Optimality conditions for truncated Kautz series

Optimality Conditions for a Specific Class of Truncated Kautz Series

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***Abstract*— Kautz functions constitute a basis in L^2 and can be used for filter synthesis starting from a prescribed transient response. In a practical situation, only truncated series are used and thus the speed of convergence of such series is of importance. This convergence speed is a function of the parameters. The Kautz functions that are considered can be generated by a line of identical Nth-order allpass sections. The optimality conditions for the parameters of this specific class of truncated Kautz series are established solely on the basis of the orthonormality and the polynomial character of the basis functions.**

I. Introduction

Kautz functions [1], [2] are a special case of orthonormal functions. These can be used in signal analysis and filter synthesis. In the case of filter synthesis, networks are designed starting from a prescribed transient response instead of prescribed frequency characteristics; for a discussion on this issue see [1]. More recently, adaptive filters on the basis of Kautz functions have been proposed [3]–[5].

In practice, a truncated Kautz series will be used and the question arises which parameters provide the best approximation to a given function by a fixed number of expansion terms. This question has already been answered for a Laguerre series [6]–[10] which can be interpreted as a Kautz series governed by a single pole. In this paper, these results are extended to the more general class of Kautz functions that is specified in Section II. Some consequences of the orthonormality and the recurrent pole set for the derivatives of the basis functions with respect to the parameters are discussed in Section III. These results are subsequently used to derive the optimality conditions (Section IV). A discussion concludes the paper.

II. The Considered Kautz Functions

The set of causal functions $\{f_i(t); i = 1, 2, \cdots\}$ having a Laplace transform

$$\mathcal{L}\{f_i(t)\} = \frac{\sqrt{-(p_i + p_i^*)}}{s - p_i} \prod_{l=1}^{i-1} \frac{s + p_l^*}{s - p_l} \tag{1}$$

with $\Re\{p_i\} < 0$ ($\Re$ denotes the real part of a complex number) is a set of orthonormal functions and are called Kautz functions [1]. If the poles p_i are all distinct, the Kautz functions form an orthonormal basis in $L^2(\boldsymbol{R}_+)$ under the Szász condition [2], [11]

$$\sum_{i=1}^{\infty} \frac{-\Re\{p_i\}}{1 + |p_i + 1/2|^2} = \infty. \tag{2}$$

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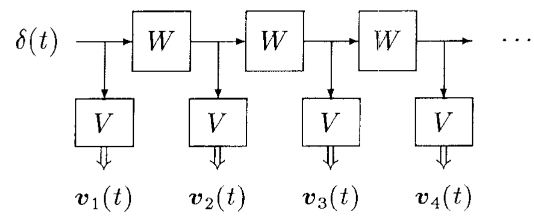

Fig. 1. A Kautz filter on the basis of Nth-order allpass sections.

We consider a specific case of Kautz functions where we have N recurrent poles $p_{i+N} = p_i$ $(i \geq 1)$ and with the restrictions that $p_i \neq p_l$ for $i \neq l$ and $1 \leq i, l \leq N$ and that the first N poles are real-valued or occur in complex-conjugated pairs. It is known that in this case the Szász condition still holds and it can be shown that these Kautz functions constitute an orthonormal basis in $L^2(\boldsymbol{R}_+)$ [12]. If $N = 1$ (i.e., all identical poles) we have as a special case the Laguerre functions.

The considered Kautz functions can be generated as the impulse responses of the filter bank shown in Fig. 1. The filter bank consists of a line of allpass filters W with transfer function

$$W(s) = \frac{\prod_{i=1}^{N}(s + p_i^*)}{\prod_{i=1}^{N}(s - p_i)}. \tag{3}$$

We repeat, all poles are distinct, and they are real-valued or occurring in complex-conjugated pairs. Furthermore, the allpass filter is stable: $\Re\{p_i\} < 0$ for all i. The filter line is tapped by filters V where V has one input and N outputs. The vector $\boldsymbol{v}_1(t)$ (see Fig. 1) contains the N impulse responses of this filter where

$$\boldsymbol{v}_1(t) = (v_{11}(t), \cdots, v_{1N}(t))^T. \tag{4}$$

The impulse responses $v_{1i}(t), i = 1, \cdots, N$, are the result of an orthonormalization procedure on the functions $\exp\{p_i t\}$ [this is slightly more general than the specification by the Laplace transform in (1)]. Thus, the first N basis functions are linear combinations of the functions $\exp\{p_i t\}$ or in matrix notation

$$\boldsymbol{v}_1(t) = \boldsymbol{Q}\begin{pmatrix}\exp\{p_1 t\}\\ \vdots \\ \exp\{p_N t\}\end{pmatrix} \tag{5}$$

where $\boldsymbol{Q}$ is a $N \times N$ matrix dependent on the poles (but not on the variable t) such that $\langle \boldsymbol{v}_1, \boldsymbol{v}_1^T\rangle = I$, where I is the identity matrix and $\langle\cdot,\cdot\rangle$ denotes an inner product matrix containing inner products with respect to the time t. For convenience, we require that all impulse responses $v_{1i}(t)$ are real-valued.

Similarly to (4), the vectors $\boldsymbol{v}_m(t), m = 2, 3, \cdots,$ contain N impulse responses with $\boldsymbol{v}_m(t) = (v_{m1}(t), \cdots, v_{mN}(t))^T = \boldsymbol{v}_{m-1}(t) * w(t)$, where $w(t)$ is the inverse Laplace transform of $W(s)$ and $*$ denotes convolution. Consequently, all basis functions $v_{mi}(t)$ are real-valued. Furthermore, the mth vector contains impulse responses that are linear combinations of products of, at most, $(m-1)$-order polynomials in t and exponential functions $\exp\{p_i t\}$.

III. Properties of the Basis Functions

The orthonormality of the basis functions can be expressed as $\langle \boldsymbol{v}_m, \boldsymbol{v}_j^T\rangle = \delta_{mj} I$, where δ_{mj} is the Kronecker delta. From the derivative of this orthonormality condition it is found that in the case of a real-valued pole

$$\left\langle \frac{\partial \boldsymbol{v}_m}{\partial p_l}, \boldsymbol{v}_j^T\right\rangle = -\left\langle \boldsymbol{v}_m, \frac{\partial \boldsymbol{v}_j^T}{\partial p_l}\right\rangle = -\left\langle \frac{\partial \boldsymbol{v}_j}{\partial p_l}, \boldsymbol{v}_m^T\right\rangle^T \tag{6}$$

and, especially, the inner product matrix $\langle \partial \boldsymbol{v}_m/\partial p_l, \boldsymbol{v}_m^T\rangle$ is skew-symmetrical. The $N \times N$ matrices $\boldsymbol{A}_{mj}^{(l)}$ are introduced as

$$\boldsymbol{A}_{mj}^{(l)} = \left\langle \frac{\partial \boldsymbol{v}_m}{\partial p_l}, \boldsymbol{v}_j^T\right\rangle = -\{\boldsymbol{A}_{jm}^{(l)}\}^T. \tag{7}$$

For the sake of clarity it is noted that the indexes m and j do not indicate entries in a matrix $\boldsymbol{A}^{(l)}$.

From (5) it is concluded that the vector $\partial \boldsymbol{v}_1/\partial p_l$ is a linear combination of products of, at most, first-order polynomials in t and exponential functions. Thus, $\partial \boldsymbol{v}_1/\partial p_l$ can be expressed in the vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$:

$$\partial \boldsymbol{v}_1/\partial p_l = \boldsymbol{A}_{11}^{(l)}\boldsymbol{v}_1 + \boldsymbol{A}_{12}^{(l)}\boldsymbol{v}_2 \tag{8}$$

and so $\boldsymbol{A}_{1j}^{(l)} = \boldsymbol{A}_{j1}^{(l)} = 0$ for $j > 2$. Extending this reasoning, it is found that $\boldsymbol{A}_{mj}^{(l)} = \boldsymbol{A}_{jm}^{(l)} = 0$ for $j > m+1$. Consequently, the derivative of the mth vector with respect to pole p_l can be written as a combination of just three vectors:

$$\partial \boldsymbol{v}_m/\partial p_l = \boldsymbol{A}_{m,m-1}^{(l)}\boldsymbol{v}_{m-1} + \boldsymbol{A}_{m,m}^{(l)}\boldsymbol{v}_m + \boldsymbol{A}_{m,m+1}^{(l)}\boldsymbol{v}_{m+1} \tag{9}$$

for $m > 1$.

Using the recurrence relation $\boldsymbol{v}_m(t) = \boldsymbol{v}_{m-1}(t) * w(t)$ we find for the derivative of the $m+1$ basis vector

$$\frac{\partial \boldsymbol{v}_{m+1}}{\partial p_l} = \frac{\partial \boldsymbol{v}_1}{\partial p_l}\{\overset{m}{*} w\} + m\boldsymbol{v}_1 * \frac{\partial w}{\partial p_l}\{\overset{m-1}{*} w\} \tag{10}$$

where $\{\overset{m}{*} w\}$ means m times a convolution by w. Taking $m = 1$ in combination with (9) it is clear that $\boldsymbol{v}_1 * \partial w/\partial p_l$ can be written as a linear combination of the first three basis vectors. To this end, the matrices $\boldsymbol{D}_{-1}^{(l)}, \boldsymbol{D}_0^{(l)}$ and $\boldsymbol{D}_1^{(l)}$ are introduced:

$$\boldsymbol{v}_1 * \frac{\partial w}{\partial p_l} = \boldsymbol{D}_{-1}^{(l)}\boldsymbol{v}_1 + \boldsymbol{D}_0^{(l)}\boldsymbol{v}_2 + \boldsymbol{D}_1^{(l)}\boldsymbol{v}_3. \tag{11}$$

Next, substitution of (8) and (11) in (9) and using (7) leads to

$$\boldsymbol{A}_{m,m-1}^{(l)} = -(m-1)\{\boldsymbol{A}_{12}^{(l)}\}^T, \tag{12}$$

$$\boldsymbol{A}_{m,m+1}^{(l)} = m\boldsymbol{A}_{12}^{(l)}, \tag{13}$$

$$\boldsymbol{A}_{m,m}^{(l)} = \boldsymbol{A}_{11}^{(l)} + (m-1)\boldsymbol{D}_0^{(l)}. \tag{14}$$

Thus the whole infinite set of matrices $\boldsymbol{A}^{(l)}$ is expressed in just the three matrices $\boldsymbol{A}_{11}^{(l)}, \boldsymbol{D}_0^{(l)}$ and $\boldsymbol{A}_{12}^{(l)}$.

The matrix $\boldsymbol{A}_{12}^{(l)}$ (and, implicitly, the matrices $\boldsymbol{A}_{m,m-1}^{(l)}$ and $\boldsymbol{A}_{m,m+1}^{(l)}$) is considered in more detail. Taking the derivative of (5) with respect to p_l yields

$$\frac{\partial \boldsymbol{v}_1}{\partial p_l} = \frac{\partial \boldsymbol{Q}}{\partial p_l}\begin{pmatrix}\exp\{p_1 t\}\\ \vdots \\ \exp\{p_N t\}\end{pmatrix} + \boldsymbol{Q}\begin{pmatrix}\vdots\\ 0\\ t\exp\{p_l t\}\\ 0\\ \vdots\end{pmatrix} = \frac{\partial \boldsymbol{Q}}{\partial p_l}\boldsymbol{Q}^{-1}\boldsymbol{v}_1 + \boldsymbol{Q}_l t\exp\{p_l t\}. \tag{15}$$

where $\boldsymbol{Q}_l$ stands for a vector identical to the lth column of $\boldsymbol{Q}$. The last term in (15) is a first-order polynomial in t multiplied by an exponential function and can be written as a linear combination of $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$:

$$\boldsymbol{Q}_l t\exp\{p_l t\} = \boldsymbol{E}_1^{(l)}\boldsymbol{v}_1 + \boldsymbol{E}_2^{(l)}\boldsymbol{v}_2 \tag{16}$$

$\boldsymbol{E}_1^{(l)}$ and $\boldsymbol{E}_2^{(l)}$ being matrices. Since $\boldsymbol{Q}_l$ does not depend on t, both $\boldsymbol{E}_1^{(l)}$ and $\boldsymbol{E}_2^{(l)}$ have rank 1. Comparing (16) to (8) gives $\boldsymbol{A}_{11}^{(l)} = \partial \boldsymbol{Q}/\partial p_l \boldsymbol{Q}^{-1} + \boldsymbol{E}_1^{(l)}$ and $\boldsymbol{A}_{12}^{(l)} = \boldsymbol{E}_2^{(l)}$. Thus, all matrices $\boldsymbol{A}_{m,m-1}^{(l)}$ and $\boldsymbol{A}_{m,m+1}^{(l)}$ have rank 1 and can be written as a dyadic product.

If p_l is a complex-valued pole, the situation is slightly different. It was assumed that in this case there is a r $(1 \le r \le N)$ such that $p_l = p_r^* = \alpha + \jmath\beta$ $(\jmath = \sqrt{-1}, \beta > 0)$. We are now no longer interested in $\boldsymbol{A}^{(l)}$ and $\boldsymbol{A}^{(r)}$ but in derivatives of the basis functions with respect to α and β. Similarly to (8), we express these derivatives in the basis functions itself: $\partial \boldsymbol{v}_1/\partial\alpha = \boldsymbol{A}_{11}^{(\alpha)}\boldsymbol{v}_1 + \boldsymbol{A}_{12}^{(\alpha)}\boldsymbol{v}_2$ and $\partial \boldsymbol{v}_1/\partial\beta = \boldsymbol{A}_{11}^{(\beta)}\boldsymbol{v}_1 + \boldsymbol{A}_{12}^{(\beta)}\boldsymbol{v}_2$. From (15) we have [13]

$$\partial \boldsymbol{v}_1/\partial\alpha = \left\{\frac{\partial \boldsymbol{Q}}{\partial p_l} + \frac{\partial \boldsymbol{Q}}{\partial p_r}\right\}\boldsymbol{Q}^{-1}\boldsymbol{v}_1 + \boldsymbol{Q}_l t\exp\{p_l t\} + \boldsymbol{Q}_r t\exp\{p_r t\}, \tag{17}$$

$$\partial \boldsymbol{v}_1/\partial\beta = \jmath\left\{\frac{\partial \boldsymbol{Q}}{\partial p_l} - \frac{\partial \boldsymbol{Q}}{\partial p_r}\right\}\boldsymbol{Q}^{-1}\boldsymbol{v}_1 + \jmath\boldsymbol{Q}_l t\exp\{p_l t\} - \jmath\boldsymbol{Q}_r t\exp\{p_r t\}. \tag{18}$$

We now introduce the vectors $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ by

$$t\exp\{p_l t\} = \boldsymbol{d}_1^T \boldsymbol{v}_1 + \boldsymbol{d}_2^T \boldsymbol{v}_2. \tag{19}$$

Since the vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ contain real-valued functions only, the vectors $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ are complex-valued. From (19) it follows that

$$t\exp\{p_r t\} = \boldsymbol{d}_1^{*T} \boldsymbol{v}_1 + \boldsymbol{d}_2^{*T} \boldsymbol{v}_2. \tag{20}$$

Since (19) and (20) describe linearly independent functions, the vectors $\boldsymbol{d}_j$ and $\boldsymbol{d}_j^*$ are linearly independent ($j = 1, 2$). Similarly to (16) we now introduce

$$\boldsymbol{Q}_l t\exp\{p_l t\} + \boldsymbol{Q}_r t\exp\{p_r t\} = \boldsymbol{E}_1^{(\alpha)} \boldsymbol{v}_1 + \boldsymbol{E}_2^{(\alpha)} \boldsymbol{v}_2 \tag{21}$$

$$\jmath\{\boldsymbol{Q}_l t\exp\{p_l t\} - \boldsymbol{Q}_r t\exp\{p_r t\}\} = \boldsymbol{E}_1^{(\beta)} \boldsymbol{v}_1 + \boldsymbol{E}_2^{(\beta)} \boldsymbol{v}_2 \tag{22}$$

where

$$\boldsymbol{E}_1^{(\alpha)} = 2\Re\{\boldsymbol{Q}_l \boldsymbol{d}_1^T\}, \quad \boldsymbol{E}_2^{(\alpha)} = 2\Re\{\boldsymbol{Q}_l \boldsymbol{d}_2^T\},$$
$$\boldsymbol{E}_1^{(\beta)} = -2\Im\{\boldsymbol{Q}_l \boldsymbol{d}_1^T\} \quad \text{and} \quad \boldsymbol{E}_2^{(\beta)} = -2\Im\{\boldsymbol{Q}_l \boldsymbol{d}_2^T\}$$

(with $\Im$ denoting the imaginary part of a complex number). Since the matrix $\boldsymbol{Q}$ is regular, the vectors $\boldsymbol{Q}_l$ and $\boldsymbol{Q}_r$ are linearly independent. This implies that the matrices $\boldsymbol{A}_{12}^{(\alpha)} = \boldsymbol{E}_2^{(\alpha)}$ and $\boldsymbol{A}_{12}^{(\beta)} = \boldsymbol{E}_2^{(\beta)}$ both have rank 2.

IV. Optimality Conditions

Since the vectors $\boldsymbol{v}_m(t)$ ($m = 1, 2, \cdots$) constitute an orthonormal basis in $L^2(\boldsymbol{R}_+)$, any causal square-integrable function $h(t)$ can be expressed as $h(t) = \Sigma_{m=1}^{\infty} \boldsymbol{c}_m^T \boldsymbol{v}_m(t)$. $\boldsymbol{c}_m$ is called the Kautz vector spectrum and can be calculated by an inner product $\boldsymbol{c}_m = \langle h, \boldsymbol{v}_m \rangle$. In the remainder only real-valued functions $h(t)$ are considered, and thus the vector spectrum is real-valued as well.

Considered are truncated Kautz series according to $h(t) \approx f_M(t) = \Sigma_{m=1}^{M} \boldsymbol{c}_m^T \boldsymbol{v}_m(t)$. Given a fixed number of terms (M) we search for an optimality condition for the set of poles, i.e., by which poles do we obtain the "best" possible representation in this finite series? The "best" representation is defined as that which maximizes the energy in the approximating function $f_M(k)$.

The energy E in $h(t)$ is given by $E = \langle h, h \rangle$ and the energy E_M in the approximating function f_M is given by $E_M = \langle f_M, f_M \rangle = \Sigma_{m=1}^{M} \boldsymbol{c}_m^T \boldsymbol{c}_m$. The last equality stems from the orthonormality relation. Optimality conditions are sought which apply to the set of poles that maximizes E_M.

If a certain set of poles $\{p_i\}$ yields this maximum then

$$\frac{\partial E_M}{\partial p_l} = 0 \tag{23}$$

for each p_l, assuming real-valued poles. The extension to complex-conjugated pole pairs can be easily established as well [13]. Naturally, (23) holds not just for the global maximum but for any other extremum or saddle point of the energy function as well.

Taking the derivative of E_M with respect to p_l gives

$$\frac{\partial E_M}{\partial p_l} = 2\sum_{m=1}^{M} \boldsymbol{c}_m^T \frac{\partial \boldsymbol{c}_m}{\partial p_l}. \tag{24}$$

From (9) and $\boldsymbol{c}_m = \langle h, \boldsymbol{v}_m \rangle$ we find

$$\frac{\partial \boldsymbol{c}_m}{\partial p_l} = \boldsymbol{A}_{m,m-1}^{(l)} \boldsymbol{c}_{m-1} + \boldsymbol{A}_{m,m}^{(l)} \boldsymbol{c}_m + \boldsymbol{A}_{m,m+1}^{(l)} \boldsymbol{c}_{m+1}. \tag{25}$$

Substituting (25) in (24) and using (7) gives

$$\frac{\partial E_M}{\partial p_l} = 2\boldsymbol{c}_M^T \boldsymbol{A}_{M,M+1}^{(l)} \boldsymbol{c}_{M+1} = 2M \boldsymbol{c}_M^T \boldsymbol{A}_{12}^{(l)} \boldsymbol{c}_{M+1}. \tag{26}$$

For real-valued poles this expression must be zero for each l. Thus, if the poles are such that $\boldsymbol{c}_M = 0$ or $\boldsymbol{c}_{M+1} = 0$, then the energy function E_M of a Kautz series of M terms has an extremum or saddle point for this particular set of poles. The same observation holds if complex-conjugated pole pairs are allowed [13]. For $N = 1$ and $N = 2$ with a complex-conjugated pole pair, $\boldsymbol{c}_M = 0$ or $\boldsymbol{c}_{M+1} = 0$ are the only solutions of (23) [6]–[10]. In general, other solutions exist; these depend on the specific orthonormalization matrix $\boldsymbol{Q}$ and are not considered here.

V. Conclusion

Approximations by Kautz functions governed by a recurrent set of poles have been considered. An explicit expression for the derivatives of the approximating energy function with respect to its free parameters (the poles) has been derived. This gradient is of a very simple form. Our analysis extends previously reported work on optimality conditions for Laguerre functions [6]–[8] and the Kautz functions considered in [9] and [10]. It was also shown that the derivatives of higher order basis vectors $\boldsymbol{v}_m$ can be easily established once the derivative of the first vector of basis functions is known, see (12)–(14). The present results can be easily adapted to the discrete-time Kautz functions [14], since the analysis in Section III only uses the orthonormality and the polynomial character of the basis functions [15].

For a given function $h(t)$ and specific values for N and M, the presented analysis gives the derivatives of the basis functions with respect to the parameters and can consequently be used to establish the optimal poles numerically by a gradient-oriented search method. For a fixed number $N \times M$, an increase in N results in more degrees of freedom and thus, in general, in a better approximation. On the other hand, an increase in N results in a more complicated optimization procedure. Furthermore, many functions can be associated with a single time-scale at which relevant changes occur. This implies that in many instances only a very limited number N of free poles suffices to obtain a good approximation.

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