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Optimization for Minimum Sensitivity to Uncertain Parameters

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OPTIMIZATION FOR MINIMUM SENSITIVITY
TO UNCERTAIN PARAMETERS

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Abstract

A procedure to design a structure for minimum sensitivity to uncertainties in problem parameters is described. The approach is to directly minimize the sensitivity derivatives of the optimum design with respect to fixed design parameters using a nested optimization procedure. The procedure is demonstrated for the design of a bimetallic beam for minimum weight with insensitivity to uncertainties in structural properties. The beam is modeled with finite elements based on two dimensional beam analysis. A sequential quadratic programming procedure used as the optimizer supplies the Lagrange multipliers that are used to calculate the optimum sensitivity derivatives. The procedure is validated by comparing the optimization results to parametric studies.

Introduction

Parameters are characteristics of an optimization problem that are held constant during the optimization process in which the design variables are changing. In practical design situations the exact values of the parameters may not be known (e.g., loads, material properties, manufacturing errors, etc.). Uncertainties in parameters may cause variations in the outcome of the design process; for example, uncertainties in the constraints may cause the design to be unacceptable. Unfortunately, in many cases the uncertainties are very difficult and costly to control. It would be advantageous to approach the design task using the key principle of quality design as formulated by Taguchi who proposed that designs be as insensitive as possible to variations in parameters that affect their behavior.[1]

Many efforts have been directed toward developing design processes wherein the performance of the final product will be minimally affected by variations in parameters. Several efforts have been made to approach the problem by imposing safety margins on the

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constraints to avoid entering the infeasible region. For example, in reference 2 a worst case approach is used wherein a specified tolerance on the uncertain design variables reduces the feasible design space. The objective is to stay as close as possible to the original optimum and to remain within the newly defined feasible region. A padding factor is added to the constraints in reference 3. The factor is proportional to the gradient of the constraint. The constraints are updated throughout the optimization process using linear and nonlinear approximations. In references 4 and 5 the authors proposed a method of designing for "feasibility robustness". Given specified tolerances on design variables and parameters, variations in the constraints are predicted using worst case and statistical analysis. This maintains a certain probability of remaining feasible throughout the design process in spite of fluctuations in the variables and parameters.

The above methods effectively reduce the size of the feasible region resulting in trading a less optimum value of the objective function for a higher degree of robustness. In references 2, 4, and 5 this concept was enhanced by adding tolerance ranges on specified controllable variations as variables in the objective function so that the values of the tolerances were chosen during the design process. Reference 6 labeled this approach a "tolerance allocation problem". The method was applied to the design of a simple two-bar truss to obtain a trade-off between the tolerances on the structural dimensions and the manufacturing cost. As the tolerances decrease, the cost of manufacturing the product increases. In reference 7 a second order tolerance model was incorporated into a nonlinear optimization procedure which improved the formulation by including a measure of function skewness in the design process.

Another approach to decreasing the effects of uncertainties in design is to directly minimize the sensitivity of the design with respect to the uncertainties. Reference 8 minimized the sensitivity derivative of the response of a system with respect to uncertain parameters using shape design variables with formal optimization techniques. Reference 9 proposed that tradeoffs be made in the design objectives to ensure a degree of insensitivity to uncontrollable parameter variations. This approach was referred to as a "design for latitude" and it was handled with a multiple objective function minimization procedure which included the sensitivity of the design with respect to the uncertain parameters in the design objective. Reference 10 also used multiobjective optimization methods to minimize sensitivities to variations in design

parameters by incorporating sensitivity derivatives of the objective and constraints into the objective function or by constraining them to be less than a specified value. Several formulations incorporating sensitivity-based constraints and/or sensitivity-based objectives were discussed. Reference 11 applied goal programming methods in a similar manner to minimize multiple objectives including the sensitivities of both fixed parameters and design variables to uncertainties. Reference 12 utilized fuzzy sets to define a multiple objective design optimization problem dealing with uncertainty.

The approach taken in this paper extends the concept of directly minimizing sensitivity derivatives to that of minimizing optimum sensitivity derivatives (OSD)[13,14] with respect to problem parameters using a nested optimization procedure. The procedure is demonstrated for the design of a bimetallic beam for minimum weight with insensitivity to uncertain structural propeties. The beam is modeled with finite elements based on two dimensional beam analysis. An IMSL[15] sequential quadratic programming procedure used for the optimization supplied the Lagrange multipliers that were used to calculate the OSD using the method described in references 13 and 14.

Approach

The overall procedure contains two nested optimization loops as shown in Figure 1.
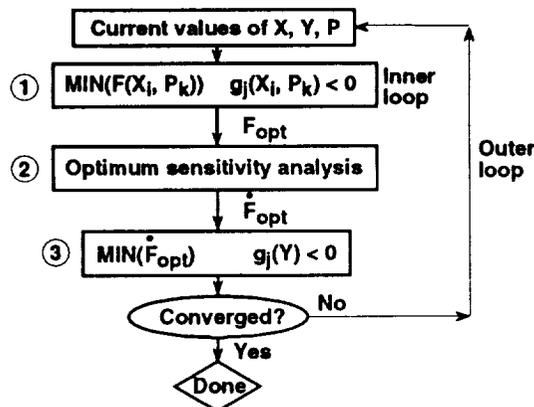
Fig. 1.   Flowchart of overall procedure.

The uncertain inputs are represented by the constant parameters, P. The usual formulation of an optimization problem is solved within the inner loop (box 1) yielding a constrained optimum comprised of the optimal values of the objective function, F and the design variables, X denoted by F_{opt} and X_{opt}. These quantities are functions of the parameters P, therefore they are sensitive to uncertainties in these parameters. That sensitivity is quantified by the optimum sensitivity derivatives in box 2. These derivatives are represented by dF_{opt}/dP or $\dot{F}_{opt}$. The derivatives may

be computed by finite differencing on the optimization repeated with a parameter perturbed one at a time (expensive, inaccurate, but simple to implement) or by using a quasi-analytical method that is cheaper, more accurate, but requires more implementation effort.[13] The success of this procedure was found to be dependent on a highly converged inner loop optimization that provides accurate Lagrange multipliers. The outer loop minimizes the sensitivity of the outcome of the inner loop to the uncertain parameters by selecting the design associated with the smallest values for the optimum sensitivity derivatives (box 3). The objective function in the outer loop can be made up of multiple terms including OSD's with respect to several parameters and a term to reflect the value of the objective function in the inner loop. Weighting factors can be applied to each of these terms to prioritize their relative importance during the outer loop optimization. Such a weighted multiobjective function allows tradeoffs between the optimum design that is achieved in the inner loop and the level of sensitivity of this design to the uncertain parameters. A user may choose to accept a reduction in the measure of goodness (objective function) of the original design to gain a degree of robustness (insensitivity to uncertain parameters).

This optimization problem could be expensive to solve because there are two nested optimization loops. The computational expense could be amplified due to the fact that to minimize a sensitivity means to minimize a function that already depends on the derivatives. Use of a gradient-driven search algorithm would imply working with the computationally expensive second derivatives. Therefore, it was decided to use Powell's method[16], a non-gradient-based search algorithm, in the outer loop.

Example Problem

A simple but cogent example is used to demonstrate the procedure and its potential. A beam of a fixed length L=100.0 inches is clamped at both ends as shown in figure 2.
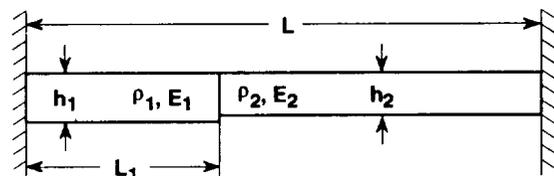
Fig. 2.   Bi-material beam problem.

The beam is made up of two sections. From the left end to a distance L_1 from the left end, the beam has Young's modulus E_1, a density ρ_1 and height h_1. The other section has E_2, ρ_2, and h_2. Both sections have rectangular cross-sections of width B_1 and B_2. The values of these material properties are given in table 1. It is assumed that there is uncertainty in the values of

ρ_1 and E_2. The objective is to design this beam using design variables L_1, h_1, and h_2 to have minimum

Table 1 - Beam material properties		
	Section I	Section 2
Rho (lbm/in³)	0.100	0.056
Youngs modulus (psi)	2.0 E+7	1.0 E+7
Width (in)	4.0	4.0
L = 100 inches		

weight with constraints on the frequencies and to design it with minimum sensitivity to the uncertainty in the parameters, ρ_1 and E_2.

Analytical model

The beam was modeled with 10 finite elements which were based on a two dimensional beam analysis using bending elements. As the value of L_1 was varied each section of the beam was divided into elements such that a joint always existed at L_1. Each section had a minimum of two elements.

Inner Loop Formulation

The inner optimization problem minimizes the weight of the beam subject to frequency constraints. The weight is given in equation 1.

$$F = \sum_{i=1}^{2} \rho_i L_i B_i h_i \qquad (1)$$

where B_i, L_i, and h_i are the width, length, and height of the ith section of the beam, respectively. In the inner loop optimization, the design variables are the heights of the beam sections, h_1 and h_2, while L_1 is a constant parameter. Constraints place lower and upper bounds on the fundamental bending frequency, ω and are given in equation (2). Satisfied constraints are represented by the following expressions.

$$1.0 - \frac{\omega}{\omega_l} \leq 0 \text{ and } \frac{\omega}{\omega_u} - 1.0 \leq 0 \qquad (2)$$

where ω_l and ω_u are the lower and upper bounds on the frequency.

An IMSL sequential quadratic programming (SQP) routine performs the optimization from which the Lagrange multipliers needed for subsequent calculation of the OSD's are output.

Outer Loop Formulation

In the outer loop the sensitivity of the inner loop optimum design is minimized with respect to the uncertainty in ρ_1 and E_2 using L_1 as the "auxiliary" design variable. In other words, of all the minimum weight designs that satisfy the constraints the objective is to find the one which is least sensitive to uncertainties in ρ_1 and E_2. Since there are two uncertain parameters, the sensitivity of the optimum weight design must be minimized with respect to both parameters. This can be done using a multiobjective function with the first two terms being the magnitudes of the two derivatives with a weighting factor β as shown in equation 3.

$$OBJ = \beta(DFOPT_1) + \beta(DFOPT_2)$$
$$+ \delta \left| \frac{W - W*}{W*} \right| \qquad (3)$$

where $DFOPT_1$ and $DFOPT_2$ are normalized optimum sensitivity derivatives as shown in equation 4.

$$DFOPT_1 = \frac{\rho_1}{F_{opt}} \frac{dF_{opt}}{d\rho_1}$$
$$DFOPT_2 = \frac{E_2}{F_{opt}} \frac{dF_{opt}}{dE_2} \qquad (4)$$

These normalized derivatives, sometimes referred to as logarithmic derivatives, are valuable in that they measure a percent change in the objective function due to a percent change in the design variables. The optimum sensitivity derivatives are calculated from equation 5 using the quasi-analytical method of references 13 and 14.

$$\frac{dF_{opt}}{dP} = \frac{\partial \overline{F}}{\partial P} + \overline{\lambda}^T \frac{\partial \overline{g}_a}{\partial P} \qquad (5)$$

The overbars are used to denote quantities at the optimum and $\overline{g}_a$ represents the constraints that are active at the optimum, λ is the Lagrange multiplier, and P represents the parameter.

The last term in equation 3 represents the tradeoff between a low weight design and the insensitivity to the uncertain parameters. The term $W*$ is a reference weight and δ is a weighting factor. The values of δ and β which are problem dependent are chosen arbitrarily. A rational method for obtaining the value of $W*$ is by optimizing the weight of the beam subject to frequency constraints using inner and outer loop design variables simultaneously. The value of $W*$ will have an effect on the location of the optimum and the nature of the curve in the vicinity of the optimum.

The objective function OBJ is minimized using the auxiliary design variable L_1. Minimization of the optimum sensitivity is done using Powell's Method which for one design variable degenerates into a one-dimensional search.

Results

Results of Optimization Procedure

In order to assess the validity of the nested optimization procedure, a study was conducted in which the parameter L_1 was changed in discrete increments in the outer loop and only the inner loop optimization was performed. A curve that represents a series of discrete calculations of OBJ versus .5 inch increments of L_1/L is shown in figure 3 for β equal to 10, W* equal to 30 and δ equal to 1. Each data point represents a minimum weight
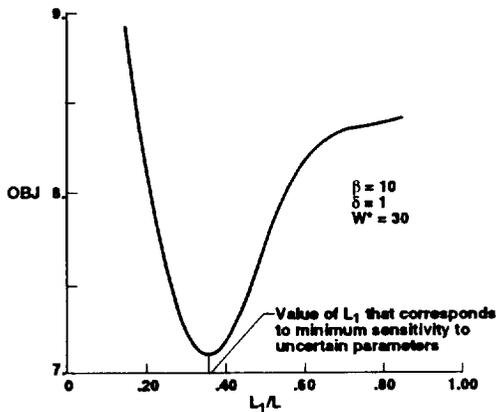
Fig. 3. Curve of discrete calculations of outer loop objective function OBJ versus L_1/L.

optimization performed in the inner loop and an optimum sensitivity analysis calculation. The minimum indicated in the figure corresponds to the value of L_1/L for which the beam has a minimum weight design and is least sensitive to uncertainties in the parameters p_1 and E_2. From the graph, the data point at minimum OBJ is L_1/L is .365. The nested optimization procedure produced a value for L_1/L equal to .364. Values associated with the initial and final designs are given in table 2. The values in the table

Table 2 - Initial and final optimum weight designs for $\delta = 1$, $\beta = 10$							
W*=30	L_1	h_1	h_2	Weight	DFOPT$_1$	DFOPT$_2$	OBJ
Initial	50.0	0.533	3.584	50.809	.379	.321	7.696
Final	35.43	0.283	4.664	70.939	.132	.442	7.096

indicate that by adding approximately 40 percent more weight to the beam, the sensitivity derivative of the

optimum weight with respect to p_1 (DFOPT$_1$) was decreased by nearly a factor of 3. The optimum sensitivity derivative with respect to E$_2$ (DFOPT$_2$) was increased slightly. This trend may be altered depending upon the choice of the weighting factors β and δ. In this case β was considerably large and weighted both optimal sensitivity derivatives equally in the outer loop objective function whereas δ was comparatively small. It may be advantageous to use a different weighting factor for each optimum sensitivity derivative to emphasize minimization of one or the other or to increase the value of δ to emphasize minimization of the weight. The following section will discuss the role of the weighting factors in tailoring the formulation of the problem to alter the optimum design.

Tradeoffs Using Weighting Factors

The weighting factors play a crucial role in the outcome of the process. Figure 4 shows four graphs, the first three representing one of the three individual terms in the outer loop objective function versus L_1/L and the fourth showing the optimum values of the beam heights versus L_1/L. The figure clearly shows that in
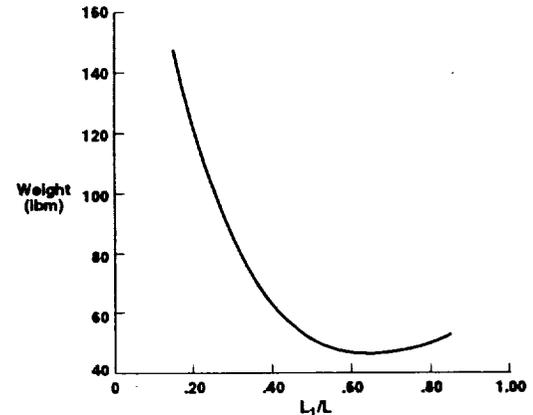
(a) Weight from inner loop optimization versus L_1/L
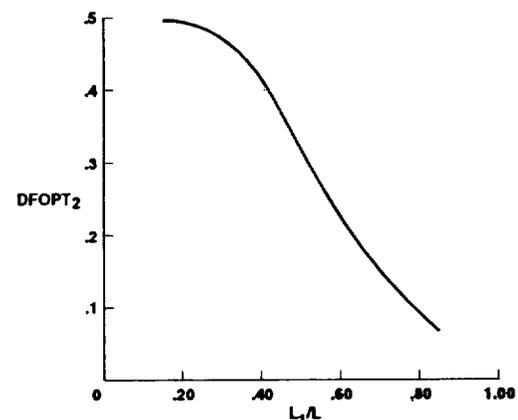
(b) Optimum sensitivity calculated at inner loop optimum versus L_1/L
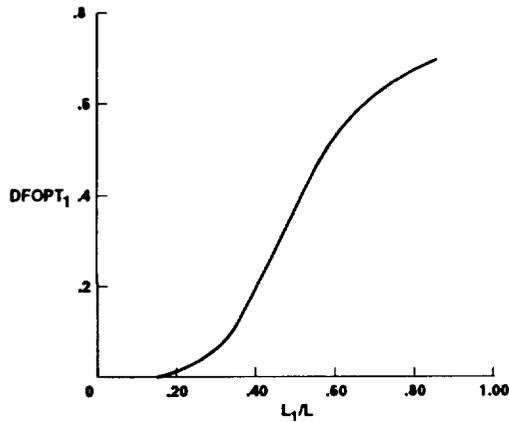
(c) Optimum sensitivity calculated at inner loop optimum versus L₁/L
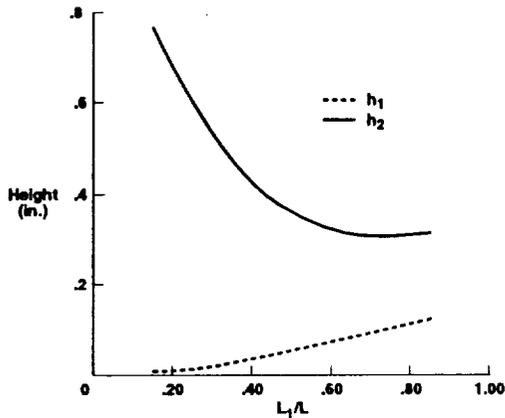
(d) Height of beam sections from inner loop optimizations versus L₁/L

Fig. 4. Graphs of the three components of the outer loop objective function and the heights of the beam sections versus L₁/L.

order to minimize each individual term, contradicting values of L_1/L are needed. For instance, in figure 4a which shows the optimum weight versus L_1/L demands that L_1/L be about 64 percent. Figure 4b indicates that the optimum sensitivity derivative with respect to E_2 (DFOPT₂) wants to push L_1/L all the way to the right end of the beam. On the other hand, figure 4c shows that in order to decrease the optimum sensitivity derivative with respect to ρ_1 (DFOPT₁), L_1/L should be located near the left end of the beam. Obviously a value for L_1/L has to be chosen that represents a compromise in minimizing all three terms simultaneously.

The following example demonstrates the effect of the weighting factors on the final design. Similar to the first example, a series of discrete calculations of the outer loop composite objective function OBJ versus 0.5 inch increments of L_1/L were carried out for two cases. In the first case β=5 and δ=1 for W* equal to 30. In the second case β=1 and δ=5 for W*=30. The plots are

shown in figure 5. Also shown in figure 5 is a curve representing the optimum weight (normalized with respect to W*) from the inner loop for each incremental value of L_1/L. Investigation of the figure shows that
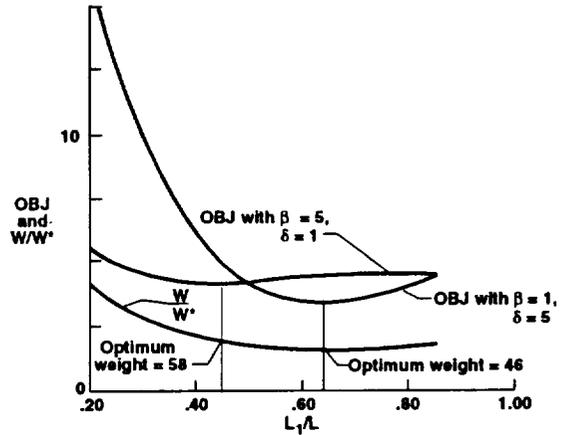
Fig. 5. Graph showing tradeoff of insensitivity versus weight.

the minima of the two OBJ curves correspond to different values of L_1/L and therefore different values of weight depending upon what values of the weighting factors are used. This is evident from figure 5 when comparing the weight values that correspond to the minima of the two curves. A large value of β allows the procedure to add weight to the structure if it will reduce the sensitivity of the beam to the uncertain parameters. On the other hand if the sensitivity to the problem parameters is considered to be less important, the value of β may be decreased and δ increased. In this case the value of L_1/L that gives the minimum value of OBJ will correspond to a lower weight.

The optimization procedure was applied to this example problem for both cases involving values of δ and β for W*=30 lbs. The procedure successfully located the values of L_1/L that correspond to the minima shown in figure 5 for each case. Table 3 lists the initial and final values of L_1/L, the heights of the sections, the weights of the beam, the values of the composite outer loop objective function (OBJ), and the optimal sensitivity derivatives for both cases. In the

Table 3 - Initial and final optimum weight designs for different values of β and δ							
W*=30	L₁	h₁	h₂	Weight	DFOPT₁	DFOPT₂	OBJ
Initial β=5 δ=1 W*=30	50	.533	3.584	50.809	.379	.321	4.195
Final β=5 δ=1 W*=30	43	.403	4.027	58.341	.247	.388	4.119
Initial β=1 δ=5 W*=30	50	.533	3.584	50.809	.379	.321	4.168
Final β=1 δ=5 W*=30	64	.805	4.154	46.053	.577	.196	3.448

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case where β was larger than δ, a 15 percent increase in the weight occurred for a 34 percent decrease in $DFOPT_1$ and a 20 percent increase in $DFOPT_2$. For the second case where δ was larger than β, a 9 percent decrease in the weight was achieved for a 52 percent increase in $DFOPT_1$ and a 38 percent reduction in $DFOPT_2$. In this example the individual optimal sensitivity derivatives had opposite trends. As one was decreased the other was increased. By assigning different weighting factors to the optimum sensitivity terms, one sensitivity or the other may be emphasized for reduction. The OBJ curve for case 1 was a relatively flat curve and therefore the reduction in the value of OBJ for case 1 was small compared to that of case 2 for any choice of the initial L_1/L value.

Concluding Remarks

A procedure based on minimizing a linear combination of optimum sensitivity derivatives has been developed and tested on a simple beam problem. The results show that the procedure has the capability to successfully perform structural optimization which also minimizes the effects of uncertain parameters. It was important in the procedure to include a term that provided a measure of the minimum weight optimization along with optimum sensitivity terms in the overall objective function to gain control over the trade-off between a low-weight design and the insensitivity to uncertainties. The procedure provides insight into design tradeoffs among various uncertainties as well as tradeoffs between insensitivity and structural weight. The design space was studied and local minima were found to exist depending upon the starting values of the inner loop design variables for each consecutive inner loop optimization.

Acknowledgment

Appendix: Examination of Local Minima in the Design Space

For the results shown in figures 3-5, the inner loop optimization is always initiated using the final values of the design variables from the previous inner loop optimization. This procedure produced smooth curves as shown in the figures. However, during the course of
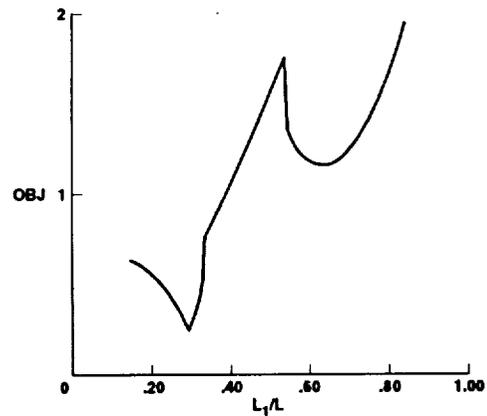
Fig. 6. Plot of outer loop objective function versus L_1/L showing discontinuities in function.

this study the inner loop optimizations were also initiated using some fixed nominal values for the heights of the beam sections. The plots of the outer loop objective function versus L_1/L exhibited discontinuities as shown for a typical case in figure 6. These discontinuities suggested that local minima were present in this example problem. While investigating

(a) $L_1/L = .25$

$L_1/L=.50$
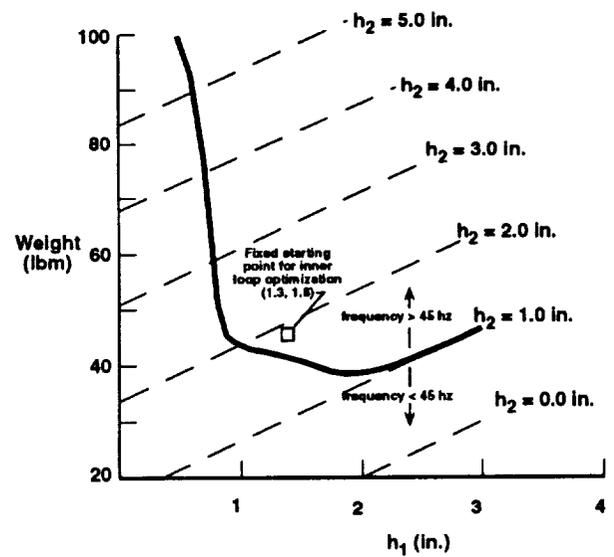
(c) $L_1/L=.75$
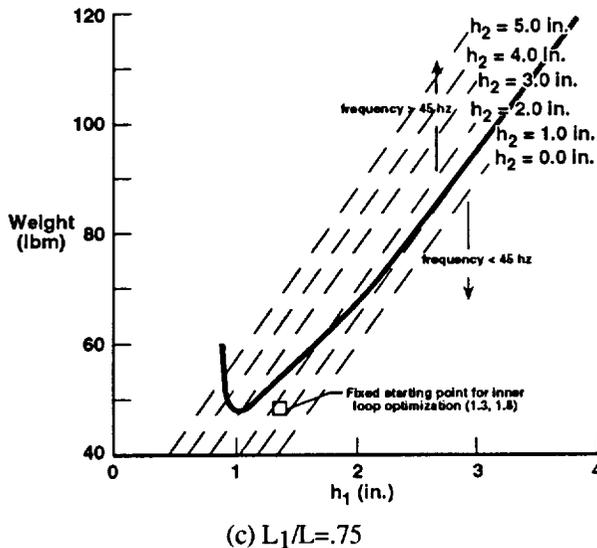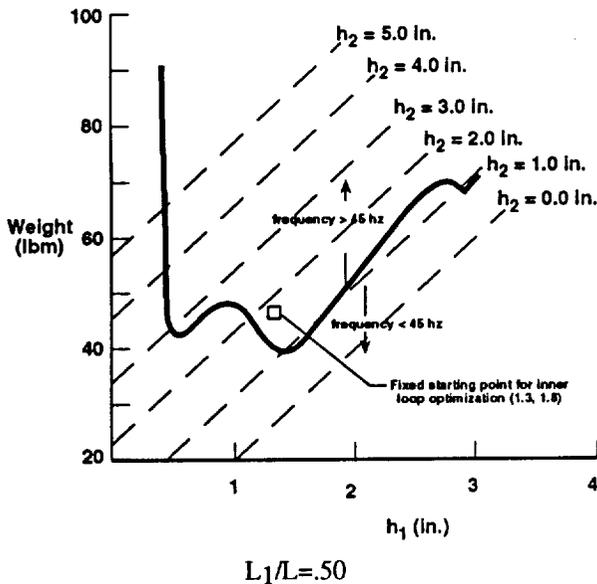
Fig. 7. Weight versus beam segment heights at lower bound constraint on frequency for $L_1/L=.25$, .50, and .75.

this local minima phenomenon, it was observed that at the inner loop optimum the lower bound frequency constraint was active for all values of L_1. This constraint boundary is shown in figure 7 where the weight is plotted versus beam segment heights for three values of L_1/L. In figure 7a and 7c which represent values of L_1/L near the left and right ends of the beam respectively there is one minimum. Figure 7b shows that for values of L_1/L in the vicinity of the beam center as many as three local minima may exist. As L_1/L changes the inner loop optimization may converge to a different local optimum depending upon the initial values of the design variables hence discontinuities will appear in the outer loop objective function. If the inner loop is initiated with design

variables that are far away from the most current inner loop optimum the optimization algorithm can cause the procedure to find different local minima as L_1/L is changed. Starting successive inner loop optimizations close to the optimum of the previous pass through the inner loop lead to convergence to a local minimum.

It should be noted that the discontinuities in the sweeps of the outer loop objective function versus L_1 shown in figure 6 did not cause the procedure to fail because the search algorithm in the outer loop did not require the calculation of any derivatives. However, the potential did exist for the overall procedure to converge to a local minimum which was not a global minimum.

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13. ABSTRACT (Maximum 200 words)
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