OPTIMAL URBAN WATER DISTRIBUTION DESIGN

by

David Rhys Morgan

A thesis
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ABSTRACT

A model for the least cost layout and design of an urban looped water distribution has been developed. The technique couples the Hardy Cross network solving technique with a linear programming based pipe diameter modification method. An algorithm is used to assign different weights to pipes in the system according to the effect that their modification will have on the pressure at each node. A large number of demand patterns can be considered simultaneuosly during the optimization. Three design examples, including an expansion of an existing system are analysed and the results compared to previous studies.

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Chapter I

INTRODUCTION

This thesis studies the problem of optimizing a looped urban water distribution network. This problem involved finding the least cost network which has the flexibility to perform adequately under various adverse conditions, such as pipes breakage and fire flow conditions.

The American Water Works Association (AWWA) estimates that $15 billion will be invested in water facilities in the coming decades. In fact, a recent issue of the Journal of the AWWA is dedicated to economic considerations in water supply and distribution. The Economic Research Committee of the AWWA has found that the problem of optimal design of looped networks is virtually unsolved(Lauria [1983]). It was these same concerns which prompted this study into optimal design of looped networks.

There is also a shortage of research which pertains to uncertainties occurring in water distribution operation. Fire flows are uncertain in their quantity and location , and pipe breakage is an additional uncertaintity. For many communities it is these uncertaintities, rather than steady state maximum day flows, which are the determining factors in system design.

The problem of designing a looped system is directly linked to uncertainties in the design and the operation of the system. A looped network cannot be optimized under steady state conditions and remain looped, since the network will be driven to branched system. The following thesis will study the problem of urban water distribution system optimization by addressing the uncertainties in design and operation of these systems.

In this method the Hardy Cross netwrk solving technique is coupled with a linear programming based pipe diameter modification technique. The results from each technique are passed to the other in an iterative process until an optimal solution is converged upon. The problem of flexibility in the design is dealt with explicitly by considering a large number of adverse conditions during the optimization. An algorithm is developed which assigns different weights to each pipe in the system according to how its modification effects the pressure at each node. The method is then tested in three design examples. The first example demonstrates the shortcomings of using a single steady state demand pattern in the design of a looped system. The second example considers the same system, however, many demand patterns are considered during the design. A final example considers the expansion of an existing and the results are compared to previous studies. It should be noted that the solutions obtained by this model may not be the true mathematical optimal, however, for the practical problem this model has obtained less expensive solutions more efficiently than previuos models.

Chapter II

OBJECTIVES

This study investigates new methods of designing the layout and pipe sizes of urban water distribution systems while considering many of the rigorous criteria described below:

1. The system must deliver a certain flow at a specified pressure to any node in the system when one of the key pipes in the system is not functioning. Therefore at least two independent and adequate paths from a source to each node must be provided.
2. The system must deliver severe fire flow demands at adequate pressures. While these fire flow demands occur infrequently at different nodes in the system, they may, however, may be the limiting factor in the design of systems.
3. Combinations of 1) and 2)
4. The methods should be efficient enough to be applied to large systems.
5. The techniques used to solve the problem should be widely available.
6. The method should be applicable to expansion of existing systems as well as design of new systems.

7. The method should incorporate a realistic cost function, preferably using standard unit costs as given by suppliers, i.e. cost per length.

Chapter III

LITERATURE REVIEW

An overview of systems analysis of water distribution networks was given by de Neufville et al. [1971]. In this overview systems analysis was studied as being an integrated part of the design process. The steps of the process were described by "five fundamental steps: (1) Definition of objectives; (2) formulation of measures of effectiveness; (3) generation of alternatives; (4) evaluation; and (5) selection."[1] It was stressed that of the five process steps only evaluation could be done by systems analysis while the other four are dependent upon the judgement of the engineer.

Many of the previous techniques used in water distribution system optimization were based upon a technique developed by Karmeli, Gadish and Meyers [1968]. In this paper linear programming was used to optimize branched water distribution systems in which the flow in each section was predetermined. Since this technique was the basis for much of the subsequent research it is reviewed.

[1] de Neufville et al. "Systems Analysis of Water Distribution Networks. Journal of the Sanitary Engineering Division. December,1971. pg. 825.

The loss of energy as described by the Hazen-Williams formula was used in the form:

$$J=1.13\times10^{12}\ Q^{1.852}\ C^{-1.852}\ D^{-4.87} \qquad \text{Eq.(1)}$$

where

J = friction loss in metres per kilometre
Q = flow in cubic metres per hour
D = pipe diameter in millimetres
C = friction factor

There are G(n) different diameter pipes to be considered in each section (n). It is clear that since the flow in each section was known each pipe j had a specific head gradient which could be calculated using the Hazen-Williams formula. Thus, by multiplying the head gradient by the length of each pipe (X_{nj})and summing for all G(n),the total friction loss in each section was calculated.

The cost of the pipe in each section n was equal to the sum of the lengths of each pipe j times the unit cost of that pipe (k_{nj}). The objective function was therefore written

$$\text{Minimize } Z = \sum_{n=1}^{N} \sum_{j=1}^{G(n)} X_{nj} k_{nj} \qquad \text{Eq.(2)}$$

An important constraint upon the system was that the pressure at each node must always be maintained above the minimum allowable pressure. This was done by ensuring that the sum of pressure losses in the pipe sections along the path between the reservoir and the particular node is equal to or less than the allowable pressure difference.

$$\sum_{n \in p}^{N} \sum_{j=1}^{G(n)} X_{nj} J_{nj} \leq E_0 - E_n \quad \forall\, n \qquad \text{Eq.(3)}$$

where

p = the set of paths from 0 to n.

E_n = the minimum allowable pressure at demand point n.

E_0 = the reservoir pressure.

The total length of pipe in each section must equal the length of that section. A length constraint was therefore needed.

$$\sum_{j=1}^{G(n)} X_{nj} = l_n \qquad \forall\, n \qquad \text{Eq.(4)}$$

Using this formulation and selecting a group of eligible pipe diameters for each section, the least cost solution can be obtained.

More recently, Bhave [1979] has indicated that since at most two different diameters are choosen in any one link the number of eligible pipe diameters could be reduced. He devised a method of selecting these two pipe diameters before the optimization proceeded.

Alperovits and Shamir [1978] expanded upon Karmeli et al. [1968] to include looped systems. The flow in each section was initially assumed and an additional constraint was needed to ensure that the summation of headlosses around each loop must be zero. This kept the problem hydraulically consistent during the iterative solution procedure.

The dual variables associated with the loop constraints were used in a gradient technique to adjust the flows in a manner which will reduce the overall cost. Using an iterative process the problem was rerun with the new flows until the solution was believed to have converged upon optimality.

In this paper Alperovits and Shamir noted the importance of designing for multiple demand patterns and presented a method of expanding the problem to consider multiple design loadings. Unfortunately the number of constraints increases rapidly with the number of demand patterns, making it impossible to design for all the severe possibilities which might occur within urban water distribution systems.

In their discussion of Alperovits and Shamir [1978], Quindry et al. [1979] noted a deficiency in the original formulation and provided the corrective measures necessary for the application of the gradient technique. This new technique took into account the dual solution of the path constraints. While this correction lead to a cheaper solution, it did however reduce to a solution in which a branched layout, based on the paths chosen for headloss constraints, formed a primary network with minimum allowable pipe diameters spanning the primary links.

It can also be shown that a cheaper solution is obtained if different paths are chosen for headloss constraints(see Appendix A). The cause of the dependence of the solution on choice of the paths in this formulation was due to an implicit assumption. Under this assumption it was assumed that a change in the resistance (pipe diameter) in a section of designated path has a one to one relationship with the change in pressure at the end of that path. This is true in a branched system because all of the flow going to the end node passes through the designated sections. In a looped network, however, there is more than one path to the end node. Consequently the effect of changing the resistance in one of these paths has less effect on the head at the end node than than it would if it was the only path. This issue will be discussed in more detail during the development of the model described in this thesis.

The linear programming approach was used in a different manner by Schaake and Lai [1969] to optimize looped water distribution systems. The decision variable in this model was derived from the Hazen-Williams formula and was proportional to the flow in each section. Hydraulic consistency was maintained by assuming an initial head at each node rather than the initial flows in the links. A continuity constraint was used to insure that the summation of flow at each node was zero. The cost function was not linear but was assumed to be a concave polynomial. While this polynomial function was not directly usable in linear programming, piecewise linearization was utilized to permit inclusion of the values in the model. A method to consider multiple loadings (up to three) was also considered. The expansion of the New York City water distribution system was used as an example to develop and demonstrate the model. There are however a number of shortcomings to this procedure. Since the head of each node was fixed it was obvious that the solution is far from optimal. Variations in the initial head pattern may provide a more 'optimal' answer.

Quindry et al. [1981] expanded upon this method by adding a gradient technique using the dual solution to adjust the head at each node. A technique was then developed to solve for the optimal flows, update the pressures, and re-iterate until no major improvement was realized. This method resulted in a less expensive alternative than Schaake and Lai's solution (63.5 million vs. 78 million). However this is far greater than the solution obtained by Gessler [1982] (41.8 million) or by the model developed in this thesis (38.8 million) (see section 5.4).

Morgan and Goulter [1982] used two linked linear programming models to determine the least cost layout and design of a water distribution system. A redundancy constraint was added to Schaake and Lai's model to ensure that at least two pipes connect every node. In this method a minimum pipe size was not required and uneconomical pipes could by eliminated from the layout. Since one linear program changed heads given flows and the other linear program changed flows for given heads the two could be interfaced to work toward a better solution. Theoretically multiple loadings could be considered, although like the previous models the additional multiple loading constraints made the problem too cumbersome to solve effectively.

Kettler and Goulter [1983] have addressed a different aspect of reliability. By analysing pipe breakage data the statistical probabilities of failures per kilometre for different diameter pipe was calculated. The probability of failure of pipes in the paths from the demand points to the source were constrained to increase the reliability of the system. This study pointed out that small diameter pipes, although lowering the cost of the system, decrease the reliability of the system.

Kally [1972] used a linear programming approach similar to Karmeli et al. [1968]. Initially a design was specified and flows and heads were solved by using the Hardy Cross method. In this model the decision was the length of pipe changed from the present diameter to either a larger or smaller diameter. If there was excess pressure at some nodes then certain pipes would be changed to smaller diameters. It was assumed that changing a length of pipe from one diameter to the next would result in a proportional (linear) head change at a junction. It was noted that in a looped system this linearity would apply only approximatley. In a branched system, however, it would be accurate. The new design was then resolved by the Hardy Cross analysis. This iterative procedure was continued for several iterations until a satisfactory solution was reached. While the results from two examples were given, one for a branched system and one for a looped system, the procedure was difficult to duplicate due to inadequate descriptions of the model and the formulation.

Another operations research technique, dynamic programming has also been used to design water distribution systems and other hydraulic networks (Mays et al.[1976]). This method has the advantage of being a nonlinear logical optimization method. A more realistic cost function (actually cost tables) can therefore be used. Since the method requires the problem to be serialized into discrete stages it is particularly appropriate for pipelines (Liang [1971],Martin [1980]) and branched networks such as irrigation systems (Yang et al.[1975]). This requirement however makes dynamic programming extremely difficult to adapt to a looped water distribution system.

Methods other than linear and dynamic programming have also been used in many studies. One of the earlier attempts at optimizing a looped water distributon system was by Jacoby [1968]. The problem was defined subject to the laws of hydraulic consistency. The pipe cost function was represented by a nonlinear continuous cubic function and pumping costs were also included. A merit function was used to move toward optimality in which a cost penalty was assigned to any constraint violation. The flows and pipe diameters were then changed to reduce the sum of the cost of the system and penalty costs. This model recognized the probability of reaching local minima and therefore suggested the use of many starting points.

Deb and Sakar[1971](and Deb[1974,1976]) have studied many aspects of optimal hydraulic network design using differrent techniques. In one paper (Deb and Sakar [1971]), a new method was developed based on an equivalent diameter concept from which the minimum cost of pipe was obtained for a particular pressure surface and reservoir height. The optimal pressure profile was found for a particular network and the cost was empirically determined as a function of pipe size , service reservoir height and horse power of pumps. An example with a single source and a steady state condition was given. There was no reference to the incorporation of multiple sources or demand patterns into the design procedure.

In a later paper Deb [1974] developed another method to achieve least cost design of branched pipe networks. The method could be used with the help of a desk calculator. In a third paper looped systems were again analysed(Deb[1976]). In this paper it was recognized that without suitable constraints the system would deteriorate to a branched system. To overcome this problem minimum pipes were specified. The location and elevation of the single reservoir was considered along with energy costs during the optimization procedure.

The optimization method used in all three papers by Deb (Deb and Sakar[1971],Deb[1974,1976]) analysed the derivative of the cost function with respect to the change in diameter, while satisfying constraints which represent the minimum flows and pressures at the nodes. Since the derivative was needed it was assumed that the cost function is continuous.The nearest commercially available pipe diameters were selected following the analysis. The continuous function used was first derived by Linaweaver [1964] for large scale pipe and tunnels. Unfortunately, it is unlikely that the cost function for small discrete pipes will follow this continuous polynomial function.

Shamir [1974] proposed a method of optimizing both design and operation of water distribution systems. A newly developed optmization technique was combined with a previous flow solving model (Shamir and Howard [1968]) to produce a new procedure. The objective function was constructed to select the optimal changes in an existing system or an existing design of a new system. Decision variables associated with operation (pump and valve settings) as well as design variables (pipe diameters and pump capacities) were considered for up to five different demand patterns. The problem was solved using a generalized reduced gradient method which used Lagrangian multipliers to construct the gradient vectors. These gradient vectors were used to 'step' towards an optimal solution while maintaining the hydraulic consistancy. A large network (20 nodes, 50 pipes) was solved to illustrate this method.

Watanatada [1973] used a more complex quadratic function to define the costs. The nonlinear function representing the system, along with constraints, was defined and solved using the Lagrangian function. Both capital and pumping costs were considered during the solving of the system for a single steady state demand pattern. Minimum diameters were specified to prevent the system from deteriorating to a branched system. A major strength of this method is that it was powerful enough to be used on a large system (50 nodes).

Rasmusen [1976] has used a heuristic technique to converge upon an optimal solution. The technique operated in a two stage fashion. The hydraulic flow problem was solved initially and then the pipe diameters were modified to reduce the cost. These two problems were linked and solved iteratively until the optimal solution was found. Within the procedure itself nodes with pressures below the minimum allowable were labelled critical nodes. All pipes in which flow may travel from the source to this node were labelled critical links. These critical links have a potential for saving energy by a reduction in pressure losses. Noncritical pipes could be reduced to save capital costs. The relationship between energy costs and capital costs dictated the degree to which pipe diameters were reduced or increased. The fact that a number of paths from source to node have an important effect on the pressure at the critical node is an important concept in designing looped water distribution systems. This concept which has been recognised by Rasmusen has been neglected in many other studies (Bhave [1978], Alperovits and Shamir [1978]). While only a single source and a single steady state demand pattern was considered during the design, Rasmusen indicated that further work would enable the model to consider multi-source networks.

Bhave [1978] suggested a simple technique which can be used without the aid of a computer. The procedure found the optimal spanning tree,which were then termed primary links, with redundant pipes connecting the end nodes to create looping. While energy costs were considered,only a single source and a single steady state demand pattern were used.

Gessler [1982] has used a complete enumeration approach to arrive at an optimal solution. Alternative designs were generated and tested. The first test calculated the cost of the new alternative and checked it against the best previous cost. If it failed this test, i.e. was more expensive, then a new alternative was generated. If it passed this test then the network was solved to check if it performed adequately. If this second test was failed then the alternative was discarded. If the test was passed then this alternative became the new best soluton. New alternatives were generated and this was continued until the choices were exhausted.

This method is very time consuming and would presently be impracticable on a system of any complexity. The method, however, was made more usable by the procedure of "grouping". By grouping certain pipes into sets (at the discretion of the design engineer) and specifying that all pipes in each set must be the same size, the number of alternatives was greatly reduced. Although this technique ensures that the solution is a global optimum for the specified groupings , there is no guarentee that the groupings are optimal. Thus the true global optimum may not be reached.

This technique has been applied successfully to the New York City water supply problem. The solution is the best published to date being over thirty per cent cheaper than the solution generated by the Quindry et al. [1981] method. While this method is applicable to multi-source and multi-demand pattern design, the time requirements which are already quite extensive increase very rapidily with the size of the system.

The problem of designing the layout as well as the size of the components has been addressed in recent years. Mays et al. [1976] incorporated the layout and design of sewer systems into one model. A heuristic technique using discrete differiential dynamic programming (DDDP) was used in a conjunctive form with two models to form a screening model. One of these models simultaneously considers system layout and design, while the other uses the layouts of the first model to compute the optimal design.

Martin [1980] used dynamic programming to optimize a water conveyance system. In this study the layout and component size of a single path large scale system was considered. A combination of facilities (pipelines, open canals and pumping stations) were considered during the optimization. Both of these dynamic programming methods, i.e. Mays et al. [1976] and Martin [1980], were limited to serial systems and are unsuitable for looped water distribution systems.

Rowell and Barnes [1982] have addressed the problem of obtaining a layout for municipal water distribution systems directly. A two level hierachically integrated system of models was developed. An economical tree layout was first developed then redundant pipes were added. A "rule of thumb" method, which estimates pipe capacity using just the diameter, and neglecting the hydraulic gradient, was used in the model. The performance of the system was not tested by a network solver and hydraulic consistency is neglected during the optimization.

The major shortcoming of these present optimization methods is they tend to design inflexible systems. Templeman [1982] in his discussion of Quindry et al. [1981] stated

> "Optimization tends to remove redundancy, and any spare capacity which is not immediately required by the design demand pattern is optimized out. Thus, all flexibility is removed. The optimization process is not at fault here, it merely extrapolates the design process to its logical limits. Such faults are inherent in the design process itself which does not directly incorporate resilience, flexibility, and reliability into the the design process."[2]

Templeman continued that the design criteria must include the flexibility to serve firefighting demands at all nodes. Most of the papers mentioned consider only one demand pattern and thus tend to design branched systems with minimum size redundant pipes connecting the end nodes. Two papers

[2] Templeman,A.B. , "Discussion of Looped Water Distribution Systems" , Journal of the Environmental Engineering Division,ASCE. pg. 599, June , 1982.

Alperovits and Shamir [1978] and Quindry et al. [1981] realized that flexibility is obtained by considering more than one demand pattern (two to three patterns were considered). This however falls far short of considering a fire flow at each node in the system.

Chapter IV

MODEL DEVELOPMENT

4.1 GENERAL DESCRIPTION

To begin the optimization procedure an initial layout and design for the system is assumed. For a given demand pattern the flows in all pipes and the pressures at all nodes is then determined. Of the many methods available (see Holloway and Chaudhry [1983]) to solve this problem the Hardy Cross technique appears most suitable due to its simplicity and general widespread acceptance. The solution provided by the Hardy Cross Method is passed to a pipe design/modification procedure. If the pressures at some demand or diversion points are below minimum it is necessary to replace sections of pipe with pipe of larger diameter. This procedure determines which sections of pipe should be replaced in order to raise the pressure to the minimum at the least cost.

Conversely,if the pressures are above the minimum allowable then some section of pipe may be replaced by pipes of smaller diameter. Again the decision is which section of pipe should be replaced in order to bring about the greatest saving while maintaining minimum pressure requirements elsewhere. In general , replacement of designated pipes (designated pipes are the pipe diameters which have been initially assumed or selected as replacement pipes in previous iterations) with both smaller and larger diameters is needed since some areas may be underdesigned while other areas may be overdesigned. An overall reduction in cost may also be obtained by increasing the diameter of one pipe, therefore allowing a number of other pipes to be replaced by pipes of smaller diameter. This new configuration of pipe sizes is passed back to the network solver to calculate true flows and pressures. The process is repeated iteratively until an optimal solution is reached. This process is described in more detail in later sections.

4.2 SINGLE DEMAND PATTERN

The procedure used in multiple demand pattern design is an extension of that used for single demand pattern design. The common parts of the formulation will therefore be described in this section and the differences will be explained in the section on multiple demand patterns.

Since the powerful simplex algorithm allows complex problems to be solved efficiently, linear programming is used to select the optimal replacement sections. In order to formulate a linear programming model the terms must have linear relationships. In this model the cost function is constructed by assuming that the cost of replacing a length of one diameter of pipe with the same length of another diameter is a linear function of the length of pipe replaced. The basis of this assumption is described as follows. The unit cost of pipe is usually defined in cost per length (.ie dollars per metre). Expressing the unit cost of a pipe as C_d for a designated pipe of the **dth** diameter and C_r as the unit cost of a replacement pipe of **rth** diameter, then the unit cost of replacing pipe d with pipe r is a constant.

$$K_{dr} = C_r - C_d \qquad \text{Eq. (5)}$$

Using the definition of cost given in the above equation the objective function can be written

$$\text{Minimize } Z = \sum_j K_{jdr} X_{jdr} + K_{jds} X_{jds} \qquad \text{Eq. (6)}$$

where

K_{jdr} = the unit cost of changing a pipe of the d^{th} diameter to a larger diameter pipe r.

$$K_{jdr} > 0$$

K_{jds} = is the cost of changing a pipe of the d^{th} diameter to a smaller diameter pipe s

$$K_{jds} < 0$$

X_{jdr} & X_{jds} = the decision variable; the length of pipe d replaced by pipe r or s.

In this part of the procedure the pressure must be constrained to maintain a feasible solution. If the flow in each section remains constant, as in a branched system , then every metre of pipe replaced will cause a proportional change in pressure across the section. For example an X_{jdr} value of 2 metres will cause twice as much change in pressure across the section as an X_{jdr} value of 1 metre. This relationship can be derived from the Hazen-Williams formula

$$J=1.13 \times 10^{12} Q^{1.852} C^{-1.852} D^{-4.87} \qquad \text{Eq (7)}$$

where

J = hydraulic gradient in m/km

Q = flow in liters/second

C = Hazen-Williams friction coefficient

D = diameter in metres

The change in pressure head ($\triangle P$)caused by replacing a section of diameter d with a section of diameter r is

$$\triangle P_{jdr}=G_{jdr}X_{jdr} \qquad \text{Eq. (8)}$$

where

$G_{jdr} = J_{jr} - J_{jd}$

J_{jd} = the hydraulic gradient in metres per kilometre for pipe diameter d in link j

All other terms are as previously described.

The change in pressure at a particular node is the sum of all changes in the flow links between that node and the source which has a fixed head. The change in pressure at each node must satisfy the minimum allowable pressure at that node. The constraint to ensure this condition can be expressed as

$$\sum_{j \in p} G_{jdr}X_{jdr} + G_{jds}X_{jds} \geq H_i - h_i \text{ for all } i \qquad \text{Eq.(9)}$$

where

G_{jdr} & G_{jds} = the change in pressure caused by replacing the designated pipe d with a larger pipe r or smaller pipe s

H_i = the minimum allowable pressure head

h_i = the existing pressure

All other terms are as previously described.

If the initial solution is not close to the optimal the model may try to replace more of a designated pipe than is available. It is obviously unrealistic to allow a pipe length greater than that of the link length to be replaced. The following constraints are therefore needed.

$$X_{jdr} \leq L_j \qquad \text{Eq. (10)}$$

$$X_{jds} \leq L_j \qquad \text{Eq. (11)}$$

where L_j is the length of link j.

If the designated pipe in the link is made up of two diameters (see Figure 1) then the length of pipe available to be replaced is less than the length of the link. The new constraints are

$$X_{jdr} \leq l_{1j} \qquad \text{Eq. (12)}$$

$$X_{jds} \leq l_{2j} \qquad \text{Eq. (13)}$$

where

l_{1j} = the length of designated pipe of smaller diameter

l_{2j} = the length of designated pipe pipe of larger diameter

and

$l_{1j}+l_{2j}=L_j$ (see Figure 1)

All other terms are as previously described.

If the system is branched it can be solved directly with the linear programming model as formulated. With a looped system, however, additional factors must be taken into account. If there are several paths to the demand node i from a source (or sources) with a fixed head then all paths to that node must be considered . It should be recognized that a change to one link, on one of these several paths, may not have the same importance to the change in pressure at node i as a change in another link in the same path or in one of the other paths. This condition is explained in reference to Figure 2 which shows a simple looped system. It can be seen that changes in links 1 and 4 will have a direct

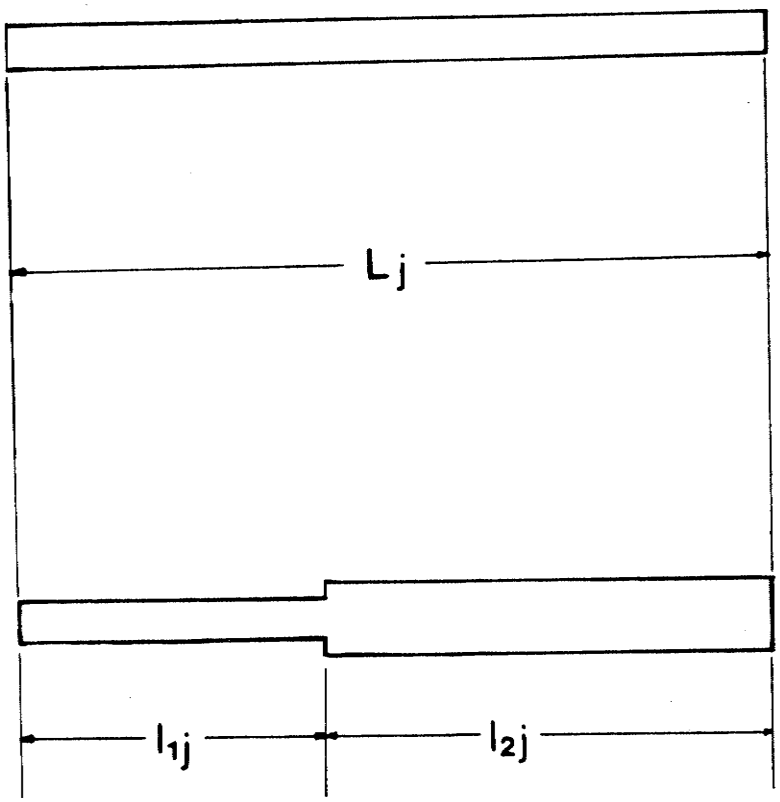

Figure 1: Definition of Length Constraints

one to one effect on the pressure at node i. The weighting attributed to link j with regard to its effect on node i is $W_{ij}=1$.

The weightings given to links 2 and 3 must be less than one since links 2 and 3 do not handle all the flow between the reservoir and the outlet separately. The method used to weight these links is to calculate the percentage of flow drawn from node i that passes through link j. For example if 65% of the flow is passing through link 2 and 35% is through link 3 then the weightings would be $W_{i2} = 0.65$ and $W_{i3}=0.35$.

Consequently, Equation 9 can be modified to consider many paths.

$$\sum_{j \in P} W_{ij}G_{jdr}X_{jdr} + W_{ij}G_{jds}X_{jds} \geq H_i-h_i \text{ for all i} \quad \text{Eq.(14)}$$

where P = the set of paths from node i to a
fixed head source.(each link is
counted only once)

All other terms as previously described.

With a looped system, however, the linearity assumption, expressed by Equation 14, only holds approximately for the changes in pressure, since the flows in the pipes do not remain constant as pipes are changed throughout the network. This is in direct contrast to the situation which exists

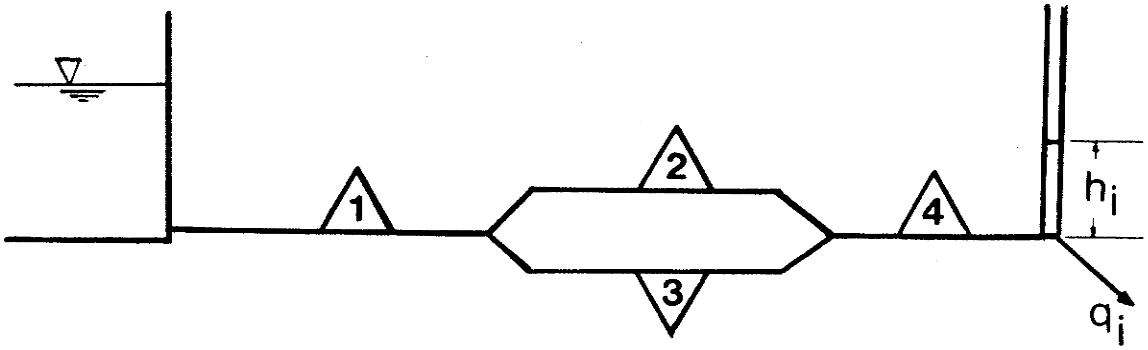

Figure 2: Simple Looped System

with branched networks. A feedback to the network solver (see Figure 3) is therefore needed to calculate the true pressures and flows. This process is repeated iteratively until an "optimal" solution is converged upon. This convergence is recognized when the change in cost becomes insignificant (0.01%) and none of the length constraints are binding.

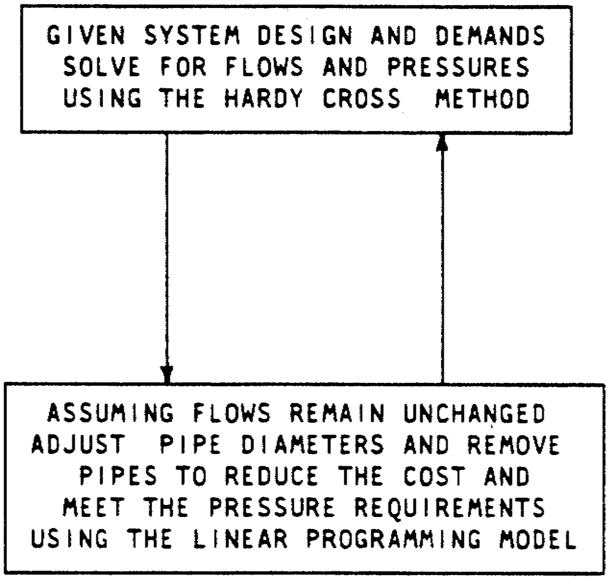
GIVEN SYSTEM DESIGN AND DEMANDS SOLVE FOR FLOWS AND PRESSURES USING THE HARDY CROSS METHOD
ASSUMING FLOWS REMAIN UNCHANGED ADJUST PIPE DIAMETERS AND REMOVE PIPES TO REDUCE THE COST AND MEET THE PRESSURE REQUIREMENTS USING THE LINEAR PROGRAMMING MODEL

Figure 3: Schematic of Model

4.3 MULTIPLE DEMAND PATTERNS

If a system is designed for one situation , then the most economical layout will be a branched system. This is due to economies of scale , where one large pipe is a less expensive method of transporting water than two smaller pipes. However, if a single pipe fails then no flow can reach the nodes downstream of this failure. This problem is alleviated by adding redundant pipes to ensure looping. These pipes have small diameters, usually the minimum available diameter, and there is no guarantee that the secondary path can deliver water at an adequate pressure in case of a primary link failure.

A solution to this problem is to design this system while considering all the worst case scenarios; fire flows and pipes breaks. One method of doing this is to add one set of constraints for each demand pattern. Formulations for multiple demand patterns are described by Schaake and Lai [1969], Alperovits and Shamir [1978] and Quindry et al. [1981]. The problem with these formulations is that the additional constraints needed to consider multiple demand patterns increase rapidly making the problem computationally impractical. If forty constraints were needed for one demand pattern then 120 would be needed for three demand patterns.

Many of these constraints can, however, be eliminated because they are non-binding. From linear programming theory (Hillier and Lieberman [1980], p. 68-117) it is known the maximum number of non-zero variables in the solution equals the number of binding constraints. Equation (6)(p.23) states that there are two variables for each link in the system, one representing the length of pipe to be replaced by a larger diameter pipe , the other representing the length of pipe to be replaced by a smaller diameter pipe. If pressures need to be increased then the larger diameter pipe will replace the designated diameter, and if the pressures can be reduced a smaller diameter pipe will replace the designated diameter. Therefore, only one of these variables will ever be non-zero in the solution. In many cases if there is no change in the pipes of that link, both variables will be zero. Therefore the maximum number of pressure constraints needed (independent of the number of demand patterns) is equal to the number of links (N).

A further condition of this model is that when the optimal solution is found none of the length constraints will be binding.They are, however, needed initially if the first assumption is far from optimal. Since the number of these constraints is not dependent on the number of demand patterns considered they are left intact for the multiple demand patterns situation.

The pressure constraints used must be selected before the linear programming stage proceeds. The Hardy Cross stage of the procedure first solves each of the demand patterns for the pressures and flows. The amount by which the actual pressure is below (or above) minimum pressure is calculated.

$$a_{it} = H_i - h_{it} \text{ for all } i \text{ and } t \qquad \text{Eq.(15)}$$

a_{it} = the amount the actual head
is below the minimum allowable
head at node i demand pattern t

Starting with the largest value of a_{it} the pressure constraints are constructed.

$$\sum_{j \in P} W_{ijt}G_{jtdr}X_{jdr} + W_{ijt}G_{jtds}X_{jds} \geq H_i - h_{it} \quad \forall\ i \qquad \text{Eq.(16)}$$

where

$G_{jtdr} = J_{jtr} - J_{jtd}$

J_{jtd} = the hydraulic gradient in metres per
kilometre for pipe d in link j when
the flow in the pipe is Q_{jt} for
demand pattern t.

All other terms have previously been described.

This construction of pressure constraints continues until N constraints have been constructed.

One aspect of this technique is the interaction between two similar methods working together to achieve optimality.

The well accepted Hardy Cross method solves the flows and pressures in an iterative manner which uses the previous solution to arrive at a better solution. The linear programming method developed modifies the previous solution to arrive at a better solution. This iterative process has been used with linear programming in previous studies by Alperovits and Shamir [1978], Quindry et al. [1981] and Kally [1972]. In the two former studies the design variable is the diameter or length of a certain diameter selected as opposed to the length of diameter to be modified used in the latter study and in this thesis. In the studies by Alperovits and Shamir[1978],and Quindry et al.[1981] therefore the simplex tableau from the previous iteration must be saved in some form to maintain efficiency. If the previous tableau is not saved, many redundant simplex iterations must be carried out at each step. This condition requires a complex simplex computer package in order to save and modify this tableau during the optimization. In Kally's technique and the one described in this thesis the previous solution is described by the zero simplex tableau. This makes it possible to use a straightfoward simplex package, while still maintaining the efficiency of the method.

4.4 WEIGHTING ALGORITHM

The weight assigned to each link j in the pressure constraint equation (Equation 16,p.33) for each node i and demand pattern t is denoted by W_{ijt}. Since i and t remain constant throughout the weighting procedure for each equation, they are omitted for clarity in this example.

$$W_{ijt} = W_j = (Q_j/I_m) \times w_m \qquad \text{Eq.(17)}$$

where

Q_j = the flow in link j

I_m = the sum of the inflow to node m

w_m = the weight of the node immediately downstream of link j.

The weight of the node m is calculated as

$$w_m = \sum_{j \in B} W_j \qquad \text{Eq.(18)}$$

where

B = the set of all outflow links from the node m.

The procedure begins at the node being constrained with m=i and w_m=1.0 . The algorithm used to calculate these weights is demonstrated by application to the simple network shown in Figure 4. Initially this network is solved for a given demand pattern and pipe configuration using the Hardy Cross technique. The Hardy Cross solution provides the flows in each pipe and the pressure at each node. Since this weighting algorithm requires only the flows in each pipe, the pipe diameters and the demands and pressures at each node are not shown in Figure 5. Only the flows in each link which are obtained from this Hardy Cross Solution, are shown in this figure. In this example the equation (Equation 16, p.33) constrains the pressure at node 5. The procedure begins immediately upstream of this node. The total flow entering this node is calculated from the summation of all inflow, i.e. links 5 and 7 (160+190=350) and the weights of links 5 and 7 are calculated from this total.

$$W_5 = (160/350) \times 1 = .46 \qquad \text{Eq.(19)}$$

$$W_7 = (190/350) \times 1 = .54 \qquad \text{Eq.(20)}$$

These weights are then assigned to the nodes upstream of the links, .i.e nodes 4 and 6. For example node 4 is assigned the weight 0.46 from the single outflow link from that node. Similarly node 6 is assigned a weight of 0.54. The process then continues upstream. Under this formulation link 8 contributes 100% of the flow to node 6 via node 7. Link 8 has its weight calculated as follows

$$W_8 = 1x(.54) = .54 \qquad \text{Eq.(21)}$$

Link 3 contributes 50% of the flow to node 4, and is therefore assigned a weight calculated thus

$$W_3 = (.5)x(.46) = .23 \qquad \text{Eq.(22)}$$

Likewise

$$W_6 = (.5)x(.46) = .23 \qquad \text{Eq.(23)}$$

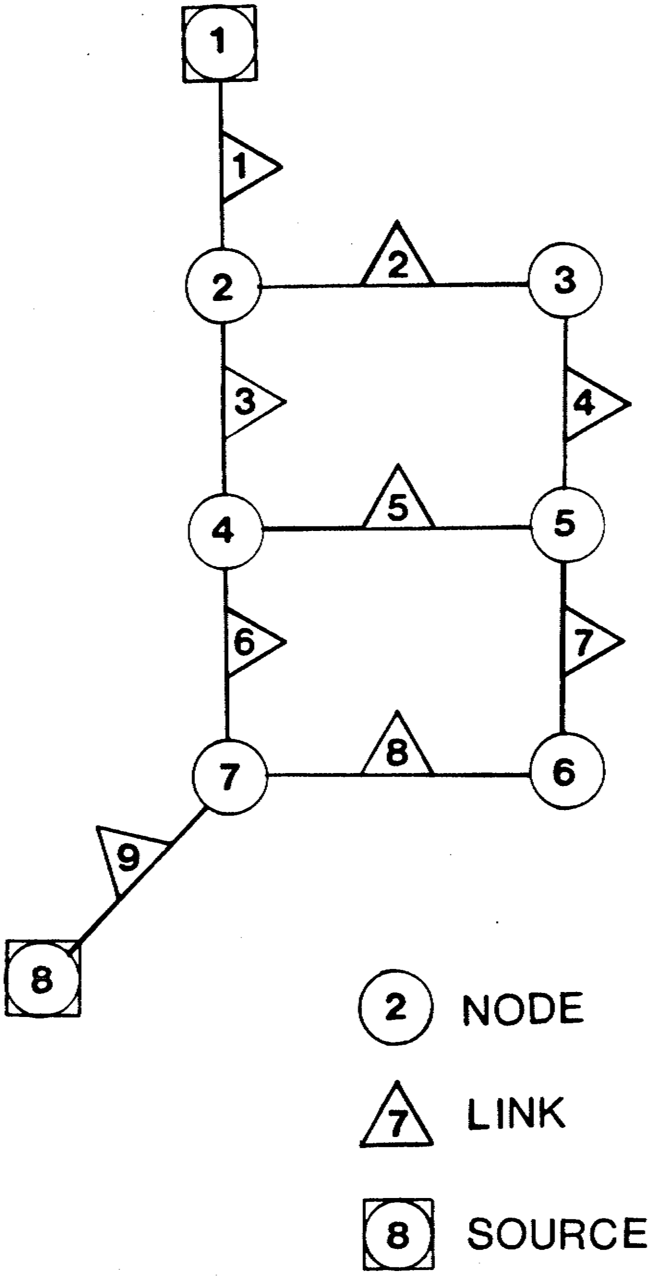

Figure 4: Network for Weighting Algorithm

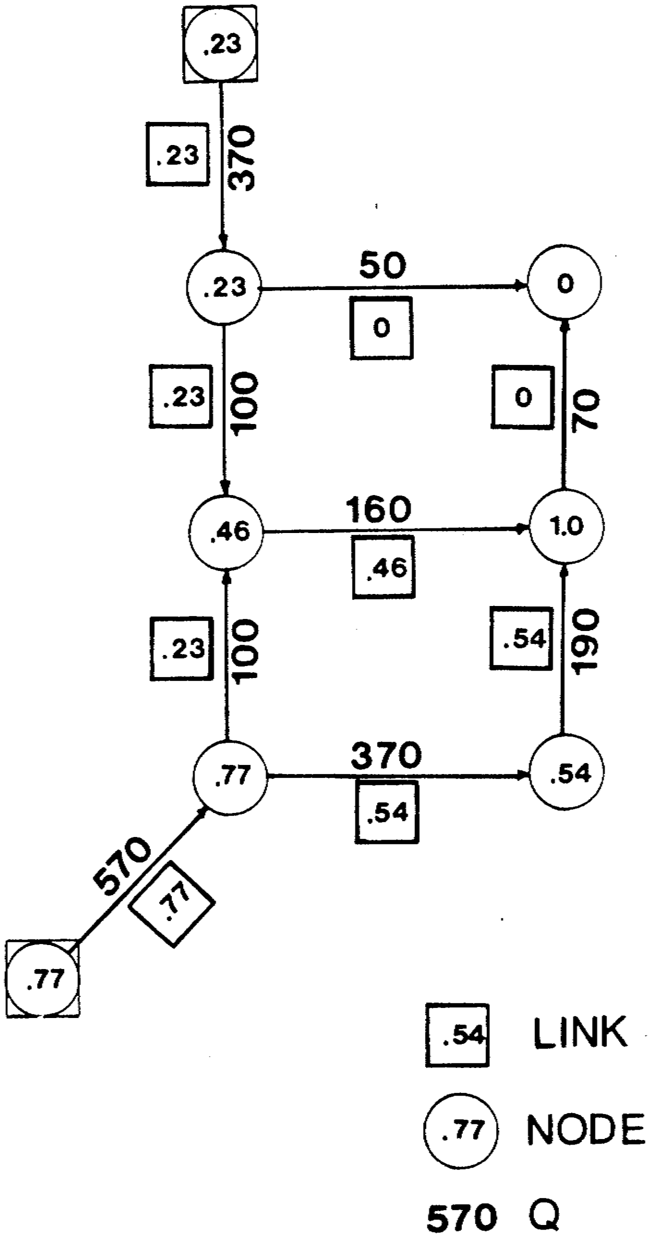

Figure 5: Flows and Weights of Network

It should be noted that the flow to node 5 from node 7 follows two distinct paths. The weighting at node 7 is therefore

$$w_7 = (.23)+(.54) = .77 \qquad \text{Eq.(24)}$$

The weights for each link and node are shown in Figure 5. Links 2 and 4 have weights of zero since none of the flow going to node 5 passes through these links.

Chapter V

USING THE MODEL IN DESIGN

5.1 INTRODUCTION

Three types of examples are illustrated in the following sections. The first example ,a large network with two reservoirs , is designed using a single demand pattern. Using one demand pattern is the standard procedure in present optimization methods. This example will demonstrate the basic approach used in the model while illustrating the shortcomings of single demand pattern design.

The second example uses the same network but with consideration for multiple demand patterns. This method will create a reliable and flexible water distribution system while eliminating pipes deemed uneconomical.

The third example uses a real problem encountered in the expansion of the New York City Water Supply Tunnels in 1969. Since this problem has been studied many times (Schaake and Lai [1969], Quindry et al. [1981] and Gessler [1982]), its solution allows the model to be compared in terms of performance and efficiency with respect to previous models. A listing of the computer program used in these examples is shown in Appendix B.

5.2 SINGLE DEMAND PATTERN DESIGN

The example network shown in figure 6 has 20 nodes and 37 possible pipe locations. The costs per metre of the available pipe sizes are shown in Table 1. The system lengths and connections are shown in Table 2 and the initial pipe size assumptions are shown in Table 3. The demands, minimum allowable pressures and initial pressure assumption for the system are shown in Table 4.

The results of the first computer run are shown in Table 5, where the final column lists the maximum weighting given to each pipe during the construction of the constraint equations. Many of the pipes have been reduced to the sub-minimum diameter and then automatically eliminated. The criteria for eliminating these pipes is that if the entire section of pipe is designated sub-minimum and the maximum weighting is less than 0.5 it is removed. The reason for its removal is that a small pipe is uneconomical due to economies of scale. A low maximum weighting also indicates that since only a small proportion of flow to any node passes through this pipe then its removal will have little effect on system performance. It can be seen that although the system tends towards branching, it is not entirely branched since many of the weightings are less than one. With a single demand pattern the most economical system should theoretically be a branched one. The model is there-

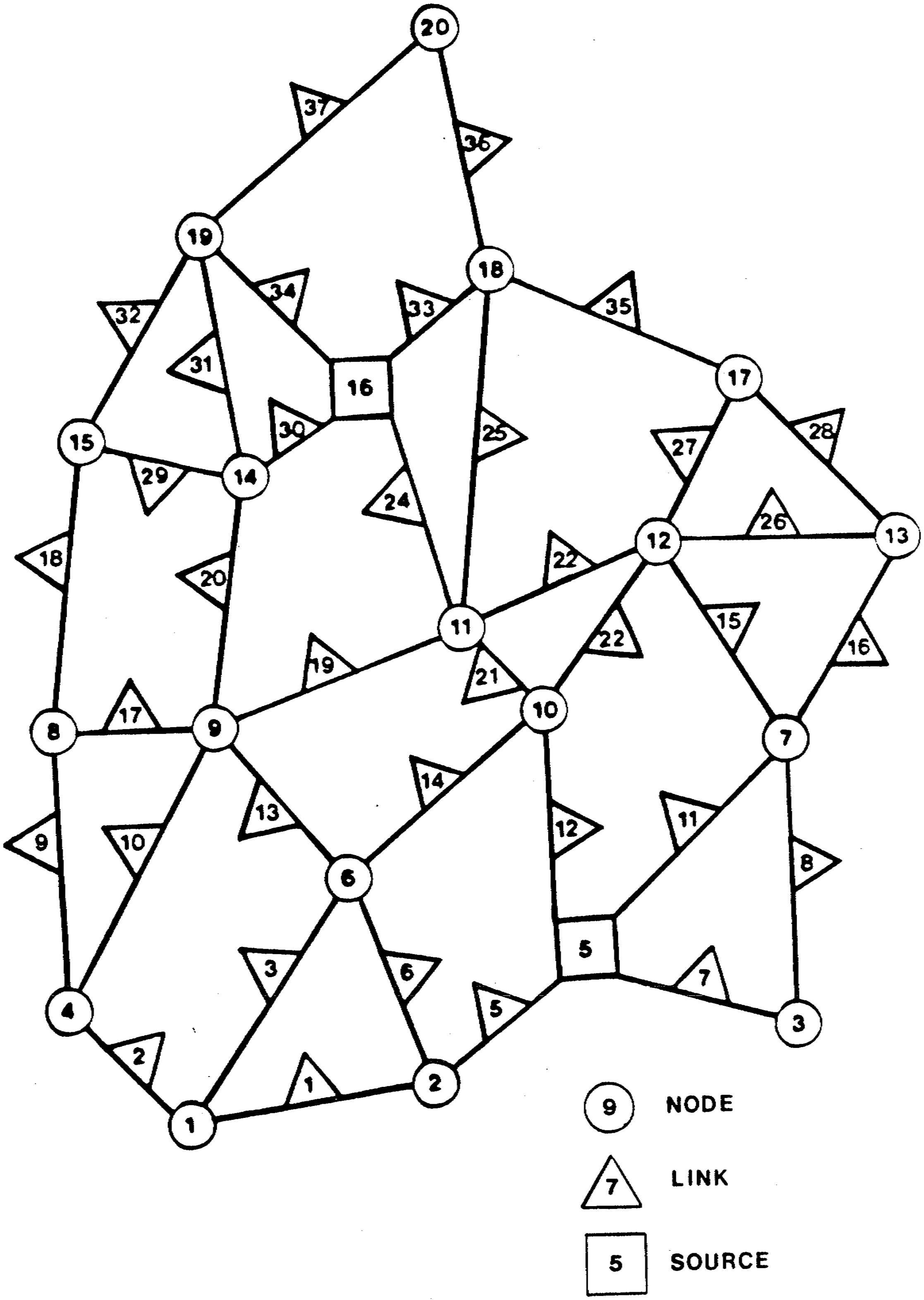

Figure 6: Possible Pipe Locations for System in Single and Multiple Demand Pattern Design

TABLE 1

Cost per Metre of Different Diameter Pipes

Diameter (metres)	Cost ($/m)
0.125	58.00
0.150	62.00
0.200	71.70
0.250	88.90
0.300	112.30
0.350	138.70
0.400	169.00
0.450	207.00
0.500	248.00
0.550	297.00
0.600	347.00
0.650	405.00
0.700	470.00

TABLE 2

Pipe Data for System

Pipe	Connecting Nodes From	To	Length (metres)
1	1	2	760.00
2	1	4	520.00
3	1	6	890.00
4	2	3	1120.00
5	2	5	610.00
6	2	6	680.00
7	3	5	680.00
8	3	7	870.00
9	4	8	860.00
10	4	9	980.00
11	5	7	890.00
12	5	10	750.00
13	6	9	620.00
14	6	10	800.00
15	7	12	730.00
16	7	13	680.00
17	8	9	480.00
18	8	15	860.00
19	9	11	800.00
20	9	14	770.00
21	10	11	350.00
22	10	12	620.00
23	11	12	670.00
24	11	16	790.00
25	11	18	1150.00
26	12	13	750.00
27	12	17	550.00
28	13	17	700.00
29	14	15	500.00
30	14	16	450.00
31	14	19	750.00
32	15	19	720.00
33	16	18	540.00
34	16	19	700.00
35	17	18	850.00
36	18	20	750.00
37	19	20	970.00

TABLE 3

Initial Pipe Size Assumption

Pipe	Diameter (m)	Length (m)
1	0.400	760.00
2	0.400	520.00
3	0.400	890.00
4	0.400	1120.00
5	0.400	610.00
6	0.400	680.00
7	0.400	680.00
8	0.400	870.00
9	0.400	860.00
10	0.400	980.00
11	0.400	890.00
12	0.400	750.00
13	0.400	620.00
14	0.400	800.00
15	0.400	730.00
16	0.400	680.00
17	0.400	480.00
18	0.400	860.00
19	0.400	800.00
20	0.400	770.00
21	0.400	350.00
22	0.400	620.00
23	0.400	670.00
24	0.400	790.00
25	0.400	1150.00
26	0.400	750.00
27	0.400	550.00
28	0.400	700.00
29	0.400	500.00
30	0.400	450.00
31	0.400	750.00
32	0.400	720.00
33	0.400	540.00
34	0.400	700.00
35	0.400	850.00
36	0.400	750.00
37	0.400	970.00

Initial Cost= $ 4,590,040.00

TABLE 4

Initial Pressure Assumptions

Node	Minimum Head (m)	Demand (l/s)	Initial Head (m)
1	75.00	165.0	80.00
2	74.00	220.0	90.00
3	73.00	145.0	90.00
4	72.00	165.0	70.00
5	102.00	-	102.00
6	73.00	140.0	80.00
7	67.00	175.0	90.00
8	72.00	180.0	70.00
9	70.00	140.0	75.00
10	69.00	160.0	90.00
11	71.00	170.0	93.00
12	70.00	160.0	85.00
13	64.00	190.0	80.00
14	73.00	200.0	90.00
15	73.00	150.0	80.00
16	96.00	-	96.00
17	67.00	165.0	80.00
18	70.00	140.0	90.00
19	70.00	185.0	90.00
20	67.00	160.0	70.00

fore neglecting some aspect of reality. As stated in the model development (Chapter 4) the flow in each pipe is assumed fixed during the solution of the linear programming substage. The flow is then corrected during the Hardy Cross procedure. Since it is assumed that the flow is fixed , replacing a small pipe with the sub-minimum pipe diameter (125mm) will cause a severe pressure drop across that link. Since in reality only a minority of the flow going to the downstream node passes through this pipe it is easily rerouted causing a much smaller pressure drop. Thus the reduction and removal of this pipe becomes economical. In addition since the cost function only compares the difference in cost between two pipes it fails to take into account the basic fixed cost of putting any pipe in that position initially i.e. trenching, transport etc.. The pipe with the lowest weighting is therefore removed and the computer program is run again. Four of these computer runs, in which the pipe with the lowest maximum weighting was removed each time, were needed to arrive at an optimal solution. An AMDAHL 580 computer was used and 22.25 seconds CPU time was needed to obtain these results.

The final configuration is shown in Figure 7 with the pipe sizes given in Table 6. As expected the cost has been minimized. However as Templeman[1982] stated the flexibily has been optimized out of the system. This can be clearly seen in Figure 7, since the system has deteriorated into two

TABLE 5

Results of First Run for Single Demand Pattern Design

Pipe	Diameter (m)	Length (m)	Diameter (m)	Length (m)	Maximumg Weighting
1	0.200	37.75	0.250	722.25	1.000
2	0.200	520.00	-	-	1.000
3	-	-	-	-	-
4	-	-	-	-	-
5	0.300	610.00	-	-	1.000
6	0.150	418.38	0.200	261.62	1.000
7	0.125	103.15	0.150	576.85	1.000
8	-	-	-	-	-
9	-	-	-	-	-
10	-	-	-	-	-
11	0.200	442.59	0.250	447.41	1.000
12	0.200	648.56	0.250	101.44	1.000
13	-	-	-	-	-
14	-	-	-	-	-
15	-	-	-	-	-
16	0.200	680.00	-	-	1.000
17	0.200	480.00	-	-	0.679
18	0.150	860.00	-	-	0.321
19	0.200	800.00	-	-	0.661
20	0.150	770.00	-	-	0.339
21	-	-	-	-	-
22	0.200	620.00	-	-	1.000
24	0.250	790.00	-	-	0.661
25	-	-	-	-	-
26	-	-	-	-	-
27	-	-	-	-	-
28	-	-	-	-	-
29	0.200	500.00	-	-	1.000
30	0.250	291.45	0.300	158.55	1.000
31	-	-	-	-	-
32	-	-	-	-	-
33	0.250	540.00	-	-	1.000
34	0.150	398.18	0.200	301.82	1.000
35	0.150	181.01	0.200	668.99	1.000
36	0.150	240.60	0.200	509.40	1.000
37	-	-	-	-	-

Cost = $ 1,026,979

separate branched systems. A pipe failure anywhere in the system will the cause water supply to be stopped in some part of the system. Clearly this configuration is not suitable for an urban water distribution system, therefore the criteria by which the system is designed must be changed.

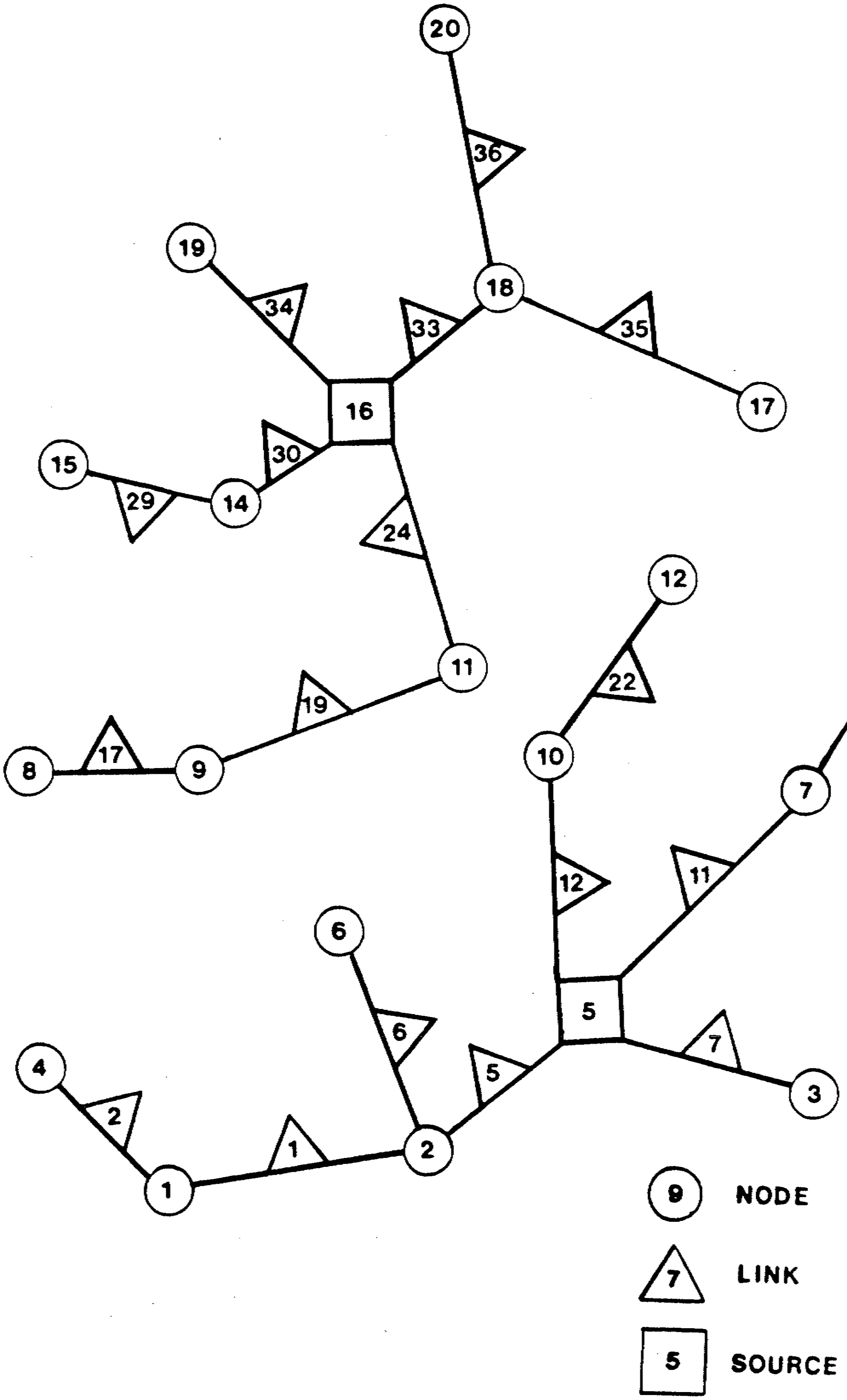

Figure 7: Single Demand Pattern Design Layout

TABLE 6

Results of Final Run for Single Demand Pattern Design

Pipe	Diameter (m)	Length (m)	Diameter (m)	Length (m)	Maximum Weighting
1	0.200	38.75	0.250	721.25	1.000
2	0.200	520.00	-	-	1.000
3	-	-	-	-	-
4	-	-	-	-	-
5	0.300	610.00	-	-	1.000
6	0.150	418.78	0.200	261.22	1.000
7	0.125	103.15	0.150	576.85	1.000
8	-	-	-	-	-
9	-	-	-	-	-
10	-	-	-	-	-
11	0.200	442.70	0.250	447.30	1.000
12	0.200	650.53	0.250	99.47	1.000
13	-	-	-	-	-
14	-	-	-	-	-
15	-	-	-	-	-
16	0.200	680.00	-	-	1.000
17	0.200	322.15	0.250	157.85	1.000
18	-	-	-	-	-
19	0.250	800.00	-	-	1.000
20	-	-	-	-	-
21	-	-	-	-	-
22	0.200	620.00	-	-	1.000
23	-	-	-	-	-
24	0.300	790.00	-	-	1.000
25	-	-	-	-	-
26	-	-	-	-	-
27	-	-	-	-	-
28	-	-	-	-	-
29	0.200	500.00	-	-	1.000
30	0.200	400.23	0.250	49.77	1.000
31	-	-	-	-	-
32	-	-	-	-	-
33	0.250	540.00	-	-	1.000
34	0.150	398.18	0.200	301.82	1.000
35	0.150	181.01	0.200	668.99	1.000
36	0.150	240.60	0.200	509.40	1.000
37	-	-	-	-	-

Cost = $ 950,230

5.3 MULTIPLE DEMAND PATTERN DESIGN

In this design example the pipe costs and system configuration remain the same as the previous example(see Tables 1 & 2). The initial assumption for pipe sizes also remains the same as in Table 3. This system is, however, designed for many situations simultaneously. The criteria of flexibility selected was that the system must give adequate pressure when one of each possible pipes is out of order and a fire flow is present at the worst possible location. This worst location will vary with the pipe that is assumed broken and should be picked by the design engineer. For this example the location and magnitude of each of these fire flows is given in Table 7. From this table and Table 4 , thirty seven different demand patterns are generated (see Appendix C). It should be noted that the demand is not evenly distributed throughout the system and the fire flow demands vary in magnitude depending upon their location. These patterns are used, as described in section 4.2, to construct the pressure constraints.

The results of the first run are shown in Table 8 (An example of the output for this type of computer run is given in Appendix C). The final column gives the maximum weighting assigned to each pipe during the construction of the constraints. The same criteria as stated in the previous example is used to remove pipes, i.e. a small pipe with the lowest maximum weighting is removed first. This low maximum weighting indicates that removal of this pipe will do the least damage to the pressure profiles. The program is then rerun and if the solution is cheaper this result is the new best result. This process continues until the cost increases or until all the maximum weightings are greater than 0.5. The final results are shown in Figure 8 and Table 9. To obtain these results six computer runs requiring 6 minutes and 4.6 seconds of CPU time were needed.

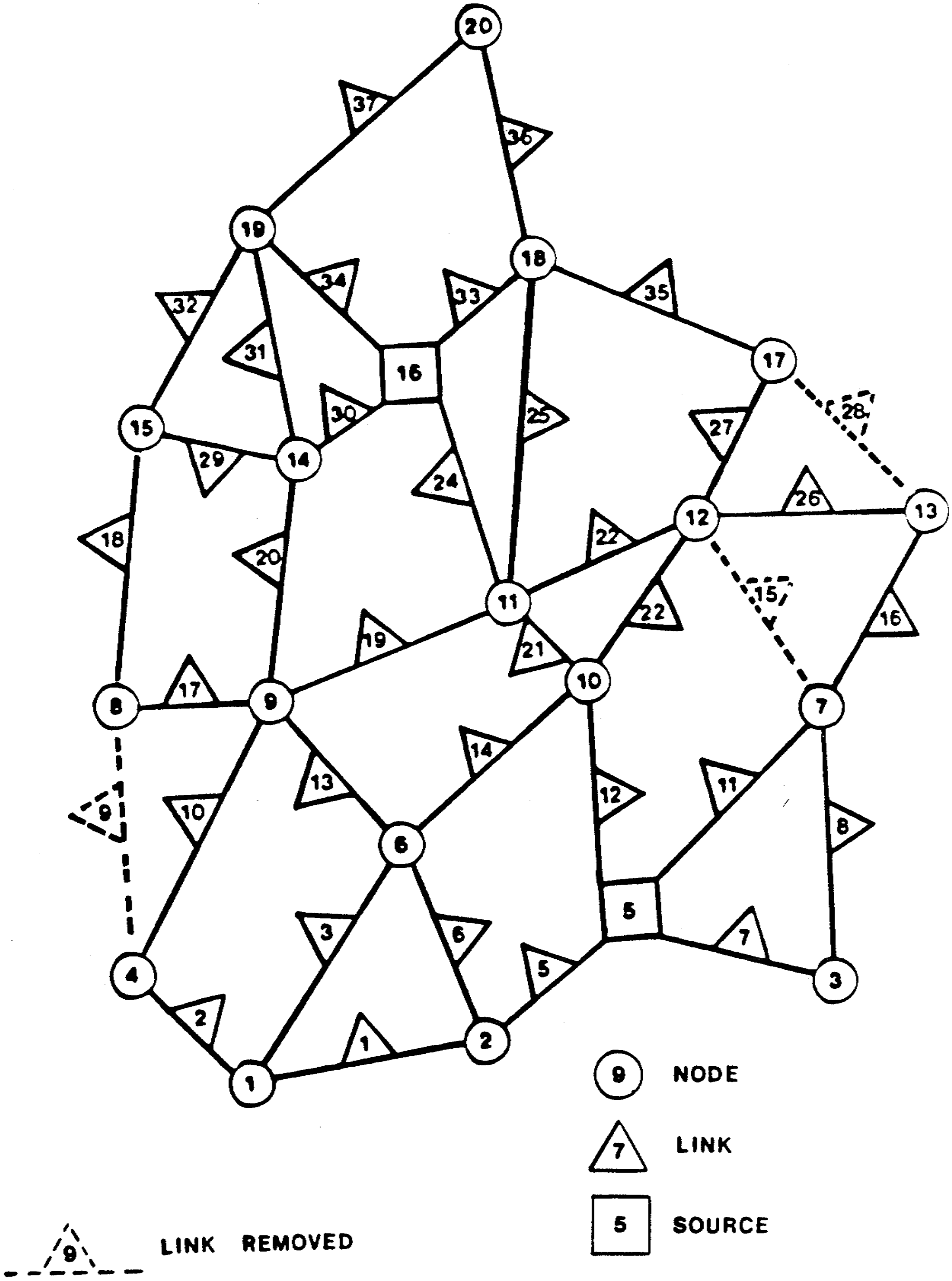

Figure 8: Multiple Demand Pattern Design Layout

TABLE 7

Pipe Breaks and Fire Flows for Each Demand Pattern Design

		Fire Flows	
Demand Pattern	Pipe Broken	at Node	Demand (1/s)
1	1	1	70.0
2	2	4	70.0
3	3	1	70.0
4	4	3	70.0
5	5	2	70.0
6	6	6	90.0
7	7	3	70.0
8	8	7	70.0
9	9	4	70.0
10	10	4	70.0
11	11	7	90.0
12	12	10	90.0
13	13	9	90.0
14	14	6	90.0
15	15	12	50.0
16	16	13	70.0
17	17	8	70.0
18	18	8	70.0
19	19	9	90.0
20	20	9	90.0
21	21	11	100.0
22	22	12	50.0
23	23	12	50.0
24	24	11	100.0
25	25	11	100.0
26	26	13	70.0
27	27	17	70.0
28	28	13	70.0
29	29	15	70.0
30	30	14	120.0
31	31	19	120.0
32	32	15	70.0
33	33	18	120.0
34	34	19	120.0
35	35	17	70.0
36	36	20	120.0
37	37	20	120.0

TABLE 8

Results of First Run of Multiple Demand Pattern Design

Pipe	Diameter (m)	Length (m)	Diameter (m)	Length (m)	Maximum Weighting
1	0.200	350.98	0.250	409.02	1.000
2	0.150	250.77	0.200	269.23	0.703
3	0.200	753.42	0.250	136.58	0.894
4	0.200	1120.00	-	-	0.550
5	0.300	610.00	-	-	0.892
6	0.200	680.00	-	-	0.777
7	0.250	680.00	-	-	0.771
8	0.150	130.92	0.200	739.08	0.867
9	0.125	89.63	0.150	770.37	0.297
10	0.200	980.00	-	-	0.759
11	0.200	890.00	-	-	0.383
12	0.350	577.84	0.400	172.16	1.000
13	0.200	620.00	-	-	0.440
14	0.300	800.00	-	-	0.885
15	-	-	-	-	-
16	0.150	507.91	0.200	172.09	0.611
17	0.150	123.33	0.200	356.67	0.754
18	0.200	860.00	-	-	0.748
19	0.200	800.00	-	-	0.338
20	0.200	677.52	0.250	92.48	0.720
21	0.200	350.00	-	-	0.483
22	0.200	620.00	-	-	1.000
23	0.200	600.07	0.250	69.93	1.000
24	0.250	790.00	-	-	0.653
25	0.150	118.84	0.200	1031.16	0.612
26	0.150	123.94	0.200	626.06	0.686
27	0.150	150.86	0.200	399.14	0.759
28	0.150	700.00	-	-	0.389
29	0.200	500.00	-	-	0.952
30	0.250	314.92	0.300	135.08	0.955
31	0.200	647.03	0.250	102.97	0.643
32	0.200	720.00	-	-	1.000
33	0.250	540.00	-	-	0.848
34	0.300	606.71	0.350	93.29	1.000
35	0.150	466.31	0.200	383.69	0.808
36	0.200	546.60	0.250	203.40	1.000
37	0.200	569.24	0.250	400.76	1.000

Cost = $ 2,074,762

TABLE 9

Results of Final Run of Multiple Demand Pattern Design

Pipe	Diameter	Length	Diameter	Length	Maximum
	(m)	(m)	(m)	(m)	Weighting
1	0.250	760.00	-	-	1.000
2	0.150	112.99	0.200	407.01	1.000
3	0.250	796.70	0.300	93.30	1.000
4	-	-	-	-	-
5	0.300	370.60	0.350	239.40	1.000
6	0.200	680.00	-	-	0.725
7	0.200	473.58	0.250	206.42	1.000
8	0.200	314.96	0.250	555.04	1.000
9	-	-	-	-	-
10	0.200	519.99	0.250	460.01	1.000
11	0.250	890.00	-	-	1.000
12	0.400	750.00	-	-	1.000
13	0.250	620.00	-	-	0.797
14	0.350	540.84	0.400	259.16	1.000
15	-	-	-	-	-
16	0.150	98.14	0.200	581.86	1.000
17	0.150	41.82	0.200	438.18	1.000
18	0.200	173.27	0.250	686.73	1.000
19	-	-	-	-	-
20	0.200	770.00	-	-	0.889
21	-	-	-	-	-
22	0.200	35.54	0.250	584.46	1.000
23	0.150	345.08	0.200	324.92	0.663
24	0.200	336.57	0.250	453.43	0.553
25	0.200	1150.00	-	-	0.588
26	0.200	750.00	-	-	1.000
27	0.150	98.81	0.200	451.19	1.000
28	-	-	-	-	-
29	0.200	500.00	-	-	1.000
30	0.250	6.36	0.300	443.64	0.973
31	0.150	81.59	0.200	668.41	0.562
32	0.200	713.83	0.250	6.17	1.000
33	0.250	540.00	-	-	1.000
34	0.300	700.00	-	-	1.000
35	0.150	39.23	0.200	810.77	1.000
36	0.200	538.20	0.250	211.80	1.000
37	0.200	625.46	0.250	344.54	1.000

Cost = $ 1,950,698

5.4 NEW YORK CITY WATER SUPPLY TUNNELS EXPANSION

This example demonstrates how the technique can be applied to expansion of an existing system and compares the results to previous studies. The imperial system of measurements was used to facilitate comparison with the previous studies. The cost and layout in the New York City problem are described in Figure 9 and Tables 10 and 11. The proposed method of expansion is the same as in the previous studies (Schaake and Lai[1969],Quindry et al.[1981] and Gessler[1982]), i.e. to reinforce the system by constructing tunnels parallel to the existing tunnels. The existing pipe locations, lengths and diameters are shown in Table 10. The corresponding parallel reinforcing pipes are shown in column 2 of this table. In the initial assumption all reinforcing tunnels were assumed to be 84 inches in diameter.

The final results, shown in Table 12 and Figure 10, indicate that only a few pipes are needed to arrive at a least cost solution of 38.9 million. Two computer runs and 14.37 seconds CPU time were needed to obtain these results. This solution has discrete pipe diameters which span an entire link length in some links and are 'split' diameters in others. To compare this solution to that of Gessler[1982], which used only discrete pipes across the entire length of the links, the split pipes were replaced by a single diameter equal to the major portion of the present solution (see

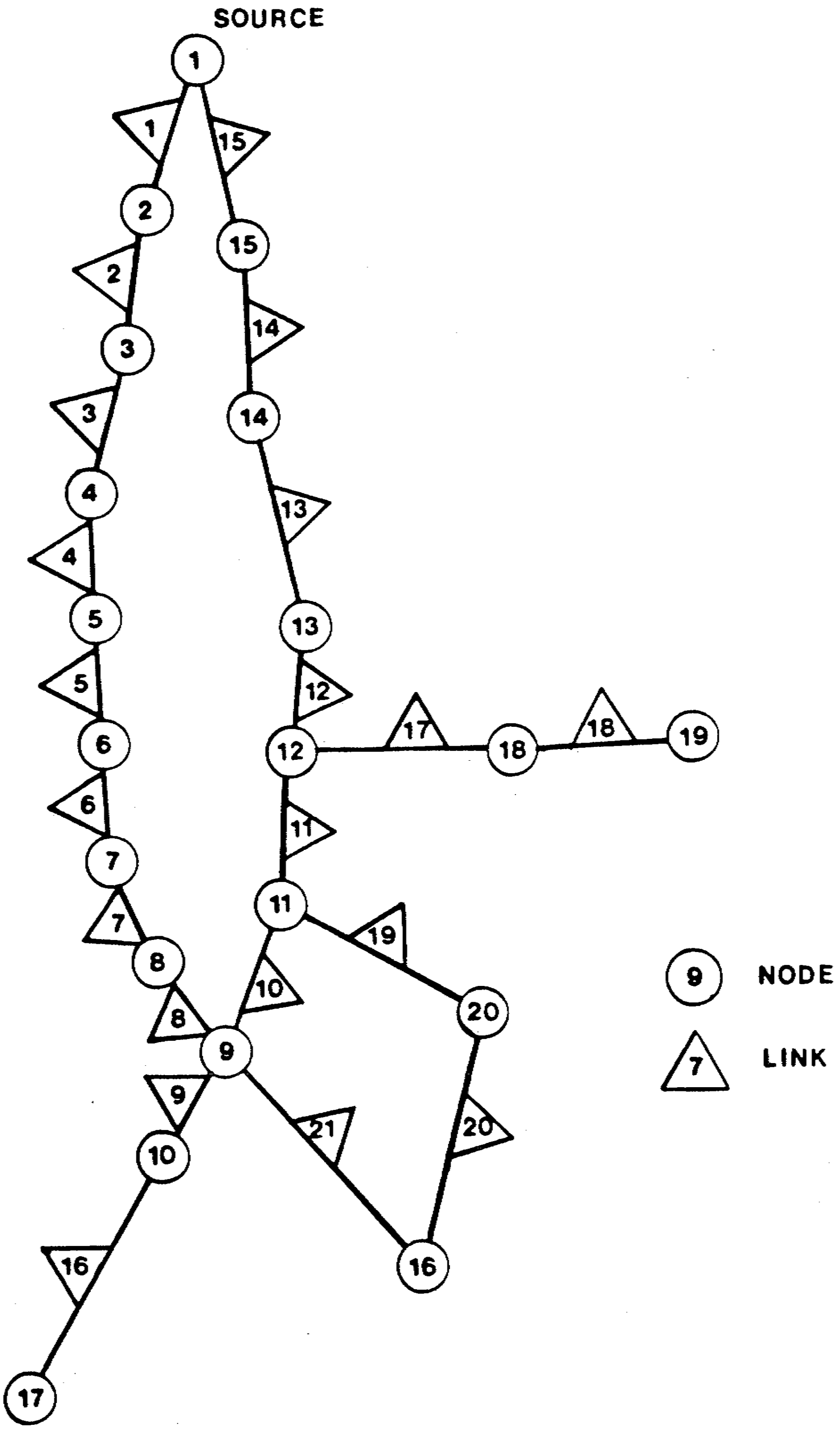

Figure 9: New York City Water Supply Tunnels

TABLE 10

Tunnel Costs for New York City

Diameter (inches)	Cost ($/ft)
36	93.50
48	134.00
60	176.00
72	221.00
84	267.00
96	316.00
108	365.00
120	417.00
132	469.00
144	522.00
156	577.00
168	632.00
180	689.00
192	746.00
204	804.00

TABLE 11

New York City Pipe Data

Pipe		Connecting Nodes		Length	Existing Diameter
Existing	Reinforcing	From	To	(feet)	(inches)
1	22	1	2	11600.	180
2	23	2	3	19800.	180
3	24	3	4	7300.	180
4	25	4	5	8300.	180
5	26	5	6	8600.	180
6	27	6	7	19100.	180
7	28	7	8	9600.	132
8	29	8	9	12500.	132
9	30	9	10	9600.	180
10	31	11	9	11200.	204
11	32	12	11	14500.	204
12	33	12	13	12200.	204
13	34	13	14	24100.	204
14	35	14	15	21100.	204
15	36	1	15	15500.	204
16	37	10	17	26400.	72
17	38	12	18	31200.	72
18	39	18	19	24000.	60
19	40	11	20	14400.	60
20	41	16	20	38400.	60
21	42	9	16	26400.	72

table 13). This configuration was tested using the Hardy Cross technique and the pressures were found to be adequate. The costs of these two solutions i.e. the split pipe solution and the discrete pipe solution, are compared with the previous studies in table 14. It can be seen that even the 'discrete' pipe solution, used for comparison with Gessler's solution, is less expensive than the previous solutions. Gessler's method, although it was a complete enumeration technique, failed to obtain this solution. This method's (decribed earlier in this thesis (p.18)) major shortcoming was that to reduce the number of combinations available pipes must be grouped together. In this problem pipes 7 and 8 are considered to be a single pipe, therefore a reinforcing pipe was added to both of these links. The solution of in this thesis is less expensive because only a reinforcing pipe parrellel to link 7 was used. A more complete record of the input and output datafor the New York City Water Supply Tunnels problem, including demands and pressures at each node, is shown in Appendix D.

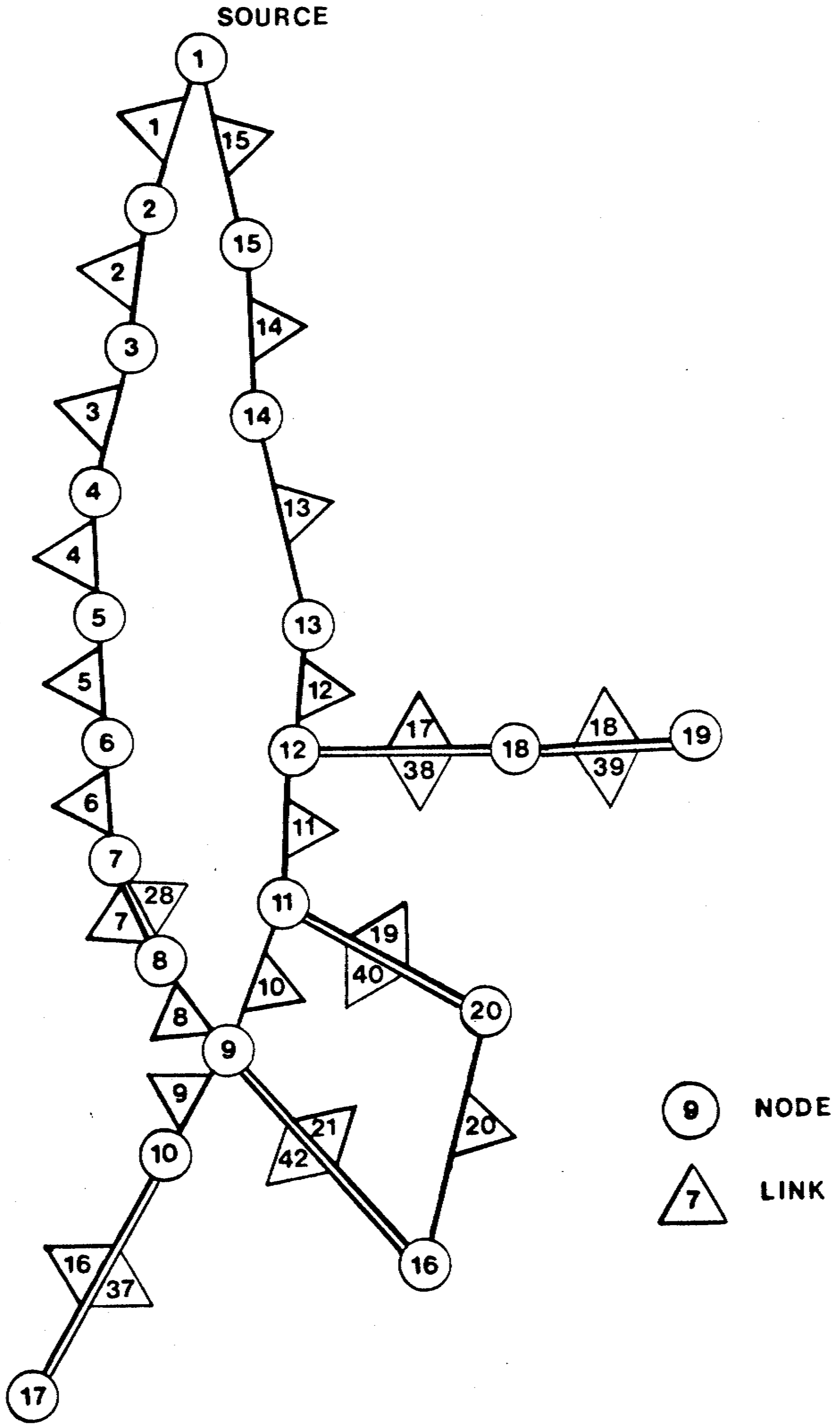

Figure 10: New York City Water Supply Tunnels Final Solution

TABLE 12

Split Pipe Solution for New York City Expansion

Pipe	Diameter (inches)	Length (feet)	Diameter (inches)	Length (feet)
28	144	9600.00	-	-
37	96	23432.08	108	2967.92
38(	96	31200.00	-	-
39	72	57.99	84	23942.01
40	48	4527.76	60	9872.24
42	72	3492.89	84	22907.11

Cost = $ 38.9 million

TABLE 13

Discrete Pipe Solution for New York City Expansion

Pipe	Diameter (inches)	Length (feet)	Diameter (inches)	Length (feet)
28	144	9600.00	-	-
37	96	26400.00	-	-
38	96	31200.00	-	-
39	84	24000.00	-	-
40	60	14400.00	-	-
42	84	26400.00	-	-

Cost = $ 39.2 million

TABLE 14

Comparison with Previous Studies

Schaake & Lai [1969]	78.1 million
Quindry et al. [1981]	63.6 million
Gessler [1982]	41.8 million
Split Pipe Solution	38.9 million
Discrete Pipe Solution	39.2 million

Chapter VI
CONCLUSIONS

A method of optimizing looped urban water distribution systems was developed. Many of the problems encountered in this type of optimization were addressed and the major conclusions are listed below.

1. Using a single demand pattern the loops are optimized out of the system thereby creating an inflexible system which will not perform adequately during an unexpected pipe failure.
2. A model has been developed which can consider many demand patterns simultaneously during the optimization procedure. Fire flow demands and pipe breakages were considered in each of the demand patterns.
3. Two widely available methods, the Hardy Cross network solver and a simple linear programming package were joined to produce the optimization technique.
4. The efficient simplex method of solving linear programming problems makes the technique applicable to large systems.
5. The technique produced a less expensive solution of the expansion of an existing system than had been obtained in all previous studies of that system.

6. The method incorporates a realistic cost function which uses standard unit costs (cost per length).

Appendix A

ANOMALIES IN ALPEROVITS AND SHAMIR'S TECHNIQUE

Since the method developed by Alperovits and Shamir strongly influenced this work, reasons requiring it to be modified are explained in detail in this appendix. In their method a single path was selected, presumably by the engineer, from each demand point to the source. The test system used to demonstrate their technique is shown in Figure 11. There are three different paths which can be used to constrain the pressure at node 5. The path choosen by Alperovits and Shamir (1-3-4) produced the result shown in Table 15. However, if an alternative path is choosen (1-2-7) then a less expensive solution, shown in Table 16, is arrived at. From these results it can be inferred that the method may choose larger pipes along the path which is selected, and reduces, to a minimum diameter, the pipes which happen to be off the paths. A different choice of paths will give a different least cost solution. Unfortunately it is not possible a priori to determine which path will lead to the least expensive solution. This is the main reason why this technique must be modified to consider all paths from the source(s) to each demand point.

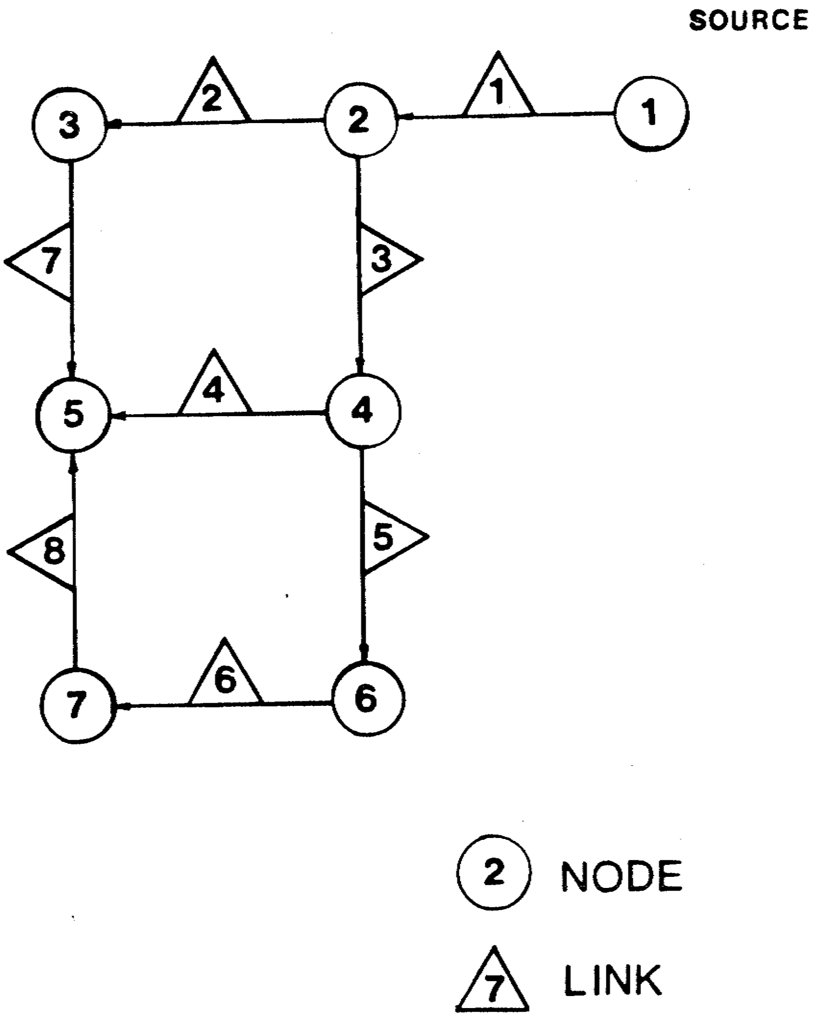

Figure 11: Example Used by Alperovits and Shamir

TABLE 15

Results Using Path 1-3-4

Pipe	Diameter (inches)	Length (feet)	Diameter (inches)	Length (feet)
1	18	783.43	20	216.57
2	6	666.60	8	333.40
3	18	1000.00	-	-
4	8	511.07	10	488.93
5	16	1000.0	-	-
6	10	902.84	12	97.16
7	1	1000.00	-	-
8	1	1000.00	-	-

Cost = $443,079

TABLE 16

Results Using Path 1-2-7

Pipe	Diameter (inches)	Length (feet)	Diameter (inches)	Length (feet)
1	18	889.54	20	110.46
2	10	653.57	12	346.43
3	16	1000.00	-	-
4	1	1000.00	-	-
5	16	1000.0	-	-
6	10	902.72	12	97.27
7	8	50.78	10	949.22
8	1	1000.00	-	-

Cost =$421,948

Appendix B

LISTING OF COMPUTER PROGRAM

```
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C:
C:                  VARIABLE LIST
C:
C: AAA(NL,I)         DIFFERENCE BETWEEN ACTUAL PRESSURE AND MINIMUN
C:                   ALLOWABLE PRESSURE AT NODE I IN DEMAND PATTERN NL
C: ALP(NC,NV)        SIMPLEX COEFFICIENT MATRIX
C: BIG               LARGE CONSTANT (10**10)
C: BLP(NC)           RIGHT HAND SIDE OF CONSTRAINT MATRIX
C: BREAK(NL,J)       0 IF PIPE J IN DEMAND PATTERN NL IN SERVICE
C:                   1 IF PIPE J IN DEMAND PATTERN NL OUT OF SERVICE
C: CHOICE(I)         NUMBER OF PIPES CONNECTING NODE I
C: CLP(NV)           COST FUNCTION IN SIMPLEX ALGORITHM
C: CONDUC(J)         CONDUCTIVITY OF LINK J
C: COSPIP(ID)        COST PER METRE OF PIPE OF DIAMETER NUMBER ID
C: COS1              TEMPORARY COST PER METRE VALUE
C: DELH              CHANGE OF PRESSURE TO MOVE TOWARD SOLUTION OF
C:                    FLOW AND PRESSURE PROBLEM
C: DELQ              AVERAGE ERROR OF FLOW IN HARDY CROSS SOLUTION
C: DEMAND
C: DENOM             DENOMINATER IN PRESSURE CHANGE CALCULATION IN
C:                    HARDY CROSS SOLUTION
C: DEQUIV            EQUIVALENT DIAMETER OF SPLIT PIPES
C: DEXP              HAZEN WILLIAMS EXPONENT = 2.63
C: DIAM(ID)          DIAMETER OF PIPE OF DIAMETER NUMBER ID
C: DIV1              DIVISOR OF COST FUNCTION TO MAKE VALUES FALL
C:                    WITHIN A CERTAIN RANGE FOR SIMPLEX PACKAGE
C: DIV2(N2)          DIVISOR OF CONSTRAINT MATRIX VALUES
C: DQEX              EXPONENT IN HAZEN WILLIAMS EQUATION (-DEXP/QEXP)
C: DSOL(NC)          DUAL SOLUTION OF SIMPLEX SOLUTION
C: DO
C: D1                TEMPORARY DIAMETER VALUE
C: D2                TEMPORARY DIAMETER VALUE
C: D3                TEMPORARY DIAMETER VALUE
C: EPSOL             CHANGE IN COST FROM ONE ITERATION TO THE NEXT
C: EQUA              NUMBER OF EQUALITY CONSTRAINTS IN LP FORMULATION = 0
C: EXIST(J)          DIAMETER NUMBER OF EXISTING PIPE
C: FACTOR(K,J)       WEIGHTING FACTOR FOR CONSTRAINT K AND PIPE J
C: FLOW(NL,J)        FLOW IN PIPE J DURING DEMAND PATTERN NL
C: F1                TEMPORARY FLOW
C: HAZEN             HAZEN-WILLIAMS COEFFICIENT
C: HCON              NUMBER OF PRESSURE CONSTRAINTS
C: HDIFF             PRESSURE DROP ACCROSS A PIPE
C: I                 NODE NUMBER
C: ID                DIAMETER NUMBER
C: IDPT1(NV)         DIAMETER OF REPLACEMENT PIPE OF VARIABLE NV
C: IDPT2(NV)         DIAMETER OF DESIGNATED PIPE
C: IER               ERROR NUMBER FOR SIMPLEX PACKAGE
C: II                COUNTER FOR READ DO LOOPS
C: IK(K)             POINTER TO NODE I WHOSE PRESSURE IS
C:                    CONSTRAINED BY CONSTRAINT K
C: INEQ              NUMBER OF INEQUALITY CONSTRAINTS
C: INFLOW(NL,I)      TOTAL INFLOW INTO NODE I DEMAND PATTERN NL
C: INHEAD(NL,I)      ACTUAL PRESSURE AT NODE I DEMAND PATTERN NL
```

```
C: IPT(NC)              POINTER TO NODE OF WHICH THE PRESSURE IS CONSTRAINED
C:                       BY CONSTRAINT NC
C: ISUM                 VALUE TO CHECK IF TWO CONSTRAINTS HAVE IDENTICAL
C:                       PATHS
C: IT                   ITERATION COUNTER FOR OPTIMIZATION PROCEDURE
C: ITEMP                TEMPORY VALUE (INTEGER)
C: ITER(NL)             NUMBER OF HARDY CROSS ITERATIONS
C: IW                   WORK VECTOR FOR SIMPLEX PACKAGE
C: I2                   TEMPORARY NODE NUMBER USED TO TRACE FLOW FROM
C:                       CONSTRAINED NODE TO THE SOURCE(S)
C: J                    PIPE NUMBER
C: JJ                   COUNTER FOR READING IN PIPE DATA
C: JNV(NV)              PIPE NUMBER ASSOCIATED WITH VARIABLE NV
C: JPT(NC)              LINK ASSOCIATED WITH LENGTH CONTRAINT NC
C: J1                   HYDRAULIC GRADIENT
C: J2                   HYDRAULIC GRADIENT
C: K                    PRESSURE CONSTRAINT NUMBER
C: KA                   CONSTANT MUST EQUAL FIRST DIMENSION OF ALP(NC,NV)
C: KEND                 NUMBER OF PRESSURE CONSTRAINTS
C: K1                   CONSTRAINT NUMBER
C: L                    COUNTER FOR LINKS CONNECTING A NODE
C: LEN(1,J)             LENGTH OF SECTION 1 OF LINK J
C: LENGTH(J)            LENGTH OF SECTION J
C: LENTES               IF =0 THEN NO LENGTH CONSTRAINTS ARE BINDING
C:                         =1 THEN A LENGTH CONSTRAINT IS BINDING
C: LL                   LENGTH OF LINK IN 1000 UNITS
C: LOADS                TOTAL NUMBER OF DEMAND PATTERNS
C: L1                   LENGTH OF SECTION 1
C: L2                   LENGTH OF SECTION 2
C: MAXFAC(J)            MAXIMUM WIEGHTING FACTOR FOR LINK J
C: MINHED(NL,I)         MINIMUM ALLOWABLE PRESSURE FOR NODE I
C:                       DEMAND PATTERN J
C: N                    LINK END NUMBER 1 OR 2
C: NC                   CONSTRAINT NUMBER
C: ND(2,J)              DIAMETER NUMBER FOR SECTION 2 OF LINK J
C: NDIAM                NUMBER OF DIFFERENT DIAMETERS
C: ND1                  DIAMETER NUMBER FOR SECTION 1
C: ND2                  DIAMETER NUMBER FOR SECTION 2
C: NFACT(I)             WEIGHT OF NODE I
C: NL                   DEMAND PATTERN NUMBER
C: NLK(K)               POINTER TO DEMAND PATTERN ASSOCIATED WITH PRESSURE
C:                       CONSTRAINT K
C: NLPT(NC)             POINTER TO DEMAND PATTERN ASSOCIATED WITH
C:                       CONTRAINT K
C: NN                   COUNTER FOR READING IN DEMAND PATTERNS
C: NODE(2,J)            NODE NUMBER CONNECTING END 2 OF LINK J
C: NODES                NUMBER OF NODES
C: NTYPE                TYPE OF PROGRAM TO RUN 0 = HARDY CROSS ANALYSIS
C:                                             1 = OPTIMIZATION
C: NUMCON               TOTAL NUMBER OF CONSTRAINTS
C: NUMVAR               TOTAL NUMBER OF VARIABLES
C: NV                   VARIABLE NUMBER
C: PATH(K,J)             IF 1 THEN LINK J IS ON ONE OF THE PATHS OF
                          CONSTRAINT K, OTHERWISE = 0
```

```
C: PENALT            VALUE OF PENALTY COST (SHOULD BE ABOUT 3X MAXIMUM
C:                    PIPE COST)
C: PIPES             TOTAL NUMBER OF LINKS
C: PSOL(NV)          PRIMAL SOLUTION
C: PT                POINTER USED TO KEEP TRACK OF POSITION WHEN
C:                    KEEPING TRACK OF PATHS FROM NODE TO SOURCE(S)
C: PT1                   "               "              "
C: PT2                   "               "              "
C: QEXP              EXPONENT USED IN THE HAZEN WILLIAMS FORMULA=0.54
C: QNEG              -0.54
C: RW                WORK VECTOR FOR SIMPLEX PACKAGE
C: S                 SIMPLEX SOLUTION
C: SELECT(I,L)       LINK NUMBERS OF LINKS CONNECTED TO NODE I
C: SIGN1,SIGN2,SIGN3 VARIABLES TO CHANGE SIGN
C: SMALL             LOWEST AAA VALUE
C: SOURCE(NL,I)      RESERVOIR IN DEMAND PATTERN NL AT NODE I
C: SUMNOD            TOTAL DEMAND IN ALL NODES
C: SUPPLY(NL,I)      DEMAND CALCULATED BY HARDY CROSS METHOD
C: TEMP              TEMPORARY VALUES
C: TOTAL1(IT)        SYSTEM COST CALCULATED FROM SUBTRACTING  S
C:                    FROM PREVIOUS TOTAL2
C: TOTAL2(IT)        SYSTEM COST AS CALCULATED BY SUMMING PIPE COSTS
C: UNITS             COEFFICIENT IN HAZEN WILLIAMS FORMULA WHICH
C:                    CHANGES WITH UNITS
C: WORKN(PT)         WORK VECTOR FOR WEIGTHING LINKS
C:
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C
      REAL*8 DIAM(23),D1,D2,UNITS,HAZEN,LENGTH(49),LEN(2,49)
     $       ,DEXP,QEXP,DQEX,L1,L2,LL,DEQUIV,QNEG,J1,J2,TEMP
     $       ,COSPIP(23)
      REAL DEMAND(40,30),CONDUC(49),SUPPLY(40,30),SUMNOD
     $     ,DELQ,HDIFF,INHEAD(40,30),TOTAL2(90),DIV1,DIV2(120)
     $     ,FLOW(40,49),AAA(40,30),MINHED(40,30),BLP(120)
     $     ,BIG,SMALL,F1,CLP(153),ALP(120,153)
     $     ,TOTAL1(90),PSOL(153),DSOL(120),RW(25000)
     $     ,FACTOR(40,49),MAXFAC(49)
      REAL NFACT(30),INFLOW(40,49),PENALT
      INTEGER PT,PT1,PT2,WORKN(40),EXIST(49)
      INTEGER SOURCE(40,30),NODE(2,49),SELECT(30,10),CHOICE(30),ND(2,49)
     $        ,NL,I,J,L,LOADS,NODES,PIPES,BREAK(40,49),SIGN1,SIGN2,SIGN3
     $        ,K,NLK(49),IK(49),N,I2,ND1,ND2,PATH(40,49)
     $        ,KEND,K1,ISUM,NV,NC,ID
     $    ,IDPT1(153),JNV(153),NUMVAR,NUMCON,IPT(120),NLPT(80),IDPT2(153)
     $        ,IER,IW(1000),INEQ,EQUA,ITER(40)
      INTEGER JPT(120),IT,NDIAM,HCON
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
C:                                                                   :
C:                        READ IN DATA                               :
C:                                                                   :
C:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
      IT=1
      REWIND 9
      READ(9,*) NTYPE
      READ (9,*)  UNITS
      READ (9,*)  HAZEN
      READ (9,*)  PENALT
      READ (9,*)  NDIAM
      IF(NTYPE.NE.1)  WRITE(6,32)
   32 FORMAT(///20X,'**** HARDY CROSS ANALYSIS ONLY ***')
      IF(NTYPE.EQ.1)  WRITE(6,34)
   34 FORMAT(///20X,'***** NEWORK OPTIMIZER *****')
      WRITE(6,36)  UNITS,HAZEN,NDIAM
   36 FORMAT(//25X,'UNITS=',F13.7,//25X,'HAZEN-WILLIAMS C=',F7.0,
     $       //25X,'NUMBER OF CANDIDATE DIAMETERS',I5)
      WRITE(6,39)
   39 FORMAT(//20X,'        DIAMETER       COST',/)
      DO 44 II=1,NDIAM
         READ(9,*)  ID,DIAM(ID),COSPIP(ID)
         WRITE(6,43)  ID,DIAM(ID),COSPIP(ID)
   43    FORMAT(17X,I5,F13.3,F12.2)
   44 CONTINUE
C
C                              READ IN SYSTEM LAYOUT DATA
      REWIND 10
      READ(10,*)  PIPES,NODES
      WRITE(6,50)  PIPES,NODES
   50 FORMAT(//12X,'NUMBER OF LINKS',I5,5X,'NUMBER OF NODES',I5)
      WRITE(6,52)
   52 FORMAT(//15X,' PIPE  FROM    TO       LENGTH',/)
      DO 57 JJ=1,PIPES
         READ(10,*)  J,NODE(1,J),NODE(2,J),LENGTH(J)
         WRITE(6,56)  J,NODE(1,J),NODE(2,J),LENGTH(J)
   56    FORMAT(13X,3I6,F14.0)
   57 CONTINUE
C
      REWIND 11
      WRITE(6,61)
   61 FORMAT('1',/23X,'INITIAL ASSUMPTION')
      WRITE(6,63)
   63 FORMAT(//3X,' PIPE    DIAMETER    LENGTH    DIAMETER    LENGTH',
     $             '    EXISTING',/)
      TOTAL2(1)=0
      DO 82 JJ=1,PIPES
         READ(11,*)  J,ND(1,J),LEN(1,J),ND(2,J),LEN(2,J),EXIST(J)
         D1=DIAM(ND(1,J))
         D2=0
         D3=0
         IF(EXIST(J).NE.0)  D3=DIAM(EXIST(J))
         IF(LEN(2,J).NE.0)  D2=DIAM(ND(2,J))
         IF(LEN(2,J).EQ.0.AND.ND(1,J).EQ.1)  D1=0
         WRITE(6,75)  J,D1,LEN(1,J),D2,LEN(2,J),D3
   75 FORMAT(I6,6X,F7.3,F12.2,F10.3,F10.2,F10.3)
      IF(EXIST(J).NE.0)  GO TO 82
      IF(D1.EQ.0)  GO TO 82
C                              CALCULATE INITIAL COST
      TOTAL2(1)=TOTAL2(1)+LEN(1,J)*COSPIP(ND(1,J))
      IF(LEN(2,J).EQ.0)  GO TO 82
      TOTAL2(1)=TOTAL2(1)+LEN(2,J)*COSPIP(ND(2,J))
   82 CONTINUE
      WRITE(6,84) TOTAL2(1)
   84 FORMAT(//25X,'INITIAL COST=',F12.0)
C
      WRITE(6,87)
   87 FORMAT('1')
      REWIND 12
      READ(12,*) LOADS
      READ(12,*) NODES
      WRITE(6,92) LOADS,NODES
   92 FORMAT(/20X,'NUMBER OF LOADING CONDITIONS',I5,
     $       /20X,'NUMBER OF NODES',I5)
      WRITE(6,61)
      WRITE(6,96)
   96 FORMAT(//15X,'NODE   MINHEAD     DEMAND        INHEAD',/)
      DO 105 NN=1,LOADS
         READ(12,*) NL
         WRITE(6,634) NL
         DO 104 II=1,NODES
            READ(12,*) I,MINHED(NL,I),DEMAND(NL,I),INHEAD(NL,I)
            WRITE(6,103) I,MINHED(NL,I),DEMAND(NL,I),INHEAD(NL,I)
  103       FORMAT(13X,I5,F10.2,F12.1,F13.2)
  104    CONTINUE
  105 CONTINUE
      DO 110 NL=1,LOADS
         DO 109 I=1,NODES
            BREAK(NL,I)=0
  109    CONTINUE
  110 CONTINUE
      WRITE(6,112)
  112 FORMAT(///10X,'PIPES OUT OF SERVICE IN EACH CONDITION')
      REWIND 14
  114 READ(14,*) NL
      IF(NL.EQ.0) GO TO 124
      WRITE(6,117) NL
  117 FORMAT(/3X,'LOAD',I6)
  118 READ(14,*) J
      IF(J.LE.0) GO TO 114
      WRITE(6,121) J
  121 FORMAT(10X,'PIPE'I5)
      BREAK(NL,J)=1
      GO TO 118
  124 CONTINUE
      WRITE(6,87)
      IF(NTYPE.EQ.1)WRITE(6,127)
  127 FORMAT(//5X,'ITERATION',10X,'COST 1',10X,'COST 2',/)
C
C                                        CALCULATE THE NUMBER OF POSSIBLE
C                                        PIPES CONNECTING EACH NODE
      LENTES=1
  132 CONTINUE
      DO 146 I=1,NODES
         DO 137 NL=1,LOADS
```

```
            SOURCE(NL,I)=0
            IF(DEMAND(NL,I).LT.0)  SOURCE(NL,I)=1
  137    CONTINUE
C
         L=0
         DO 144 J=1,PIPES
            IF(NODE(1,J).NE.I  .AND.  NODE(2,J).NE.I)  GO TO 144
            L=L+1
            SELECT(I,L)=J
  144    CONTINUE
         CHOICE(I)=L
  146 CONTINUE
C
C                             HARDY CROSS FLOW BALANCING METHOD
C
      QEXP=0.54
      DEXP=2.63
      DQEX=(-1.0D0)*DEXP/QEXP
      QNEG=(-1.0D0)*QEXP
C     WRITE(6,155)  QEXP,DEXP,DQEX,QNEG
C 155 FORMAT(//3X,'N=',F12.7,//3X,'M=',F12.7,//3X,'-M/N=',F12.7,
C    $        //3X,'-N=',F12.7)
C     WRITE(6,158)
C 158 FORMAT(//3X,'PIPE      CONDUCTIVITY')
      DO 179 J=1,PIPES
         CONDUC(J)=0
         IF(ND(1,J).EQ.1.AND.LEN(2,J).EQ.0)  GO TO 176
         L1=LEN(1,J)/1000.00
         L2=LEN(2,J)/1000.00
         LL=LENGTH(J)/1000.00
         IF(DIAM(ND(1,J)).EQ.0)  GO TO 176
         IF(LEN(2,J).EQ.0)  GO TO 175
         D1=DIAM(ND(1,J))
         D2=DIAM(ND(2,J))
         DEQUIV=(D1**DQEX*L1+D2**DQEX*L2)**QNEG
C        WRITE(6,*)  DEQUIV
         CONDUC(J)=UNITS*HAZEN*DEQUIV
C        CONDUC(J)=UNITS*HAZEN*((DIAM(ND(1,J))**DQEX)*L1
C    $   (DIAM(ND(2,J))**DQEX)*L2)**(-1.0D0/QEXP)
         GO TO 176
  175    CONDUC(J)=UNITS*HAZEN*DIAM(ND(1,J))**DEXP/LL**QEXP
  176    CONTINUE
C        WRITE(6,178)  J,CONDUC(J)
C 178    FORMAT(I5,F15.2)
  179 CONTINUE
C
      DO 239 NL=1,LOADS
C        WRITE(6,634)  NL
       ITER(NL)=0
  184    DELQ=0
         DO 217  I=1,NODES
            SUPPLY(NL,I)=0
            SUMNOD=0
            DENOM=0
            ITEMP=CHOICE(I)
            DO 208 L=1,ITEMP
            J=SELECT(I,L)
               IF(BREAK(NL,J).EQ.1)  GO TO 208
               SIGN1=1
               IF(NODE(2,J).EQ.I)  GO TO 197
C              IF(NODE(1,J).NE.I)  GO TO 208
               SIGN1=-1
  197          HDIFF=INHEAD(NL,NODE(1,J))-INHEAD(NL,NODE(2,J))
               SIGN2=1
               IF(HDIFF.LT.0)  SIGN2=-1
               HDIFF=ABS(HDIFF)
               FLOW(NL,J)=CONDUC(J)*(HDIFF**QEXP)*SIGN2
               SUPPLY(NL,I)=SUPPLY(NL,I)  + FLOW(NL,J)*SIGN1
               SUMNOD=SUMNOD+ABS(FLOW(NL,J))
               IF(HDIFF.EQ.0)  GO TO 208
               DENOM = DENOM + FLOW(NL,J)*SIGN2/HDIFF
C             WRITE(6,207)  NL,I,J,HDIFF,FLOW(NL,J),SUPPLY(NL,I),DENOM
C 207          FORMAT(3X,'7001',3I5,F8.1,2F8.0,F8.1)
  208       CONTINUE
            IF(SOURCE(NL,I).NE.0)  GO TO 217
            IF(SUMNOD.EQ.0)SUMNOD=0.1
            IF(DENOM.EQ.0)  DENOM=0.1
            DELQ=DELQ+ABS(DEMAND(NL,I)-SUPPLY(NL,I))/SUMNOD
            DELH=(DEMAND(NL,I)-SUPPLY(NL,I))/(QEXP*DENOM)
            INHEAD(NL,I)=INHEAD(NL,I)-DELH
C           WRITE(6,216)  DELQ,DELH,INHEAD(NL,I)
C 216       FORMAT(3X,'7002',F8.0,F8.2,F8.2)
  217    CONTINUE
C        WRITE(6,219)  DELQ
C 219    FORMAT(3X,'DELQ=',F10.4)
         ITER(NL)=ITER(NL)+1
         IF(ITER(NL).GT.500)  GO TO 223
         IF(DELQ.GT.0.01)  GO TO 184
  223    CONTINUE
C        WRITE(6,225)  ITER(NL)
C 225    FORMAT(/3X,'NUMBER OF  ITERATIONS',I5)
         DO 238 I=1,NODES
            ITEMP=CHOICE(I)
            INFLOW(NL,I)=0
            DO 237 L=1,ITEMP
               J=SELECT(I,L)
               SIGN1=1
               IF(NODE(2,J).EQ.I)  GO TO 234
               SIGN1=-1
  234          F1=FLOW(NL,J)  *SIGN1
               IF(F1.LE.0)  GO TO 237
               INFLOW(NL,I)=INFLOW(NL,I)+F1
  237       CONTINUE
  238    CONTINUE
  239 CONTINUE
      IF(NTYPE.NE.1)  GO TO 580
      DO 243 NL=1,LOADS
         IF(ITER(NL).GE.3)  GO TO 248
  243 CONTINUE
      GO TO 580
C                               CALCULATE CHANGE IN COST
C                               IF LESS THAN 0.01 % AND LENGTH
C                               CONSTRAINTS NONBINDING
  248 CONTINUE
      EPSOL=ABS(TOTAL2(IT)-TOTAL2(IT-1))/TOTAL2(IT)
      IF(EPSOL.LT.0.0001.AND.LENTES.EQ.0) GO TO 580
      IF(IT.GE.65) GO TO 580
C                               FIND PATHS OF BINDING CONSTRAINTS
      DO 258 J=1,PIPES
         DO 257 K=1,PIPES
            PATH(K,J)=0
            FACTOR(K,J)=0
  257    CONTINUE
  258 CONTINUE
      BIG=10**10
      DO 265 I=1,NODES
         DO 264 NL=1,LOADS
            AAA(NL,I)=INHEAD(NL,I)-MINHED(NL,I)
            IF(SOURCE(NL,I).EQ.1)AAA(NL,I)=BIG
  264    CONTINUE
  265 CONTINUE
C
      K=0
  268 K=K+1
      IF(K.GT.PIPES) GO TO 355
C                               SEARCH FOR LOWEST PRESSURE
      SMALL=BIG
      DO 279 I=1,NODES
         DO 278 NL=1,LOADS
            IF(AAA(NL,I).GE.SMALL) GO TO 278
            SMALL=AAA(NL,I)
            NLK(K)=NL
            IK(K)=I
  278    CONTINUE
  279 CONTINUE
      IF(SMALL.EQ.BIG) GO TO 355
      IF(SMALL.GT.10) GO TO 355
C                               TRACE PATH TO A SOURCE
      DO 285 I=1,NODES
         NFACT(I)=0
  285 CONTINUE
      NL=NLK(K)
      I=IK(K)
      NFACT(I)=1
      PT1=1
      PT2=1
C
C                               SEARCH FOR PATHS FROM NODE TO SOURCE(S)
  293 L=0
  294 L=L+1
      IF(L.GT.CHOICE(I)) GO TO 327
      J=SELECT(I,L)
      IF(ND(1,J).EQ.1.AND.LEN(2,J).EQ.0) GO TO 294
      IF(BREAK(NL,J).EQ.1) GO TO 294
      N=1
      SIGN1=1
      IF(NODE(N,J).NE.I) GO TO 304
      N=2
      SIGN1=-1
  304 I2=NODE(N,J)
      F1=FLOW(NL,J)*SIGN1
      IF(F1.LE.0.01) GO TO 294
      PATH(K,J)=1
      TEMP=ABS(F1/INFLOW(NL,I))*NFACT(I)
      NFACT(I2)=NFACT(I2)+(TEMP-FACTOR(K,J))
      FACTOR(K,J)=TEMP
      PT=PT1
  312 PT=PT+1
      IF(PT.GT.40) PT=1
      IF(PT.EQ.(PT2+1)) GO TO 320
      IF(PT1.EQ.1.AND.PT2.EQ.1) GO TO 320
      IF(PT.EQ.1.AND.PT2.EQ.40) GO TO 320
      IF(WORKN(PT).EQ.I2) GO TO 294
      GO TO 312
C
  320 CONTINUE
      PT2=PT2+1
      IF(PT2.GT.40)PT2=1
      WORKN(PT2)=I2
C     WRITE(6,325) PT1,PT2,WORKN(PT2),I2
C 325 FORMAT(7X,'4001',4I7)
      GO TO 294
  327 PT1=PT1+1
      IF(PT1.GT.40) PT1=1
      I=WORKN(PT1)
      IF(PT1.EQ.(PT2+1)) GO TO 334
      IF(PT1.EQ.1.AND.PT2.EQ.40) GO TO 334
      IF(SOURCE(NL,I).EQ.1) GO TO 327
      GO TO 293
  334 CONTINUE
C                             CHECK IF PATH HAS BEEN TRACED ALREADY
      AAA(NL,IK(K))=BIG
      KEND=K-1
      IF(KEND.EQ.0) GO TO 268
      DO 353 K1=1,KEND
         ISUM=0
         DO 344 J=1,PIPES
            ISUM=PATH(K,J)-PATH(K1,J)
            IF(ISUM.NE.0) GO TO 353
  344    CONTINUE
C                             IF IDENTICAL PATH HAS BEEN TRACED DON'T
C                             SAVE THIS ONE
         DO 350 J=1,PIPES
            PATH(K,J)=0
            FACTOR(K,J)=0
  350    CONTINUE
```

```
         K=K-1
         GO TO 268
  353 CONTINUE
      GO TO 268
  355 HCON=K-1
      DO 359 K=1,HCON
C        WRITE(6,358)  (FACTOR(K,J),J=1,PIPES)
C 358    FORMAT(15F7.3)
  359 CONTINUE
C     IF(IT.GT.3)STOP
C
C                       COST FUNCTION
      NV=0
      DO 394 J=1,PIPES
         IF(LEN(2,J).EQ.0.AND.ND(1,J).EQ.1)  GO TO 394
         COS1=COSPIP(ND(1,J))
         IF(EXIST(J).EQ.ND(1,J))GO TO 394
         IF(LEN(2,J).EQ.0)  GO TO 380
         NV=NV+1
         CLP(NV)=COS1-COSPIP(ND(2,J))
         IDPT1(NV)=ND(1,J)
         IDPT2(NV)=ND(2,J)
         JNV(NV)=J
         NV=NV+1
         CLP(NV)=COSPIP(ND(2,J))-COS1
         IDPT1(NV)=ND(2,J)
         IDPT2(NV)=ND(1,J)
         JNV(NV)=J
         GO TO 394
  380    ID=ND(1,J)-1
         IF(ID.LT.1)  GO TO 387
         NV=NV+1
         CLP(NV)=COSPIP(ID)-COS1
         IDPT1(NV)=ID
         IDPT2(NV)=ND(1,J)
         JNV(NV)=J
  387    ID=ND(1,J)+1
         IF(ID.GT.NDIAM)  GO TO 394
         NV=NV+1
         CLP(NV)=COSPIP(ID)-COS1
         IDPT1(NV)=ID
         IDPT2(NV)=ND(1,J)
         JNV(NV)=J
  394 CONTINUE
      NUMPIP=NV
      DO 399 I=1,PIPES
         NV=NV+1
         CLP(NV)=PENALT
  399 CONTINUE
      NUMVAR=NV
      DO 403  NV=1,NUMVAR
         CLP(NV)=CLP(NV)*(-1)
  403 CONTINUE
C     WRITE(6,405)  (CLP(NV),NV=1,NUMVAR)
```

```
C 405 FORMAT(7F9.2)
C                            CONSTRUCT HEADLOSS CONSTRAINTS
C                            VERSION MAY 23,83
      NUMCON=HCON+PIPES*2
      DO 413 NV=1,NUMVAR
         DO 412 NC=1,NUMCON
            ALP(NC,NV)=0
  412    CONTINUE
  413 CONTINUE

      NC=0
      IF(HCON.EQ.0) GO TO 444
      DO 443 K=1,HCON
         NL=NLK(K)
         I=IK(K)
         NC=NC+1
         NV=0
         TEST=0
         DO 435 NV=1,NUMPIP
            J=JNV(NV)
            IF(PATH(K,J).NE.1) GO TO 435
            IF(EXIST(J).NE.0) GO TO 435
            F1=ABS(FLOW(NL,J))
  428       CONTINUE
            J1=(F1/(HAZEN*UNITS))**(1/QEXP)*DIAM(IDPT1(NV))
     $          **DQEX
            J2=(F1/(HAZEN*UNITS))**(1/QEXP)*DIAM(IDPT2(NV))
     $          **DQEX
            ALP(NC,NV)=(J1-J2)*FACTOR(K,J)
            IF(ALP(NC,NV).GE.TEST) TEST=ALP(NC,NV)
  435    CONTINUE
         NV=NUMPIP+K
         ALP(NC,NV)=(-0.001)*TEST
         BLP(NC)=(INHEAD(NL,I)-MINHED(NL,I))
C        WRITE(6,405) (ALP(NC,NV),NV=1,NUMVAR)
C        WRITE(6,405) BLP(NC)
         NLPT(NC)=NL
         IPT(NC)=I
  443 CONTINUE
  444 CONTINUE
C                                       LENGTH CONTRAINTS
      DO 463 NV=1,NUMPIP
         IF(CLP(NV).LT.0) GO TO 456
         J=JNV(NV)
         NC=NC+1
         JPT(NC)=J
         ALP(NC,NV)=1
         IF(LEN(2,J).EQ.0) TEMP=0
         IF(LEN(2,J).NE.0) TEMP=LEN(1,J)
         BLP(NC)=(LENGTH(J)-TEMP)/1000
         GO TO 463
  456    J=JNV(NV)
         NC=NC+1
         JPT(NC)=J
```

```
            ALP(NC,NV)=1
            IF(LEN(2,J).EQ.0)  TEMP=0
            IF(LEN(2,J).NE.0)  TEMP=LEN(2,J)
            BLP(NC)=(LENGTH(J)-TEMP)/1000
  463    CONTINUE
         NUMCON=NC
C
C                            ADJUST COEFFICIENTS TO ORDER OF MAGNITUDE
C                            OF .01 TO 10
         DIV1=0
         DO 472 NV=1,NUMVAR
            TEST=ABS(CLP(NV))/10
            IF(TEST.GT.DIV1)  DIV1=TEST
  472    CONTINUE
C
         DO 476 NV=1,NUMVAR
            CLP(NV)=CLP(NV)/DIV1
  476    CONTINUE
C
C        WRITE(6,405)  (CLP(NV),NV=1,NUMVAR)
         DO 493 NC=1,NUMCON
            DIV2(NC)=0
            DO 484 NV=1,NUMVAR
               TEST=ABS(ALP(NC,NV))/10
               IF(TEST.GT.DIV2(NC))  DIV2(NC)=TEST
  484       CONTINUE
C
            DO 489 NV=1,NUMVAR
               ALP(NC,NV)=ALP(NC,NV)/DIV2(NC)
C              IF(ALP(NC,NV).LT.0.001)  ALP(NC,NV)=0
  489       CONTINUE
            BLP(NC)=BLP(NC)/DIV2(NC)
C           WRITE(6,405)  (ALP(NC,NV),NV=1,NUMVAR)
C           WRITE(6,405)  BLP(NC)
  493    CONTINUE
         INEQ=NUMCON
C        WRITE(6,496)  INEQ,NUMVAR
C 496    FORMAT(3X,'NUM INEQ=',I5,3X,'NUMVAR=',I5)
         KA=120
         EQUA=0
C
         CALL ZX3LP(ALP,KA,BLP,CLP,NUMVAR,INEQ,EQUA,S,PSOL,DSOL,RW,IW,IER)
C
         IT=IT+1
C                            CALCULATE CHANGE IN COST
         TOTAL1(IT)=TOTAL2(IT-1)-S*1000 *DIV1
C        DO 507  NV=1,NUMVAR
C           WRITE(6,*)  NV,JNV(NV),PSOL(NV)
C 507    CONTINUE
C                            COMPUTE THE CHANGE TO THE NEW SYSTEM
         NV=0
         DO 542 J=1,PIPES
            IF(LEN(2,J).EQ.0.AND.ND(1,J).EQ.1)  GO TO 542
            IF(EXIST(J).NE.0)  GO TO 542
```

```
        IF (LEN (2,J) .EQ.0)  GO TO 525
        NV=NV+1
        LEN (1,J) =LEN (1,J) +PSOL (NV) *1000
        LEN (2,J) =LEN (2,J) -PSOL (NV) *1000
        NV=NV+1
        LEN (1,J) =LEN (1,J) -PSOL (NV) *1000
        LEN (2,J) =LEN (2,J) +PSOL (NV) *1000
        IF (LEN (1,J) .LT.0.01)  LEN (1,J) =0
        IF (LEN (2,J) .LT.0.01)  LEN (2,J) =0
        IF (LEN (1,J) .GT.LENGTH (J))  LEN (1,J) =LENGTH (J)
        IF (LEN (2,J) .GT.LENGTH (J))  LEN (2,J) =LENGTH (J)
        GO TO 542
 525    IF (ND (1,J) .LE.1)  GO TO 534
        NV=NV+1
        IF (PSOL (NV) .EQ.0)  GO TO 534
        ND (2,J) =ND (1,J)
        ND (1,J) =ND (2,J) -1
        LEN (2,J) =LEN (1,J) -PSOL (NV) *1000
        LEN (1,J) =PSOL (NV) *1000
        IF (LEN (2,J) .LT.0.01)  LEN (2,J) =0
        IF (LEN (1,J) .GT.LENGTH (J))  LEN (1,J) =LENGTH (J)
 534    IF (ND (1,J) .GE.NDIAM)  GO TO 542
        NV=NV+1
        IF (PSOL (NV) .EQ.0)  GO TO 542
        ND (2,J) =ND (1,J) +1
        LEN (2,J) =LEN (2,J) +PSOL (NV) *1000
        LEN (1,J) =LEN (1,J) -PSOL (NV) *1000
        IF (LEN (1,J) .LT.0.01)  LEN (1,J) =0
        IF (LEN (2,J) .GT.LENGTH (J))  LEN (2,J) =LENGTH (J)
 542  CONTINUE
      NTEMP=HCON+1
C                     CHECK TO SEE IF LENGTH CONSTRAINT ARE
C                     BINDING
      SUM=0
      LENTES=0
      DO 550 NC=NTEMP,NUMCON
        SUM=SUM+DSOL (NC)
 550  CONTINUE
      IF (SUM.NE.0)  LENTES=1
C                     CALCULATE MAXIMUM WEIGHTING FOR EACH
C                     LINK
      DO 559 J=1,PIPES
        MAXFAC (J) =0
        DO 558 K=1,HCON
          IF (FACTOR (K,J) .GT.MAXFAC (J))  MAXFAC (J) =FACTOR (K,J)
 558    CONTINUE
 559  CONTINUE
C                     CALCULATE TRUE COST AND REMOVE SUB-
C                     MINIMUM PIPES WITH LOW WEIGHTING
      DO 575 J=1,PIPES
        IF (LEN (2,J) .EQ.0.AND.ND (1,J) .EQ.1.AND.MAXFAC (J) .LT.0.5)
     $   GO TO 575
        IF (LEN (1,J) .GE.0.01)  GO TO 570
        LEN (1,J) =LEN (2,J)
```

```
         LEN(2,J)=0
         ND(1,J)=ND(2,J)
         ND(2,J)=ND(1,J)+1
  570    COS1=COSPIP(ND(1,J))
         IF(ND(1,J).EQ.EXIST(J))  COS1=0
         TOTAL2(IT)=TOTAL2(IT)+LEN(1,J)*COS1
         IF(LEN(2,J).EQ.0)  GO TO 575
         TOTAL2(IT)=TOTAL2(IT)+LEN(2,J)*COSPIP(ND(2,J))
  575 CONTINUE
      WRITE(6,577)   IT,TOTAL1(IT),TOTAL2(IT)
  577 FORMAT(5X,I5,6X,F15.0,F15.0)
      GO TO 132
C                            WRITE PRIMAL SOLUTION
  580 WRITE(6,87)
      WRITE(6,582)
  582 FORMAT(//30X,'FINAL RESULTS')
      IF(NTYPE.NE.1)  GO TO 623
      WRITE(6,585)
  585 FORMAT(//3X,' PIPE    DIAMETER    LENGTH    DIAMETER    LENGTH',
     $             '  EXISTING   MAX %',/)
      REWIND11
      DO 599 J=1,PIPES
         WRITE(11,590)  J,ND(1,J),LEN(1,J),ND(2,J),LEN(2,J),EXIST(J)
  590    FORMAT(2I8,F12.2,I8,F12.2,I8)
         IF(LEN(2,J).EQ.0.AND.ND(1,J).EQ.1)  GO TO 599
         D1=DIAM(ND(1,J))
         D2=0
         D3=0
         IF(EXIST(J).NE.0)D3=DIAM(EXIST(J))
         IF(LEN(2,J).NE.0)  D2=DIAM(ND(2,J))
         WRITE(6,598)  J,D1,LEN(1,J),D2,LEN(2,J),D3,MAXFAC(J)
  598 FORMAT(I6,6X,F7.3,F12.2,F10.3,F10.2,2F10.3)
  599 CONTINUE
C                            WRITE DUAL CONSTRAINTS
      WRITE(6,602)
  602 FORMAT(///25X,'DUAL SOLUTION',/)
      WRITE(6,604)
  604 FORMAT(///15X,'CONSTRAINTS   LOAD',8X,'NODE',12X,'DUAL SOL.')
      DO 610 NC=1,HCON
         DSOL(NC)=DSOL(NC)*DIV1
         DSOL(NC)=DSOL(NC)/DIV2(NC)
         WRITE(6,609)  NC,NLPT(NC),IPT(NC),DSOL(NC)
  609    FORMAT(15X,I5,I11,3X,I9,3X,5X,F15.8)
  610 CONTINUE
         WRITE(6,612)
  612    FORMAT('1',///15X,'CONSTRAINT     LINK       DUAL SOL.')
         NTEMP=HCON+1
         DO 619  NC=NTEMP,NUMCON
         DSOL(NC)=DSOL(NC)*DIV1
         DSOL(NC)=DSOL(NC)/DIV2(NC)
         WRITE(6,618)  NC,JPT(NC),DSOL(NC)
  618    FORMAT(15X,I5,3X,6X,I3,6X,F10.2)
  619 CONTINUE
C
```

```
C                             WRITE PRESSURES FOR EACH DEMAND PATTERN
      WRITE(6,87)
  623  WRITE(6,624)
  624 FORMAT(///15X,'CORRECTED PRESSURES AND FLOWS')
      REWIND 12
      WRITE(12,628) LOADS
      WRITE(12,628) NODES
  628 FORMAT(I7)
      WRITE(6,630)
  630 FORMAT(//15X,'NODE   DEMAND   SUPPLY     MINHED     INHEAD')
      DO 641 NL=1,LOADS
         WRITE(6,634) NL
         WRITE(12,628) NL
  634 FORMAT(/25X,'LOAD',I5)
         DO 640 I=1,NODES
            WRITE(6,639) I,DEMAND(NL,I),SUPPLY(NL,I),MINHED(NL,I)
     $                      ,INHEAD(NL,I)
            WRITE(12,103) I,MINHED(NL,I),DEMAND(NL,I),INHEAD(NL,I)
  639 FORMAT(12X,I5,2X,2F9.0,2F10.2)
  640    CONTINUE
  641 CONTINUE
C
C                             WRITE PIPE FLOWS AND HYDRAULIC GRADIENTS
      WRITE(6,645)
  645 FORMAT(//15X,'PIPE          FLOWS         HGL')
      DO 653 NL=1,LOADS
         WRITE(6,634) NL
         DO 652 J=1,PIPES
         J1=(INHEAD(NL,NODE(1,J))-INHEAD(NL,NODE(2,J)))/LENGTH(J)*1000
            WRITE(6,651) J,FLOW(NL,J),J1
  651 FORMAT(12X,I5,F15.0,F15.6)
  652    CONTINUE
  653 CONTINUE
C       IF(IT.LE.15)GO TO 132
      STOP
      END
```

Appendix C

OUTPUT FOR MULTIPLE DEMAND PATTERN DESIGN EXAMPLE

```
***** NEWORK OPTIMIZER *****

       UNITS=   24.0000000

       HAZEN-WILLIAMS C=   130.

       NUMBER OF CANDIDATE DIAMETERS   13

              DIAMETER       COST

   1            0.125        58.00
   2            0.150        62.00
   3            0.200        71.70
   4            0.250        88.90
   5            0.300       112.30
   6            0.350       138.70
   7            0.400       169.00
   8            0.450       207.00
   9            0.500       248.00
  10            0.550       297.00
  11            0.600       347.00
  12            0.650       405.00
  13            0.700       470.00

NUMBER OF LINKS   37     NUMBER OF NODES   20

     PIPE   FROM    TO       LENGTH

      1      1      2          760.
      2      1      4          520.
      3      1      6          890.
      4      2      3         1120.
      5      2      5          610.
      6      2      6          680.
      7      3      5          680.
      8      3      7          870.
      9      4      8          860.
     10      4      9          980.
     11      5      7          890.
     12      5     10          750.
     13      6      9          620.
     14      6     10          800.
     15      7     12          730.
     16      7     13          680.
     17      8      9          480.
     18      8     15          860.
     19      9     11          800.
```

```
                    20       9      14              770.
                    21      10      11              350.
                    22      10      12              620.
                    23      11      12              670.
                    24      11      16              790.
                    25      11      18             1150.
                    26      12      13              750.
                    27      12      17              550.
                    28      13      17              700.
                    29      14      15              500.
                    30      14      16              450.
                    31      14      19              750.
                    32      15      19              720.
                    33      16      18              540.
                    34      16      19              700.
                    35      17      18              850.
                    36      18      20              750.
                    37      19      20              970.
1
                         INITIAL ASSUMPTION

      PIPE     DIAMETER     LENGTH     DIAMETER     LENGTH     EXISTING

        1        0.400       760.00      0.0          0.0        0.0
        2        0.400       520.00      0.0          0.0        0.0
        3        0.400       890.00      0.0          0.0        0.0
        4        0.400      1120.00      0.0          0.0        0.0
        5        0.400       610.00      0.0          0.0        0.0
        6        0.400       680.00      0.0          0.0        0.0
        7        0.400       680.00      0.0          0.0        0.0
        8        0.400       870.00      0.0          0.0        0.0
        9        0.400       860.00      0.0          0.0        0.0
       10        0.400       980.00      0.0          0.0        0.0
       11        0.400       890.00      0.0          0.0        0.0
       12        0.400       750.00      0.0          0.0        0.0
       13        0.400       620.00      0.0          0.0        0.0
       14        0.400       800.00      0.0          0.0        0.0
       15        0.400       730.00      0.0          0.0        0.0
       16        0.400       680.00      0.0          0.0        0.0
       17        0.400       480.00      0.0          0.0        0.0
       18        0.400       860.00      0.0          0.0        0.0
       19        0.400       800.00      0.0          0.0        0.0
       20        0.400       770.00      0.0          0.0        0.0
       21        0.400       350.00      0.0          0.0        0.0
       22        0.400       620.00      0.0          0.0        0.0
       23        0.400       670.00      0.0          0.0        0.0
       24        0.400       790.00      0.0          0.0        0.0
       25        0.400      1150.00      0.0          0.0        0.0
       26        0.400       750.00      0.0          0.0        0.0
       27        0.400       550.00      0.0          0.0        0.0
       28        0.400       700.00      0.0          0.0        0.0
       29        0.400       500.00      0.0          0.0        0.0
       30        0.400       450.00      0.0          0.0        0.0
```

```
 31          0.400       750.00      0.0         0.0        0.0
 32          0.400       720.00      0.0         0.0        0.0
 33          0.400       540.00      0.0         0.0        0.0
 34          0.400       700.00      0.0         0.0        0.0
 35          0.400       850.00      0.0         0.0        0.0
 36          0.400       750.00      0.0         0.0        0.0
 37          0.400       970.00      0.0         0.0        0.0

                         INITIAL COST=    4590040.
1
                     NUMBER OF LOADING CONDITIONS   37
                     NUMBER OF NODES   20
1
                        INITIAL ASSUMPTION


                 NODE   MINHEAD      DEMAND        INHEAD


                          LOAD     1
                   1     75.00        235.0         80.00
                   2     74.00        220.0         90.00
                   3     73.00        145.0         90.00
                   4     72.00        165.0         70.00
                   5    102.00       -800.0        102.00
                   6     73.00        140.0         80.00
                   7     67.00        175.0         90.00
                   8     72.00        180.0         70.00
                   9     70.00        140.0         75.00
                  10     69.00        160.0         90.00
                  11     71.00        170.0         93.00
                  12     70.00        160.0         85.00
                  13     64.00        190.0         80.00
                  14     73.00        200.0         90.00
                  15     73.00        150.0         80.00
                  16     96.00       -800.0         96.00
                  17     67.00        165.0         80.00
                  18     70.00        140.0         90.00
                  19     70.00        185.0         90.00
                  20     67.00        160.0         70.00

                          LOAD     2
                   1     75.00        165.0         80.00
                   2     74.00        220.0         90.00
                   3     73.00        145.0         90.00
                   4     72.00        235.0         70.00
                   5    102.00       -800.0        102.00
                   6     73.00        140.0         80.00
                   7     67.00        175.0         90.00
                   8     72.00        180.0         70.00
                   9     70.00        140.0         75.00
                  10     69.00        160.0         90.00
```

```
11      71.00      170.0       93.00
12      70.00      160.0       85.00
13      64.00      190.0       80.00
14      73.00      200.0       90.00
15      73.00      150.0       80.00
16      96.00     -800.0       96.00
17      67.00      165.0       80.00
18      70.00      140.0       90.00
19      70.00      185.0       90.00
20      67.00      160.0       70.00

           LOAD      3
 1      75.00      235.0       80.00
 2      74.00      220.0       90.00
 3      73.00      145.0       90.00
 4      72.00      165.0       70.00
 5     102.00     -800.0      102.00
 6      73.00      140.0       80.00
 7      67.00      175.0       90.00
 8      72.00      180.0       70.00
 9      70.00      140.0       75.00
10      69.00      160.0       90.00
11      71.00      170.0       93.00
12      70.00      160.0       85.00
13      64.00      190.0       80.00
14      73.00      200.0       90.00
15      73.00      150.0       80.00
16      96.00     -800.0       96.00
17      67.00      165.0       80.00
18      70.00      140.0       90.00
19      70.00      185.0       90.00
20      67.00      160.0       70.00

           LOAD      4
 1      75.00      165.0       80.00
 2      74.00      220.0       90.00
 3      73.00      215.0       90.00
 4      72.00      165.0       70.00
 5     102.00     -800.0      102.00
 6      73.00      140.0       80.00
 7      67.00      175.0       90.00
 8      72.00      180.0       70.00
 9      70.00      140.0       75.00
10      69.00      160.0       90.00
11      71.00      170.0       93.00
12      70.00      160.0       85.00
13      64.00      190.0       80.00
14      73.00      200.0       90.00
15      73.00      150.0       80.00
16      96.00     -800.0       96.00
17      67.00      165.0       80.00
18      70.00      140.0       90.00
19      70.00      185.0       90.00
20      67.00      160.0       70.00
```

```
           LOAD     5
   1     75.00        165.0          80.00
   2     74.00        290.0          90.00
   3     73.00        145.0          90.00
   4     72.00        165.0          70.00
   5    102.00       -800.0         102.00
   6     73.00        140.0          80.00
   7     67.00        175.0          90.00
   8     72.00        180.0          70.00
   9     70.00        140.0          75.00
  10     69.00        160.0          90.00
  11     71.00        170.0          93.00
  12     70.00        160.0          85.00
  13     64.00        190.0          80.00
  14     73.00        200.0          90.00
  15     73.00        150.0          80.00
  16     96.00       -800.0          96.00
  17     67.00        165.0          80.00
  18     70.00        140.0          90.00
  19     70.00        185.0          90.00
  20     67.00        160.0          70.00

           LOAD     6
   1     75.00        165.0          80.00
   2     74.00        220.0          90.00
   3     73.00        145.0          90.00
   4     72.00        165.0          70.00
   5    102.00       -800.0         102.00
   6     73.00        230.0          80.00
   7     67.00        175.0          90.00
   8     72.00        180.0          70.00
   9     70.00        140.0          75.00
  10     69.00        160.0          90.00
  11     71.00        170.0          93.00
  12     70.00        160.0          85.00
  13     64.00        190.0          80.00
  14     73.00        200.0          90.00
  15     73.00        150.0          80.00
  16     96.00       -800.0          96.00
  17     67.00        165.0          80.00
  18     70.00        140.0          90.00
  19     70.00        185.0          90.00
  20     67.00        160.0          70.00

           LOAD     7
   1     75.00        165.0          80.00
   2     74.00        220.0          90.00
   3     73.00        215.0          90.00
   4     72.00        165.0          70.00
   5    102.00       -800.0         102.00
   6     73.00        140.0          80.00
   7     67.00        175.0          90.00
   8     72.00        180.0          70.00
```

```
 9      70.00        140.0        75.00
10      69.00        160.0        90.00
11      71.00        170.0        93.00
12      70.00        160.0        85.00
13      64.00        190.0        80.00
14      73.00        200.0        90.00
15      73.00        150.0        80.00
16      96.00       -800.0        96.00
17      67.00        165.0        80.00
18      70.00        140.0        90.00
19      70.00        185.0        90.00
20      67.00        160.0        70.00

           LOAD      8
 1      75.00        165.0        80.00
 2      74.00        220.0        90.00
 3      73.00        145.0        90.00
 4      72.00        165.0        70.00
 5     102.00       -800.0       102.00
 6      73.00        140.0        80.00
 7      67.00        265.0        90.00
 8      72.00        180.0        70.00
 9      70.00        140.0        75.00
10      69.00        160.0        90.00
11      71.00        170.0        93.00
12      70.00        160.0        85.00
13      64.00        190.0        80.00
14      73.00        200.0        90.00
15      73.00        150.0        80.00
16      96.00       -800.0        96.00
17      67.00        165.0        80.00
18      70.00        140.0        90.00
19      70.00        185.0        90.00
20      67.00        160.0        70.00

           LOAD      9
 1      75.00        165.0        80.00
 2      74.00        220.0        90.00
 3      73.00        145.0        90.00
 4      72.00        235.0        70.00
 5     102.00       -800.0       102.00
 6      73.00        140.0        80.00
 7      67.00        175.0        90.00
 8      72.00        180.0        70.00
 9      70.00        140.0        75.00
10      69.00        160.0        90.00
11      71.00        170.0        93.00
12      70.00        160.0        85.00
13      64.00        190.0        80.00
14      73.00        200.0        90.00
15      73.00        150.0        80.00
16      96.00       -800.0        96.00
17      67.00        165.0        80.00
18      70.00        140.0        90.00
```

```
 19      70.00         185.0          90.00
 20      67.00         160.0          70.00

           LOAD    10
  1      75.00         165.0          80.00
  2      74.00         220.0          90.00
  3      73.00         145.0          90.00
  4      72.00         235.0          70.00
  5     102.00        -800.0         102.00
  6      73.00         140.0          80.00
  7      67.00         175.0          90.00
  8      72.00         180.0          70.00
  9      70.00         140.0          75.00
 10      69.00         160.0          90.00
 11      71.00         170.0          93.00
 12      70.00         160.0          85.00
 13      64.00         190.0          80.00
 14      73.00         200.0          90.00
 15      73.00         150.0          80.00
 16      96.00        -800.0          96.00
 17      67.00         165.0          80.00
 18      70.00         140.0          90.00
 19      70.00         185.0          90.00
 20      67.00         160.0          70.00

           LOAD    11
  1      75.00         165.0          80.00
  2      74.00         220.0          90.00
  3      73.00         145.0          90.00
  4      72.00         165.0          70.00
  5     102.00        -800.0         102.00
  6      73.00         140.0          80.00
  7      67.00         265.0          90.00
  8      72.00         180.0          70.00
  9      70.00         140.0          75.00
 10      69.00         160.0          90.00
 11      71.00         170.0          93.00
 12      70.00         160.0          85.00
 13      64.00         190.0          80.00
 14      73.00         200.0          90.00
 15      73.00         150.0          80.00
 16      96.00        -800.0          96.00
 17      67.00         165.0          80.00
 18      70.00         140.0          90.00
 19      70.00         185.0          90.00
 20      67.00         160.0          70.00

           LOAD    12
  1      75.00         165.0          80.00
  2      74.00         220.0          90.00
  3      73.00         145.0          90.00
  4      72.00         165.0          70.00
  5     102.00        -800.0         102.00
  6      73.00         140.0          80.00
  7      67.00       175.0        90.00
  8      72.00       180.0        70.00
  9      70.00       140.0        75.00
 10      69.00       250.0        90.00
 11      71.00       170.0        93.00
 12      70.00       160.0        85.00
 13      64.00       190.0        80.00
 14      73.00       200.0        90.00
 15      73.00       150.0        80.00
 16      96.00      -800.0        96.00
 17      67.00       165.0        80.00
 18      70.00       140.0        90.00
 19      70.00       185.0        90.00
 20      67.00       160.0        70.00

           LOAD    13
  1      75.00       165.0        80.00
  2      74.00       220.0        90.00
  3      73.00       145.0        90.00
  4      72.00       165.0        70.00
  5     102.00      -800.0       102.00
  6      73.00       140.0        80.00
  7      67.00       175.0        90.00
  8      72.00       180.0        70.00
  9      70.00       230.0        75.00
 10      69.00       160.0        90.00
 11      71.00       170.0        93.00
 12      70.00       160.0        85.00
 13      64.00       190.0        80.00
 14      73.00       200.0        90.00
 15      73.00       150.0        80.00
 16      96.00      -800.0        96.00
 17      67.00       165.0        80.00
 18      70.00       140.0        90.00
 19      70.00       185.0        90.00
 20      67.00       160.0        70.00

           LOAD    14
  1      75.00       165.0        80.00
  2      74.00       220.0        90.00
  3      73.00       145.0        90.00
  4      72.00       165.0        70.00
  5     102.00      -800.0       102.00
  6      73.00       230.0        80.00
  7      67.00       175.0        90.00
  8      72.00       180.0        70.00
  9      70.00       140.0        75.00
 10      69.00       160.0        90.00
 11      71.00       170.0        93.00
 12      70.00       160.0        85.00
 13      64.00       190.0        80.00
 14      73.00       200.0        90.00
 15      73.00       150.0        80.00
 16      96.00      -800.0        96.00
 17      67.00          165.0          80.00
 18      70.00          140.0          90.00
 19      70.00          185.0          90.00
 20      67.00          160.0          70.00

           LOAD     15
  1      75.00          165.0          80.00
  2      74.00          220.0          90.00
  3      73.00          145.0          90.00
  4      72.00          165.0          70.00
  5     102.00         -800.0         102.00
  6      73.00          140.0          80.00
  7      67.00          175.0          90.00
  8      72.00          180.0          70.00
  9      70.00          140.0          75.00
 10      69.00          160.0          90.00
 11      71.00          170.0          93.00
 12      70.00          210.0          85.00
 13      64.00          190.0          80.00
 14      73.00          200.0          90.00
 15      73.00          150.0          80.00
 16      96.00         -800.0          96.00
 17      67.00          165.0          80.00
 18      70.00          140.0          90.00
 19      70.00          185.0          90.00
 20      67.00          160.0          70.00

           LOAD     16
  1      75.00          165.0          80.00
  2      74.00          220.0          90.00
  3      73.00          145.0          90.00
  4      72.00          165.0          70.00
  5     102.00         -800.0         102.00
  6      73.00          140.0          80.00
  7      67.00          175.0          90.00
  8      72.00          180.0          70.00
  9      70.00          140.0          75.00
 10      69.00          160.0          90.00
 11      71.00          170.0          93.00
 12      70.00          160.0          85.00
 13      64.00          260.0          80.00
 14      73.00          200.0          90.00
 15      73.00          150.0          80.00
 16      96.00         -800.0          96.00
 17      67.00          165.0          80.00
 18      70.00          140.0          90.00
 19      70.00          185.0          90.00
 20      67.00          160.0          70.00

           LOAD     17
  1      75.00          165.0          80.00
  2      74.00          220.0          90.00
  3      73.00          145.0          90.00
  4      72.00          165.0          70.00
 5    102.00      -800.0       102.00
 6     73.00       140.0        80.00
 7     67.00       175.0        90.00
 8     72.00       250.0        70.00
 9     70.00       140.0        75.00
10     69.00       160.0        90.00
11     71.00       170.0        93.00
12     70.00       160.0        85.00
13     64.00       190.0        80.00
14     73.00       200.0        90.00
15     73.00       150.0        80.00
16     96.00      -800.0        96.00
17     67.00       165.0        80.00
18     70.00       140.0        90.00
19     70.00       185.0        90.00
20     67.00       160.0        70.00

          LOAD     18
 1     75.00       165.0        80.00
 2     74.00       220.0        90.00
 3     73.00       145.0        90.00
 4     72.00       165.0        70.00
 5    102.00      -800.0       102.00
 6     73.00       140.0        80.00
 7     67.00       175.0        90.00
 8     72.00       250.0        70.00
 9     70.00       140.0        75.00
10     69.00       160.0        90.00
11     71.00       170.0        93.00
12     70.00       160.0        85.00
13     64.00       190.0        80.00
14     73.00       200.0        90.00
15     73.00       150.0        80.00
16     96.00      -800.0        96.00
17     67.00       165.0        80.00
18     70.00       140.0        90.00
19     70.00       185.0        90.00
20     67.00       160.0        70.00

          LOAD     19
 1     75.00       165.0        80.00
 2     74.00       220.0        90.00
 3     73.00       145.0        90.00
 4     72.00       165.0        70.00
 5    102.00      -800.0       102.00
 6     73.00       140.0        80.00
 7     67.00       175.0        90.00
 8     72.00       180.0        70.00
 9     70.00       230.0        75.00
10     69.00       160.0        90.00
11     71.00       170.0        93.00
12     70.00       160.0        85.00
13     64.00       190.0        80.00
14     73.00       200.0        90.00
```

```
15      73.00        150.0          80.00
16      96.00       -800.0          96.00
17      67.00        165.0          80.00
18      70.00        140.0          90.00
19      70.00        185.0          90.00
20      67.00        160.0          70.00

           LOAD    20
 1      75.00        165.0          80.00
 2      74.00        220.0          90.00
 3      73.00        145.0          90.00
 4      72.00        165.0          70.00
 5     102.00       -800.0         102.00
 6      73.00        140.0          80.00
 7      67.00        175.0          90.00
 8      72.00        180.0          70.00
 9      70.00        230.0          75.00
10      69.00        160.0          90.00
11      71.00        170.0          93.00
12      70.00        160.0          85.00
13      64.00        190.0          80.00
14      73.00        200.0          90.00
15      73.00        150.0          80.00
16      96.00       -800.0          96.00
17      67.00        165.0          80.00
18      70.00        140.0          90.00
19      70.00        185.0          90.00
20      67.00        160.0          70.00

           LOAD    21
 1      75.00        165.0          80.00
 2      74.00        220.0          90.00
 3      73.00        145.0          90.00
 4      72.00        165.0          70.00
 5     102.00       -800.0         102.00
 6      73.00        140.0          80.00
 7      67.00        175.0          90.00
 8      72.00        180.0          70.00
 9      70.00        140.0          75.00
10      69.00        160.0          90.00
11      71.00        270.0          93.00
12      70.00        160.0          85.00
13      64.00        190.0          80.00
14      73.00        200.0          90.00
15      73.00        150.0          80.00
16      96.00       -800.0          96.00
17      67.00        165.0          80.00
18      70.00        140.0          90.00
19      70.00        185.0          90.00
20      67.00        160.0          70.00

           LOAD    22
 1      75.00        165.0          80.00
 2      74.00        220.0          90.00
```

```
  3      73.00        145.0          90.00
  4      72.00        165.0          70.00
  5     102.00       -800.0         102.00
  6      73.00        140.0          80.00
  7      67.00        175.0          90.00
  8      72.00        180.0          70.00
  9      70.00        140.0          75.00
 10      69.00        160.0          90.00
 11      71.00        170.0          93.00
 12      70.00        210.0          85.00
 13      64.00        190.0          80.00
 14      73.00        200.0          90.00
 15      73.00        150.0          80.00
 16      96.00       -800.0          96.00
 17      67.00        165.0          80.00
 18      70.00        140.0          90.00
 19      70.00        185.0          90.00
 20      67.00        160.0          70.00

           LOAD    23
  1      75.00        165.0          80.00
  2      74.00        220.0          90.00
  3      73.00        145.0          90.00
  4      72.00        165.0          70.00
  5     102.00       -800.0         102.00
  6      73.00        140.0          80.00
  7      67.00        175.0          90.00
  8      72.00        180.0          70.00
  9      70.00        140.0          75.00
 10      69.00        160.0          90.00
 11      71.00        170.0          93.00
 12      70.00        210.0          85.00
 13      64.00        190.0          80.00
 14      73.00        200.0          90.00
 15      73.00        150.0          80.00
 16      96.00       -800.0          96.00
 17      67.00        165.0          80.00
 18      70.00        140.0          90.00
 19      70.00        185.0          90.00
 20      67.00        160.0          70.00

           LOAD    24
  1      75.00        165.0          80.00
  2      74.00        220.0          90.00
  3      73.00        145.0          90.00
  4      72.00        165.0          70.00
  5     102.00       -800.0         102.00
  6      73.00        140.0          80.00
  7      67.00        175.0          90.00
  8      72.00        180.0          70.00
  9      70.00        140.0          75.00
 10      69.00        160.0          90.00
 11      71.00        270.0          93.00
 12      70.00        160.0          85.00
```

```
13      64.00       190.0       80.00
14      73.00       200.0       90.00
15      73.00       150.0       80.00
16      96.00      -800.0       96.00
17      67.00       165.0       80.00
18      70.00       140.0       90.00
19      70.00       185.0       90.00
20      67.00       160.0       70.00

          LOAD    25
 1      75.00       165.0       80.00
 2      74.00       220.0       90.00
 3      73.00       145.0       90.00
 4      72.00       165.0       70.00
 5     102.00      -800.0      102.00
 6      73.00       140.0       80.00
 7      67.00       175.0       90.00
 8      72.00       180.0       70.00
 9      70.00       140.0       75.00
10      69.00       160.0       90.00
11      71.00       270.0       93.00
12      70.00       160.0       85.00
13      64.00       190.0       80.00
14      73.00       200.0       90.00
15      73.00       150.0       80.00
16      96.00      -800.0       96.00
17      67.00       165.0       80.00
18      70.00       140.0       90.00
19      70.00       185.0       90.00
20      67.00       160.0       70.00

          LOAD    26
 1      75.00       165.0       80.00
 2      74.00       220.0       90.00
 3      73.00       145.0       90.00
 4      72.00       165.0       70.00
 5     102.00      -800.0      102.00
 6      73.00       140.0       80.00
 7      67.00       175.0       90.00
 8      72.00       180.0       70.00
 9      70.00       140.0       75.00
10      69.00       160.0       90.00
11      71.00       170.0       93.00
12      70.00       160.0       85.00
13      64.00       260.0       80.00
14      73.00       200.0       90.00
15      73.00       150.0       80.00
16      96.00      -800.0       96.00
17      67.00       165.0       80.00
18      70.00       140.0       90.00
19      70.00       185.0       90.00
20      67.00       160.0       70.00

          LOAD    27
```

```
  1      75.00        165.0         80.00
  2      74.00        220.0         90.00
  3      73.00        145.0         90.00
  4      72.00        165.0         70.00
  5     102.00       -800.0        102.00
  6      73.00        140.0         80.00
  7      67.00        175.0         90.00
  8      72.00        180.0         70.00
  9      70.00        140.0         75.00
 10      69.00        160.0         90.00
 11      71.00        170.0         93.00
 12      70.00        160.0         85.00
 13      64.00        190.0         80.00
 14      73.00        200.0         90.00
 15      73.00        150.0         80.00
 16      96.00       -800.0         96.00
 17      67.00        235.0         80.00
 18      70.00        140.0         90.00
 19      70.00        185.0         90.00
 20      67.00        160.0         70.00

           LOAD    28
  1      75.00        165.0         80.00
  2      74.00        220.0         90.00
  3      73.00        145.0         90.00
  4      72.00        165.0         70.00
  5     102.00       -800.0        102.00
  6      73.00        140.0         80.00
  7      67.00        175.0         90.00
  8      72.00        180.0         70.00
  9      70.00        140.0         75.00
 10      69.00        160.0         90.00
 11      71.00        170.0         93.00
 12      70.00        160.0         85.00
 13      64.00        260.0         80.00
 14      73.00        200.0         90.00
 15      73.00        150.0         80.00
 16      96.00       -800.0         96.00
 17      67.00        165.0         80.00
 18      70.00        140.0         90.00
 19      70.00        185.0         90.00
 20      67.00        160.0         70.00

           LOAD    29
  1      75.00        165.0         80.00
  2      74.00        220.0         90.00
  3      73.00        145.0         90.00
  4      72.00        165.0         70.00
  5     102.00       -800.0        102.00
  6      73.00        140.0         80.00
  7      67.00        175.0         90.00
  8      72.00        180.0         70.00
  9      70.00        140.0         75.00
 10      69.00        160.0         90.00
```

11	71.00	170.0	93.00
12	70.00	160.0	85.00
13	64.00	190.0	80.00
14	73.00	200.0	90.00
15	73.00	220.0	80.00
16	96.00	-800.0	96.00
17	67.00	165.0	80.00
18	70.00	140.0	90.00
19	70.00	185.0	90.00
20	67.00	160.0	70.00
	LOAD	30	
1	75.00	165.0	80.00
2	74.00	220.0	90.00
3	73.00	145.0	90.00
4	72.00	165.0	70.00
5	102.00	-800.0	102.00
6	73.00	140.0	80.00
7	67.00	175.0	90.00
8	72.00	180.0	70.00
9	70.00	140.0	75.00
10	69.00	160.0	90.00
11	71.00	170.0	93.00
12	70.00	160.0	85.00
13	64.00	190.0	80.00
14	73.00	320.0	90.00
15	73.00	150.0	80.00
16	96.00	-800.0	96.00
17	67.00	165.0	80.00
18	70.00	140.0	90.00
19	70.00	185.0	90.00
20	67.00	160.0	70.00
	LOAD	31	
1	75.00	165.0	80.00
2	74.00	220.0	90.00
3	73.00	145.0	90.00
4	72.00	165.0	70.00
5	102.00	-800.0	102.00
6	73.00	140.0	80.00
7	67.00	175.0	90.00
8	72.00	180.0	70.00
9	70.00	140.0	75.00
10	69.00	160.0	90.00
11	71.00	170.0	93.00
12	70.00	160.0	85.00
13	64.00	190.0	80.00
14	73.00	200.0	90.00
15	73.00	150.0	80.00
16	96.00	-800.0	96.00
17	67.00	165.0	80.00
18	70.00	140.0	90.00
19	70.00	305.0	90.00
20	67.00	160.0	70.00

```
             LOAD    32
   1       75.00         165.0          80.00
   2       74.00         220.0          90.00
   3       73.00         145.0          90.00
   4       72.00         165.0          70.00
   5      102.00        -800.0         102.00
   6       73.00         140.0          80.00
   7       67.00         175.0          90.00
   8       72.00         180.0          70.00
   9       70.00         140.0          75.00
  10       69.00         160.0          90.00
  11       71.00         170.0          93.00
  12       70.00         160.0          85.00
  13       64.00         190.0          80.00
  14       73.00         200.0          90.00
  15       73.00         220.0          80.00
  16       96.00        -800.0          96.00
  17       67.00         165.0          80.00
  18       70.00         140.0          90.00
  19       70.00         185.0          90.00
  20       67.00         160.0          70.00

             LOAD    33
   1       75.00         165.0          80.00
   2       74.00         220.0          90.00
   3       73.00         145.0          90.00
   4       72.00         165.0          70.00
   5      102.00        -800.0         102.00
   6       73.00         140.0          80.00
   7       67.00         175.0          90.00
   8       72.00         180.0          70.00
   9       70.00         140.0          75.00
  10       69.00         160.0          90.00
  11       71.00         170.0          93.00
  12       70.00         160.0          85.00
  13       64.00         190.0          80.00
  14       73.00         200.0          90.00
  15       73.00         150.0          80.00
  16       96.00        -800.0          96.00
  17       67.00         165.0          80.00
  18       70.00         260.0          90.00
  19       70.00         185.0          90.00
  20       67.00         160.0          70.00

             LOAD    34
   1       75.00         165.0          80.00
   2       74.00         220.0          90.00
   3       73.00         145.0          90.00
   4       72.00         165.0          70.00
   5      102.00        -800.0         102.00
   6       73.00         140.0          80.00
   7       67.00         175.0          90.00
   8       72.00         180.0          70.00
  9      70.00        140.0         75.00
 10      69.00        160.0         90.00
 11      71.00        170.0         93.00
 12      70.00        160.0         85.00
 13      64.00        190.0         80.00
 14      73.00        200.0         90.00
 15      73.00        150.0         80.00
 16      96.00       -800.0         96.00
 17      67.00        165.0         80.00
 18      70.00        140.0         90.00
 19      70.00        305.0         90.00
 20      67.00        160.0         70.00

          LOAD    35
  1      75.00        165.0         80.00
  2      74.00        220.0         90.00
  3      73.00        145.0         90.00
  4      72.00        165.0         70.00
  5     102.00       -800.0        102.00
  6      73.00        140.0         80.00
  7      67.00        175.0         90.00
  8      72.00        180.0         70.00
  9      70.00        140.0         75.00
 10      69.00        160.0         90.00
 11      71.00        170.0         93.00
 12      70.00        160.0         85.00
 13      64.00        190.0         80.00
 14      73.00        200.0         90.00
 15      73.00        150.0         80.00
 16      96.00       -800.0         96.00
 17      67.00        235.0         80.00
 18      70.00        140.0         90.00
 19      70.00        185.0         90.00
 20      67.00        160.0         70.00

          LOAD    36
  1      75.00        165.0         80.00
  2      74.00        220.0         90.00
  3      73.00        145.0         90.00
  4      72.00        165.0         70.00
  5     102.00       -800.0        102.00
  6      73.00        140.0         80.00
  7      67.00        175.0         90.00
  8      72.00        180.0         70.00
  9      70.00        140.0         75.00
 10      69.00        160.0         90.00
 11      71.00        170.0         93.00
 12      70.00        160.0         85.00
 13      64.00        190.0         80.00
 14      73.00        200.0         90.00
 15      73.00        150.0         80.00
 16      96.00       -800.0         96.00
 17      67.00        165.0         80.00
 18      70.00        140.0         90.00
```

```
                19     70.00        185.0         90.00
                20     67.00        280.0         70.00

                         LOAD    37
                 1     75.00        165.0         80.00
                 2     74.00        220.0         90.00
                 3     73.00        145.0         90.00
                 4     72.00        165.0         70.00
                 5    102.00       -800.0        102.00
                 6     73.00        140.0         80.00
                 7     67.00        175.0         90.00
                 8     72.00        180.0         70.00
                 9     70.00        140.0         75.00
                10     69.00        160.0         90.00
                11     71.00        170.0         93.00
                12     70.00        160.0         85.00
                13     64.00        190.0         80.00
                14     73.00        200.0         90.00
                15     73.00        150.0         80.00
                16     96.00       -800.0         96.00
                17     67.00        165.0         80.00
                18     70.00        140.0         90.00
                19     70.00        185.0         90.00
                20     67.00        280.0         70.00


          PIPES OUT OF SERVICE IN EACH CONDITION

LOAD      1
          PIPE     1

LOAD      2
          PIPE     2

LOAD      3
          PIPE     3

LOAD      4
          PIPE     4

LOAD      5
          PIPE     5

LOAD      6
          PIPE     6

LOAD      7
          PIPE     7

LOAD      8
          PIPE     8

LOAD      9
```

```
          PIPE    9

LOAD       10
          PIPE   10

LOAD       11
          PIPE   11

LOAD       12
          PIPE   12

LOAD       13
          PIPE   13

LOAD       14
          PIPE   14

LOAD       15
          PIPE   15

LOAD       16
          PIPE   16

LOAD       17
          PIPE   17

LOAD       18
          PIPE   18

LOAD       19
          PIPE   19

LOAD       20
          PIPE   20

LOAD       21
          PIPE   21

LOAD       22
          PIPE   22

LOAD       23
          PIPE   23

LOAD       24
          PIPE   24

LOAD       25
          PIPE   25

LOAD       26
          PIPE   26

LOAD       27
```

```
          PIPE    27

    LOAD     28
          PIPE    28

    LOAD     29
          PIPE    29

    LOAD     30
          PIPE    30

    LOAD     31
          PIPE    31

    LOAD     32
          PIPE    32

    LOAD     33
          PIPE    33

    LOAD     34
          PIPE    34

    LOAD     35
          PIPE    35

    LOAD     36
          PIPE    36

    LOAD     37
          PIPE    37
1

      ITERATION          COST 1         COST 2

          2             3767093.       3767064.
          3             3050041.       3050042.
          4             2414499.       2414501.
          5             2133649.       2133649.
          6             2101664.       2101664.
          7             2069679.       2175942.
          8             2113005.       2113006.
          9             2110889.       2110887.
         10             2108599.       2066259.
         11             2068086.       2068084.
         12             2068111.       2068115.
         13             2070268.       2070266.
         14             2072391.       2072391.
         15             2074021.       2074025.
         16             2075025.       2075023.
         17             2075265.       2075268.
         18             2075058.       2075055.
         19             2074888.       2074886.
      20               2074646.        2074649.
      21               2074458.        2074457.
      22               2074589.        2074594.
      23               2074763.        2074762.
1


                            FINAL RESULTS

   PIPE     DIAMETER     LENGTH    DIAMETER     LENGTH  EXISTING    MAX %

    1         0.200      350.98      0.250      409.02     0.0       1.000
    2         0.150      250.77      0.200      269.23     0.0       0.703
    3         0.200      753.42      0.250      136.58     0.0       0.894
    4         0.200     1120.00      0.0          0.0      0.0       0.550
    5         0.300      610.00      0.0          0.0      0.0       0.892
    6         0.200      680.00      0.0          0.0      0.0       0.777
    7         0.250      680.00      0.0          0.0      0.0       0.771
    8         0.150      130.92      0.200      739.08     0.0       0.867
    9         0.125       89.63      0.150      770.37     0.0       0.297
   10         0.200      980.00      0.0          0.0      0.0       0.759
   11         0.200      890.00      0.0          0.0      0.0       0.383
   12         0.350      577.84      0.400      172.16     0.0       1.000
   13         0.200      620.00      0.0          0.0      0.0       0.440
   14         0.300      800.00      0.0          0.0      0.0       0.885
   16         0.150      507.91      0.200      172.09     0.0       0.611
   17         0.150      123.33      0.200      356.67     0.0       0.754
   18         0.200      860.00      0.0          0.0      0.0       0.748
   19         0.200      800.00      0.0          0.0      0.0       0.338
   20         0.200      677.52      0.250       92.48     0.0       0.720
   21         0.200      350.00      0.0          0.0      0.0       0.483
   22         0.200      620.00      0.0          0.0      0.0       1.000
   23         0.200      600.07      0.250       69.93     0.0       1.000
   24         0.250      790.00      0.0          0.0      0.0       0.653
   25         0.150      118.84      0.200     1031.16     0.0       0.612
   26         0.150      123.94      0.200      626.06     0.0       0.686
   27         0.150      150.86      0.200      399.14     0.0       0.759
   28         0.150      700.00      0.0          0.0      0.0       0.389
   29         0.200      500.00      0.0          0.0      0.0       0.952
   30         0.250      314.92      0.300      135.08     0.0       0.955
   31         0.200      647.03      0.250      102.97     0.0       0.643
   32         0.200      720.00      0.0          0.0      0.0       1.000
   33         0.250      540.00      0.0          0.0      0.0       0.848
   34         0.300      606.71      0.350       93.29     0.0       1.000
   35         0.150      466.31      0.200      383.69     0.0       0.808
   36         0.200      546.60      0.250      203.40     0.0       1.000
   37         0.200      569.24      0.250      400.76     0.0       1.000


                        DUAL SOLUTION
```

CONSTRAINTS	LOAD	NODE	DUAL SOL.
1	12	1	0.79868931
2	12	4	3.87035275
3	34	15	0.08093327
4	10	4	0.19100928
5	11	7	0.13621670
6	17	8	0.73620003
7	26	13	0.40397751
8	27	17	0.39013046
9	18	8	0.30809450
10	37	20	0.87695795
11	16	13	0.25092107
12	35	17	0.22638708
13	30	14	2.39629555
14	36	20	0.80073583
15	1	1	0.72842526
16	33	18	0.36467516
17	5	1	2.36910343
18	30	15	0.09528291
19	22	12	0.76084524
20	2	4	0.0
21	12	12	0.0
22	30	8	0.0
23	12	6	0.0
24	34	8	0.0
25	12	8	0.0
26	5	2	0.0
27	34	19	0.0
28	12	17	0.0
29	22	17	0.0
30	30	4	0.0
31	5	4	0.0
32	23	12	0.0
33	1	4	0.0
34	33	17	0.0
35	12	10	0.0
36	12	9	0.0
37	29	15	0.0

1

CONSTRAINT	LINK	DUAL SOL.
38	1	0.0
39	1	0.0
40	2	0.0
41	2	0.0
42	3	0.0
43	3	0.0
44	4	0.0
45	4	0.0
46	5	0.0
47	5	0.0

48	6	0.0
49	6	0.0
50	7	0.0
51	7	0.0
52	8	0.0
53	8	0.0
54	9	0.0
55	9	0.0
56	10	0.0
57	10	0.0
58	11	0.0
59	11	0.0
60	12	0.0
61	12	0.0
62	13	0.0
63	13	0.0
64	14	0.0
65	14	0.0
66	16	0.0
67	16	0.0
68	17	0.0
69	17	0.0
70	18	0.0
71	18	0.0
72	19	0.0
73	19	0.0
74	20	0.0
75	20	0.0
76	21	0.0
77	21	0.0
78	22	0.0
79	22	0.0
80	23	0.0
81	23	0.0
82	24	0.0
83	24	0.0
84	25	0.0
85	25	0.0
86	26	0.0
87	26	0.0
88	27	0.0
89	27	0.0
90	28	0.0
91	28	0.0
92	29	0.0
93	29	0.0
94	30	0.0
95	30	0.0
96	31	0.0
97	31	0.0
98	32	0.0
99	32	0.0
100	33	0.0
101	33	0.0

```
          102              34               0.0
          103              34               0.0
          104              35               0.0
          105              35               0.0
          106              36               0.0
          107              36               0.0
          108              37               0.0
          109              37               0.0
1


       CORRECTED PRESSURES AND FLOWS


       NODE    DEMAND    SUPPLY      MINHED      INHEAD

                 LOAD       1
         1       235.      235.      75.00       75.01
         2       220.      220.      74.00       97.10
         3       145.      145.      73.00       97.03
         4       165.      165.      72.00       75.44
         5      -800.    -1635.     102.00      102.00
         6       140.      140.      73.00       88.73
         7       175.      175.      67.00       91.19
         8       180.      180.      72.00       79.54
         9       140.      140.      70.00       83.80
        10       160.      160.      69.00       92.71
        11       170.      170.      71.00       89.11
        12       160.      160.      70.00       84.26
        13       190.      190.      64.00       81.05
        14       200.      200.      73.00       88.60
        15       150.      150.      73.00       84.61
        16      -800.    -1443.      96.00       96.00
        17       165.      165.      67.00       81.15
        18       140.      140.      70.00       89.22
        19       185.      185.      70.00       89.61
        20       160.      160.      67.00       87.52

                 LOAD       2
         1       165.      165.      75.00       91.66
         2       220.      220.      74.00       95.81
         3       145.      145.      73.00       96.07
         4       235.      235.      72.00       72.09
         5      -800.    -1654.     102.00      102.00
         6       140.      140.      73.00       91.86
         7       175.      175.      67.00       90.89
         8       180.      180.      72.00       78.98
         9       140.      140.      70.00       83.99
        10       160.      160.      69.00       93.99
        11       170.      170.      71.00       89.54
        12       160.      160.      70.00       84.87
        13       190.      190.      64.00       81.46
        14       200.      200.      73.00       88.61
```

```
 15        150.      150.       73.00       84.47
 16       -800.    -1425.       96.00       96.00
 17        165.      165.       67.00       81.60
 18        140.      140.       70.00       89.54
 19        185.      185.       70.00       89.63
 20        160.      160.       67.00       87.70

           LOAD        3
  1        235.      235.       75.00       81.50
  2        220.      220.       74.00       94.24
  3        145.      145.       73.00       95.29
  4        165.      165.       72.00       80.97
  5       -800.    -1720.      102.00      102.00
  6        140.      140.       73.00       92.28
  7        175.      175.       67.00       90.61
  8        180.      180.       72.00       82.22
  9        140.      140.       70.00       86.30
 10        160.      160.       69.00       94.25
 11        170.      170.       71.00       89.98
 12        160.      160.       70.00       85.17
 13        190.      190.       64.00       81.66
 14        200.      200.       73.00       89.43
 15        150.      150.       73.00       85.88
 16       -800.    -1360.       96.00       96.00
 17        165.      165.       67.00       81.85
 18        140.      140.       70.00       89.91
 19        185.      185.       70.00       90.16
 20        160.      160.       67.00       88.14

           LOAD        4
  1        165.      165.       75.00       88.92
  2        220.      220.       74.00       94.76
  3        215.      215.       73.00       94.34
  4        165.      165.       72.00       84.22
  5       -800.    -1744.      102.00      102.00
  6        140.      140.       73.00       91.39
  7        175.      175.       67.00       90.16
  8        180.      180.       72.00       84.08
  9        140.      140.       70.00       87.14
 10        160.      160.       69.00       93.89
 11        170.      170.       71.00       90.03
 12        160.      160.       70.00       85.02
 13        190.      190.       64.00       81.48
 14        200.      200.       73.00       89.81
 15        150.      150.       73.00       86.68
 16       -800.    -1335.       96.00       96.00
 17        165.      165.       67.00       81.71
 18        140.      140.       70.00       89.96
 19        185.      185.       70.00       90.42
 20        160.      160.       67.00       88.29

           LOAD        5
  1        165.      165.       75.00       75.04
  2        290.      290.       74.00       75.37
  3        145.     145.     73.00     89.73
  4        165.     165.     72.00     74.96
  5       -800.   -1535.    102.00    102.00
  6        140.     140.     73.00     81.38
  7        175.     175.     67.00     87.35
  8        180.     180.     72.00     77.72
  9        140.     140.     70.00     80.72
 10        160.     160.     69.00     89.93
 11        170.     170.     71.00     87.44
 12        160.     160.     70.00     82.05
 13        190.     190.     64.00     78.67
 14        200.     200.     73.00     87.77
 15        150.     150.     73.00     83.60
 16       -800.   -1543.     96.00     96.00
 17        165.     165.     67.00     78.99
 18        140.     140.     70.00     88.27
 19        185.     185.     70.00     89.08
 20        160.     160.     67.00     86.75

            LOAD      6
  1        165.     165.     75.00     87.10
  2        220.     220.     74.00     96.42
  3        145.     145.     73.00     96.46
  4        165.     165.     72.00     82.68
  5       -800.   -1700.    102.00    102.00
  6        230.     230.     73.00     87.49
  7        175.     175.     67.00     90.94
  8        180.     180.     72.00     82.66
  9        140.     140.     70.00     85.43
 10        160.     160.     69.00     92.33
 11        170.     170.     71.00     89.25
 12        160.     160.     70.00     84.17
 13        190.     190.     64.00     80.95
 14        200.     200.     73.00     89.26
 15        150.     150.     73.00     85.91
 16       -800.   -1400.     96.00     96.00
 17        165.     165.     67.00     81.08
 18        140.     140.     70.00     89.35
 19        185.     185.     70.00     90.04
 20        160.     160.     67.00     87.78

            LOAD      7
  1        165.     165.     75.00     86.73
  2        220.     220.     74.00     91.34
  3        215.     215.     73.00     77.95
  4        165.     165.     72.00     83.04
  5       -800.   -1684.    102.00    102.00
  6        140.     140.     73.00     89.75
  7        175.     175.     67.00     79.06
  8        180.     180.     72.00     83.04
  9        140.     140.     70.00     86.14
 10        160.     160.     69.00     92.95
 11        170.     169.     71.00     89.23
 12        160.     160.     70.00     82.73
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13         190.     190.      64.00      76.87
14         200.     200.      73.00      89.44
15         150.     150.      73.00      86.12
16        -800.   -1394.      96.00      96.00
17         165.     165.      67.00      78.64
18         140.     139.      70.00      89.24
19         185.     185.      70.00      90.12
20         160.     160.      67.00      87.75

            LOAD     8
 1         165.     165.      75.00      89.31
 2         220.     220.      74.00      95.79
 3         145.     145.      73.00      98.04
 4         165.     165.      72.00      84.23
 5        -800.   -1730.     102.00     102.00
 6         140.     139.      73.00      91.44
 7         265.     265.      67.00      75.31
 8         180.     180.      72.00      84.02
 9         140.     140.      70.00      87.00
10         160.     160.      69.00      93.60
11         170.     170.      71.00      89.43
12         160.     160.      70.00      82.27
13         190.     190.      64.00      74.86
14         200.     200.      73.00      89.74
15         150.     150.      73.00      86.60
16        -800.   -1369.      96.00      96.00
17         165.     165.      67.00      77.76
18         140.     140.      70.00      89.39
19         185.     185.      70.00      90.32
20         160.     160.      67.00      87.92

            LOAD     9
 1         165.     165.      75.00      87.98
 2         220.     220.      74.00      94.95
 3         145.     145.      73.00      95.61
 4         235.     235.      72.00      80.41
 5        -800.   -1714.     102.00     102.00
 6         140.     140.      73.00      90.99
 7         175.     175.      67.00      90.71
 8         180.     180.      72.00      83.07
 9         140.     140.      70.00      86.10
10         160.     160.      69.00      93.69
11         170.     170.      71.00      89.79
12         160.     160.      70.00      84.90
13         190.     190.      64.00      81.47
14         200.     200.      73.00      89.47
15         150.     150.      73.00      86.16
16        -800.   -1364.      96.00      96.00
17         165.     165.      67.00      81.65
18         140.     140.      70.00      89.78
19         185.     185.      70.00      90.20
20         160.     160.      67.00      88.09

            LOAD    10
 1        165.      165.      75.00      86.15
 2        220.      220.      74.00      94.62
 3        145.      145.      73.00      95.46
 4        235.      235.      72.00      71.99
 5       -800.    -1729.     102.00     102.00
 6        140.      140.      73.00      90.93
 7        175.      175.      67.00      90.67
 8        180.      180.      72.00      80.92
 9        140.      140.      70.00      87.57
10        160.      160.      69.00      93.72
11        170.      170.      71.00      90.05
12        160.      160.      70.00      85.04
13        190.      190.      64.00      81.58
14        200.      200.      73.00      89.62
15        150.      150.      73.00      85.59
16       -800.    -1350.      96.00      96.00
17        165.      165.      67.00      81.77
18        140.      140.      70.00      89.95
19        185.      185.      70.00      90.19
20        160.      160.      67.00      88.18

           LOAD      11
 1        165.      165.      75.00      87.96
 2        220.      220.      74.00      93.70
 3        145.      145.      73.00      92.75
 4        165.      165.      72.00      83.44
 5       -800.    -1682.     102.00     102.00
 6        140.      139.      73.00      90.40
 7        265.      265.      67.00      66.99
 8        180.      180.      72.00      83.37
 9        140.      140.      70.00      86.32
10        160.      160.      69.00      92.88
11        170.      169.      71.00      88.74
12        160.      160.      70.00      79.70
13        190.      190.      64.00      68.38
14        200.      200.      73.00      89.48
15        150.      150.      73.00      86.23
16       -800.    -1416.      96.00      96.00
17        165.      165.      67.00      74.41
18        140.      139.      70.00      88.74
19        185.      185.      70.00      90.10
20        160.      160.      67.00      87.45

           LOAD      12
 1        165.      165.      75.00      74.86
 2        220.      220.      74.00      89.58
 3        145.      145.      73.00      92.88
 4        165.      165.      72.00      71.90
 5       -800.    -1209.     102.00     102.00
 6        140.      141.      73.00      73.96
 7        175.      175.      67.00      86.88
 8        180.      180.      72.00      73.06
 9        140.      141.      70.00      74.15
10        250.      250.      69.00      72.57
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11        170.      170.      71.00      76.43
12        160.      160.      70.00      70.21
13        190.      190.      64.00      69.11
14        200.      200.      73.00      85.92
15        150.      150.      73.00      81.14
16       -800.    -1892.      96.00      96.00
17        165.      165.      67.00      69.20
18        140.      140.      70.00      84.48
19        185.      185.      70.00      87.65
20        160.      160.      67.00      83.82

           LOAD     13
 1        165.      165.      75.00      89.01
 2        220.      220.      74.00      95.62
 3        145.      145.      73.00      95.96
 4        165.      165.      72.00      79.57
 5       -800.    -1633.     102.00     102.00
 6        140.      140.      73.00      93.36
 7        175.      175.      67.00      90.84
 8        180.      180.      72.00      79.30
 9        230.      230.      70.00      80.47
10        160.      160.      69.00      94.59
11        170.      170.      71.00      89.25
12        160.      160.      70.00      84.90
13        190.      190.      64.00      81.44
14        200.      200.      73.00      87.95
15        150.      150.      73.00      84.19
16       -800.    -1467.      96.00      96.00
17        165.      165.      67.00      81.58
18        140.      140.      70.00      89.30
19        185.      185.      70.00      89.36
20        160.      160.      67.00      87.44

           LOAD     14
 1        165.      166.      75.00      81.17
 2        220.      220.      74.00      92.02
 3        145.      145.      73.00      94.44
 4        165.      165.      72.00      78.42
 5       -800.    -1673.     102.00     102.00
 6        230.      231.      73.00      81.14
 7        175.      175.      67.00      90.35
 8        180.      180.      72.00      79.02
 9        140.      140.      70.00      81.29
10        160.      160.      69.00      96.80
11        170.      170.      71.00      89.97
12        160.      160.      70.00      85.92
13        190.      190.      64.00      82.11
14        200.      200.      73.00      88.11
15        150.      150.      73.00      84.22
16       -800.    -1428.      96.00      96.00
17        165.      165.      67.00      82.32
18        140.      140.      70.00      89.86
19        185.      185.      70.00      89.48
20        160.      160.      67.00      87.78
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           LOAD    15
  1       165.      165.       75.00      89.06
  2       220.      220.       74.00      95.20
  3       145.      145.       73.00      95.69
  4       165.      165.       72.00      84.19
  5      -800.    -1705.      102.00     102.00
  6       140.      140.       73.00      91.36
  7       175.      175.       67.00      90.47
  8       180.      180.       72.00      84.02
  9       140.      140.       70.00      87.03
 10       160.      160.       69.00      93.69
 11       170.      170.       71.00      89.69
 12       210.      210.       70.00      83.32
 13       190.      190.       64.00      80.30
 14       200.      200.       73.00      89.77
 15       150.      150.       73.00      86.63
 16      -800.    -1353.       96.00      96.00
 17       165.      165.       67.00      80.48
 18       140.      140.       70.00      89.68
 19       185.      185.       70.00      90.36
 20       160.      160.       67.00      88.10

           LOAD    16
  1       165.      165.       75.00      88.81
  2       220.      220.       74.00      95.38
  3       145.      145.       73.00      96.88
  4       165.      165.       72.00      83.76
  5      -800.    -1656.      102.00     102.00
  6       140.      140.       73.00      90.87
  7       175.      175.       67.00      95.66
  8       180.      180.       72.00      83.59
  9       140.      140.       70.00      86.48
 10       160.      160.       69.00      92.94
 11       170.      170.       71.00      88.56
 12       160.      160.       70.00      78.31
 13       260.      260.       64.00      64.01
 14       200.      200.       73.00      89.53
 15       150.      150.       73.00      86.32
 16      -800.    -1422.       96.00      96.00
 17       165.      165.       67.00      72.48
 18       140.      140.       70.00      88.56
 19       185.      185.       70.00      90.11
 20       160.      160.       67.00      87.35

           LOAD    17
  1       165.      165.       75.00      88.50
  2       220.      220.       74.00      95.15
  3       145.      145.       73.00      95.72
  4       165.      165.       72.00      81.46
  5      -800.    -1689.      102.00     102.00
  6       140.      139.       73.00      91.52
  7       175.      175.       67.00      90.81
  8       250.      250.       72.00      72.00
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   9        140.     140.      70.00      87.76
  10        160.     160.      69.00      93.97
  11        170.     170.      71.00      90.13
  12        160.     160.      70.00      85.17
  13        190.     190.      64.00      81.70
  14        200.     200.      73.00      89.04
  15        150.     150.      73.00      82.85
  16       -800.   -1389.      96.00      96.00
  17        165.     165.      67.00      81.88
  18        140.     140.      70.00      89.93
  19        185.     185.      70.00      89.61
  20        160.     160.      67.00      87.89

             LOAD     18
   1        165.     165.      75.00      87.39
   2        220.     220.      74.00      94.72
   3        145.     145.      73.00      95.49
   4        165.     165.      72.00      79.27
   5       -800.   -1743.     102.00     102.00
   6        140.     140.      73.00      90.37
   7        175.     175.      67.00      90.59
   8        250.     250.      72.00      72.00
   9        140.     140.      70.00      83.93
  10        160.     160.      69.00      93.37
  11        170.     170.      71.00      89.43
  12        160.     160.      70.00      84.59
  13        190.     190.      64.00      81.21
  14        200.     200.      73.00      90.00
  15        150.     150.      73.00      88.82
  16       -800.   -1336.      96.00      96.00
  17        165.     165.      67.00      81.38
  18        140.     140.      70.00      89.55
  19        185.     185.      70.00      90.80
  20        160.     160.      67.00      88.22

             LOAD     19
   1        165.     165.      75.00      87.46
   2        220.     220.      74.00      94.71
   3        145.     145.      73.00      95.52
   4        165.     166.      72.00      79.97
   5       -800.   -1723.     102.00     102.00
   6        140.     140.      73.00      90.21
   7        175.     175.      67.00      90.80
   8        180.     180.      72.00      79.87
   9        230.     230.      70.00      81.34
  10        160.     160.      69.00      93.73
  11        170.     170.      71.00      91.15
  12        160.     160.      70.00      85.56
  13        190.     190.      64.00      82.02
  14        200.     200.      73.00      88.24
  15        150.     150.      73.00      84.55
  16       -800.   -1378.      96.00      96.00
  17        165.     165.      67.00      82.23
  18        140.     140.      70.00      90.36
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 19       185.     185.      70.00      89.63
 20       160.     160.      67.00      88.11

           LOAD    20
  1       165.     165.      75.00      87.35
  2       220.     220.      74.00      94.62
  3       145.     145.      73.00      95.43
  4       165.     165.      72.00      80.29
  5      -800.   -1763.     102.00     102.00
  6       140.     139.      73.00      89.86
  7       175.     175.      67.00      90.50
  8       180.     180.      72.00      80.29
  9       230.     230.      70.00      81.51
 10       160.     160.      69.00      93.08
 11       170.     170.      71.00      88.97
 12       160.     160.      70.00      84.25
 13       190.     190.      64.00      80.93
 14       200.     199.      73.00      90.66
 15       150.     150.      73.00      85.99
 16      -800.   -1335.      96.00      96.00
 17       165.     165.      67.00      81.08
 18       140.     140.      70.00      89.29
 19       185.     184.      70.00      90.68
 20       160.     160.      67.00      88.02

           LOAD    21
  1       165.     165.      75.00      89.20
  2       220.     220.      74.00      95.36
  3       145.     145.      73.00      95.77
  4       165.     165.      72.00      83.21
  5      -800.   -1602.     102.00     102.00
  6       140.     140.      73.00      92.09
  7       175.     175.      67.00      90.44
  8       180.     180.      72.00      82.96
  9       140.     140.      70.00      85.49
 10       160.     160.      69.00      95.49
 11       270.     270.      71.00      85.66
 12       160.     160.      70.00      83.20
 13       190.     190.      64.00      80.02
 14       200.     200.      73.00      89.22
 15       150.     150.      73.00      85.94
 16      -800.   -1507.      96.00      96.00
 17       165.     165.      67.00      80.10
 18       140.     140.      70.00      88.03
 19       185.     185.      70.00      89.88
 20       160.     160.      67.00      86.93

           LOAD    22
  1       165.     165.      75.00      89.50
  2       220.     220.      74.00      95.33
  3       145.     145.      73.00      95.45
  4       165.     165.      72.00      84.15
  5      -800.   -1627.     102.00     102.00
  6       140.     140.      73.00      92.36
  7       175.     175.     67.00     88.02
  8       180.     180.     72.00     83.89
  9       140.     140.     70.00     86.81
 10       160.     160.     69.00     95.55
 11       170.     169.     71.00     88.26
 12       210.     210.     70.00     70.03
 13       190.     190.     64.00     69.26
 14       200.     200.     73.00     89.61
 15       150.     150.     73.00     86.45
 16      -800.   -1432.     96.00     96.00
 17       165.     165.     67.00     69.44
 18       140.     139.     70.00     88.26
 19       185.     185.     70.00     90.13
 20       160.     160.     67.00     87.17

         LOAD    23
  1       165.     165.     75.00     88.98
  2       220.     220.     74.00     95.04
  3       145.     145.     73.00     95.34
  4       165.     165.     72.00     84.43
  5      -800.   -1755.    102.00    102.00
  6       140.     140.     73.00     91.18
  7       175.     175.     67.00     88.56
  8       180.     180.     72.00     84.28
  9       140.     140.     70.00     87.45
 10       160.     160.     69.00     93.28
 11       170.     170.     71.00     91.39
 12       210.     210.     70.00     73.22
 13       190.     190.     64.00     72.11
 14       200.     200.     73.00     89.90
 15       150.     150.     73.00     86.78
 16      -800.   -1304.     96.00     96.00
 17       165.     165.     67.00     72.36
 18       140.     140.     70.00     89.79
 19       185.     185.     70.00     90.44
 20       160.     160.     67.00     88.20

         LOAD    24
  1       165.     165.     75.00     87.06
  2       220.     220.     74.00     94.38
  3       145.     145.     73.00     95.18
  4       165.     165.     72.00     80.88
  5      -800.   -1857.    102.00    102.00
  6       140.     140.     73.00     89.12
  7       175.     175.     67.00     89.67
  8       180.     180.     72.00     80.80
  9       140.     140.     70.00     82.79
 10       160.     160.     69.00     91.51
 11       270.     270.     71.00     82.60
 12       160.     160.     70.00     80.33
 13       190.     190.     64.00     77.69
 14       200.     200.     73.00     88.38
 15       150.     150.     73.00     84.78
 16      -800.   -1253.     96.00     96.00
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17        165.      165.       67.00       77.76
18        140.      140.       70.00       86.96
19        185.      185.       70.00       89.29
20        160.      160.       67.00       86.03

           LOAD     25
 1        165.      165.       75.00       88.83
 2        220.      220.       74.00       95.12
 3        145.      145.       73.00       95.67
 4        165.      165.       72.00       83.81
 5       -800.    -1719.      102.00      102.00
 6        140.      140.       73.00       91.11
 7        175.      175.       67.00       90.61
 8        180.      180.       72.00       83.65
 9        140.      140.       70.00       86.52
10        160.      160.       69.00       93.45
11        270.      270.       71.00       88.38
12        160.      160.       70.00       84.16
13        190.      190.       64.00       80.92
14        200.      200.       73.00       89.61
15        150.      150.       73.00       86.42
16       -800.    -1389.       96.00       96.00
17        165.      165.       67.00       81.09
18        140.      140.       70.00       89.65
19        185.      185.       70.00       90.27
20        160.      160.       67.00       88.05

           LOAD     26
 1        165.      165.       75.00       89.10
 2        220.      220.       74.00       95.03
 3        145.      145.       73.00       95.15
 4        165.      165.       72.00       84.31
 5       -800.    -1735.      102.00      102.00
 6        140.      140.       73.00       91.54
 7        175.      175.       67.00       86.85
 8        180.      180.       72.00       84.13
 9        140.      140.       70.00       87.20
10        160.      160.       69.00       94.03
11        170.      170.       71.00       90.06
12        160.      160.       70.00       85.84
13        260.      260.       64.00       64.00
14        200.      200.       73.00       89.82
15        150.      150.       73.00       86.69
16       -800.    -1343.       96.00       96.00
17        165.      165.       67.00       76.57
18        140.      140.       70.00       89.65
19        185.      185.       70.00       90.39
20        160.      160.       67.00       88.10

           LOAD     27
 1        165.      165.       75.00       89.13
 2        220.      220.       74.00       95.18
 3        145.      145.       73.00       95.51
 4        165.      165.       72.00       84.25
 5      -800.   -1705.    102.00     102.00
 6       140.     140.     73.00      91.53
 7       175.     175.     67.00      89.31
 8       180.     180.     72.00      84.06
 9       140.     140.     70.00      87.11
10       160.     160.     69.00      93.97
11       170.     170.     71.00      89.84
12       160.     160.     70.00      85.69
13       190.     190.     64.00      75.38
14       200.     200.     73.00      89.74
15       150.     150.     73.00      86.60
16      -800.   -1374.     96.00      96.00
17       235.     235.     67.00      67.00
18       140.     140.     70.00      88.92
19       185.     185.     70.00      90.27
20       160.     160.     67.00      87.63

          LOAD    28
 1       165.     165.     75.00      89.01
 2       220.     220.     74.00      95.11
 3       145.     145.     73.00      95.45
 4       165.     165.     72.00      84.17
 5      -800.   -1726.    102.00     102.00
 6       140.     140.     73.00      91.33
 7       175.     175.     67.00      89.13
 8       180.     180.     72.00      84.01
 9       140.     140.     70.00      87.02
10       160.     160.     69.00      93.67
11       170.     169.     71.00      89.70
12       160.     160.     70.00      83.28
13       260.     260.     64.00      74.65
14       200.     200.     73.00      89.77
15       150.     150.     73.00      86.62
16      -800.   -1353.     96.00      96.00
17       165.     165.     67.00      80.94
18       140.     139.     70.00      89.70
19       185.     185.     70.00      90.36
20       160.     160.     67.00      88.12

          LOAD    29
 1       165.     165.     75.00      88.14
 2       220.     220.     74.00      94.95
 3       145.     145.     73.00      95.61
 4       165.     165.     72.00      81.38
 5      -800.   -1718.    102.00     102.00
 6       140.     140.     73.00      90.88
 7       175.     175.     67.00      90.68
 8       180.     180.     72.00      77.72
 9       140.     140.     70.00      85.41
10       160.     160.     69.00      93.61
11       170.     170.     71.00      89.64
12       160.     160.     70.00      84.78
13       190.     190.     64.00      81.37
14       200.     200.     73.00      90.22
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15        220.     220.      73.00      77.66
16       -800.   -1360.      96.00      96.00
17        165.     165.      67.00      81.54
18        140.     139.      70.00      89.63
19        185.     185.      70.00      89.87
20        160.     160.      67.00      87.86

           LOAD     30
 1        165.     165.      75.00      85.24
 2        220.     220.      74.00      93.91
 3        145.     145.      73.00      95.06
 4        165.     165.      72.00      74.78
 5       -800.   -1847.     102.00     102.00
 6        140.     140.      73.00      88.18
 7        175.     175.      67.00      90.08
 8        180.     180.      72.00      72.33
 9        140.     140.      70.00      76.39
10        160.     160.      69.00      92.10
11        170.     170.      71.00      87.45
12        160.     160.      70.00      83.01
13        190.     190.      64.00      79.76
14        320.     320.      73.00      73.01
15        150.     149.      73.00      73.03
16       -800.   -1282.      96.00      96.00
17        165.     165.      67.00      79.84
18        140.     140.      70.00      87.55
19        185.     185.      70.00      84.23
20        160.     160.      67.00      83.71

           LOAD     31
 1        165.     165.      75.00      88.96
 2        220.     220.      74.00      95.20
 3        145.     145.      73.00      95.74
 4        165.     165.      72.00      83.77
 5       -800.   -1693.     102.00     102.00
 6        140.     140.      73.00      91.37
 7        175.     175.      67.00      90.77
 8        180.     180.      72.00      83.43
 9        140.     140.      70.00      86.65
10        160.     160.      69.00      93.85
11        170.     170.      71.00      89.83
12        160.     160.      70.00      84.96
13        190.     190.      64.00      81.52
14        200.     200.      73.00      88.79
15        150.     150.      73.00      85.64
16       -800.   -1435.      96.00      96.00
17        165.     165.      67.00      81.68
18        140.     140.      70.00      89.68
19        305.     305.      70.00      88.94
20        160.     160.      67.00      87.42

           LOAD     32
 1        165.     165.      75.00      88.09
 2        220.     220.      74.00      94.93
 3        145.      145.      73.00      95.60
 4        165.      165.      72.00      81.35
 5       -800.    -1720.     102.00     102.00
 6        140.      140.      73.00      90.81
 7        175.      175.      67.00      90.69
 8        180.      180.      72.00      78.40
 9        140.      140.      70.00      85.04
10        160.      160.      69.00      93.60
11        170.      170.      71.00      89.71
12        160.      160.      70.00      84.83
13        190.      190.      64.00      81.42
14        200.      200.      73.00      88.17
15        220.      220.      73.00      78.42
16       -800.    -1360.      96.00      96.00
17        165.      165.      67.00      81.60
18        140.      140.      70.00      89.78
19        185.      185.      70.00      91.30
20        160.      160.      67.00      88.56

           LOAD     33
 1        165.      165.      75.00      87.62
 2        220.      220.      74.00      94.54
 3        145.      145.      73.00      95.10
 4        165.      165.      72.00      81.91
 5       -800.    -1837.     102.00     102.00
 6        140.      139.      73.00      89.75
 7        175.      175.      67.00      88.68
 8        180.      180.      72.00      81.60
 9        140.      140.      70.00      84.41
10        160.      160.      69.00      92.03
11        170.      170.      71.00      85.51
12        160.      160.      70.00      78.58
13        190.      190.      64.00      73.18
14        200.      199.      73.00      87.64
15        150.      150.      73.00      84.22
16       -800.    -1290.      96.00      96.00
17        165.      165.      67.00      70.46
18        260.      260.      70.00      70.02
19        185.      184.      70.00      87.61
20        160.      160.      67.00      71.88

           LOAD     34
 1        165.      165.      75.00      86.24
 2        220.      220.      74.00      94.24
 3        145.      145.      73.00      95.18
 4        165.      165.      72.00      77.28
 5       -800.    -1821.     102.00     102.00
 6        140.      140.      73.00      89.04
 7        175.      175.      67.00      90.03
 8        180.      180.      72.00      73.03
 9        140.      140.      70.00      80.27
10        160.      160.      69.00      92.34
11        170.      170.      71.00      87.21
12        160.      160.      70.00      82.73
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13       190.     190.      64.00      79.30
14       200.     200.      73.00      80.63
15       150.     150.      73.00      72.98
16      -800.   -1309.      96.00      96.00
17       165.     165.      67.00      79.30
18       140.     140.      70.00      85.66
19       305.     305.      70.00      71.87
20       160.     160.      67.00      73.37

          LOAD    35
 1       165.     165.      75.00      88.79
 2       220.     220.      74.00      94.97
 3       145.     145.      73.00      95.30
 4       165.     165.      72.00      84.04
 5      -800.   -1762.     102.00     102.00
 6       140.     139.      73.00      91.00
 7       175.     175.      67.00      88.56
 8       180.     180.      72.00      83.91
 9       140.     140.      70.00      86.87
10       160.     160.      69.00      93.19
11       170.     170.      71.00      89.46
12       160.     160.      70.00      79.04
13       190.     190.      64.00      72.19
14       200.     200.      73.00      89.80
15       150.     150.      73.00      86.65
16      -800.   -1316.      96.00      96.00
17       235.     235.      67.00      67.01
18       140.     140.      70.00      90.94
19       185.     185.      70.00      90.51
20       160.     160.      67.00      88.84

          LOAD    36
 1       165.     165.      75.00      88.94
 2       220.     220.      74.00      95.22
 3       145.     145.      73.00      95.77
 4       165.     165.      72.00      83.51
 5      -800.   -1681.     102.00     102.00
 6       140.     139.      73.00      91.43
 7       175.     175.      67.00      90.92
 8       180.     180.      72.00      82.96
 9       140.     140.      70.00      86.44
10       160.     160.      69.00      94.04
11       170.     170.      71.00      90.53
12       160.     160.      70.00      85.53
13       190.     190.      64.00      82.11
14       200.     199.      73.00      88.00
15       150.     150.      73.00      84.78
16      -800.   -1447.      96.00      96.00
17       165.     165.      67.00      82.38
18       140.     140.      70.00      91.23
19       185.     185.      70.00      87.58
20       280.     280.      67.00      67.01

          LOAD    37
 1       165.     165.     75.00     89.01
 2       220.     220.     74.00     95.16
 3       145.     145.     73.00     95.66
 4       165.     165.     72.00     84.24
 5      -800.   -1716.    102.00    102.00
 6       140.     140.     73.00     91.25
 7       175.     175.     67.00     90.37
 8       180.     180.     72.00     84.17
 9       140.     140.     70.00     86.94
10       160.     160.     69.00     93.53
11       170.     170.     71.00     88.76
12       160.     160.     70.00     83.74
13       190.     190.     64.00     79.90
14       200.     200.     73.00     90.24
15       150.     150.     73.00     87.21
16      -800.   -1413.     96.00     96.00
17       165.     165.     67.00     79.89
18       140.     140.     70.00     84.96
19       185.     185.     70.00     91.58
20       280.     280.     67.00     67.00


PIPE          FLOWS          HGL

          LOAD     1
 1               0.    -29.054561
 2             -25.     -0.817900
 3            -210.    -15.405959
 4              10.      0.058774
 5            -405.     -8.039005
 6             176.     12.309063
 7            -238.     -7.308264
 8             103.      6.709886
 9             -46.     -4.765018
10            -144.     -8.532528
11             174.     12.142944
12             817.     12.388855
13             139.      7.942126
14            -313.     -4.977894
15               0.      9.495252
16             102.     14.921458
17            -108.     -8.883254
18            -118.     -5.898214
19            -126.     -6.636105
20            -127.     -6.235554
21             159.     10.278713
22             185.     13.624376
23             137.      7.238155
24            -262.     -8.720495
25             -11.     -0.097484
26              80.      4.286743
27              83.      5.651634
28              -8.     -0.152370
29             139.      7.985748
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30          -410.      -16.437107
31           -56.       -1.346883
32          -129.       -6.948662
33           319.       12.550156
34           451.        9.123622
35           -90.       -9.494198
36            78.        2.270996
37            81.        2.158552

          LOAD     2
 1          -144.       -5.449737
 2             0.       37.646162
 3           -21.       -0.222950
 4           -21.       -0.233378
 5          -460.      -10.153924
 6           117.        5.799080
 7          -262.       -8.724280
 8            97.        5.945982
 9           -61.       -8.015105
10          -174.      -12.149313
11           177.       12.478106
12           755.       10.684469
13           179.       12.690489
14          -223.       -2.654896
15             0.        8.249863
16            99.       13.875557
17          -117.      -10.444450
18          -123.       -6.376488
19          -129.       -6.927490
20          -125.       -5.993236
21           179.       12.714364
22           193.       14.700908
23           134.        6.961993
24          -253.       -8.181492
25            -1.       -0.000146
26            82.        4.550639
27            85.        5.940191
28            -9.       -0.208391
29           142.        8.288696
30          -410.      -16.423509
31           -56.       -1.363363
32          -131.       -7.176208
33           311.       11.968909
34           451.        9.097225
35           -89.       -9.331539
36            82.        2.454936
37            78.        1.996243

          LOAD     3
 1          -263.      -16.757021
 2            28.        1.014680
 3             0.      -12.113250
 4           -44.       -0.941072
 5          -519.      -12.728932
 6          80.       2.874330
 7        -280.      -9.868599
 8          91.       5.374198
 9         -24.      -1.451324
10        -113.      -5.438871
11         179.      12.793483
12         741.      10.334920
13         154.       9.642472
14        -214.      -2.460003
15           0.       7.460419
16          96.      13.170871
17        -105.      -8.504073
18         -99.      -4.256741
19        -103.      -4.602413
20        -101.      -4.063832
21         175.      12.184012
22         193.      14.646961
23         137.       7.189122
24        -244.      -7.614677
25           9.       0.060929
26          84.       4.680115
27          86.       6.030690
28         -11.      -0.276010
29         131.       7.100616
30        -385.     -14.596388
31         -47.      -0.968709
32        -118.      -5.940056
33         301.      11.269746
34         430.       8.345490
35         -90.      -9.486497
36          80.       2.361491
37          80.       2.077256

        LOAD     4
 1        -173.      -7.683784
 2          91.       9.032880
 3         -83.      -2.772488
 4           0.       0.376252
 5        -500.     -11.867786
 6         107.       4.959062
 7        -301.     -11.265811
 8          86.       4.801345
 9           8.       0.171307
10         -82.      -2.973159
11         183.      13.301035
12         759.      10.809021
13         128.       6.856315
14        -244.      -3.130932
15           0.       7.037312
16          94.      12.769699
17         -90.      -6.377125
18         -82.      -3.024345
19         -91.      -3.614426
20         -93.      -3.476547
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21             166.        11.040344
22             190.        14.303860
23             139.         7.469063
24            -243.        -7.558084
25               8.         0.059443
26              84.         4.728210
27              86.         6.026362
28             -12.        -0.330941
29             122.         6.274048
30            -372.       -13.745524
31             -42.        -0.803446
32            -110.        -5.193901
33             300.        11.183788
34             420.         7.975573
35             -91.        -9.706367
36              78.         2.230977
37              82.         2.195441

           LOAD     5
 1             -37.        -0.433069
 2              10.         0.156227
 3            -139.        -7.130912
 4            -180.       -12.827478
 5               0.       -43.660523
 6            -147.        -8.849088
 7            -388.       -18.038447
 8              64.         2.745530
 9             -37.        -3.210538
10            -118.        -5.876767
11             205.        16.466016
12             942.        16.093933
13              47.         1.078255
14            -473.       -10.681362
15               0.         7.259839
16              94.        12.757761
17             -89.        -6.246185
18            -128.        -6.841793
19            -143.        -8.406448
20            -157.        -9.164825
21             130.         7.109855
22             179.        12.716109
23             145.         8.053042
24            -295.       -10.834049
25             -33.        -0.716619
26              82.         4.500793
27              82.         5.555337
28             -14.        -0.457371
29             142.         8.342285
30            -434.       -18.282539
31             -65.        -1.747803
32            -136.        -7.613881
33             343.        14.323680
34             471.         9.880415
35             -97.       -10.911865
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36           74.        2.016947
37           86.        2.403306

          LOAD     6
 1         -222.      -12.263047
 2           88.        8.505278
 3          -31.       -0.440310
 4           -8.       -0.036975
 5         -435.       -9.147994
 6            0.       13.129470
 7         -253.       -8.145388
 8          100.        6.346323
 9            2.        0.014815
10          -79.       -2.811837
11          177.       12.427152
12          836.       12.899801
13           87.        3.321002
14         -347.       -6.041832
15            0.        9.267206
16          102.       14.685844
17          -85.       -5.767377
18          -93.       -3.770997
19         -105.       -4.774513
20         -113.       -4.964873
21          146.        8.779646
22          182.       13.145767
23          140.        7.578357
24         -259.       -8.541426
25          -10.       -0.082371
26           80.        4.295085
27           83.        5.619673
28           -9.       -0.186419
29          126.        6.696472
30         -390.      -14.987522
31          -49.       -1.042643
32         -116.       -5.736415
33          316.       12.320370
34          435.        8.517718
35          -91.       -9.721231
36           75.        2.091349
37           85.        2.328979

          LOAD     7
 1         -152.       -6.064706
 2           80.        7.093283
 3          -93.       -3.392595
 4          173.       11.956610
 5         -616.      -17.475366
 6           72.        2.337893
 7            0.      -35.369671
 8          -42.       -1.274354
 9           -1.       -0.002910
10          -84.       -3.165872
11          262.       25.778301
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12            806.       12.063110
13            117.        5.815100
14           -278.       -4.003010
15              0.       -5.028795
16             45.        3.215251
17            -91.       -6.458441
18            -90.       -3.579393
19            -94.       -3.861046
20           -104.       -4.281933
21            162.       10.625523
22            206.       16.490862
23            161.        9.709555
24           -260.       -8.564894
25             -3.       -0.004299
26            110.        7.809855
27             96.        7.427396
28            -35.       -2.531891
29            126.        6.637726
30           -384.      -14.573364
31            -45.       -0.907511
32           -114.       -5.554856
33            319.       12.520967
34            432.        8.396258
35           -104.      -12.465192
36             73.        1.979390
37             87.        2.441737

            LOAD     8
 1           -183.       -8.537212
 2             95.        9.761077
 3            -77.       -2.400533
 4            -66.       -2.003670
 5           -460.      -10.174235
 6            123.        6.399716
 7           -211.       -5.826726
 8              0.       26.125625
 9              9.        0.245879
10            -79.       -2.822347
11            284.       29.990413
12            774.       11.203532
13            131.        7.171508
14           -225.       -2.694302
15              0.       -9.534256
16             19.        0.666450
17            -89.       -6.202825
18            -82.       -3.003852
19            -83.       -3.043175
20            -94.       -3.566455
21            172.       11.906390
22            217.       18.272277
23            169.       10.688918
24           -256.       -8.316310
25              6.        0.036780
26            125.        9.884257
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27          101.      8.205566
28          -46.     -4.143045
29          122.      6.280426
30         -375.    -13.907233
31          -41.     -0.768311
32         -110.     -5.161730
33          315.     12.244783
34          424.      8.117174
35         -109.    -13.685105
36           72.      1.953084
37           88.      2.469045

         LOAD    9
 1         -190.     -9.172219
 2          118.     14.563722
 3          -93.     -3.389046
 4          -34.     -0.591087
 5         -493.    -11.558308
 6          117.      5.815641
 7         -273.     -9.394926
 8           94.      5.636790
 9            0.     -3.097818
10         -117.     -5.812478
11          179.     12.688266
12          769.     11.074849
13          138.      7.892190
14         -254.     -3.373833
15            0.      7.958357
16           97.     13.580031
17          -90.     -6.316884
18          -90.     -3.598022
19         -103.     -4.615231
20         -105.     -4.372574
21          166.     11.142970
22          190.     14.187130
23          138.      7.307434
24         -248.     -7.855920
25            3.      0.011146
26           82.      4.566427
27           85.      5.907593
28          -10.     -0.250920
29          125.      6.609375
30         -383.    -14.514398
31          -47.     -0.973348
32         -115.     -5.603748
33          305.     11.516656
34          429.      8.287811
35          -90.     -9.567458
36           78.      2.254557
37           82.      2.173654

         LOAD   10
 1         -211.    -11.150099
 2          165.     27.223499
```

```
  3        -119.       -5.370048
  4         -39.       -0.749016
  5        -505.      -12.096593
  6         113.        5.433430
  7        -276.       -9.617682
  8          93.        5.507090
  9         -70.      -10.384706
 10           0.      -15.892714
 11         179.       12.731676
 12         768.       11.033488
 13         113.        5.420488
 14        -259.       -3.498173
 15           0.        7.716537
 16          97.       13.363154
 17        -137.      -13.841693
 18        -113.       -5.429574
 19         -84.       -3.107929
 20         -80.       -2.663828
 21         161.       10.493992
 22         188.       14.014755
 23         140.        7.486941
 24        -242.       -7.529131
 25          10.        0.086272
 26          83.        4.605164
 27          85.        5.932090
 28         -11.       -0.273176
 29         140.        8.051453
 30        -379.      -14.184909
 31         -41.       -0.760966
 32        -123.       -6.383959
 33         300.       11.198567
 34         429.        8.303550
 35         -90.       -9.623162
 36          80.        2.367147
 37          80.        2.072269

          LOAD    11
  1        -171.       -7.552498
  2          89.        8.680373
  3         -83.       -2.750817
  4          41.        0.849533
  5        -539.      -13.612116
  6         106.        4.840694
  7        -333.      -13.610099
  8         230.       29.601332
  9           5.        0.081280
 10         -81.       -2.936336
 11           0.       39.334861
 12         809.       12.164062
 13         125.        6.587761
 14        -242.       -3.090019
 15           0.      -17.403778
 16         -35.       -2.047191
 17         -88.       -6.140645
```

```
18            -87.       -3.326398
19            -82.       -3.019485
20           -102.       -4.108043
21            172.       11.830967
22            236.       21.258422
23            192.       13.491616
24           -270.       -9.194792
25             -2.       -0.002468
26            157.       15.083557
27            111.        9.619917
28            -68.       -8.602448
29            124.        6.500000
30           -383.      -14.480625
31            -43.       -0.820699
32           -112.       -5.368784
33            331.       13.446384
34            432.        8.429653
35           -122.      -16.862560
36             67.        1.716248
37             92.        2.729357

           LOAD   12
 1           -285.      -19.361918
 2             71.        5.699451
 3             48.        1.014538
 4            -81.       -2.945709
 5           -669.      -20.361728
 6            246.       22.967641
 7           -331.      -13.413912
 8            105.        6.897253
 9            -23.       -1.350261
10            -71.       -2.295000
11            209.       16.991090
12              0.       39.244080
13            -24.       -0.303748
14            178.        1.743011
15              0.       22.831308
16            139.       26.122194
17            -51.       -2.266407
18           -152.       -9.397746
19            -80.       -2.855339
20           -206.      -15.285779
21           -166.      -11.048584
22             93.        3.799783
23            157.        9.287865
24           -461.      -24.767159
25           -112.       -6.993846
26             45.        1.461650
27             45.        1.838129
28             -7.       -0.121809
29            153.        9.551727
30           -485.      -22.400614
31            -75.       -2.310954
32           -149.       -9.040387
```

```
33           425.       21.339134
34           522.       11.924373
35          -127.      -17.972663
36            47.        0.877726
37           113.        3.952954

          LOAD   13
 1          -185.       -8.695542
 2           133.       18.143228
 3          -113.       -4.887407
 4           -24.       -0.305217
 5          -467.      -10.465778
 6            87.        3.321794
 7          -265.       -8.885709
 8            96.        5.876756
 9            11.        0.321286
10           -43.       -0.914048
11           177.       12.533775
12           723.        9.876139
13             0.       20.787910
14          -166.       -1.544800
15             0.        8.140439
16            98.       13.827425
17           -54.       -2.441820
18          -116.       -5.693320
19          -165.      -10.973225
20          -162.       -9.717867
21           197.       15.273612
22           200.       15.629799
23           129.        6.484644
24          -259.       -8.547935
25            -7.       -0.046015
26            83.        4.613505
27            86.        6.048418
28            -9.       -0.190713
29           135.        7.517151
30          -429.      -17.885980
31           -67.       -1.878662
32          -131.       -7.177183
33           317.       12.407317
34           461.        9.485277
35           -88.       -9.087363
36            82.        2.477132
37            78.        1.977429

          LOAD   14
 1          -241.      -14.268735
 2            68.        5.286760
 3             8.        0.032472
 4           -69.       -2.166339
 5          -595.      -16.364001
 6           202.       15.989909
 7          -299.      -11.111383
 8            85.        4.700549
 9            -16.       -0.689733
10            -81.       -2.920672
11            182.       13.084514
12            598.        6.939880
13            -20.       -0.229103
14              0.      -19.562836
15              0.        6.079791
16             92.       12.121358
17            -77.       -4.727268
18           -120.       -6.045213
19           -164.      -10.848312
20           -154.       -8.855963
21            225.       19.513070
22            213.       17.546057
23            124.        6.043255
24           -244.       -7.638588
25             11.        0.094857
26             87.        5.072367
27             90.        6.531677
28            -11.       -0.302647
29            137.        7.778595
30           -425.      -17.542318
31            -66.       -1.833008
32           -133.       -7.311185
33            303.       11.376981
34            456.        9.313267
35            -86.       -8.861551
36             87.        2.762756
37             73.        1.748815

           LOAD   15
 1           -178.       -8.080272
 2             93.        9.357159
 3            -80.       -2.589588
 4            -29.       -0.439603
 5           -484.      -11.150373
 6            115.        5.641578
 7           -271.       -9.278488
 8             97.        5.998511
 9              8.        0.199961
10            -80.       -2.901365
11            181.       12.952895
12            770.       11.082092
13            129.        6.979222
14           -234.       -2.908039
15              0.        9.793802
16            103.       14.955745
17            -89.       -6.281885
18            -82.       -3.029562
19            -87.       -3.319397
20            -94.       -3.549412
21            169.       11.422947
22            207.       16.719326
23            159.        9.504404
```

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24          -250.       -7.986836
25             3.        0.013083
26            77.        4.027242
27            79.        5.161715
28           -10.       -0.259269
29           122.        6.285858
30          -374.      -13.849046
31           -42.       -0.791768
32          -110.       -5.189938
33           307.       11.712307
34           422.        8.054635
35           -96.      -10.813940
36            75.        2.093811
37            85.        2.326556

          LOAD   16
 1          -184.       -8.643060
 2            95.        9.706820
 3           -76.       -2.315153
 4           -53.       -1.339844
 5          -477.      -10.858329
 6           126.        6.629764
 7          -242.       -7.533758
 8            44.        1.400827
 9             8.        0.196554
10           -79.       -2.774749
11           131.        7.125477
12           806.       12.080444
13           130.        7.078675
14          -220.       -2.589359
15             0.       23.764352
16             0.       46.540361
17           -87.       -6.017272
18           -85.       -3.176685
19           -76.       -2.601776
20          -100.       -3.962133
21           177.       12.510986
22           250.       23.595675
23           205.       15.299213
24          -273.       -9.416682
25            -1.        0.000013
26           178.       19.065959
27           117.       10.596397
28           -82.      -12.102073
29           123.        6.414368
30          -381.      -14.377238
31           -42.       -0.773112
32          -111.       -5.259747
33           336.       13.776285
34           432.        8.414176
35          -130.      -18.915854
36            65.        1.616842
37            95.        2.847322
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          LOAD   17
  1         -185.        -8.749470
  2          113.        13.532815
  3          -93.        -3.397704
  4          -32.        -0.509875
  5         -486.       -11.231619
  6          112.         5.331825
  7         -270.        -9.235629
  8           94.         5.648856
  9           72.        11.006448
 10         -124.        -6.427562
 11          178.        12.578351
 12          755.        10.700928
 13          120.         6.067756
 14         -241.        -3.064041
 15            0.         7.721846
 16           97.        13.393492
 17            0.       -32.842827
 18         -178.       -12.617493
 19          -81.        -2.966156
 20          -62.        -1.657481
 21          165.        10.972334
 22          190.        14.203201
 23          139.         7.411444
 24         -240.        -7.425332
 25           15.         0.177413
 26           83.         4.627502
 27           86.         5.983970
 28          -10.        -0.256348
 29          176.        12.379547
 30         -397.       -15.472616
 31          -41.        -0.766663
 32         -152.        -9.395515
 33          301.        11.240811
 34          451.         9.125257
 35          -90.        -9.473913
 36           87.         2.723816
 37           73.         1.778577

          LOAD   18
  1         -195.        -9.645161
  2          122.        15.613791
  3          -92.        -3.343732
  4          -37.        -0.687490
  5         -502.       -11.935250
  6          123.         6.403530
  7         -276.        -9.574284
  8           94.         5.626003
  9           62.         8.448508
 10         -105.        -4.756554
 11          179.        12.814760
 12          786.        11.505920
 13          160.        10.376887
 14         -269.        -3.756828
```

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15           0.      8.226661
16          98.     13.802584
17        -188.    -24.848207
18           0.    -19.557243
19        -128.     -6.868210
20        -144.     -7.878252
21         167.     11.270185
22         189.     14.163159
23         137.      7.218796
24        -256.     -8.321525
25         -12.     -0.108908
26          82.      4.507060
27          84.      5.841814
28         -10.     -0.238996
29          72.      2.348328
30        -366.    -13.338487
31         -50.     -1.072408
32         -78.     -2.747875
33         311.     11.942150
34         404.      7.425733
35         -90.     -9.617453
36          69.      1.775167
37          91.      2.661982

          LOAD   19
 1        -194.     -9.540016
 2         117.     14.412660
 3         -88.     -3.081435
 4         -38.     -0.722354
 5        -502.    -11.945230
 6         126.      6.629316
 7        -275.     -9.525815
 8          92.      5.432182
 9           6.      0.119569
10         -54.     -1.401707
11         178.     12.588261
12         768.     11.030558
13         190.     14.295787
14        -293.     -4.402008
15           0.      7.170272
16          95.     12.910663
17         -61.     -3.076045
18        -113.     -5.451185
19           0.    -12.261505
20        -155.     -8.952133
21         133.      7.359401
22         182.     13.169246
23         148.      8.342003
24        -217.     -6.137606
25          32.      0.692391
26          84.      4.726603
27          86.      6.057850
28         -11.     -0.304478
29         133.      7.363251
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30          -421.     -17.255046
31           -67.      -1.865987
32          -130.      -7.057105
33           289.      10.453627
34           451.       9.093257
35           -90.      -9.558482
36            91.       2.999552
37            69.       1.576642

        LOAD   20
 1          -194.      -9.557624
 2           114.      13.579677
 3           -84.      -2.820339
 4           -38.      -0.725760
 5          -506.     -12.105748
 6           129.       6.990724
 7          -277.      -9.664199
 8            94.       5.659573
 9            -1.      -0.001118
10           -51.      -1.242813
11           180.      12.916273
12           800.      11.895426
13           184.      13.473511
14          -279.      -4.020786
15             0.       8.563107
16            99.      14.083144
17           -55.      -2.535407
18          -126.      -6.629447
19          -151.      -9.325428
20             0.     -11.882574
21           171.      11.742467
22           190.      14.233841
23           135.       7.037490
24          -265.      -8.900548
25           -20.      -0.282646
26            81.       4.433960
27            84.       5.764354
28           -10.      -0.221536
29           151.       9.330505
30          -344.     -11.871541
31            -7.      -0.027181
32          -125.      -6.507831
33           317.      12.419241
34           409.       7.602583
35           -91.      -9.659478
36            67.       1.703328
37            93.       2.744411

        LOAD   21
 1          -178.      -8.113199
 2           104.      11.502251
 3           -91.      -3.255968
 4           -26.      -0.362369
 5          -477.     -10.883194
```

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  6          106.       4.806204
  7         -269.      -9.166022
  8           98.       6.124141
  9           10.       0.293572
 10          -71.      -2.326701
 11          181.      12.989773
 12          674.       8.674744
 13          162.      10.643251
 14         -287.      -4.251137
 15            0.       9.913468
 16          104.      15.324222
 17          -81.      -5.276330
 18          -89.      -3.461119
 19          -19.      -0.206261
 20         -111.      -4.838503
 21            0.      28.099191
 22          227.      19.825277
 23           95.       3.667097
 24         -326.     -13.089588
 25          -58.      -2.061953
 26           79.       4.244853
 27           83.       5.637429
 28           -7.      -0.118648
 29          125.       6.563446
 30         -391.     -15.066969
 31          -45.      -0.882996
 32         -113.      -5.477736
 33          348.      14.758386
 34          441.       8.739842
 35          -89.      -9.327985
 36           62.       1.463643
 37           98.       3.040589

           LOAD   22
  1         -173.      -7.665895
  2           98.      10.290733
  3          -90.      -3.215113
  4          -14.      -0.114659
  5         -479.     -10.941327
  6          100.       4.359750
  7         -277.      -9.626164
  8          118.       8.542922
  9           10.       0.305832
 10          -78.      -2.716142
 11          200.      15.705769
 12          671.       8.594076
 13          148.       8.952922
 14         -278.      -3.991604
 15            0.      24.649382
 16          143.      27.589641
 17          -88.      -6.093407
 18          -82.      -2.986571
 19          -62.      -1.811066
 20          -95.      -3.642154
 21          233.       20.843549
 22            0.       41.171978
 23          280.       27.211021
 24         -279.       -9.798480
 25           -1.        0.000902
 26           37.        1.022542
 27           34.        1.062123
 28          -10.       -0.261056
 29          123.        6.321686
 30         -379.      -14.189317
 31          -39.       -0.685689
 32         -109.       -5.104319
 33          343.       14.336734
 34          431.        8.387037
 35         -142.      -22.134723
 36           62.        1.454122
 37           98.        3.053095

           LOAD   23
  1         -176.       -7.974745
  2           90.        8.751209
  3          -78.       -2.468306
  4          -22.       -0.268132
  5         -490.      -11.407321
  6          116.        5.682373
  7         -279.       -9.791408
  8          112.        7.800047
  9            8.        0.172761
 10          -83.       -3.081357
 11          196.       15.105841
 12          790.       11.627584
 13          119.        6.012406
 14         -222.       -2.627239
 15            0.       21.013924
 16          133.       24.182780
 17          -92.       -6.600634
 18          -81.       -2.908307
 19         -107.       -4.922543
 20          -88.       -3.178485
 21          113.        5.404140
 22          296.       32.360766
 23            0.       27.122725
 24         -211.       -5.838148
 25           47.        1.392928
 26           45.        1.472168
 27           41.        1.552318
 28          -12.       -0.357644
 29          122.        6.229187
 30         -370.      -13.561639
 31          -40.       -0.729289
 32         -109.       -5.085500
 33          305.       11.507416
 34          419.        7.936816
 35         -136.      -20.498980
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36           75.        2.110372
37           85.        2.310322

         LOAD   24
 1         -195.       -9.632572
 2          106.       11.889032
 3          -75.       -2.309786
 4          -38.       -0.713839
 5         -514.      -12.488093
 6          137.        7.742713
 7         -283.      -10.026820
 8          100.        6.333239
 9            5.        0.087401
10          -65.       -1.947473
11          187.       13.851860
12          873.       13.992737
13          159.       10.208844
14         -237.       -2.985287
15            0.       12.796083
16          112.       17.626774
17          -71.       -4.132684
18         -104.       -4.624762
19           21.        0.236435
20         -138.       -7.266830
21          260.       25.448172
22          216.       18.023780
23           91.        3.384900
24            0.      -16.963814
25          -80.       -3.793189
26           72.        3.526754
27           75.        4.678456
28           -6.       -0.102735
29          131.        7.203705
30         -417.      -16.926236
31          -53.       -1.214864
32         -122.       -6.268056
33          373.       16.739344
34          463.        9.579511
35          -96.      -10.827296
36           56.        1.234375
37          104.        3.360174

         LOAD   25
 1         -180.       -8.268758
 2           94.        9.666267
 3          -80.       -2.554887
 4          -31.       -0.495611
 5         -487.      -11.282274
 6          118.        5.897657
 7         -272.       -9.304563
 8           96.        5.824122
 9            8.        0.182165
10          -78.       -2.766948
11          179.       12.802347
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12           782.        11.401510
13           133.         7.401128
14          -235.        -2.926826
15             0.         8.826582
16           100.        14.241589
17           -87.        -5.975564
18           -85.        -3.222532
19           -72.        -2.328644
20          -100.        -4.017560
21           192.        14.477844
22           195.        14.978003
23           127.         6.297188
24          -277.        -9.643516
25             0.        -1.099389
26            80.         4.321167
27            82.         5.579307
28           -10.        -0.246081
29           123.         6.380829
30          -379.       -14.195048
31           -45.        -0.880310
32          -112.        -5.348121
33           308.        11.766815
34           426.         8.182199
35           -93.       -10.061215
36            76.         2.129089
37            84.         2.292122

            LOAD   26
 1          -174.        -7.805473
 2            92.         9.219478
 3           -83.        -2.742879
 4           -14.        -0.108215
 5          -490.       -11.422604
 6           109.         5.133820
 7          -283.       -10.068512
 8           125.         9.539707
 9             8.         0.201966
10           -81.        -2.953354
11           209.        17.018127
12           753.        10.631694
13           129.         7.001618
14          -243.        -3.106270
15             0.         1.394256
16           159.        33.610108
17           -90.        -6.391621
18           -82.        -2.969626
19           -90.        -3.578396
20           -92.        -3.396666
21           168.        11.323722
22           182.        13.209952
23           127.         6.308758
24          -242.        -7.515282
25            23.         0.362390
26             0.        29.116089
```

```
27          150.      16.844011
28         -101.     -17.961230
29          122.       6.259064
30         -372.     -13.743015
31          -41.      -0.759277
32         -110.      -5.137486
33          308.      11.766335
34          421.       8.021284
35         -116.     -15.381559
36           74.       2.063090
37           85.       2.356948

           LOAD   27
 1         -176.      -7.951837
 2           93.       9.391902
 3          -82.      -2.692422
 4          -24.      -0.299999
 5         -484.     -11.183292
 6          112.       5.363442
 7         -275.      -9.537955
 8          107.       7.126995
 9            9.       0.222335
10          -81.      -2.920439
11          190.      14.254264
12          755.      10.700907
13          131.       7.125830
14         -240.      -3.054085
15            0.       4.957643
16          122.      20.493137
17          -90.      -6.360912
18          -81.      -2.955308
19          -88.      -3.413811
20          -92.      -3.417097
21          172.      11.800668
22          184.      13.354345
23          126.       6.193224
24         -247.      -7.792297
25           35.       0.801325
26          150.      13.755005
27            0.      33.990895
28           81.      11.969626
29          122.       6.285675
30         -375.     -13.901774
31          -39.      -0.700480
32         -109.      -5.094719
33          327.      13.106367
34          426.       8.186340
35         -154.     -25.791680
36           68.       1.723348
37           92.       2.721145

           LOAD   28
 1         -177.      -8.025882
 2           93.       9.299498
```

```
  3         -81.        -2.609562
  4         -24.        -0.307669
  5        -487.       -11.299108
  6         114.         5.554648
  7        -277.        -9.629216
  8         108.         7.263201
  9           8.         0.192296
 10         -81.        -2.910536
 11         192.        14.457137
 12         770.        11.100769
 13         129.         6.945038
 14        -235.        -2.930050
 15           0.         8.017595
 16         124.        21.298689
 17         -89.        -6.286875
 18         -83.        -3.038841
 19         -87.        -3.345985
 20         -94.        -3.559737
 21         168.        11.351929
 22         207.        16.764708
 23         159.         9.583499
 24        -250.        -7.973104
 25           2.         0.004140
 26         136.        11.507019
 27          71.         4.260670
 28           0.        -8.981280
 29         122.         6.290588
 30        -374.       -13.854540
 31         -42.        -0.795898
 32        -110.        -5.197525
 33         307.        11.673171
 34         422.         8.053741
 35         -94.       -10.305355
 36          75.         2.105876
 37          85.         2.314743

          LOAD   29
  1        -188.        -8.967791
  2         111.        12.997114
  3         -88.        -3.077372
  4         -34.        -0.585188
  5        -493.       -11.551203
  6         119.         5.995088
  7        -273.        -9.398270
  8          94.         5.663010
  9          43.         4.251116
 10         -97.        -4.115389
 11         179.        12.716452
 12         774.        11.182719
 13         147.         8.813354
 14        -256.        -3.419819
 15           0.         8.083249
 16          98.        13.695616
 17        -148.       -16.018836
```

```
18          11.       0.075265
19        -111.      -5.280247
20        -127.      -6.243243
21         168.      11.359820
22         190.      14.244153
23         137.       7.246923
24        -251.      -8.054400
25           1.       0.002216
26          82.       4.549662
27          85.       5.900269
28         -10.      -0.238713
29           0.      25.122131
30        -359.     -12.844170
31          32.       0.471781
32        -209.     -16.954486
33         309.      11.788008
34         442.       8.762447
35         -90.      -9.527103
36          80.       2.368958
37          80.       2.070649

        LOAD   30
 1        -214.     -11.407230
 2         140.      20.123379
 3         -91.      -3.299790
 4         -46.      -1.024546
 5        -531.     -13.262364
 6         143.       8.430414
 7        -285.     -10.209633
 8          95.       5.720187
 9          35.       2.842074
10         -59.      -1.643481
11         184.      13.392262
12         846.      13.201925
13         222.      19.016709
14        -310.      -4.901600
15           0.       9.692571
16         103.      15.176526
17        -105.      -8.447488
18         -41.      -0.812513
19        -187.     -13.834248
20         105.       4.389815
21         183.      13.269261
22         193.      14.666527
23         131.       6.640306
24        -295.     -10.817323
25         -10.      -0.084985
26          80.       4.325948
27          84.       5.761136
28          -6.      -0.108337
29          -9.      -0.048248
30           0.     -51.096090
31        -205.     -14.964315
32        -199.     -15.554322
```

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33            359.        15.644356
34            628.        16.814292
35            -88.        -9.076897
36            122.         5.118612
37             38.         0.532894

         LOAD   31
 1           -179.        -8.203968
 2             96.         9.984442
 3            -82.        -2.700909
 4            -31.        -0.480924
 5           -484.       -11.147621
 6            115.         5.634128
 7           -270.        -9.207961
 8             95.         5.709137
 9             12.         0.401732
10            -81.        -2.931400
11            178.        12.616138
12            762.        10.869141
13            136.         7.617655
14           -242.        -3.099251
15              0.         7.957082
16             98.        13.606756
17            -92.        -6.704712
18            -75.        -2.571088
19            -96.        -3.986282
20            -82.        -2.782173
21            169.        11.466631
22            191.        14.330932
23            137.         7.271428
24           -247.        -7.804021
25             13.         0.134755
26             83.         4.591899
27             85.         5.963801
28            -10.        -0.234048
29            122.         6.298798
30           -405.       -16.026510
31              0.        -0.202209
32           -103.        -4.584800
33            307.        11.703972
34            476.        10.086103
35            -89.        -9.408210
36             91.         3.013468
37             69.         1.566983

         LOAD   32
 1           -188.        -8.997967
 2            111.        12.970029
 3            -88.        -3.046409
 4            -34.        -0.596523
 5           -494.       -11.584823
 6            120.         6.069340
 7           -273.        -9.409759
 8             94.         5.643858
 9          38.        3.428561
10         -93.       -3.764281
11         179.       12.706508
12         774.       11.196167
13         151.        9.301167
14        -259.       -3.495960
15           0.        8.031181
16          98.       13.632157
17        -137.      -13.828246
18          -5.       -0.017920
19        -117.       -5.836391
20        -101.       -4.063138
21         166.       11.126796
22         189.       14.152305
23         137.        7.283658
24        -250.       -7.963929
25          -9.       -0.062853
26          82.        4.542806
27          85.        5.873413
28         -10.       -0.252468
29         225.       19.501526
30        -423.      -17.404446
31        -103.       -4.177429
32           0.      -17.894215
33         305.       11.517080
34         382.        6.712755
35         -90.       -9.626716
36          66.        1.631897
37          94.        2.829090

         LOAD   33
 1        -189.       -9.104116
 2         101.       10.988588
 3         -77.       -2.388343
 4         -31.       -0.495352
 5        -508.      -12.223341
 6         130.        7.049269
 7        -285.      -10.149182
 8         109.        7.382588
 9          11.        0.360427
10         -75.       -2.555022
11         195.       14.971118
12         849.       13.296326
13         145.        8.606080
14        -232.       -2.846870
15           0.       13.831841
16         129.       22.783594
17         -86.       -5.862268
18         -83.       -3.040828
19         -54.       -1.367779
20        -103.       -4.185922
21         220.       18.625837
22         238.       21.692411
23         166.       10.343660
```

```
24          -329.      -13.280110
25           159.       13.468960
26           105.        7.194132
27           139.       14.757025
28            44.        3.886806
29           128.        6.843872
30          -438.      -18.583001
31             8.        0.041219
32          -105.       -4.709752
33             0.       48.112205
34           523.       11.990378
35            19.        0.520809
36           -82.       -2.477661
37           242.       16.215547

          LOAD    34
 1          -205.      -10.530211
 2           129.       17.223446
 3           -89.       -3.152877
 4           -41.       -0.842244
 5          -519.      -12.721252
 6           136.        7.642499
 7          -283.      -10.024486
 8            96.        5.923129
 9            47.        4.942765
10           -83.       -3.050824
11           184.       13.449182
12           835.       12.874674
13           189.       14.149106
14          -283.       -4.126072
15             0.        9.996325
16           106.       15.785397
17          -143.      -15.084553
18            10.        0.058179
19          -145.       -8.667889
20           -31.       -0.466602
21           193.       14.682835
22           199.       15.501748
23           131.        6.674764
24          -299.      -11.132909
25            46.        1.347603
26            83.        4.582336
27            88.        6.240845
28            -2.       -0.006125
29           198.       15.299805
30          -609.      -34.155613
31           180.       11.680379
32            57.        1.542197
33           401.       19.156929
34             0.       34.471872
35           -79.       -7.476250
36           228.       16.381836
37           -68.       -1.545558
```

```
            LOAD   35
 1            -178.        -8.136047
 2              92.         9.142391
 3             -78.        -2.480873
 4             -23.        -0.294794
 5            -492.       -11.516709
 6             117.         5.846203
 7            -280.        -9.845621
 8             112.         7.757411
 9               7.         0.145899
10             -80.        -2.892662
11             196.        15.105584
12             794.        11.743551
13             126.         6.656794
14            -227.        -2.741184
15               0.        13.035437
16             133.        24.071570
17             -88.        -6.167253
18             -85.        -3.179772
19             -85.        -3.231888
20             -97.        -3.796763
21             163.        10.670428
22             245.        22.826090
23             207.        15.548547
24            -255.        -8.281408
25             -45.        -1.289580
26             120.         9.137065
27             172.        21.875166
28              63.         7.397919
29             122.         6.298370
30            -373.       -13.787367
31             -47.        -0.956604
32            -112.        -5.370331
33             272.         9.369066
34             416.         7.838375
35               0.       -28.155159
36              88.         2.797750
37              72.         1.722418

            LOAD   36
 1            -180.        -8.261530
 2              99.        10.445932
 3             -84.        -2.804154
 4             -31.        -0.495447
 5            -483.       -11.121756
 6             114.         5.563332
 7            -269.        -9.160838
 8              93.         5.571089
 9              15.         0.634695
10             -82.        -2.995721
11             177.        12.445188
12             752.        10.616109
13             140.         8.051251
14            -249.        -3.256569
```

```
15              0.        7.394451
16             95.       12.955138
17            -97.       -7.253424
18            -68.       -2.121220
19           -109.       -5.110283
20            -69.       -2.025931
21            157.       10.025155
22            186.       13.729169
23            139.        7.467583
24           -232.       -6.925172
25            -30.       -0.608414
26             82.        4.548726
27             83.        5.711670
28            -13.       -0.385895
29            124.        6.434723
30           -428.      -17.775879
31             35.        0.563822
32            -94.       -3.881243
33            264.        8.835574
34            524.       12.031446
35            -94.      -10.405148
36              0.       32.291646
37            280.       21.204054

          LOAD   37
 1           -178.       -8.097839
 2             92.        9.175110
 3            -79.       -2.519740
 4            -29.       -0.441442
 5           -485.      -11.209006
 6            116.        5.752631
 7           -272.       -9.328057
 8             98.        6.080364
 9              5.        0.080073
10            -78.       -2.758088
11            181.       13.070782
12            778.       11.294210
13            129.        6.952741
14           -232.       -2.848282
15              0.        9.082052
16            104.       15.388466
17            -85.       -5.774562
18            -90.       -3.539329
19            -71.       -2.278767
20           -104.       -4.284827
21            185.       13.618033
22            201.       15.793930
23            140.        7.501380
24           -269.       -9.160720
25             74.        3.304496
26             88.        5.112345
27             93.        6.989163
28              2.        0.013973
29            120.        6.054565
```

30	-358.	-12.801480
31	-65.	-1.787130
32	-120.	-6.066153
33	415.	20.439148
34	370.	6.314741
35	-70.	-5.964463
36	280.	23.950846
37	0.	25.340161

Appendix D

NEW YORK CITY WATER SUPPLY TUNNELS OUTPUT

```
          **** HARDY CROSS ANALYSIS ONLY ***

               UNITS=     0.0000151

               HAZEN-WILLIAMS C=    100.

               NUMBER OF CANDIDATE DIAMETERS    15

                  DIAMETER        COST

           1       36.000         93.50
           2       48.000        134.00
           3       60.000        176.00
           4       72.000        221.00
           5       84.000        267.00
           6       96.000        316.00
           7      108.000        365.00
           8      120.000        417.00
           9      132.000        469.00
          10      144.000        522.00
          11      156.000        577.00
          12      168.000        632.00
          13      180.000        689.00
          14      192.000        746.00
          15      204.000        804.00

NUMBER OF LINKS    42     NUMBER OF NODES    20

    PIPE   FROM    TO        LENGTH

       1      1     2        11600.
       2      2     3        19800.
       3      3     4         7300.
       4      4     5         8300.
       5      5     6         8600.
       6      6     7        19100.
       7      7     8         9600.
       8      8     9        12500.
       9      9    10         9600.
      10     11     9        11200.
      11     12    11        14500.
      12     12    13        12200.
      13     13    14        24100.
      14     14    15        21100.
      15      1    15        15500.
      16     10    17        26400.
      17     12    18        31200.
      18     18    19        24000.
      19     11    20        14400.
```

```
                    20     16     20         38400.
                    21      9     16         26400.
                    22      1      2         11600.
                    23      2      3         19800.
                    24      3      4          7300.
                    25      4      5          8300.
                    26      5      6          8600.
                    27      6      7         19100.
                    28      7      8          9600.
                    29      8      9         12500.
                    30      9     10          9600.
                    31     11      9         11200.
                    32     12     11         14500.
                    33     12     13         12200.
                    34     13     14         24100.
                    35     14     15         21100.
                    36      1     15         15500.
                    37     10     17         26400.
                    38     12     18         31200.
                    39     18     19         24000.
                    40     11     20         14400.
                    41     16     20         38400.
                    42      9     16         26400.
1
                          INITIAL ASSUMPTION

         PIPE     DIAMETER     LENGTH     DIAMETER     LENGTH     EXISTING

           1      180.000    11600.00      0.0         0.0      180.000
           2      180.000    19800.00      0.0         0.0      180.000
           3      180.000     7300.00      0.0         0.0      180.000
           4      180.000     8300.00      0.0         0.0      180.000
           5      180.000     8600.00      0.0         0.0      180.000
           6      180.000    19100.00      0.0         0.0      180.000
           7      132.000     9600.00      0.0         0.0      132.000
           8      132.000    12500.00      0.0         0.0      132.000
           9      180.000     9600.00      0.0         0.0      180.000
          10      204.000    11200.00      0.0         0.0      204.000
          11      204.000    14500.00      0.0         0.0      204.000
          12      204.000    12200.00      0.0         0.0      204.000
          13      204.000    24100.00      0.0         0.0      204.000
          14      204.000    21100.00      0.0         0.0      204.000
          15      204.000    15500.00      0.0         0.0      204.000
          16       72.000    26400.00      0.0         0.0       72.000
          17       72.000    31200.00      0.0         0.0       72.000
          18       60.000    24000.00      0.0         0.0       60.000
          19       60.000    14400.00      0.0         0.0       60.000
          20       60.000    38400.00      0.0         0.0       60.000
          21       72.000    26400.00      0.0         0.0       72.000
          22        0.0      11600.00      0.0         0.0        0.0
          23        0.0      19800.00      0.0         0.0        0.0
          24        0.0       7300.00      0.0         0.0        0.0
          25        0.0       8300.00      0.0         0.0        0.0
```

```
 26       0.0       8600.00     0.0       0.0      0.0
 27       0.0      19100.00     0.0       0.0      0.0
 28     144.000     9600.00     0.0       0.0      0.0
 29       0.0      12500.00     0.0       0.0      0.0
 30       0.0       9600.00     0.0       0.0      0.0
 31       0.0      11200.00     0.0       0.0      0.0
 32       0.0      14500.00     0.0       0.0      0.0
 33       0.0      12200.00     0.0       0.0      0.0
 34       0.0      24100.00     0.0       0.0      0.0
 35       0.0      21100.00     0.0       0.0      0.0
 36       0.0      15500.00     0.0       0.0      0.0
 37      96.000    26400.00     0.0       0.0      0.0
 38      96.000    31200.00     0.0       0.0      0.0
 39      84.000    24000.00     0.0       0.0      0.0
 40      60.000    14400.00     0.0       0.0      0.0
 41       0.0      38400.00     0.0       0.0      0.0
 42      84.000    26400.00     0.0       0.0      0.0

                INITIAL COST=   39204000.
1

             NUMBER OF LOADING CONDITIONS    1
             NUMBER OF NODES   20
1
                INITIAL ASSUMPTION


           NODE   MINHEAD     DEMAND       INHEAD


                    LOAD     1
             1    300.00    -2020.0       300.00
             2    255.00       92.4       294.19
             3    255.00       92.4       286.11
             4    255.00       88.2       283.74
             5    255.00       88.2       281.64
             6    255.00       88.2       280.01
             7    255.00       88.2       277.44
             8    255.00       88.2       276.59
             9    255.00      170.0       273.68
            10    255.00        1.0       273.65
            11    255.00      170.0       273.81
            12    255.00      117.1       275.09
            13    255.00      117.1       278.06
            14    255.00       92.4       285.54
            15    255.00       92.4       293.31
            16    260.00      170.0       261.55
            17    272.80       57.5       272.77
            18    255.00      117.0       261.14
            19    255.00      117.1       255.01
            20    255.00      170.0       258.02
```

```
        PIPES OUT OF SERVICE IN EACH CONDITION
 1
 1


                           FINAL RESULTS


            CORRECTED PRESSURES AND FLOWS


            NODE    DEMAND    SUPPLY      MINHED      INHEAD

                       LOAD     1
              1     -2020.    -2019.     300.00     300.00
              2        92.       92.     255.00     294.19
              3        92.       92.     255.00     286.11
              4        88.       88.     255.00     283.74
              5        88.       88.     255.00     281.64
              6        88.       88.     255.00     280.01
              7        88.       88.     255.00     277.44
              8        88.       88.     255.00     276.59
              9       170.      171.     255.00     273.68
             10         1.        2.     255.00     273.65
             11       170.      171.     255.00     273.81
             12       117.      117.     255.00     275.09
             13       117.      117.     255.00     278.06
             14        92.       92.     255.00     285.54
             15        92.       92.     255.00     293.31
             16       170.      170.     260.00     261.55
             17        58.       58.     272.80     272.78
             18       117.      117.     255.00     261.14
             19       117.      117.     255.00     255.01
             20       170.      170.     255.00     258.02

            PIPE          FLOWS          HGL

                       LOAD     1
              1            885.       0.500762
              2            792.       0.408245
              3            700.       0.324439
              4            612.       0.252788
              5            524.       0.189436
              6            435.       0.134571
              7            154.       0.088679
              8            259.       0.232520
              9             59.       0.003281
             10            160.       0.011575
             11            481.       0.088143
             12           -833.      -0.243200
             13           -950.      -0.310322
```

14	-1042.	-0.368548
15	1135.	0.431357
16	18.	0.033218
17	75.	0.447122
18	34.	0.255432
19	75.	1.096785
20	20.	0.091966
21	76.	0.459567
22	0.	0.500762
23	0.	0.408245
24	0.	0.324439
25	0.	0.252788
26	0.	0.189436
27	0.	0.134571
28	193.	0.088679
29	0.	0.232520
30	0.	0.003281
31	0.	0.011575
32	0.	0.088143
33	0.	-0.243200
34	0.	-0.310322
35	0.	-0.368548
36	0.	0.431357
37	39.	0.033218
38	159.	0.447122
39	83.	0.255432
40	75.	1.096785
41	0.	0.091966
42	114.	0.459567

Appendix E

RUNNING THE PROGRAM ON THE AMDAHL 580

The following JCL is needed to run the program on the University of Manitoba's AMDAHL 580.

```
// JOB ',,,T=8M,L=10','MORGAN',CLASS=1
// EXEC FORTXCLG,PARM='SIZE=312K'
//FORT.SYSIN DD *
           .
           .
           .
        Program
           .
           .
           .
//GO.FT09F001 DD DSN=MORGAN.COST.DATA,
//  DISP=SHR
//GO.FT10F001 DD DSN=MORGAN.SYSTEM.DATA,
//  DISP=SHR
//GO.FT11F001 DD DSN=MORGAN.PIPES.DATA,
//  DISP=SHR
//GO.FT12F001 DD DSN=MORGAN.DEMAND.DATA,
//  DISP=SHR
//GO.FT14F001 DD DSN=MORGAN.BREAK.DATA,
//  DISP=SHR
```

The data sets assigned input/output units above contain all the input data. Data sets 'PIPES' and 'DEMAND' are updated each time the program is run. To delete a pipe which is considered uneconomical due to its weighting 'manually', use 'FETCH DA=PIPES.DATA'. When the data et has been fetched change the left most diameter number to 1 and change the left most length to the total length. The right hand diameter number and length should be 0.

The following pages are examples of how the data is entered into the data sets. The data set 'DEMAND.DATA' contains the data for single demand pattern design to save space. The data set 'BREAK.DATA' contains the data for multiple demand patterns design.

COST.DATA

```
1
24.00
130
1000
13
                  1            0.125            58.0
                  2            0.150            62.0
                  3            0.200            71.7
                  4            0.250            88.9
                  5            0.300           112.3
                  6            0.350           138.7
                  7            0.400           169.0
                  8            0.450           207.0
                  9            0.500           248.0
                 10            0.550           297.0
                 11            0.600           347.0
                 12            0.650           405.0
                 13            0.700           470.0
```

SYSTEM.DATA

```
37  20
1       1       2       760
2       1       4       520
3       1       6       890
4       2       3       1120
5       2       5       610
6       2       6       680
7       3       5       680
8       3       7       870
9       4       8       860
10      4       9       980
11      5       7       890
12      5       10      750
13      6        9      620
14      6       10      800
15      7       12      730
16      7       13      680
17      8        9      480
18      8       15      860
19      9       11      800
20      9       14      770
21      10      11      350
22      10      12      620
23      11      12      670
24      11      16      790
25      11      18      1150
26      12      13      750
27      12      17      550
28      13      17      700
29      14      15      500
30      14      16      450
31      14      19      750
32      15      19      720
33      16      18      540
34      16      19      700
35      17      18      850
36      18      20      750
37      19      20      970
```

PIPE.DATA

1	7	760	0	0	0
2	7	520	0	0	0
3	7	890	0	0	0
4	7	1120	0	0	0
5	7	610	0	0	0
6	7	680	0	0	0
7	7	680	0	0	0
8	7	870	0	0	0
9	7	860	0	0	0
10	7	980	0	0	0
11	7	890	0	0	0
12	7	750	0	0	0
13	7	620	0	0	0
14	7	800	0	0	0
15	7	730	0	0	0
16	7	680	0	0	0
17	7	480	0	0	0
18	7	860	0	0	0
19	7	800	0	0	0
20	7	770	0	0	0
21	7	350	0	0	0
22	7	620	0	0	0
23	7	670	0	0	0
24	7	790	0	0	0
25	7	1150	0	0	0
26	7	750	0	0	0
27	7	550	0	0	0
28	7	700	0	0	0
29	7	500	0	0	0
30	7	450	0	0	0
31	7	750	0	0	0
32	7	720	0	0	0
33	7	540	0	0	0
34	7	700	0	0	0
35	7	850	0	0	0
36	7	750	0	0	0
37	7	970	0	0	0

DEMAND.DATA

```
 1
20
     1
       1        75.0       165.0       80.0
       2        74.0       220.0       90.0
       3        73.0       145.0       90.0
       4        72.0       165.0       70.0
       5       102.0      -800.0      102.0
       6        73.0       140.0       80.0
       7        67.0       175.0       90.0
       8        72.0       180.0       70.0
       9        70.0       140.0       75.0
      10        69.0       160.0       90.0
      11        71.0       170.0       93.0
      12        70.0       160.0       85.0
      13        64.0       190.0       80.0
      14        73.0       200.0       90.0
      15        73.0       150.0       80.0
      16        96.0      -800.0       96.0
      17        67.0       165.0       80.0
      18        70.0       140.0       90.0
      19        70.0       185.0       90.0
      20        67.0       160.0       70.0
```

BREAK.DATA

```
1
1
0
2
2
0
3
3
0
4
4
0
5
5
0
6
6
0
7
7
0
8
8
0
9
9
0
10
10
0
11
11
0
12
12
0
13
13
0
14
14
0
15
15
0
16
16
0
17
17
0
18
18
```

```
0
19
19
0
20
20
0
21
21
0
22
22
0
23
23
0
24
24
0
25
25
0
26
26
0
27
27
0
28
28
0
29
29
0
30
30
0
31
31
0
32
32
0
33
33
0
34
34
0
35
35
0
36
36
```

```
0
37
37
0
0
```

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