Optimal Transport Theory for Cell Association in UAV-Enabled Cellular Networks

Mohammad Mozaffari, Walid Saad, Mehdi Bennis, and Mérouane Debbah

***Abstract*— In this letter, a novel framework for delay-optimal cell association in unmanned aerial vehicle (UAV)-enabled wireless cellular networks is proposed. In particular, to minimize the average network delay under any arbitrary spatial distribution of the ground users, the optimal cell partitions of the UAVs and terrestrial base stations are determined. To this end, using the powerful mathematical tools of optimal transport theory, the existence of the solution to the optimal cell association problem is proved and the solution space is completely characterized. The analytical and simulation results show that the proposed approach yields substantial improvements in terms of the average network delay.**

***Index Terms*— UAV, cell association, transport theory, delay, unmanned aerial vehicles, cellular networks.**

I. Introduction

THE use of unmanned aerial vehicles (UAVs) such as drones and balloons is an effective technique for improving the quality-of-service (QoS) of wireless cellular networks due to their inherent ability to create line-of-sight (LoS) communication links [1]–[6]. One of the important challenges in UAV-based communications is cell (or user) association. Sharma *et al.* [7] analyzed the user-UAV assignment for capacity enhancement of heterogeneous networks. However, this work is limited to the case in which users are uniformly distributed within a geographical area. Silva *et al.* [8] proposed a power-efficient cell association scheme while satisfying the rate requirement of users in cellular networks. However, the work in [8] does not consider the presence of UAVs and the adopted objective function does not account for network delay. In [9], the optimal deployment and cell association of UAVs are determined with the goal of minimizing the UAVs' transmit power while satisfying the users' rate requirements. However, the work in [9] mainly focused on the optimal deployment of the UAVs and does not address the cell association problem. In fact, none of the previous studies in [1]–[9] addressed the delay-optimal cell association problem considering both UAVs and terrestrial base stations, for any arbitrary distribution of users.

The main contribution of this letter is to introduce a novel framework for delay-optimal cell association in a cellular network in which both UAVs and terrestrial base stations (BSs) co-exist. In particular, given the locations of the UAVs and terrestrial BSs as well as any general spatial distribution of users, we find the optimal cell association by exploiting the framework of *optimal transport theory* [10]. Within the framework of optimal transport theory, one can address cell association problems for any general spatial distribution of users. For instance, the main advantage of optimal transport theory is that it allows deriving tractable solutions for a variety of cell association problems in wireless networks. In our problem, using optimal transport theory, we first prove the existence of the optimal solution to the cell association problem, and, then, we characterize the solution space. The results show that, our approach results in a significantly lower delay compared to a conventional signal strength-based association. Here, we note that, our work is different from [9] in terms of the system model, the objective function, the problem formulation as well as analytical results.

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II. System Model and Problem Formulation

Consider a geographical area $\mathcal{D} \subset \mathbb{R}^2$ in which K terrestrial BSs in set $\mathcal{K}$ are deployed to provide service for ground users that are spatially distributed according to a distribution $f(x, y)$ over the two-dimensional plane. In addition to the terrestrial BSs, M UAVs in set $\mathcal{M}$ are deployed as aerial base stations to enhance the capacity of the network. We consider a downlink scenario in which the BSs and the UAVs use a frequency division multiple access (FDMA) technique to service the ground users. The locations of BS $i \in \mathcal{K}$ and UAV $j \in \mathcal{M}$ are, respectively, given by (x_i, y_i, h_i) and $(x_j^{\text{uav}}, y_j^{\text{uav}}, h_j^{\text{uav}})$, with h_i and h_j^{uav} being the heights of BS i and UAV j. The maximum transmit powers of BS i and UAV j are P_i and P_j^{uav}. Let W_i and W_j be the total bandwidth available for each BS i and UAV j. Our performance metric is the *transmission delay*, which is defined as the time needed for transmitting a given number of bits. In this case, the delay is inversely proportional to the transmission rate. We use A_i and B_j to denote, respectively, the area (cell) partitions in which the ground users are assigned to BS i and UAV j. Hence, the geographical area is divided into $M+K$ disjoint partitions each of which is served by one of the BSs or the UAVs.

Given this model, our goal is to minimize the average network delay by optimal partitioning of the area. Based on the spatial distribution of the users, we determine the optimal cell associations to minimize the average network delay. Note that, the network delay significantly depends on the cell partitions due to the following reasons. First, the cell partitions determine the service area of each UAV and BS thus impacting the channel gain that each user experiences. Second, the number of users in each partition depends on the cell partitioning. In this case, since the total bandwidth is limited, the amount of bandwidth per user decreases as the number of users in a cell partition increases. Thus, users in crowded cell partitions achieve a lower throughput which results in a higher delay. Next, we present the channel models.

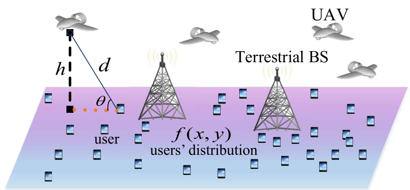


Fig. 1. Network model.

A. UAV-User and BS-User Path Loss Models

In UAV-to-ground communications, the probability of having LoS links to users depends on the locations, heights, and the number of obstacles, as well as the elevation angle between a given UAV and its served ground user. In our model, we consider a commonly used probabilistic path loss model provided by International Telecommunication Union (ITU-R), and the work in [6]. The path loss between UAV j and a user located at (x, y) is [6]:

$$\Lambda_j = \begin{cases} K_o^2 (d_j/d_o)^2 \mu_{\text{LoS}}, & \text{LoS link,} \\ K_o^2 (d_j/d_o)^2 \mu_{\text{NLoS}}, & \text{non-line-of-sight (NLoS) link,} \end{cases} \tag{1}$$

where $K_o = \left(\frac{4\pi f_c d_o}{c}\right)^2$, f_c is the carrier frequency, c is the speed of light, and d_o is the free-space reference distance. Also, μ_{LoS} and μ_{NLoS} are different attenuation factors considered for LoS and NLoS links. $d_j = \sqrt{(x - x_j^{\text{uav}})^2 + (y - y_j^{\text{uav}})^2 + h_j^{\text{uav}2}}$ is the distance between UAV j and an arbitrary ground user located at (x, y). For the UAV-user link, the LoS probability is [6]:

$$\mathbb{P}_{\text{LoS},j} = \alpha \left(\frac{180}{\pi} \theta_j - 15 \right)^{\gamma}, \quad \theta_j > \frac{\pi}{12}, \tag{2}$$

where $\theta_j = \sin^{-1}(\frac{h_j}{d_j})$ is the elevation angle (in radians) between the UAV and the ground user. Also, α and γ are constant values reflecting the environment impact. Note that, the NLoS probability is $\mathbb{P}_{\text{NLoS},j} = 1 - \mathbb{P}_{\text{LoS},j}$.

Considering $d_o = 1\,\text{m}$, the average path loss is $K_o {d_j}^2 \left[\mathbb{P}_{\text{LoS},j} \mu_{\text{LoS}} + \mathbb{P}_{\text{NLoS},j} \mu_{\text{NLoS}} \right]$. Therefore, the received signal power from UAV j considering an equal power allocation among its associated users will be:

$$\bar{P}_{r,j}^{\text{uav}} = P_j^{\text{uav}} / \left(N_j^{\text{uav}} K_o {d_j}^2 \left[\mathbb{P}_{\text{LoS},j} \mu_{\text{LoS}} + \mathbb{P}_{\text{NLoS},j} \mu_{\text{NLoS}} \right] \right), \tag{3}$$

where P_j^{uav} is the UAV's total transmit power, and $N_j^{\text{uav}} = N \iint_{B_j} f(x, y) \mathrm{d}x\mathrm{d}y$ is the average number of users associated with UAV j, with N being the total number of users. For the BS-user link, we use the traditional path loss model. In this case, the received signal power from BS i at user's location (x, y) will be:

$$P_{r,i} = P_i K_o^{-1} d_i^{-n} / N_i, \tag{4}$$

where d_i is the distance between BS i and a given user, $N_i = N \iint_{A_i} f(x, y) \mathrm{d}x\mathrm{d}y$ is the average number of users associated with BS i, and n is the path loss exponent.

B. Problem Formulation

Given the average received signal power in the UAV-user communication, the average throughput of a user located at (x, y) connecting to a UAV j can be approximated by:

$$C_j^{\text{uav}} = \frac{W_j}{N_j^{\text{uav}}} \log_2 \left(1 + \frac{\bar{P}_{r,j}^{\text{uav}}}{\sigma^2}\right), \tag{5}$$

where σ^2 is the noise power for each user which is linearly proportional to the bandwidth allocated to the user.

The throughput of the user if it connects to BS i is:

$$C_i = \frac{W_i}{N_i} \log_2 \left(1 + \frac{P_{r,i}}{\sigma^2}\right). \tag{6}$$

Now, let $\mathcal{L} = \mathcal{K} \cup \mathcal{M}$ be the set of all BSs and UAVs. Here, the location of each BS or UAV is denoted by $\boldsymbol{s}_k$, $k \in \mathcal{L}$. We also consider $D_k = \begin{cases} A_k, & \text{if } k \in \mathcal{K}, \\ B_k, & \text{if } k \in \mathcal{M}, \end{cases}$ denoting all the cell partitions, and $Q(\boldsymbol{v}, \boldsymbol{s}_k, D_k) = \begin{cases} b/C_k, & \text{if } k \in \mathcal{K}, \\ b/C_k^{\text{uav}}, & \text{if } k \in \mathcal{M}, \end{cases}$ where $\boldsymbol{v} = (x, y)$ is the 2D location of a given ground user, and b is the number of bits that must be transmitted to location $\boldsymbol{v}$. Then, our optimization problem that seeks to minimize the average network delay over the entire area will be:

$$\min_{D_k} \sum_{k \in \mathcal{L}} \int_{D_k} Q(\boldsymbol{v}, \boldsymbol{s}_k, D_k) f(x, y) \mathrm{d}x\mathrm{d}y, \tag{7}$$

$$\text{s.t.} \bigcup_{k \in \mathcal{L}} D_k = \mathcal{D}, \quad D_l \cap D_m = \emptyset \quad \forall l \neq m \in \mathcal{L}, \tag{8}$$

where both constraints in (8) guarantee that the cell partitions are disjoint and their union covers the entire area, $\mathcal{D}$.

III. Optimal Transport Theory for Cell Association

Given the locations of the BSs and the UAVs as well as the distribution of the ground users, we find the optimal cell association for which the average delay of the network is minimized. Let $g_k(z) = \frac{Nz}{W_k}$, with W_k being the bandwidth for each BS or UAV k and z is a generic argument. Also, we consider:

$$F(\boldsymbol{v}, \boldsymbol{s}_k) = \begin{cases} b/\log_2\left(1 + P_{r,k}(\boldsymbol{v}, \boldsymbol{s}_k)/\sigma^2\right), & \text{if } k \in \mathcal{K}, \\ b/\log_2\left(1 + \bar{P}_{r,j}^{\text{uav}}(\boldsymbol{v}, \boldsymbol{s}_k)/\sigma^2\right), & \text{if } k \in \mathcal{M}. \end{cases} \tag{9}$$

Now, the optimization problem in (7) can be rewritten as:

$$\min_{D_k} \sum_{k \in \mathcal{L}} \int_{D_k} \left[g_k \left(\int_{D_k} f(x, y) \mathrm{d}x\mathrm{d}y \right) F(\boldsymbol{v}, \boldsymbol{s}_k) \right] f(x, y) \mathrm{d}x\mathrm{d}y, \tag{10}$$

$$\text{s.t.} \bigcup_{k \in \mathcal{L}} D_k = \mathcal{D}, \quad D_l \cap D_m = \emptyset \quad \forall l \neq m \in \mathcal{L}, \tag{11}$$

where D_k is the cell partition of each BS or UAV k.

Solving the optimization problem in (10) is challenging and intractable due to various reasons. First, the optimization variables D_k, $\forall k \in \mathcal{L}$, are sets of continuous partitions which are mutually dependent. Second, $f(x, y)$ can be any generic function of x and y that leads to the complexity of the given two-fold integrations. To overcome these challenges, next, we model this problem by exploiting *optimal transport theory* [10] in order to characterize the solution.

Optimal transport theory [10] allows analyzing complex problems in which, for two probability measures f_1 and f_2

on $\Omega \subset \mathbb{R}^n$, one must find the optimal transport map T from f_1 to f_2 that minimizes the following function:

$$\min_{T} \int_{\Omega} c\left(x, T(x)\right) f_1(x) \mathrm{d}x; \;\; T: \Omega \rightarrow \Omega, \tag{12}$$

where $c(x, T(x))$ denotes the cost of transporting a unit mass from a location x to a location $T(x)$.

Our cell association problem can be modeled as a semi-discrete optimal transport problem. Here, the users follow a continuous distribution, and the base stations can be considered as discrete points. Then, we need to map the users to the BSs and UAVs such that the total cost function is minimized. In this case, the optimal cell partitions are directly determined by the optimal transport map [11]. Next, we prove the existence of the optimal solution to the problem in (10).

Theorem 1: The optimization problem in (10) admits an optimal solution given $N \neq 0$, and $\sigma \neq 0$.

Proof: Let $a_k = \int_{D_k} f(x, y)\mathrm{d}x\mathrm{d}y$, and for $\forall k \in \mathcal{L}$,

$$E = \left\{ \boldsymbol{a} = (a_1, a_2, ..., a_{K+M}) \in \mathbb{R}^{K+M}; a_k \geq 0, \sum_{k=1}^{K+M} a_k = 1 \right\}.$$

Now, considering $f(x, y) = f(\boldsymbol{v})$ and $c\left(\boldsymbol{v}, \boldsymbol{s}_k\right) = g_k(a_k) F\left(\boldsymbol{v}, \boldsymbol{s}_k\right)$, for any given vector $\boldsymbol{a}$, problem (10) can be considered as a classical semi-discrete optimal transport problem. First, we prove that $c\left(\boldsymbol{v}, \boldsymbol{s}\right)$ is a semi-continuous function. Considering the fact that $\boldsymbol{s}_k$ is discrete, we have: $\lim\limits_{(\boldsymbol{v},\boldsymbol{s}) \rightarrow (\boldsymbol{v}^*,\boldsymbol{s}_k)} F\left(\boldsymbol{v}, \boldsymbol{s}\right) = \lim\limits_{\boldsymbol{v} \rightarrow \boldsymbol{v}^*} F\left(\boldsymbol{v}, \boldsymbol{s}_k\right)$. Note that, given any $\boldsymbol{s}_k$, k belongs to only of $\mathcal{K}$ and $\mathcal{M}$ sets. Given $\boldsymbol{s}_k$, $F(\boldsymbol{v}, \boldsymbol{s}_k)$ is a continuous function of $\boldsymbol{v}$. Then, considering the fact that given a_k, $g_k(a_k)$ is constant, we have $\lim\limits_{(\boldsymbol{v},\boldsymbol{s}) \rightarrow (\boldsymbol{v}^*,\boldsymbol{s}_k)} g_k(a_k) F\left(\boldsymbol{v}, \boldsymbol{s}\right) = g_k(a_k) F\left(\boldsymbol{v}^*, \boldsymbol{s}_k\right)$. Therefore, $c(\boldsymbol{v}, \boldsymbol{s})$ is a continuous function and, hence, is also a lower semi-continuous function. Now, we use the following lemma from optimal transport theory:

Lemma 1: Consider two probability measures f and λ on $\mathcal{D} \subset \mathbb{R}^n$. Let f be continuous and $\lambda = \sum\limits_{k \in \mathbb{N}} a_k \delta_{s_k}$ be a discrete probability measure. Then, for any lower semi-continuous cost function, there exists an optimal transport map from f to λ for which $\int_{\mathcal{D}} c\left(x, T(x)\right) f(x) \mathrm{d}x$ is minimized [11].

Considering Lemma 1, for any $\boldsymbol{a} \in E$, the problem in (10) admits an optimal solution. Since E is a unit simplex in $\mathbb{R}^{M+K}$ which is a non-empty and compact set, the problem admits an optimal solution over the entire E. ■

Next, we characterize the solution space of (10).

Theorem 2: To acheive the delay-optimal cell partitions in (10), each user located at (x, y) must be assigned to the following BS (or UAV):

$$k = \underset{l \in \mathcal{L}}{\arg\min} \left\{ \frac{a_l}{W_l} F(\boldsymbol{v}_o, \boldsymbol{s}_l) \right\}. \tag{13}$$

Given (13), the optimal cell partition D_k includes all the points which are assigned to BS (or UAV) k.

Proof: As shown in Theorem 1, there exist optimal cell partitions D_k, $k \in \mathcal{L}$ which are the solutions to (10). Now, consider two partitions D_l and D_m, and a point $\boldsymbol{v}_o = (x_o, y_o) \in D_l$. Also, let $B_\epsilon(\boldsymbol{v}_o)$ be a ball with a center $\boldsymbol{v}_o$ and radius $\epsilon > 0$. We then generate the following new cell partitions $\widehat{D}_k$ (which are variants of the optimal partitions):

$$\begin{cases} \widehat{D}_l = D_l \backslash B_\varepsilon(\boldsymbol{v}_o), \\ \widehat{D}_m = D_m \cup B_\varepsilon(\boldsymbol{v}_o), \\ \widehat{D}_k = D_k, \quad k \neq l, m. \end{cases} \tag{14}$$

Let $a_\varepsilon = \int_{B_\varepsilon(\boldsymbol{v}_o)} f(x, y)\mathrm{d}x\mathrm{d}y$, and $\widehat{a}_k = \int_{\widehat{D}_k} f(x, y)\mathrm{d}x\mathrm{d}y$. Considering the optimality of D_k, $k \in \mathcal{L}$, we have:

$$\begin{aligned} &\sum_{k \in \mathcal{K}} \int_{D_k} g_k\left(a_k\right) F(\boldsymbol{v}, \boldsymbol{s}_k) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\qquad \overset{(a)}{\leq} \sum_{k \in \mathcal{K}} \int_{\widehat{D}_k} g_k\left(\widehat{a}_k\right) F(\boldsymbol{v}, \boldsymbol{s}_k) f(x, y)\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{15}$$

Now, canceling out the common terms in (15) leads to:

$$\begin{aligned} &\int_{D_l} g_l\left(a_l\right) F(\boldsymbol{v}, \boldsymbol{s}_l) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\quad + \int_{D_m} g_m\left(a_m\right) F(\boldsymbol{v}, \boldsymbol{s}_m) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\quad \leq \int_{D_m \cup B_\varepsilon(\boldsymbol{v}_o)} g_m\left(a_m + a_\varepsilon\right) F(\boldsymbol{v}, \boldsymbol{s}_m) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\quad + \int_{D_l \backslash B_\varepsilon(\boldsymbol{v}_o)} g_l\left(a_l - a_\varepsilon\right) F(\boldsymbol{v}, \boldsymbol{s}_l) f(x, y)\mathrm{d}x\mathrm{d}y, \\ &\int_{D_l} \left(g_l\left(a_l\right) - g_l\left(a_l - a_\varepsilon\right)\right) F(\boldsymbol{v}, \boldsymbol{s}_l) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\quad + \int_{B_\varepsilon(\boldsymbol{v}_o)} g_l\left(a_l - a_\varepsilon\right) F(\boldsymbol{v}, \boldsymbol{s}_l) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\quad \leq \int_{D_m} \left(g_m\left(a_m + a_\varepsilon\right) - g_m\left(a_m\right)\right) F(\boldsymbol{v}, \boldsymbol{s}_m) f(x, y)\mathrm{d}x\mathrm{d}y \\ &\quad + \int_{B_\varepsilon(\boldsymbol{v}_o)} g_m\left(a_m + a_\varepsilon\right) F(\boldsymbol{v}, \boldsymbol{s}_m) f(x, y)\mathrm{d}x\mathrm{d}y, \end{aligned} \tag{16}$$

where (a) comes from the fact that D_k, $\forall k \in \mathcal{L}$ are optimal and, hence, any variation of such optimal partitions, shown by $\widehat{D}_k$, cannot lead to a better solution. Now, we multiply both sides of the inequality in (16) by $\frac{1}{a_\epsilon}$, take the limit when $\epsilon \rightarrow 0$, and use the following equalities:

$$\lim_{\varepsilon \rightarrow 0} a_\varepsilon = 0, \tag{17}$$

$$\lim_{a_\varepsilon \rightarrow 0} \frac{g_l(a_l) - g_l(a_l - a_\varepsilon)}{a_\varepsilon} = g_l'(a_l), \tag{18}$$

$$\lim_{a_\varepsilon \rightarrow 0} \frac{g_m(a_m + a_\varepsilon) - g_m(a_m)}{a_\varepsilon} = g_m'(a_m), \tag{19}$$

then we have:

$$\begin{aligned} &g_l'\left(a_l\right) \int_{D_l} F(\boldsymbol{v}_o, \boldsymbol{s}_l) f(x, y)\mathrm{d}x\mathrm{d}y + g_l\left(a_l\right) F(\boldsymbol{v}_o, \boldsymbol{s}_l) \\ &\quad \leq g_m'\left(a_m\right) \int_{D_m} F(\boldsymbol{v}_o, \boldsymbol{s}_m) f(x, y)\mathrm{d}x\mathrm{d}y + g_m\left(a_m\right) F(\boldsymbol{v}_o, \boldsymbol{s}_m). \end{aligned} \tag{20}$$

Now, given $g_k(z) = \frac{Nz}{W_k}$, we can compute $g_l'(a_l) = \left.\frac{dg_l(z)}{dz}\right|_{z=a_l} = \frac{N}{W_k}$, then, using $a_k = \int_{D_k} f(x, y)\mathrm{d}x\mathrm{d}y$ leads to:

$$\begin{aligned} &\frac{N}{W_l} a_l F(\boldsymbol{v}_o, \boldsymbol{s}_l) + \frac{N a_l}{W_l} F(\boldsymbol{v}_o, \boldsymbol{s}_l) \\ &\qquad \leq \frac{N}{W_m} a_m F(\boldsymbol{v}_o, \boldsymbol{s}_m) + \frac{N a_m}{W_m} F(\boldsymbol{v}_o, \boldsymbol{s}_m), \\ &\text{as a result: } \frac{a_l}{W_l} F(\boldsymbol{v}_o, \boldsymbol{s}_l) \leq \frac{a_m}{W_m} F(\boldsymbol{v}_o, \boldsymbol{s}_m). \end{aligned} \tag{21}$$

Finally, (21) leads to (13) that completes the proof. ■

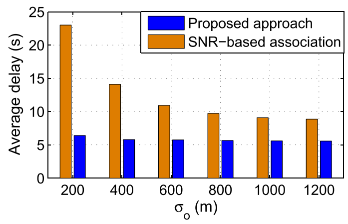


Fig. 2. Average network delay per 1Mb data transmission.

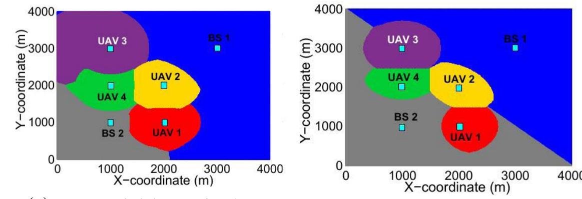
Theorem 2 provides a precise cell association rule for ground users that are distributed following any general distribution $f(x, y)$. In fact, the inequality given in (21) represents the condition under which the user is assigned to a BS or UAV l. For the special case of a uniform distribution of the users, the result in Theorem 2 leads to the classical SNR-based association in which users are assigned to base stations that provide strongest signal. From Theorem 2, we can see that there is a mutual dependence between a_l and D_l (i.e. cell association), $\forall l \in \mathcal{L}$. To solve the equation given in Theorem 2, we adopt an iterative approach which is shown to converge to the global optimal solution [11]. In this case, we start with an initial cell partition (e.g. Voronoi diagram), and iteratively update the cell partition based on Theorem 2.

IV. Simulation Results and Analysis

For our simulations, we consider an area of size 4 km × 4 km in which 4 UAVs and 2 macrocell base stations are deployed based on a traditional grid-based deployment. The ground users are distributed according to a truncated Gaussian distribution with a standard deviation σ_o. This type of distribution is suitable to model a hotspot area. The simulation parameters are given as follows. f_c =2 GHz, transmit power of each BS is 40 W, and transmit power of each UAV is 1 W. Also, $N = 300$, $W_j = W_i = 1$ MHz, and the noise power spectral density is -170 dBm/Hz. We consider a dense urban environment with $n = 3$, $\mu_{\text{LoS}} = 3$ dB, $\mu_{\text{NLoS}} = 23$ dB, $\alpha = 0.36$, and $\gamma = 0.21$ [6]. The heights of each UAV and BS are, respectively, 200 m and 20 m [4], [6], [7]. All statistical results are averaged over a large number of independent runs.

In Fig. 2, we compare the delay of the proposed cell association with the traditional SNR-based association. We consider a truncated Gaussian distribution with a center (1300 m,1300 m), and σ_o varying from 200 m to 1200 m. Lower values of σ_o correspond to scenarios in which users are more concentrated around the hotspot center. Fig. 2 shows that the proposed cell association significantly outperforms the SNR-based association in terms of the average delay. For low σ_o values, the average delay decreases by 72% compared to the SNR-based association. This is due to the fact that, in the proposed approach, the impact of network congestion is also taken into account. Hence, the proposed approach avoids creating highly loaded cells. In contrast, an SNR-based association can yield highly loaded cells. As a result, in the congested cells, each user will receive a low amount of bandwidth that leads a low transmission rate or equivalently high delay. In fact, compared to the SNR-based association case, our approach is more robust against network congestion and its performance is significantly less affected by changing σ_o.

As an illustrative example, Fig. 3 shows the locations of the BSs and UAVs as well as the cell partitions obtained using the SNR-based association and the proposed delay-optimal association. In this case, users are distributed based



(a) Proposed delay-optimal cell partitions.

(b) SNR-based association.

Fig. 3. Cell partitions associated to UAVs and BSs given the non-uniform spatial distribution of users.

on a 2D truncated Gaussian distribution with mean values of (1300 m,1300 m), and $\sigma_o = 1000$ m. As shown in Fig. 3, the size and shape of cells are different in these two association approaches. For instance, the gray cell partition in the proposed approach is smaller than the SNR-based case. In fact, the gray partition in the SNR-based approach is highly congested and, consequently, its size is reduced in the proposed approach so as to decrease the congestion as well as the delay.

V. Conclusion

In this letter, we have proposed a novel framework for delay-optimal cell association in UAV-enabled cellular networks. In particular, to minimize the average network delay based on the users' distribution, we have exploited optimal transport theory to derive the optimal cell associations for UAVs and terrestrial BSs. Our results have shown that, the proposed cell association approach results in a significantly lower network delay compared to an SNR-based association.

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