag{18}$$

The goals of the proposed technique is to determine where a fuse saving scheme should be applied, while minimizing the changes in the number of permanent and temporary outages. The first

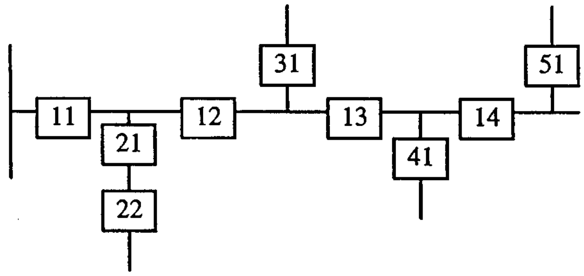

Fig. 2. Simple 9-load point radial system.

goal constraint then is to force changes in the momentary outage rates to be small. Since the deviation must be positive only the following goal is needed

$$\frac{\sum_{i=1}^{n}\hat{x}_i\left(\hat{N}_i\sum_{q\in I_1}(\hat{\gamma}_q+\hat{\lambda}_q)-\sum_{q\in I_1}\hat{N}_q\hat{\lambda}_q\right)}{N_T}-y_3^+=0 \qquad (19)$$

where $\hat{x}_i$ is 1 if recloser or breaker i has a fuse saving scheme. The second goal constraint is to achieve the maximum possible improvement in the number of the permanent outages. Here, the deviation must be negative so only the following is needed

$$\frac{\sum_{i=1}^{n}\hat{x}_i\sum_{q\in I_1}\hat{N}_q\hat{\gamma}_q}{N_T}+y_4^-=\Delta\text{SAIFI}^{\max}. \qquad (20)$$

Constraints for coordination problems and design limitations are also included. As a part of the design limitation, utilities may define an acceptable level of trade off between improvement of the permanent outage frequency and degrading the temporary outage frequency. The constraint for this criterion can be stated as

$$\sum_{i=1}^{n}\hat{x}_i\left(\hat{N}_i\sum_{q\in I_1}(\hat{\gamma}_q+\hat{\lambda}_q)-\sum_{q\in I_1}\hat{N}_q\hat{\lambda}_q\right)-\varepsilon\sum_{i=1}^{n}\hat{x}_i\sum_{q\in I_1}\hat{N}_q\hat{\gamma}_q\geq 0 \qquad (21)$$

where ε should be set to the maximum acceptable ratio between percentages of improvement in the *SAIFI* index and degradation in the *MAIFI* index as determined by the utility objectives. Finally, the objective for this problem using a normalized goal programming scheme is

$$z=w_3^+y_3^+ + w_4^-y_4^-. \qquad (22)$$

III. Test Case

Consider the simple overhead radial system shown in Fig. 2 with nine possible locations to install protective devices. (Note, a detailed analysis of a larger feeder can be found in [9]). The assumed permanent and temporary failure rates, the number of customers, and average connected load to each section are shown in Table I. This example illustrates identifying type and location of protective devices to achieve an acceptable compromise between *SAIFI* and *ASIFI* indices. Fuse saving schemes are applied to minimize the impact on the *MAIFI* index. The system has the following limitations, which translate into 25 constraints:

- There are only three line reclosers available.
- There is an unlimited number of fuses.
- A fuse cannot be installed on the main feeder.
- A fuse or a three phase device such as a line recloser or a sectionalizer must be installed on the tap points.
- Proper coordination between line reclosers in locations #12 and #13 is not possible.
- All laterals are treated as category three.
- There will be a breaker with its associated relays in position #11.

TABLE I
Component Data for the System

Section	λ_{qi}(f/yr)	γ_{qi} (f/yr)	N_{qi} (cust.)	L_{qi} (KVA)
11	0.80	1.20	1600	4100
12	0.80	1.40	1600	1000
13	0.90	1.60	1000	1600
14	0.70	1.00	800	800
21	0.90	2.00	500	2250
31	0.80	2.80	400	300
41	1.00	3.20	200	1000
51	0.50	0.70	200	450
22	0.70	1.70	300	350

TABLE II
SAIFI and ASIFI for Fig. 2 System

Cases	SAIFI	ASIFI	Reclosers	Fuses	None
1	2.71	3.33	13,21,31	41,51	12,14,22
2	3.10	2.66	12,21,41	31,51,22	13,14
3	2.86	2.85	12,14,21	31,41,51,22	13
4	2.84	2.88	13,21,41	22,31,51	12,14

TABLE III
SAIFI and MAIFI for Case #3 Solution from Table II

Cases	Fuse Saving Device	ΔSAIFI	ΔMAIFI
1	None	-0.000	+0.000
2	12	-0.352	+5.200
3	14	-0.028	+0.172
4	21	-0.102	+0.342
5	12,14,21	-0.482	+5.714
6	12,21	-0.454	+5.542
7	14,21	-0.130	+0.514
8	12,14	-0.380	+5.372

The first step is to establish numerical goals for both *SAIFI* and *ASIFI* indices by solving two different optimization problems. The solution for minimizing the *SAIFI* index (Case #1) and the *ASIFI* index (Case #2) is shown in Table II. The second step is to find the optimal tradeoff between the *SAIFI* and *ASIFI* indices. The goal is to find a solution that achieves the established goals. Two methods are applied: weighting by the Euclidean norm and the Fuzzy Programming methods. Using the Euclidean norm, the total number of the constraints is now 27, and the solution is shown as case #3 in Table III. The Fuzzy Programming which requires 29 constraints method solution is shown as case #4.

Finally case #3 from the above is used to identify where a fuse saving scheme should be applied based on the optimal trade-offs between decreases in the *SAIFI* index and increases in the *MAIFI* index. Here, the maximum acceptable trade-off in

SAIFI and *MAIFI* is specified as 4, i.e., $\varepsilon = 4$. In addition, w_4 was assigned twice the value of ε. Table III shows that both cases #4 and #7 meet acceptable trade off criteria but, as improvement in *SAIFI* carries more weight, case #7 is the solution.

IV. CONCLUSION

This paper has presented a technique to optimize the effectiveness of protective devices. The technique finds a Pareto optimal solution to minimize the *SAIFI* and *ASIFI* reliability indices by identify types and locations of protective devices. At the same time, a reasonable trade-off between a decrease in the *SAIFI* index and an increase in the *MAIFI* index for fuse saving schemes is found.

APPENDIX

This appendix summarizes the equations for calculating the *SAIFI* index based on the development in [4]. In equation (7), A_q for each main feeder or lateral q is

$$\begin{aligned} A_q = &\sum_{i=1}^{q_n}(\lambda_{qi}+\gamma_{qi})\sum_{j=i}^{q_n}N_{qj} - \sum_{i=1}^{q_n}\gamma_{qi}x_{qi2}\sum_{j=i}^{q_n}N_{qj} \\ &+ \sum_{i=2}^{q_n}\lambda_{qi}\sum_{j=1}^{i-1}N_{qj}\prod_{k=j+1}^{i}x_{qk1}x_{qk2} \\ &+ \sum_{i=2}^{q_n}\gamma_{qi}\sum_{j=1}^{i-1}(1-x_{qj2})\sum_{k=j}^{q_n}N_{qk}\prod_{l=j+1}^{i}x_{ql1}x_{ql2} \end{aligned} \quad \text{(A.1)}$$

where

q_n is the number of the possible locations on the main feeder or lateral,

λ_{qi} is the permanent failure rate and

γ_{qi} is the temporary failure rate for section i of q, respectively, and

N_{qj} is the number of customers for section j of q.

Note, if there is a three phase device at location qk, then the variable $x_{qk1} = 0$, and otherwise $x_{qk1} = 1$. The subscript 1 is used to represent a three phase device and the subscript 2 represents a fuse. For the main feeder (A.1) reduces to

$$A_q = \sum_{i=1}^{q_n}\lambda_{qi}\sum_{j=i}^{q_n}N_{qj} + \sum_{i=2}^{q_n}\lambda_{qi}\sum_{j=1}^{i-1}N_{qj}\prod_{k=j+1}^{i}x_{qk1}, \qquad q = 1. \quad \text{(A.2)}$$

Since a fuse will be installed at the tap and no other protective device will be installed on category two laterals

$$A_q = \sum_{i=1}^{q_n}(\lambda_{qi}+\gamma_{qi})\sum_{i=1}^{q_n}N_{qi} \qquad q \in \alpha+2\cdots\alpha+\beta+1. \quad \text{(A.3)}$$

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