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## Optimization and scale up of industrial fermentation processes

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As not all proof corrections were carried out, Table 1 contained errors. The correct version appears below.

**Table 1** Fermentation parameters, coefficients and terms, implicated in mixing, aeration, oxygen and heat transfer, suitable as scale-up variables to be kept constant alone or combined with each other or other process-relevant variables, preferably but not necessarily, as dimensionless groups as described and discussed, e.g. by Hubbard (1987) and Wang and Cooney (1997)

Parameter/coefficient	Mathematical characterisation	Symbol explanation
Power input ( $P$ ); volumetric power input ( $P/V$ )	(1) $P = 2\pi nM = N_{Po}\rho n^3 d_I^5 [\text{kgm}^2\text{s}^{-2} = \text{W}]$	$P$ =power input; $n$ =stirrer speed; $M$ =momentum; $N_{Po}$ =dimensionless power number (impeller-specific); $\rho$ =density of the medium; $n$ =stirrer speed; $d_I$ =impeller diameter
Dimensionless power number ( $N_{Po}$ )	(2) $N_{Po} = P/\rho n^3 d_I^5$	$n$ =stirrer speed; $d_I$ =impeller diameter
Impeller tip speed ( $v_{tip}$ )	(3) $v_{tip} = 2\pi n d_I [\text{m/sec}]$	$n$ =stirrer speed; $d_I$ =impeller diameter
Reynolds number ( $Re$ )	(4) $Re = n d_I^2 \rho / \eta$	$Re$ =Reynolds number; $n$ =stirrer speed; $d_I$ =impeller diameter; $\rho$ =density of the medium; $\eta$ =dynamic viscosity
Mixing time ( $T_m$ )	(5) $T_m = f(n, d_I, \nu) = V/Q = V/N_{\Pi} n d_I^3 [\text{sec}]$	$T_m$ =mixing time; $n$ =stirrer speed; $d_I$ =impeller diameter; $\nu$ =kinematic viscosity; $V$ =volume; $Q$ =volumetric flow rate; $N_{\Pi}$ =pumping number
Dimensionless mixing time ( $T_m n$ )	(6) $T_m n = f(Re, N_{Po})$	$T_m$ =mixing time; $n$ =stirrer speed; $T_m n$ =dimensionless mixing time; $Re$ =Reynolds number; $N_{Po}$ =dimensionless power number

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Table 1 (continued)

Parameter/coefficient	Mathematical characterisation	Symbol explanation
Modified dimensionless power number	(7) $N'_{Po} = N_{Po} Re^3 d_F / d_I = P d_F \rho^2 / \eta^3$	$N'_{Po}$ =modified dimensionless power number; $T'_m n$ =modified dimensionless mixing time; $N_{Po}$ =dimensionless power number; $Re$ =Reynolds number;
Modified dimensionless mixing time	(8) $T'_m n = n T_m / \left[ (d_F / d_I)^2 Re \right] = T_m \eta / d_F^2 \rho$	$d_F$ =inside vessel diameter; $d_I$ =impeller diameter; $P$ =power input; $n$ =stirrer speed; $t_m$ =mixing time; $\rho$ =fluid density; $\eta$ =dynamic viscosity
Aeration rate (vol vol <sup>-1</sup> min <sup>-1</sup> , vvm)	(9) $A_R = F_G / V_R [\text{m}^3 / \text{m}^3 \text{ min}]$	$A_R$ =aeration rate; $F_G$ =volumetric gas flow rate; $V_R$ =fermenter reaction volume
Superficial gas velocity ( $v_s$ )	(10) $v_s = F_G / A [\text{m/sec}]$	$v_s$ =superficial gas velocity; $F_G$ =volumetric gas flow rate; $A$ =fermenter cross section
Gas hold up ( $\tau$ )	(11) $\tau = V_R / F_G$	$\tau$ =gas hold up; $V_R$ =fermenter reaction volume; $F_G$ =volumetric gas flow rate;
Fractional gas hold up ( $\varepsilon$ )	(12) $\varepsilon = V_G / V_R$	$\varepsilon$ =fractional gas hold up; $V_G$ =dispersed gas volume
Gassing number ( $N_{Qg}$ )	(13) $N_{Qg} = f(Fr, Re, S, (\rho_g / \rho_0), (\eta_g / \eta_0)) = q_g / n d_I^3$	$N_{Qg}$ =gassing number; $Fr$ =Froude number; $Re$ =Reynolds number; $S$ =fluid constant; $\rho$ =density; $\eta$ =dynamic viscosity; $g$ =aerated; $0$ =not aerated; $q_g$ =gas throughput; $n$ =stirrer speed; $d_I$ =impeller diameter
Oxygen transfer rate (OTR)	(14) $OTR = k_L a (C_G - C_L) = k_L a L_{O_2} (p_{O_{2G}} - p_{O_{2L}}) [\text{kg O}_2 / \text{m}^3 \text{ h}]$ (15) $C_G = 0.526 p_i / 36 + T [\text{mg/l}]$	OTR=oxygen transfer rate from gas to liquid phase; $k_L$ =mass transfer coefficient; $a$ =specific interfacial surface area; $C_G$ =oxygen saturation concentration in the gas phase; $C_L$ =measured oxygen saturation concentration in the liquid phase; $L_{O_2}$ =oxygen solubility in the liquid phase; $p_{O_{2G}}$ =partial pressure of oxygen in the gas phase; $p_{O_{2L}}$ =partial pressure of oxygen in the liquid phase; $p_i$ =vessel back pressure [bar]; $T$ =temperature [°C]
Volumetric oxygen mass transfer coefficient ( $k_L a$ )	(16) $k_L a = a' (P / V_R)^b v_s^c [\text{s}^{-1}]$	$k_L a$ =oxygen transfer coefficient; $P$ =power input; $V_R$ =fermenter reaction volume; $v_s$ =superficial gas velocity; $a'$ , $b$ and $c$ as specific fluid constants to be determined experimentally
$k_L a$ scale-dependent according to Wang–Cooney	(17) $k_L a = k' (P_g / V_R)^a (v_s)^b (B/6)^{0.8} (j/d_I)^{0.3} [\text{s}^{-1}]$	$k'$ , $a$ , $b$ =vessel-specific coefficients; $P_g$ =power input in aerated vessel; $V$ =volume of culture broth; $V_s$ =superficial gas velocity; $B$ =number of stirrers; $j$ =baffle width; $d_I$ =impeller diameter
Respiratory quotient (RQ)	(18) $RQ = CER / OUR = pCO_{2E} - pCO_{2i} / pO_{2i} - pO_{2E} [\text{mol CO}_2 / \text{mol O}_2]$	RQ=respiratory quotient; CER=carbon dioxide emission rate; OUR=oxygen uptake rate; $pCO_2$ =partial pressure of carbon dioxide; $pO_2$ =partial pressure of oxygen; $i$ =air feed; $E$ =exhaust air

**Table 1** (continued)

Parameter/coefficient	Mathematical characterisation	Symbol explanation
Energy balance ( $Q_{\text{ges}}$ )	(19) $Q_t = Q_{\text{met}} + Q_{\text{ag}} + Q_{\text{aer}} - Q_{\text{evap}} - Q_{\text{hch}}$ [kWh = kJ] with (20) $Q_{\text{met}} = Y_{\text{ho}} R_o$ [kWh = kJ]	$Q_t$ =total energy; $Q_{\text{met}}$ =energy generated by metabolic activities; $Q_{\text{ag}}$ =energy generated by agitation; $Q_{\text{aer}}$ =energy generated by aeration; $Q_{\text{evap}}$ =energy losses through evaporation; $Q_{\text{hch}}$ =energy losses through cooling; $Y_{\text{ho}}$ =approximate metabolic constant (460 kJ/mol O <sub>2</sub> ); $R_o$ =molar oxygen uptake rate (mol O <sub>2</sub> /s)
Heat transfer coefficient	(21) $\alpha = (dQ/dt)/AdT$	$\alpha$ =heat transfer coefficient; $dQ/dt$ =heat transfer; $A$ =surface area; $dT$ =temperature difference

For instance, the  $k_L a$  value—as the most frequently applied physical scale-up variable—has been combined with (1) the vessel back pressure  $p$  by maintaining the product of  $k_L a p$  constant through variation of pressure, agitation and aeration for scale up of *Bacillus thuringiensis* fermentations (Flores et al 1997) or (2) the aeration rate vvm under variation of power input, working volume and aeration according to the Wang–Cooney equation (Wong et al. 2002). For details and further explanations, see text