Optimal Turbo-BLAST Detection of MIMO-OFDM Systems With Imperfect Channel Estimation

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***Abstract*— Under imperfect channel estimation, we propose an improved iterative detector for multiple-input multiple-output (MIMO) systems employing the simple spatial multiplexing or V-BLAST space-time scheme and orthogonal frequency-division multiplexing (OFDM). The detector is based on soft interference cancellation and linear minimum mean-square error filtering which takes into account the error on channel estimation. Numerical results, presented for the case of Rayleigh block fading, demonstrate that our proposed detector can achieve significant performance improvement compared to the classically-used *mismatched* detector. This improvement is obtained with almost no increase in the receiver complexity.**

I. Introduction

It is well known that multiple-input multiple-output (MIMO) systems are a promising solution for high-speed, spectrally efficient, and reliable wireless communication. In practice, in order to combat the frequency selectivity of wireless channels occurred at high data-rate transmissions, orthogonal frequency-division multiplexing (OFDM) is employed. The combination of MIMO and OFDM (MIMO-OFDM) has been proposed as a promising technology for the future generation of wireless communication systems [1]. Several recent standards such as IEEE 802.11a and IEEE 802.16, that use OFDM in a packet based communication, employ bit-interleaving combined with convolutional channel coding. In the literature, this scheme is referred to as bit-interleaved coded modulation (BICM) [2], which is an efficient and simple technique for profiting from the channel frequency diversity.

For coherent signal detection at the receiver, we should acquire the information on the transmission channel. In most applications in wireless communication, the channel time variations are slowly enough so that we can consider it almost time invariant during the transmission of a frame. To obtain the channel state information at the receiver (CSIR), a usually-used approach is to send in each frame, some known training (also called pilot) symbols from the transmitter, based on which the receiver estimates the channel before proceeding to the detection of data symbols. This method of obtaining CSIR is usually called pilot symbol assisted modulation (PSAM) [3]. Obviously, due to the finite number of pilot symbols and noise, in practice, the receiver can only obtain an *imperfect* estimate of the channel.

It is well known that the performance of coherent data detection is greatly affected by the quality of channel estimation. Notice that in this work, we consider pilot-only-based channel estimation. Instead of using a semi-blind estimator that may increase considerably the receiver complexity, we propose to modify the signal detection by taking into account the channel estimation errors (CEE). Actually, the *classical* signal detection consists in assuming the estimated channel as to be perfect and to use it for signal detection. However, although simple, this is a sub-optimal approach that does not take into account the CEE in signal detection [4]. We will refer to it in this paper as the *mismatched* approach.

Here, we consider iterative (turbo) detection at the receiver that has been shown to be an efficient technique for signal detection in the presence of channel coding. This scheme has been employed, for instance, in [5], [6] for coded MIMO systems and consists of the combination of a MIMO detector (also called demapper) and a soft-input soft-output (SISO) channel decoder, exchanging soft information with each other through several iterations. A practical concern for the implementation of turbo-detectors for MIMO systems is the receiver complexity. For instance, for the maximum a posteriori (MAP) detection, that is the optimal solution under perfect CSIR [7], this complexity grows exponentially with the number of transmit antennas and the signal constellation size. For this reason, suboptimal detection techniques are usually preferred to MAP detection. One interesting suboptimal detector is that based on soft parallel interference cancellation (PIC) and linear minimum mean-square error (MMSE) filtering. This scheme was first proposed in [8] in the context of multiuser detection, and later applied to MIMO systems in [9], [10], for instance.

Our aim in this work, is to propose a modified iterative detector, based on soft-PIC, for the case of imperfect channel estimation obtained by PSAM. To this end, we propose a Bayesian framework based on the *a posteriori* probability density function (pdf) of the perfect channel, conditioned on its estimate. In this way, we can formulate any detection problem by considering the average of the cost function that would be used if the channel was perfectly known, over the channel uncertainty. Using this approach, we propose a modified PIC detector that takes into account the imperfect channel estimate in the formulation of the instantaneous linear MMSE filter. We will refer to it as *improved* turbo-PIC in this paper.

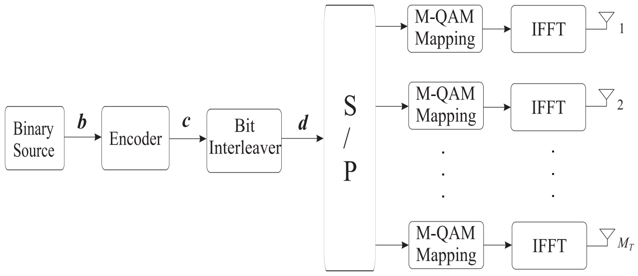

Fig. 1. Coded layered space-time OFDM transmission scheme.

The outline of this paper is as follows. In Section II, we describe the MIMO-OFDM channel and its pilot based estimation. In Section III, we formulate a general Bayesian framework for improved detection under imperfect channel estimation. Using this in Section IV, we derive the improved turbo-PIC detector, in the presence of partial CSIR. Section V illustrates, via simulations, a comparative performance study of the proposed detector, and Section VI concludes the paper.

Notational conventions are as follows. Upper and lower case bold symbols are used to denote matrices and vectors, respectively; $\mathbb{I}_N$ represents an $(N \times N)$ identity matrix; $\mathbb{E}_{\mathbf{x}}[.]$ refers to expectation with respect to the random vector $\mathbf{x}$; $|.|$, $||.||$, and $\mathrm{Tr}(.)$ denote matrix determinant, Frobenius norm, and matrix trace, respectively; $(.)^T$, $(.)^{\mathcal{H}}$, and $(.)^*$ denote vector transpose, Hermitian transpose, and conjugation, respectively.

II. System Model And Channel Estimation

We consider a single-user MIMO-OFDM transmission system with the number of subcarriers equal to M. We assume perfect interleaving and frequency non-selective channel corresponding to each subcarrier. The system uses M_T transmit and M_R receive antennas. Figure 1 depicts the BICM coding scheme performed at the transmitter. The binary data sequences $\boldsymbol{b}$ are encoded by a non-recursive non-systematic convolutional (NRNSC) code before being interleaved by a quasi-random interleaver. The output bits $\boldsymbol{d}$ are multiplexed to M_T sub-streams and mapped to complex $\mathrm{M_c}$-QAM symbols before passing to the OFDM modulator and being transmitted on the M_T antennas.

Let $\boldsymbol{s}$ be the $(MM_T \times 1)$ vector containing the OFDM symbols transmitted simultaneously over the M_T antennas. The symbols are assumed to be independent identically distributed (i.i.d.) with zero mean and unit covariance matrix $\mathbf{\Sigma}_s = \mathbb{E}[\boldsymbol{s}\boldsymbol{s}^{\mathcal{H}}] = \mathbb{I}_{MM_T}$. The received vector $\boldsymbol{y}$ at a given time index can be written as

$$\boldsymbol{y} = \mathbf{H}\,\boldsymbol{s} + \boldsymbol{z}, \tag{1}$$

where $\mathbf{H}$ is a $(MM_R \times MM_T)$ block diagonal channel matrix, containing the frequency responses of the M MIMO channels, and the noise vector $\boldsymbol{z}$ is assumed to be a zero-mean circularly symmetric complex Gaussian (ZMCSCG) random vector with the covariance matrix $\mathbf{\Sigma}_z \triangleq \mathbb{E}(\boldsymbol{z}\boldsymbol{z}^{\mathcal{H}}) = \sigma_z^2\,\mathbb{I}_{MM_R}$. We assume that the channel is invariant over a frame of L OFDM symbols. The interleaver size N_I is then equal to LMM_TB, where B is the number of bits per symbol according to the signal constellation, i.e., $B = \log_2 M_c$. With this assumption, corresponding to each frame, we consider a new and independent realization of $\mathbf{H}$.

The MIMO-OFDM channel can, in fact, be decoupled into M frequency flat MIMO channels by exploiting the block diagonal structure in (1). Now, corresponding to a subcarrier k, the channel input-output relationship can be written as follows:

$$\boldsymbol{y}_k = \mathbf{H}_k\,\boldsymbol{s}_k + \boldsymbol{z}_k \qquad k = 1, ..., M. \tag{2}$$

We have in fact, $\mathbf{H} = \mathrm{diag}[\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_M]$, $\boldsymbol{y}^T = [\boldsymbol{y}_1^T, \ldots, \boldsymbol{y}_M^T]$, $\boldsymbol{s}^T = [\boldsymbol{s}_1^T, \ldots, \boldsymbol{s}_M^T]$, and $\boldsymbol{z}^T = [\boldsymbol{z}_1^T, \ldots, \boldsymbol{z}_M^T]$. Later, we will make the assumption of uncorrelated Rayleigh distribution for $\mathbf{H}_k$.

A. Pilot Based Channel Estimation:

Consider the estimation of the channel matrix $\mathbf{H}_k$. We devote a number of N_P channel-uses to the transmission of some pilot vectors $\boldsymbol{s}_{P,i}$, $(i = 1, ..., N_P)$. Let us constitute the $(M_T \times N)$ matrix $\mathbf{S}_P$ by stacking in its columns the pilot vectors, i.e., $\mathbf{S}_P = [\boldsymbol{s}_{P,1}|...|\boldsymbol{s}_{P,N_P}]$. According to (2), corresponding to the channel training interval, we receive:

$$\mathbf{Y}_P = \mathbf{H}_k\,\mathbf{S}_P + \mathbf{Z}_P. \tag{3}$$

The definition of $\mathbf{Y}_P$ and $\mathbf{Z}_P$ is similar to that of $\mathbf{S}_P$. We denote by E_P the average energy of the training symbols. We have:

$$E_P = \frac{1}{N_P M_T}\mathrm{Tr}\big(\mathbf{S}_P\mathbf{S}_P^{\mathcal{H}}\big). \tag{4}$$

The least-square (LS) estimate of $\mathbf{H}_k$ is obtained by minimizing $||\mathbf{Y}_P - \mathbf{H}_k\,\mathbf{S}_P||^2$ with respect to $\mathbf{H}_k$ and coincides with the maximum-likelihood (ML) estimate. We have:

$$\widehat{\mathbf{H}}_k^{\mathrm{ML}} = \mathbf{Y}_P\,\mathbf{S}_P^{\mathcal{H}}\,(\mathbf{S}_P\mathbf{S}_P^{\mathcal{H}})^{-1}. \tag{5}$$

Let us denote by $\mathcal{E}$ the matrix of the estimation errors. We have:

$$\widehat{\mathbf{H}}_k^{\mathrm{ML}} = \mathbf{H}_k + \mathcal{E}\ , \quad \text{with} \quad \mathcal{E} = \mathbf{Z}_P\mathbf{S}_P^{\mathcal{H}}\,(\mathbf{S}_P\mathbf{S}_P^{\mathcal{H}})^{-1}. \tag{6}$$

It is known that the best channel estimate is obtained with mutually orthogonal training sequences that results in uncorrelated estimation errors. In other words, we choose $\mathbf{S}_P$ with orthogonal rows such that:

$$\mathbf{S}_P\mathbf{S}_P^{\mathcal{H}} = N_P E_P\ \mathbb{I}_{M_T}. \tag{7}$$

In this way, the j-th row $\mathcal{E}_j$ of the estimation error matrix $\mathcal{E}$ has the covariance matrix:

$$\mathbf{\Sigma}_{\mathcal{E},j} = \mathbb{E}\big[\boldsymbol{\mathcal{E}}_j^{\mathcal{H}}\boldsymbol{\mathcal{E}}_j\big] = \sigma_{\mathcal{E},k}^2\ \mathbb{I}_{M_T}, \text{ where } \sigma_{\mathcal{E},k}^2 = \frac{\sigma_z^2}{N_P E_P}\ \cdot \tag{8}$$

Let us now make the *a priori* assumption of uncorrelated Rayleigh distribution for the channel $\mathbf{H}_k$, according to which,

$$\mathbf{H}_k \sim \mathcal{CN}(\mathbf{0}, \mathbb{I}_{M_T} \otimes \mathbf{\Sigma}_{H,k}).$$

Here, $\mathcal{CN}$ denotes complex Gaussian distribution, $\otimes$ stands for the Kronecker product, and $\mathbf{\Sigma}_{H,k}$ is an $(M_R \times M_R)$ diagonal matrix with equal diagonal entries of σ_h^2. Based on this model, we can derive the *posterior* distribution of the perfect channel matrix, conditioned on its ML estimate, as [11]:

$$p\left(\mathbf{H}_k|\widehat{\mathbf{H}}_k^{\mathrm{ML}}\right) = \mathcal{CN}(\mathbf{\Sigma}_\Delta \widehat{\mathbf{H}}_{\mathrm{ML}}, \mathbb{I}_{M_T} \otimes \mathbf{\Sigma}_\Delta \mathbf{\Sigma}_\mathcal{E}), \tag{9}$$

where

$$\mathbf{\Sigma}_\Delta = \mathbf{\Sigma}_{H,k}(\mathbf{\Sigma}_\mathcal{E} + \mathbf{\Sigma}_{H,k})^{-1} = \delta \; \mathbb{I}_{M_R} \tag{10}$$

and

$$\delta = \frac{\sigma_h^2}{(\sigma_h^2 + \sigma_{\mathcal{E},k}^2)} \; . \tag{11}$$

The availability of the estimation error distribution is an interesting feature of pilot assisted channel estimation that we used to derive the posterior distribution (9). This distribution constitutes a Bayesian framework which will be exploited in the following, for the design of an appropriate detector under imperfect channel estimation.

III. Detector Design In The Presence Of Channel Estimation Errors

We now provide an improved detection rule that takes into account the imperfect available CSIR. To this end, we consider the model (2) and denote by $f(\boldsymbol{y}_k, \boldsymbol{s}_k, \mathbf{H}_k)$ the cost function that would let us to decide in favor of a particular $\boldsymbol{s}_k$ at the receiver if the channel was perfectly known. We note that depending on the detection criterion, $f(\boldsymbol{y}_k, \boldsymbol{s}_k, \mathbf{H}_k)$ can be the posterior pdf $p(\boldsymbol{s}_k|\boldsymbol{y}_k, \mathbf{H}_k)$, the likelihood function $W(\boldsymbol{y}_k|\mathbf{H}_k, \boldsymbol{s}_k)$, the mean-square error $\mathbb{E}[||\boldsymbol{y}_k - \mathbf{H}_k\boldsymbol{s}_k||^2]$, etc. Under a pilot based channel estimation characterized by the posterior pdf of (9), we propose a detector based on the minimization of a new cost function defined as:

$$\widetilde{f}(\boldsymbol{y}_k, \boldsymbol{s}_k, \widehat{\mathbf{H}}_k) = \mathbb{E}_{\mathbf{H}_k|\widehat{\mathbf{H}}_k}\left\{f(\boldsymbol{y}_k, \boldsymbol{s}_k, \mathbf{H}_k)\big|\widehat{\mathbf{H}}_k\right\}. \tag{12}$$

We note that the detector minimizing (12) is an alternative to the sub-optimal mismatched detector, which is based on the minimization of the cost function $f(\boldsymbol{y}_k, \boldsymbol{s}_k, \widehat{\mathbf{H}}_k)$. This latter is obtained by using the estimated channel $\widehat{\mathbf{H}}_k$ in the same metric that would be applied if the channel was perfectly known, i.e., $f(\boldsymbol{y}_k, \boldsymbol{s}_k, \mathbf{H}_k)$. The proposed approach in (12), differs from the mismatched detection on the conditional expectation $\mathbb{E}_{\mathbf{H}|\widehat{\mathbf{H}}}[.]$, which provides a robust design by averaging the cost function $f(\boldsymbol{y}_k, \boldsymbol{s}_k, \mathbf{H}_k)$ over all realizations of channel uncertainty.

IV. Iterative Detector Formulation

Now, we consider soft iterative detection of BICM MIMO-OFDM under imperfect CSIR. As shown in Fig. 2, the receiver principally consists of a bunch of demappers and a SISO channel decoder. Let $d_k^{i,m}$ be the m-th bit corresponding to the symbol s_k, transmitted from the i-th antenna and on the k-th subcarrier ; $m = 1, 2, ..., B$. We denote by $L(d_k^{i,m})$ the log-likelihood ratio (LLR) of the bit $d_k^{i,m}$ at the output of the MIMO demapper. Conditioned to perfect CSIR, $L(d_k^{i,m})$ is given by:

$$L(d_k^{i,m}) = \log \frac{P_{\mathrm{dem}}(d_k^{i,m} = 1|\boldsymbol{y}_k, \mathbf{H}_k)}{P_{\mathrm{dem}}(d_k^{i,m} = 0|\boldsymbol{y}_k, \mathbf{H}_k)}, \tag{13}$$

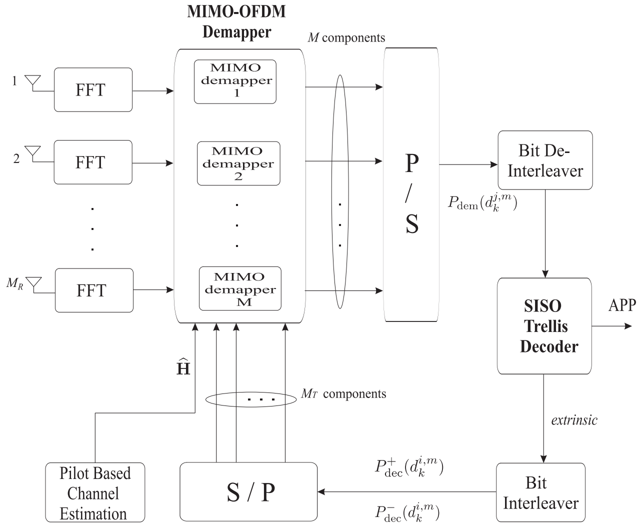

Fig. 2. Structure of MIMO-OFDM BICM receiver.

where the soft information $P_{\mathrm{dem}}(d_k^{i,m}|\boldsymbol{y}_k, \mathbf{H}_k)$ is the probability of transmission of $d_k^{i,m}$, evaluated at the demapper.

A. MAP detection

Let us first recall the formulation of the MAP detector that is the optimal detector in the sense of the error probability. This is provided for the sake of performance and complexity comparison with PIC detection which is presented in the next subsection. Let $\mathcal{S}$ be the set of all possible symbols $\boldsymbol{s}_k$, that we partition into $\mathcal{S}_0^m$ and $\mathcal{S}_1^m$, containing the symbols corresponding to the m-th bit of "0" and "1", respectively. We have

$$L(d_k^{i,m}) = \log \frac{\displaystyle\sum_{\boldsymbol{s}_k \in \mathcal{S}_1^m} W(\boldsymbol{y}_k|\mathbf{H}_k, \boldsymbol{s}_k) \prod_{\substack{n=1\\ n\neq m}}^{BM_T} P_{\mathrm{dec}}^1(d_k^{i,n})}{\displaystyle\sum_{\boldsymbol{s}_k \in \mathcal{S}_0^m} W(\boldsymbol{y}_k|\mathbf{H}_k, \boldsymbol{s}_k) \prod_{\substack{n=1\\ n\neq m}}^{BM_T} P_{\mathrm{dec}}^0(d_k^{i,n})} \tag{14}$$

where $P_{\mathrm{dec}}^1(d_k^{i,n})$ and $P_{\mathrm{dec}}^0(d_k^{i,n})$ are *extrinsic* information coming from the SISO decoder.

The computational complexity of the MAP detector becomes prohibitively large for large size constellations and/or a large number of transmit antennas, as the sets $\mathcal{S}_1^m$ and $\mathcal{S}_0^m$ in (14) contain $2^{(BM_T-1)}$ vectors $\boldsymbol{s}_k$, each. For such cases, the suboptimal soft-PIC detector would make a good compromise between complexity and performance. In what follows, we first recall the formulation of soft-PIC and then adapt it to the case of imperfect channel estimation at the receiver.

B. Soft-PIC Detection

The general block diagram of Fig. 2 still applies to the turbo-PIC detector. Here, to detect the symbol transmitted from a given antenna, we first make use of the soft information available from the SISO channel decoder to cancel (or to say better, to reduce) the interfering signals corresponding to other transmit antennas. At the first iteration where this information is not available, we perform a classical MMSE filtering.

1) Soft-PIC detection under perfect CSIR: Let us consider the transmitted vector $\boldsymbol{s}_k = [s_k(1), ..., s_k(M_T)]^T$ and assume that we are interested in the detection of the i-th symbol $s_k(i)$. We start by evaluating soft-estimates of the interfering symbols $s_k(j)$ from the SISO decoder as

$$\tilde{s}_k(j) = \sum_{j=1}^{2^B} s_k(j)\, P[s_k(j)] \quad \text{for all } j \neq i \tag{15}$$

where $P[s_k(j)]$, the probability of the transmission of $s_k(j)$, is calculated from the probabilities of its corresponding bits $P_{\text{dec}}(d_k^{j,n})$ at the decoder output: $P[s_k(j)] \propto \prod_{n=1}^{B} P_{\text{dec}}(d_k^{j,n})$. We further define

$$\underline{\tilde{\boldsymbol{s}}}_k(i) \triangleq [\tilde{s}_k(1), ..., \tilde{s}_k(i-1), 0, \tilde{s}_k(i+1), ..., \tilde{s}_k(M_T)]^T, \tag{16}$$

$$\underline{\hat{\boldsymbol{s}}}_k(i) \triangleq [e_k(1), ..., e_k(i-1), s_k(i), e_k(i+1), ..., e_k(M_T)]^T, \tag{17}$$

where $e_k(i) = s_k(i) - \tilde{s}_k(i)$. For each constellation symbol $s_k(i)$, a soft interference cancellation can be performed on the received signal $\boldsymbol{y}_k$ as

$$\begin{aligned} \underline{\boldsymbol{y}}_k(i) &= \boldsymbol{y}_k - \mathbf{H}_k\, \underline{\tilde{\boldsymbol{s}}}_k(i) \\ &= \mathbf{H}_k\, \underline{\hat{\boldsymbol{s}}}_k(i) + \boldsymbol{z}_k(i), \quad \text{for } i = 1, ..., M_T. \end{aligned} \tag{18}$$

Except under perfect prior information on the symbols which leads to $\tilde{s}_k(j) = s_k(j)$ for all $j \neq i$, there is always a residual interference in the signal $\underline{\boldsymbol{y}}_k(i)$. In order to reduce this interference, an instantaneous linear MMSE filter $\boldsymbol{w}_k(i)$ is applied to $\underline{\boldsymbol{y}}_k(i)$ that minimizes the mean square error between the symbol $s_k(i)$ and the filter output $r_k(i)$:

$$r_k(i) = \boldsymbol{w}_k(i)^{\mathcal{H}} \underline{\boldsymbol{y}}_k(i), \tag{19}$$

where

$$\boldsymbol{w}_k(i) = \underset{\boldsymbol{w} \in \mathbb{C}^{M_R}}{\arg\min}\ \mathbb{E}_{\boldsymbol{s}_k, \boldsymbol{z}_k} \left[\left| s_k(i) - \boldsymbol{w}_k(i)^{\mathcal{H}} \underline{\boldsymbol{y}}_k(i) \right|^2 \right]. \tag{20}$$

It is easy to see that the minimization problem in (20) leads to the filter [8], [9]:

$$\boldsymbol{w}_k(i) = \left(\mathbf{H}_k\, \boldsymbol{\Lambda}_{\underline{\hat{\boldsymbol{s}}}_k(i)}\, \mathbf{H}_k^{\mathcal{H}} + \sigma_z^2 \mathbb{I}_{M_R} \right)^{-1} \sigma_s^2\, \mathbf{h}_k(i) \tag{21}$$

where $\boldsymbol{\Lambda}_{\underline{\hat{\boldsymbol{s}}}_k(i)} = \mathbb{E}[\hat{\boldsymbol{s}}_k(i) \hat{\boldsymbol{s}}_k(i)^{\mathcal{H}}]$ is the covariance matrix of $\underline{\hat{\boldsymbol{s}}}_k(i)$, $\sigma_s^2 = \mathbb{E}[|s_k(i)|^2]$, and $\mathbf{h}_k(i)$ is the i-th column of the channel matrix $\mathbf{H}_k$. In (21), the covariance matrix $\boldsymbol{\Lambda}_{\underline{\hat{\boldsymbol{s}}}_k(i)}$ is given by

$$\begin{aligned} \boldsymbol{\Lambda}_{\underline{\hat{\boldsymbol{s}}}_k(i)} = \text{diag}\Big(& \mathbb{E}[|e_k(1)|^2], ..., \mathbb{E}[|e_k(i-1)|^2], \sigma_s^2, \\ & \mathbb{E}[|e_k(i+1)|^2], ..., \mathbb{E}[|e_k(M_T)|^2] \Big), \end{aligned} \tag{22}$$

where $\mathbb{E}[|e_k(j)|^2]$ can be computed from the SISO decoder as:

$$\mathbb{E}[|e_k(j)|^2] = \sum_{s_k(j) \in \mathcal{S}} |s_k(j) - \tilde{s}_k(j)|^2\, P[s_k(j)] \quad \text{for } j \neq i. \tag{23}$$

Note that the off-diagonal entries in (22) have been neglected to reduce the complexity without generating significant performance loss [10].

2) PIC detection under imperfect CSIR: As we see from (18) and (21), the interference suppression and filtering require the channel $\mathbf{H}_k$. As the receiver has only an imperfect channel estimate $\widehat{\mathbf{H}}_k$, a sub-optimal mismatched solution would consist in replacing $\mathbf{H}_k$ and $\mathbf{h}_k(i)$ in (18) and (21) by their estimates $\widehat{\mathbf{H}}_k$ and $\widehat{\mathbf{h}}_k(i)$, respectively. In what follows, we propose a novel *improved* PIC detector under imperfect CSIR. To this end, we use the Bayesian framework of (12) and make two modifications to the detector described above.

The first proposed modification concerns the design of the filter $\boldsymbol{w}_k(i)$ in (20). Since the cost function $f(\boldsymbol{y}_k, s_k(i), \mathbf{H}_k) = \mathbb{E}\left[\left| s_k(i) - \boldsymbol{w}_k(i)^{\mathcal{H}}\ \underline{\boldsymbol{y}}_k(i) \right|^2 \right]$ is a function of the perfect channel $\mathbf{H}_k$ via $\boldsymbol{y}_k$, we propose a modified filter $\widetilde{\boldsymbol{w}}_k(i)$, chosen to minimize the average of the mean square error over all realizations of the channel uncertainty. According to (12), we propose the following filter design:

$$\begin{aligned} \widetilde{\boldsymbol{w}}_k(i) &= \underset{\widetilde{\boldsymbol{w}} \in \mathbb{C}^{M_T}}{\arg\min}\ \mathbb{E}_{\mathbf{H}_k, \boldsymbol{s}_k, \boldsymbol{z}_k} \left\{ \left| s_k(i) - \widetilde{\boldsymbol{w}}_k(i)^{\mathcal{H}} \underline{\boldsymbol{y}}_k(i) \right|^2 \middle| \widehat{\mathbf{H}}_k \right\} \\ &= \underset{\widetilde{\boldsymbol{w}} \in \mathbb{C}^{M_T}}{\arg\min}\ \mathbb{E}_{\mathbf{H}_k | \widehat{\mathbf{H}}_k} \left\{ \mathbb{E}_{\boldsymbol{s}_k, \boldsymbol{z}_k} \left[\left| s_k(i) - \widetilde{\boldsymbol{w}}_k(i)^{\mathcal{H}} \underline{\boldsymbol{y}}_k(i) \right|^2 \right] \right\} \end{aligned} \tag{24}$$

where in the latter expression, we have assumed the independence between $\mathbf{H}_k$, $\boldsymbol{s}_k$ and $\boldsymbol{z}_k$. After some algebraic manipulations, we get the modified filter $\widetilde{\boldsymbol{w}}_k(i)$, directly as a function of $\widehat{\mathbf{H}}_k$ and $\widehat{\mathbf{h}}_k$:

$$\widetilde{\boldsymbol{w}}_k(i) = \left(\delta\, \sigma_{\mathcal{E},k}^2 \text{Tr}(\boldsymbol{\Lambda}_{\underline{\hat{\boldsymbol{s}}}_k(i)}) + \delta^2 \widehat{\mathbf{H}}_k\, \boldsymbol{\Lambda}_{\underline{\hat{\boldsymbol{s}}}_k(i)}\, \widehat{\mathbf{H}}_k^{\mathcal{H}} + \sigma_z^2 \mathbb{I}_{M_R} \right)^{-1} \sigma_s^2 \delta\, \widehat{\mathbf{h}}_k(i). \tag{25}$$

To get more insight on the proposed detector, let us consider the ideal case where perfect channel knowledge is available at the receiver, i.e., $\widehat{\mathbf{H}}_k = \mathbf{H}_k$ and $\sigma_{\mathcal{E},k}^2 = 0$. We note that in this case, the posterior pdf (9) reduces to a Dirac delta function, and consequently from (25), the two filters $\widetilde{\boldsymbol{w}}_k(i)$ and $\boldsymbol{w}_k(i)$ coincide. Similarly, under near-perfect CSIR, obtained either when $\sigma_{\mathcal{E},k}^2 \to 0$ or when $N_P \to \infty$, we have $\delta \to 1$, $\sigma_{\mathcal{E},k}^2 \to 0$, and the filter $\widetilde{\boldsymbol{w}}_k(i)$ gives a similar expression as $\boldsymbol{w}_k(i)$ in (21). However, in the presence of CEE, the proposed improved and mismatched detectors become different due to the inherent averaging in (24), which provides a robust design that adapts itself to the channel estimate available at the receiver.

Our second modification concerns the application of the derived filter $\widetilde{\boldsymbol{w}}_k(i)$ to the received signal $\underline{\boldsymbol{y}}_k(i)$. Since the latter is a function of the perfect channel, we propose to apply the MMSE filter of (25) to a modified received signal, evaluated from (18) as

$$\begin{aligned}\widetilde{\underline{\boldsymbol{y}}}_k(i) &= \widetilde{\mathbf{H}}_k\,\hat{\underline{\boldsymbol{s}}}_k(i) + \boldsymbol{z}_k(i), \\ &= \delta\,\widehat{\mathbf{H}}_k\,\hat{\underline{\boldsymbol{s}}}_k(i) + \boldsymbol{z}_k(i), \quad \text{for } i = 1, ..., M_T, \end{aligned} \tag{26}$$

where

$$\widetilde{\mathbf{H}}_k = \mathbb{E}_{\mathbf{H}_k|\widehat{\mathbf{H}}_k}[\mathbf{H}_k] = \delta\,\widehat{\mathbf{H}}_k \ . \tag{27}$$

Now, by applying the modified filter $\widetilde{\boldsymbol{w}}_k(i)$ to $\widetilde{\underline{\boldsymbol{y}}}_k(i)$ in (26), the output of the improved MMSE detector is obtained as

$$\widetilde{r}_k(i) = \widetilde{\boldsymbol{w}}_k(i)^{\mathcal{H}}\ \widetilde{\underline{\boldsymbol{y}}}_k(i) = \underbrace{\delta\,\widetilde{\boldsymbol{w}}_k(i)^{\mathcal{H}}\widehat{\mathbf{h}}_k(i)}_{\mu_{k,i}}\,s_k(i) + \underbrace{\sum_{j\neq i}\delta\,\widetilde{\boldsymbol{w}}_k(i)^{\mathcal{H}}\widehat{\mathbf{h}}_k(j)e_k(j) + \widetilde{\boldsymbol{w}}_k(i)^{\mathcal{H}}\boldsymbol{z}_k(i)}_{\eta_{k,i}} \tag{28}$$

where $\eta_{k,i}$ is the interference-plus-noise affecting the output of the soft instantaneous MMSE filter $\widetilde{r}_k(i)$. It is shown in [8] that this quantity is well approximated by a Gaussian distribution with variance $\sigma_\eta^2 = \sigma_s^2[\mu_{k,i} - \mu_{k,i}^2]$.

From (28), we can calculate the LLRs on the corresponding bits of the detected symbols at the output of the instantaneous MMSE filter, that will be used by the SISO channel decoder:

$$L(d_k^{i,m}) = \log\frac{P_{\text{dem}}(\,d_k^{i,m} = 1|\,\widetilde{r}_k(i), \mu_k(i)\,)}{P_{\text{dem}}(\,d_k^{i,m} = 0|\,\widetilde{r}_k(i), \mu_k(i)\,)}$$

$$= \log\frac{\displaystyle\sum_{s_k(i)\in\underline{\mathcal{S}}_1^m}\exp\left\{-\frac{|\widetilde{r}_k(i)-\mu_{k,i}s_k(i)|^2}{\eta_{k,i}{}^2}\right\}\prod_{\substack{n=1\\n\neq m}}^{B}P_{\text{dec}}^1(d_k^{i,n})}{\displaystyle\sum_{s_k(i)\in\underline{\mathcal{S}}_0^m}\exp\left\{-\frac{|\widetilde{r}_k(i)-\mu_{k,i}s_k(i)|^2}{\eta_{k,i}{}^2}\right\}\prod_{\substack{n=1\\n\neq m}}^{B}P_{\text{dec}}^0(d_k^{i,n})}. \tag{29}$$

Note that the cardinality of the sets $\underline{\mathcal{S}}_1^m$ and $\underline{\mathcal{S}}_0^m$ is now equal to 2^{B-1}.

V. Numerical Results

In this section, we provide a comparative performance study the proposed detector in terms of bit error rate (BER). The binary information data are encoded by a rate $1/2$ NRNSC code with constraint length $K = 7$ defined in octal form by (133,171). Throughout the simulations, each frame is composed of 16 OFDM symbols with 16 subcarriers and symbols belonging to QPSK or 16-QAM constellations with Gray labeling. Uncorrelated Rayleigh fading channel is considered and channel coefficients are kept constant during each frame and changed to new independent realizations for the next frame. The interleaver is pseudo-random, operating over the entire frame of size $N_I = LMM_TB$ bits.

Mutually orthogonal QPSK pilot sequences are used for channel estimation, and the average pilot-symbol power is set equal to the average data-symbol power. Moreover, the number of decoding iterations are set to 5. The SNR is considered in the form of E_b/N_0 and includes the antenna array gain at receive, M_R.

Fig. 3 shows BER curves of the mismatched and improved turbo-PIC receiver for the case of QPSK modulation and $M_T = 2$ and $M_R = 2$ that we denote by (2×2) MIMO system. The number of channel uses for pilot transmission is $N_P \in \{2, 4, 8\}$. As a reference, we have also presented the BER curve in the case of perfect CSIR for turbo-MAP and turbo-PIC detectors. We notice that the required SNR to attain the BER of 10^{-5} with $N_P = 2$ pilots is reduced by about 0.5 dB for the improved detector, as compared to the mismatched detector. By increasing N_P, CEE become less important and the difference of the performances of the two detectors decreases: the achieved gain in SNR at BER $= 10^{-5}$ is about 0.2 dB and 0.05 dB, for $N_P = 4$ and $N_P = 8$, respectively. Actually, the performance loss of the mismatched receiver with respect to the improved receiver becomes insignificant for $N_P \geq 8$. Furthermore, we observe that the performance of the turbo-PIC detector is very close to that provided by the turbo-MAP detector.

We also consider two other cases in the following for which we do not present the turbo-MAP performance as it becomes computationally too complex. Let us consider the case of (2×2) system and 16-QAM modulation. Results are shown in Fig. 4. We notice that the gain in SNR by using the improved detector at BER $= 10^{-5}$ is about 0.6 dB, 0.3 dB, and 0.2 dB, for $N_P = 2$, 4, and 8, respectively. The obtained gain by using the improved detector is a little more important for 16-QAM than for QPSK modulation. This was predictable as the detector performance is more sensitive to channel estimation errors for 16-QAM modulation.

Finally, results for the case of (4×4) system and QPSK modulation are shown in Fig. 5, for $N_P = 4, 8$. For this case, the gain in SNR by using the improved detector at BER $= 10^{-5}$ is about 0.24 dB and 0.1 dB, for $N_P = 4$ and 8, respectively. As a matter of fact, for the (4×4) system, the diversity order larger, as compared to the previous case. The increased spatial diversity helps improve the detection performance and the receiver is less sensitive to estimation errors, although there are twice as many subchannel coefficients to estimate [7]. As a result, the obtained gain my using the improved detector is less considerable, compared to the (2×2) case.

VI. Conclusion

We proposed a novel turbo-PIC detector for MIMO systems operating under imperfect channel estimation. By introducing a Bayesian approach characterizing the channel estimation process, we proposed a general detector decision criterion that takes into account the imperfect channel. Using this, we derived an improved simple iterative MIMO detector based on soft-PIC and MMSE filtering that mitigates the impact of channel uncertainty on the detection performance. This improvement comes from the inherent averaging of the detection rule that would be used if the channel was perfectly known, over the all realizations of the CEE. The performance of the proposed detector was compared to that of the mismatched receiver via simulations. Numerical results indicate that for short training sequence lengths, the proposed detector can enhance the detection performance while imposing almost no additional complexity.

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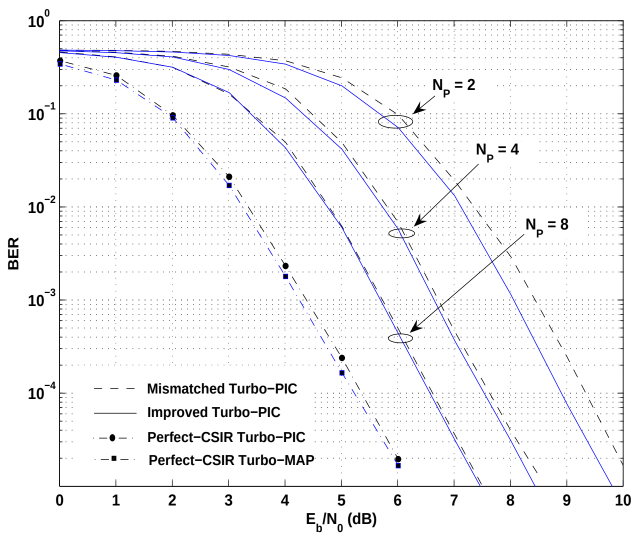

Fig. 3. BER performance of improved and mismatched turbo-PIC; (2×2) MIMO-OFDM with $M = 16$ subcarriers, i.i.d. Rayleigh fading, QPSK modulation, training sequence length $N_P \in \{2, 4, 8\}$.

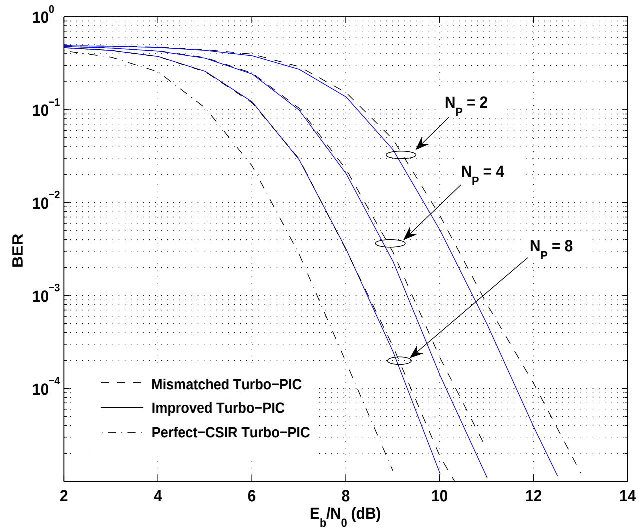

Fig. 4. BER performance of improved and mismatched turbo-PIC; (2×2) MIMO-OFDM with $M = 16$ subcarriers, i.i.d. Rayleigh fading, 16-QAM modulation, training sequence length $N_P \in \{2, 4, 8\}$.

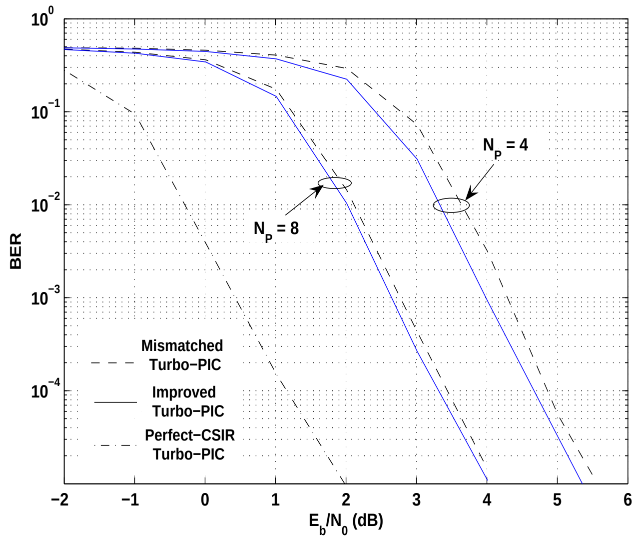

Fig. 5. BER performance of improved and mismatched turbo-PIC; (4×4) MIMO-OFDM with $M = 16$ subcarriers, i.i.d. Rayleigh fading, QPSK modulation, training sequence length $N_P \in \{4, 8\}$.