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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An almost linear optimization problem of importance in vibration isolation has been identified and algorithms were developed to minimize the forced vibrational response of structural systems. The constraints can be either displacements or accelerations. These algorithms have been studied for transient response, frequency response and stationary random using the direct dynamic solution. Multiple response points and loading conditions may be used.		

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INTRODUCTION

This study considers only passive vibration isolation by an optimization algorithm. Most physical structures designed for dynamic environments have isolator elements to attenuate the response. Examples of engineering problems that could benefit from this work are ground vehicle response and equipment or instrument vibrational response. Passive damping might be of use to damp out the vibrations of large space structures and these algorithms could be used for the selection of damping parameters.

The constraints considered are displacements or accelerations. The frequency constraint has not been used since

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it tends to destroy the linear character of the algorithm as discussed in reference (8). The design variables are linear changes to mass, stiffness or damping matrices. However, only numerical examples are presented for stiffness changes. The constraints can be expressed in either the time or frequency domain and the cumulative constraint is used to measure the amount of constraint violation. The objective function represents a design variable that restrains displacements or accelerations to be less than a maximum value at a single response point.

It is shown that the variation of displacement or acceleration constraints are shallow in reciprocal design variables. The optimization problem formulated in this space is almost linear. The weight minimization problem has not been considered. However, the form of the displacement or acceleration constraints could be used with weight minimization algorithms, but this is a nonlinear optimization problem.

The developed algorithms have been studied for transient response, frequency response and stationary random using the direct dynamic solution. The algorithms could be used with a reduced basis of old eigenvectors as well. Multiple response points and loading conditions may be used.

TRANSIENT RESPONSE

The minimization of displacements or accelerations can be formulated as a MIN-MAX optimization problem for a single response point \ddot{x}_i .

$$(1) \quad \text{MIN} \left(\text{MAX} \left| \ddot{X}_i \right| \right)$$

$$(2) \quad M\ddot{X} + C\dot{X} + KX = P$$

$$(3) \quad \text{MAX} |X_i(t) - X_j(t)| \leq X_u$$

$$(4) \quad K = K_0 + \sum \alpha_i K_i$$

$$(5) \quad M = M_0 + \sum \alpha_i M_i$$

$$(6) \quad C = C_0 + \sum \alpha_i C_i$$

$$(7) \quad \alpha_{iL} \leq \alpha_i \leq \alpha_{iu}$$

Only the direct method of solution has been considered in this study.

Equation (1) minimizes the maximum acceleration in the time domain. The objective function could be displacements instead and the present algorithms could also be used. Equation (2) is the structural dynamic equations in matrix form which describe the displacement response $X(t)$. Equation (3) is the so called relative displacement or rattle-space constraint. The present algorithms can include this type of constraint in the analysis. However, no specific numerical examples are presented using the rattle-space constraint. Equations (4), (5), and (6) show the linear

changes to the stiffness, mass or viscous damping matrix with the design variables α_i . The design variables could contain differing sets in equations (4), (5), and (6). Equation (7) lists the constraint limits on the design variables α_i .

FREQUENCY RESPONSE

Sometimes, it is convenient to solve vibration problems in the driving frequency ω domain. This is true for problems which have experimentally available results for transfer functions. Also, for stationary random analysis, the frequency domain transfer function must be determined. Equation (2) is transformed to the steady state frequency domain by,

$$(8) \quad X = \text{RE}\{X_0 e^{i\omega t}\}, \quad P = \text{RE}\{P_0 e^{i\omega t}\}$$

where RE: denotes real part of

$$i = \sqrt{-1}$$

ω : driving frequency

X_0 : amplitude of harmonic response

P_0 : amplitude of harmonic loading

For the harmonic substitution, equation (2) becomes,

$$(9) \quad (-\omega^2 M + i\omega C + K)X_0 = P_0$$

The amplitudes X_0 , P_0 are complex numbers. Equation (9) may be solved repeatedly for X_0 given P_0 and ω using complex arithmetic. It is more convenient to use the real displacement components in the analysis. The method of reference (1)

is used to work with the real and imaginary components of x_o .

$$x_o = U - iV$$

$$(10) \begin{vmatrix} -\omega^2 M + K & \omega C \\ \omega C & \omega^2 M + K \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} P_o \\ 0 \end{vmatrix}$$

The optimization becomes in the frequency domain for one response point U_i, V_i .

$$(11) \text{MIN (MAX } (\omega^2 \sqrt{U_i^2 + V_i^2}))$$

$$(12) \begin{vmatrix} -\omega^2 M + K & \omega C \\ \omega C & \omega^2 M + K \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} P_o \\ 0 \end{vmatrix}$$

$$(13) \text{MAX } |x_{oi}(\omega) - x_{oj}(\omega)| \leq x_u$$

$$(14) K = K_o + \sum \alpha_i K_i$$

$$(15) M = M_o + \sum \alpha_i M_i$$

$$(16) C = C_o + \sum \alpha_i C_i$$

$$(17) \alpha_{iL} \leq \alpha_i \leq \alpha_{iu}$$

Equation (11) is the amplitude of steady state acceleration at one point and equation (12) are the structural dynamic equations to be solved.

STATIONARY RANDOM

A Frequency Response solution is first analyzed to determine the transfer function $H(\omega)$ which is either the displacement or acceleration at a response point of interest. The spectral density of the output is given in terms of the spectral density of the input for a single input/output system is given in reference (2).

$$S_o(\omega) = |H(\omega)|^2 S_I(\omega)$$

The same reference also lists techniques for analyzing multiple input/output systems. The mean square value can be calculated for any frequency interval,

$$\bar{z}^2 = \int_{\omega_1}^{\omega_2} S_o(\omega) d\omega.$$

Various performance measures have been proposed for random analysis such as using either the spectral density or mean square value. The optimization problem for stationary random becomes,

$$(18) \quad \text{MIN (MAX } S_o)$$

$$(19) \quad \begin{vmatrix} -\omega^2 M + K & \omega C \\ \omega C & \omega^2 M + K \end{vmatrix} \begin{vmatrix} U \\ V \end{vmatrix} = \begin{vmatrix} P_o \\ 0 \end{vmatrix}$$

$$(20) \quad S_o = |H(\omega)|^2 S_I$$

$$(21) \quad K = K_o + \sum \alpha_i K_i$$

$$(22) \quad M = M_0 + \sum \alpha_i K_i$$

$$(23) \quad C = C_0 + \sum \alpha_i M_i$$

$$(24) \quad \alpha_{iL} \leq \alpha_i \leq \alpha_{iU}$$

The maximum displacement or acceleration spectral density is the objective function to be minimized in equation (18). Only the single input/output case is used to calculate the spectral density by equation (20). The objective function is converted to a set of equivalent integral constraints and the minimization is done then on the mean square response in effect. However, the algorithm can also be used for multiple input/output with minor modifications.

MIN-MAX PROBLEM

The objective function (1), (11) or (18) can be converted to a simpler algebraic form. Consider equation (1),

$$\text{MIN}(\text{MAX} |\ddot{X}_i(t)|)$$

This minimization is equivalent to minimizing an additional design variable α such that

$$\text{MIN } \alpha$$

$$|\ddot{X}_i(t)| - \alpha \leq 0 \text{ for all } t.$$

The cumulative constraint has been used in the optimal control literature (3) to convert many discrete constraints in the time domain into one equivalent integral constraint which measures the total amount of constraint violation. In terms of the cumulative constraint, the MIN-MAX part of the

optimization becomes,

$$(25) \quad \text{MIN } \alpha \\ \int < |\ddot{x}_i(t)| - \alpha > dt = 0$$

The objective functions (11) or (18) in the frequency domain are computed in the same manner with frequency replacing time in the integral.

The inner problem or the maximization in this research was done by function evaluation. This is efficient for the transient problem, but the frequency response problem requires a decomposition for each driving frequency in equation (12). It would be required to reduce the basis of equation (12) by using the real normal modes for efficient solution in locating the maximum. Reference (1) recommends performing a one dimensional search on the variable ω to locate the maximum. This one dimensional search would require several initial starting points to insure convergence to the maximum of the nonlinear problem in ω .

ANALYTIC DERIVATIVES

For statically determinate structures, stresses and deflections are proportional to design variables that are linear changes to stiffness such as areas of rods in truss members. For indeterminate structures, this is only an approximation. It was investigated in references (4,5) and found that high quality explicit expressions for stresses and deflections could be generated using a first order Taylor series expansion in reciprocal design variables.

That is, the design variable space for stress and static deflection is shallow in reciprocal design variables. The linearized Taylor series expansions represent lines that are very good approximations to the exact constraints. The expansion of a response quantity ϕ is done in the reciprocal design variables β_i of the direct design variables α_i .

$$\phi = \phi_0 + \sum \frac{\partial \phi}{\partial \beta_i} \delta \beta_i$$

$$\beta_i = \frac{1}{\alpha_i}$$

The direct solution of the dynamic response equations in the time domain uses an efficient implicit equation solver such as Newmark integration.

This would be the most general capability for solution of the dynamic equations. The Newmark integration equations presented in reference (6) are listed for one set of integration parameters, $\delta = \frac{1}{2}$ and $\alpha = \frac{1}{4}$. Given the response at t_1 , the response at t_2 can be calculated from the following:

$$(26) \quad M\ddot{X}_{t_2} + C\dot{X}_{t_2} + KX_{t_2} = P_{t_2}$$

$$(27) \quad KK = K + \frac{4}{\Delta t^2} M + \frac{2}{\Delta t} C$$

$$(28) \quad PP_{t_2} = P_{t_2} + M \left[\frac{4}{\Delta t^2} X_{t_1} + \frac{4}{\Delta t} \dot{X}_{t_1} + \ddot{X}_{t_1} \right] \\ + C \left[\frac{2}{\Delta t} X_{t_1} + \dot{X}_{t_1} \right]$$

$$(29) \quad KK \cdot X_{t_2} = PP_{t_2}$$

$$(30) \quad \ddot{X}_{t_2} = \frac{4}{\Delta t^2} (X_{t_2} - X_{t_1}) - \frac{4}{\Delta t} \dot{X}_{t_1} - \ddot{X}_{t_1},$$

$$(31) \quad \dot{X}_{t_2} = \dot{X}_{t_1} + \frac{\Delta t}{2} \ddot{X}_{t_1} + \frac{\Delta t}{2} \ddot{X}_{t_2}$$

The displacements at the next time step are calculated by equation (29). The matrix KK is only factored when the time step Δt changes. The acceleration and displacement are recovered by equations (30) and (31). The derivatives of the response quantities are found by differentiating equations (26) through (31) as was done in reference (7). This is the pseudo loads technique. The derivatives with respect to the reciprocal variables are,

$$(32) \quad KK \frac{\partial X_{t_2}}{\partial \beta_i} = - \frac{\partial KK}{\partial \beta_i} X_{t_2} + \frac{\partial PP_{t_2}}{\partial \beta_i}$$

$$(33) \quad \frac{\partial KK}{\partial \beta_i} = \frac{\partial K}{\partial \beta_i} + \frac{4}{\Delta t^2} \frac{\partial M}{\partial \beta_i} + \frac{2}{\Delta t} \frac{\partial C}{\partial \beta_i}$$

$$(34) \quad \frac{\partial PP_{t_2}}{\partial \beta_i} = \frac{\partial P_{t_2}}{\partial \beta_i} + M \left[\frac{4}{\Delta t^2} \frac{\partial X_{t_1}}{\partial \beta_i} + \frac{4}{\Delta t} \frac{\partial \dot{X}_{t_1}}{\partial \beta_i} + \frac{\partial \ddot{X}_{t_1}}{\partial \beta_i} \right]$$

$$+ C \left[\frac{2}{\Delta t} \frac{\partial X_{t_1}}{\partial \beta_i} + \frac{\partial \dot{X}_{t_1}}{\partial \beta_i} \right]$$

$$(35) \quad \frac{\partial \ddot{X}_{t_2}}{\partial \beta_i} = \frac{4}{\Delta t^2} \left[\frac{\partial X_{t_2}}{\partial \beta_i} - \frac{\partial X_{t_1}}{\partial \beta_i} \right] - \frac{4}{\Delta t} \frac{\partial \dot{X}_{t_1}}{\partial \beta_i} - \frac{\partial \ddot{X}_{t_1}}{\partial \beta_i}$$

$$(36) \quad \frac{\dot{\partial X}_{t_2}}{\partial \beta_i} = \frac{\dot{\partial X}_{t_1}}{\partial \beta_i} + \frac{\Delta t}{2} \frac{\ddot{\partial X}_{t_1}}{\partial \beta_i} + \frac{\Delta t}{2} \frac{\ddot{\partial X}_{t_2}}{\partial \beta_i}$$

Using this technique, the derivatives of displacement, velocity and acceleration must be calculated and saved for all degrees of freedom in the finite element model at two neighboring points in time. The KK matrix in (32) was decomposed in the response calculations and would not be factored again in this step.

The pseudo loads technique was applied to the structural equations (12) in the frequency domain. The required derivatives are,

$$(37) \quad \begin{bmatrix} -\omega^2 M & \omega C \\ \omega C & \omega^2 M - K \end{bmatrix} \begin{bmatrix} \frac{\partial U}{\partial \beta_i} \\ - \\ \frac{\partial V}{\partial \beta_i} \end{bmatrix} = - \begin{bmatrix} -\omega^2 \frac{\partial M}{\partial \beta_i} + \frac{\partial K}{\partial \beta_i} & \omega \frac{\partial C}{\partial \beta_i} \\ \omega \frac{\partial C}{\partial \beta_i} & \omega^2 \frac{\partial M}{\partial \beta_i} - \frac{\partial K}{\partial \beta_i} \end{bmatrix} \begin{bmatrix} U \\ - \\ V \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial P_0}{\partial \beta_i} \\ - \\ - \\ 0 \end{bmatrix}$$

LINEARIZED CONSTRAINTS

The acceleration cumulative constraint has the following first order Taylor series expansion in the reciprocal variables.

$$(38) \quad \int_0^T I dt = 0$$

where $I = \ddot{X}_{o_j} + \sum \frac{\partial \ddot{X}_j}{\partial \beta_i} \delta \beta_i - \beta$, for $\ddot{X} \geq \beta$

$$I = -\ddot{X}_{O_j} - \sum \frac{\partial \ddot{X}_j}{\partial \beta_i} \delta \beta_i - \beta, \text{ for } \ddot{X} \leq \beta$$

$$I = 0 \quad \text{For } \ddot{X} \text{ otherwise}$$

This constraint is numerically integrated by a modified trapezoid law which finds those response points above a line for which the constraints are violated. The algorithm interpolates to find the points where the actual constraint is violated.

The steady state acceleration amplitude in the frequency domain is,

$$(39) \quad A = \omega^2 \sqrt{U_j^2 + V_j^2}$$

The first order Taylor series expansion for the acceleration amplitude magnitude is,

$$(40) \quad \int_0^\omega I \, d\omega = 0$$

$$\text{where } I = A_0 + \frac{\omega^2}{\sqrt{U_j^2 + V_j^2}} \sum \left(\frac{\partial U_j}{\partial \beta_i} + \frac{\partial V_j}{\partial \beta_i} \right) \delta \beta_i - \beta$$

The spectral density acceleration is calculated like equation (40). The square root spectral density of the output S is minimized,

$$S = \omega^2 |H(\omega)| \sqrt{S_I(\omega)}$$

This is the form of equation (39) and equation (40) would be modified by multiplying ω^2 by $\sqrt{S_I(\omega)}$. A formula could be derived for multiple sources with cross correlation

correlation in a similar manner.

SEQUENTIAL LINEAR PROGRAMMING

The problem considered in this study of minimizing a linear design variable subject to constraints on displacements or accelerations in the time or frequency domain is an almost linear problem in reciprocal design space. It is only natural to use sequential linear programming as the optimization algorithm. A primal-dual linear program which is listed in reference (10) was used as the optimizer. Sequential linear programming is described in reference (11).

NUMERICAL APPLICATIONS

Transient Response:

The model of reference (1) shown in figure 1 was subjected to the displacements inputs $f_1(t)$ and $f_2(t)$ shown in figure 2. This model represents a vehicle running over a bump. The transient step size used was .1 sec and 39 time intervals were calculated. The five springs were used as design variables with limits shown on figure 1. The objective function was a design variable which represented the maximum acceleration at point 1 in the model over the 3.9 sec time of response. The acceleration constraints were made active

when the acceleration bound was 99% of the maximum. Figure 3 presents the decrease in acceleration at point 1 in the model versus the required number of structural analyses. The linear program uses design variables that are changes from a reference and the change can be positive or negative which requires two variables be subtracted to keep all variables positive. So the total number of design variables used in the linear program was eleven. Initially, the reciprocal variables were constrained by a move limit to lie within $\pm 25\%$ of the initial values. Convergence was obtained at iteration three. The spring rates found at the optimum were,

$$k_1 = 51.2 \text{ lb/in}$$

$$k_2 = 200. \text{lb/in}$$

$$k_3 = 200. \text{lb/in}$$

$$k_4 = 1600. \text{lb/in}$$

$$k_5 = 1000. \text{lb/in}$$

The minimum acceleration obtained was 228.8 in/sec^2 . Figure 4 presents the initial response versus the optimal one.

Frequency Response:

The model shown in figure 1 was subjected to equal in-phase displacement inputs $f_1(t) = f_2(t) = 5 \cos \omega t$ at the

tire. This would represent a vehicle on a shaker table. The five springs were used as design variables with the limits shown of figure 1. The acceleration amplitudes were evaluated between 5 RAD/SEC and 44 RAD/SEC in steps of 1 RAD/SEC as the driving frequencies. The objective function was the maximum steady state acceleration amplitude at point 1 over the range of driving frequencies. The acceleration constraints were made active when the acceleration was 99% of the maximum. Figure 6 presents the decrease in acceleration amplitude at point 1 in the model versus the required number of structural analyses. Initially, the reciprocal variables were constrained by a move limit to lie within $\pm 25\%$ of the initial values. When a step was not minimizing, the percent move limit on the reciprocal variables was decreased by 50% and the linear program was resolved at the previous design point. Convergence was achieved at iteration 7 which was close to the value found at iteration 5.

The spring rates at the optimum were as follows:

$$k_1 = 52.9 \text{ LB/IN}$$

$$k_2 = 231.2 \text{ LB/IN}$$

$$k_3 = 215.1 \text{ LB/IN}$$

$$k_4 = 1000. \text{LB/IN}$$

$$k_5 = 1311. \text{LB/IN}$$

The minimum acceleration amplitude was found to be 318.7 IN/SEC². Figure 7 compares the initial acceleration amplitude in the frequency domain versus the optimized response.

Stationary Random Response:

The model shown in figure 1 was subjected to a random displacement at the tire patches as discussed in reference (13) with parameters that correspond to a smooth highway. A Frequency Response solution is first completed with a unit harmonic displacement with phase lag $e^{i(\omega t - \phi)}$ at the rear tire with phase angle $\phi = \frac{L}{V}$. The spectral density of the output in terms of the spectral density of the input and transfer function is,

$$S_O(\omega) = |H(\omega)|^2 S_I(\omega).$$

The transfer function is determined by using the acceleration output of the frequency response solution due to the unit harmonic input. The spectral density acceleration was evaluated between 5 RAD/SEC and 44 RAD/SEC in steps of 1 RAD/SEC. The objective function was the design variable representing the maximum acceleration spectral density.

The acceleration constraint was made active when it was 99% of the maximum. Figure 8 presents the decrease in the objective function versus the required number of structural analyses. The reciprocal spring rates were constrained by $\pm 25\%$ of the current value as move limits. At the detection of each infeasibility, the move limit was reduced by 50% of the current percent.

Convergence was achieved at iteration 3.

The spring rates at the optimum were,

$$K_1 = 51.2 \text{ LB/IN}$$

$$K_2 = 200. \text{ LB/IN}$$

$$K_3 = 200. \text{ LB/IN}$$

$$K_4 = 1000. \text{ LB/IN}$$

$$K_5 = 2000. \text{ LB/IN}$$

The minimum spectral density was $49.61 \text{ (IN/SEC}^2\text{)}^2/\text{Hz}$.
The initial and optimized spectral densities are presented
in figure 9.

REDUCED BASIS

The dynamic equations are usually reduced from physical degrees of freedom to some set of generalized freedoms. The following transformation is used.

$$X = YZ$$
$$(Y^TMY)\ddot{Z} + (Y^TCY)Z + (Y^TKY)Z = Y^TP$$

Most solutions use the matrix Y as the collection of eigenvectors. When the equations are differentiated, the derivative of the eigenvector must be calculated. An alternate approach would use direct Ritz vectors as in reference (15). When small changes are made to a structure as would be the case in vibration

isolation an old eigenspace could be used as in reference (9). When old Ritz vectors are used, the eigenvector derivative is not calculated and the previously developed algorithm can be applied with the same sequential linearity. Sparse matrix multiplications should be used if small changes are made ΔG to a matrix G .

$$Y^T G_0 Y + Y^T \Delta G Y$$

where G_0 is the unchanged part of the matrix and is calculated initially.

LOCAL MINIMA

The algorithms presented coverage only to a local minima and it is necessary to use several initial designs to identify all minima. The method tends to converge to the minima which is the strongest resonant peak that is closer to the initial design than other strong peaks. The following table lists the initial design and optimized with the minimum transient response of figure 1.

TABLE I

	INITIAL	OPTIMUM	INITIAL	OPTIMUM	INITIAL	OPTIMUM
(LB/IN) K_1	100	51.2	300	204.8	200	161.8
K_2	300	200	800	200	500	200
K_3	300	200	800	200	500	200
K_4	1500	1600	1200	1024	1800	1138
K_5	1500	1000	1200	2000	1800	2000
MIN \ddot{x} (IN/SEC ²)		228.8		238.4		238.4

MULTICRITERIA OPTIMUM

For multiple response points and loading conditions, the techniques of multiobjective optimization is required and multiobjective programming is described in reference (14). The simultaneous minimization of all objectives is in general not possible. Individual optimization is done on one objective function at a time with the remaining objectives treated as constraints and bounds determined by the analyst. A multitude of solutions are generated depending on the constraint bounds on the objectives.

As an example, the minimization of response for the model was considered using all of the three previous loading conditions. This problem has three different objective functions with each having different units. Each objective should be minimized subject to constraints on the other two. To illustrate the method, the transient response was minimized subject to constraints on frequency response and stationary random. The constraints were made active at 99% of the initial design or the minimum of the maximum response of any previous iteration during the sequential linear programming. Convergence was achieved at iteration 6 which was very close to the results of iteration 4. The results can be presented in a table.

TABLE II

	Iteration 4	Iteration 6
K_1	102.8	65.8
K_2	200	200
K_3	200	200
K_4	1280	1217
K_5	1000	1778
Transient	237.5	235.7
Frequency Response	377.2	322.0
Random	49.4	50.1

An infinite number of solutions can be generated in this same manner and results can be found minimizing the other objectives. Some engineering judgment is required to interpret the generated solutions.

CONCLUSIONS

An almost linear optimization problem of importance in vibration isolation has been identified and algorithms were developed to minimize the forced vibrational response of structural systems. These algorithms should replace the very inefficient one presented in reference (9) which solves a series of practical problems. The linearity depends on using displacement or acceleration as the only constraints in the time or frequency domain. The frequency constraint is inherently nonlinear as discussed in reference (8) and it has not been considered in this study.

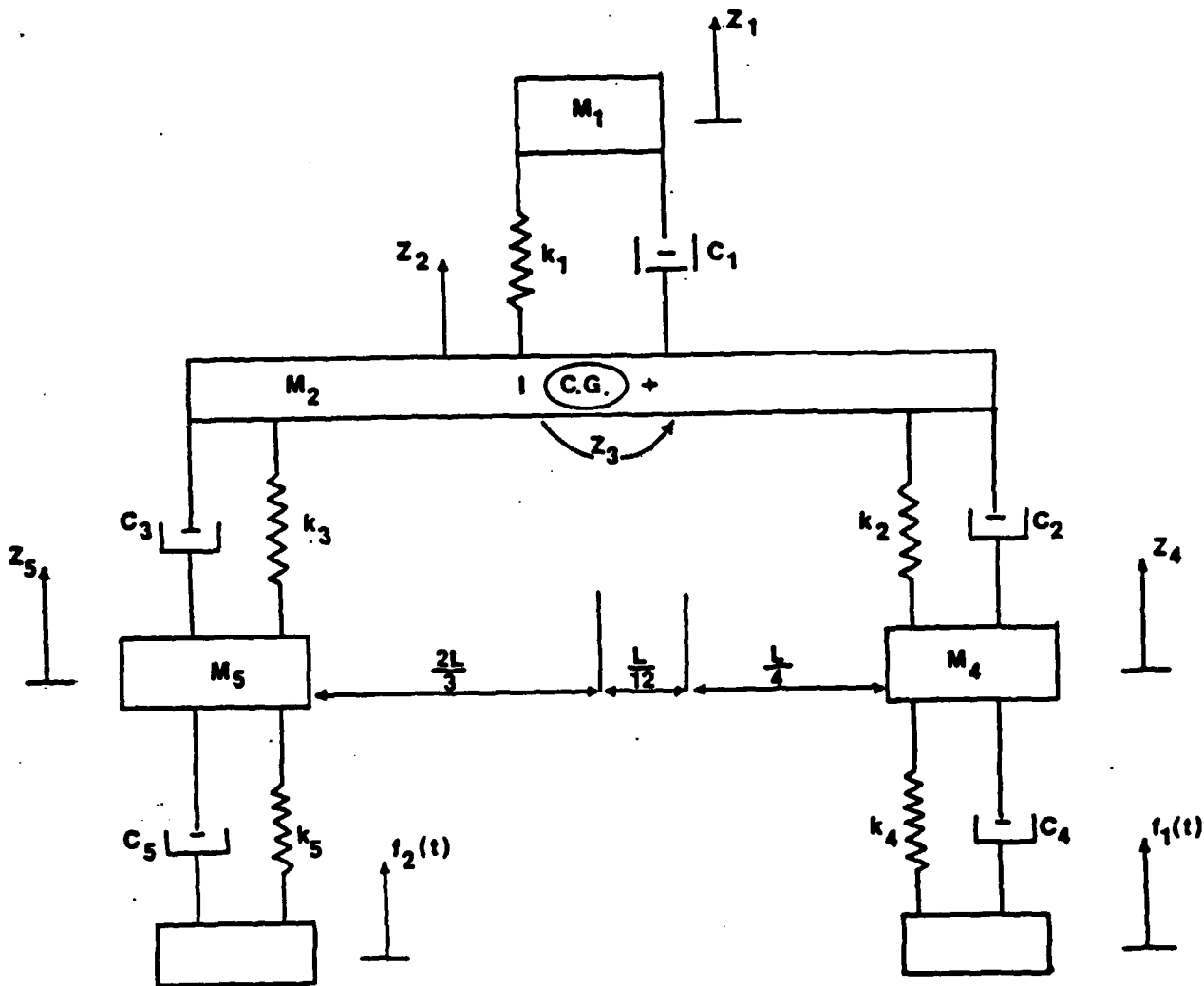
Only the direct dynamic solution has been used, but a reduced basis of old eigenvectors could be implemented as well. Only local convergence has been shown and several initial design points should be used to search out other local minima. Multiple response points and loading conditions may be used.

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$$M_1 G = 290 \text{ lb.}$$

$$M_2 G = 4,500 \text{ lb.}$$

$$I = 41,000 \text{ lb.-in-sec}$$

$$M_4 G = 96.6 \text{ lb.}$$

$$M_5 G = 96.6 \text{ lb.}$$

$$C_1 = 10 \text{ lb.-sec/in}$$

$$C_2 = 25 \text{ lb.-sec/in}$$

$$C_3 = 25 \text{ lb.-sec/in}$$

$$C_4 = 5 \text{ lb.-sec/in}$$

$$C_5 = 5 \text{ lb.-sec/in}$$

LOWER LIMITS

$$K_{L1} = 50 \text{ lb./in}$$

$$K_{L2} = 200 \text{ lb./in}$$

$$K_{L3} = 200 \text{ lb./in}$$

$$K_{L4} = 1000 \text{ lb./in}$$

$$K_{L5} = 1000 \text{ lb./in}$$

INITIAL DESIGN

$$K_1 = 100 \text{ lb./in}$$

$$K_2 = 300 \text{ lb./in}$$

$$K_3 = 300 \text{ lb./in}$$

$$K_4 = 1500 \text{ lb./in}$$

$$K_5 = 1500 \text{ lb./in}$$

UPPER LIMITS

$$K_{U1} = 500 \text{ lb./in}$$

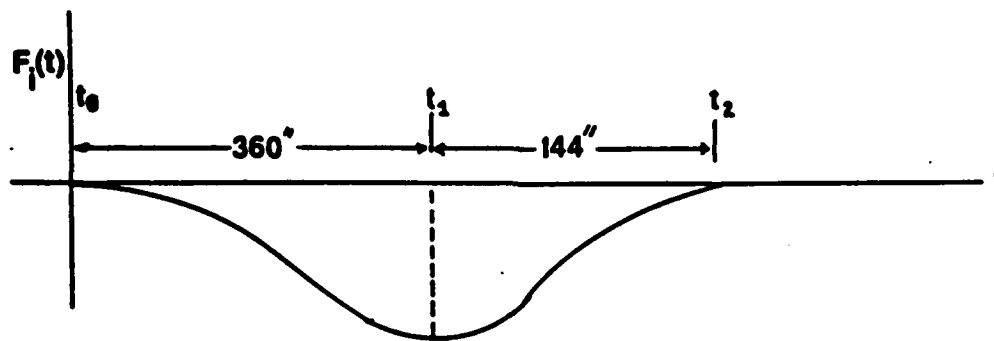
$$K_{U2} = 1000 \text{ lb./in}$$

$$K_{U3} = 1000 \text{ lb./in}$$

$$K_{U4} = 2000 \text{ lb./in}$$

$$K_{U5} = 2000 \text{ lb./in}$$

FIGURE 1: OPTIMIZATION MODEL



AMPLITUDE $X_0 = 5''$

VEHICLE SPEED $S = 450$ IN/SEC

WHEEL BASE $L = 120$ IN

$d_1 = 360''$, $d_2 = 144''$

$w_1 = \frac{\pi S}{d_1} = 1.25 \pi$

$w_2 = \frac{\pi S}{d_2} = 3.125 \pi$

$t_1 = d_1/S = .8$ SEC, $t_2 = (d_1 + d_2)/S = 1.12$ SEC

TIME LAG FRONT TO REAR t_L

$t_L = L/S = .2667$ SEC

FRONT WHEEL DISPLACEMENT

$$f_1(t) = X_0(1 - \cos wt) \quad 0 \leq t \leq t_1$$

$$f_1(t) = X_0(1 + \cos w(t - t_2)) \quad t_1 \leq t \leq t_2$$

REAR WHEEL DISPLACEMENT

$$f_2(t) = f_1(t - t_L) \quad 0 \leq t - t_L \leq t_2$$

FIGURE 2: TRANSIENT DISPLACEMENT INPUT FOR FIGURE 1

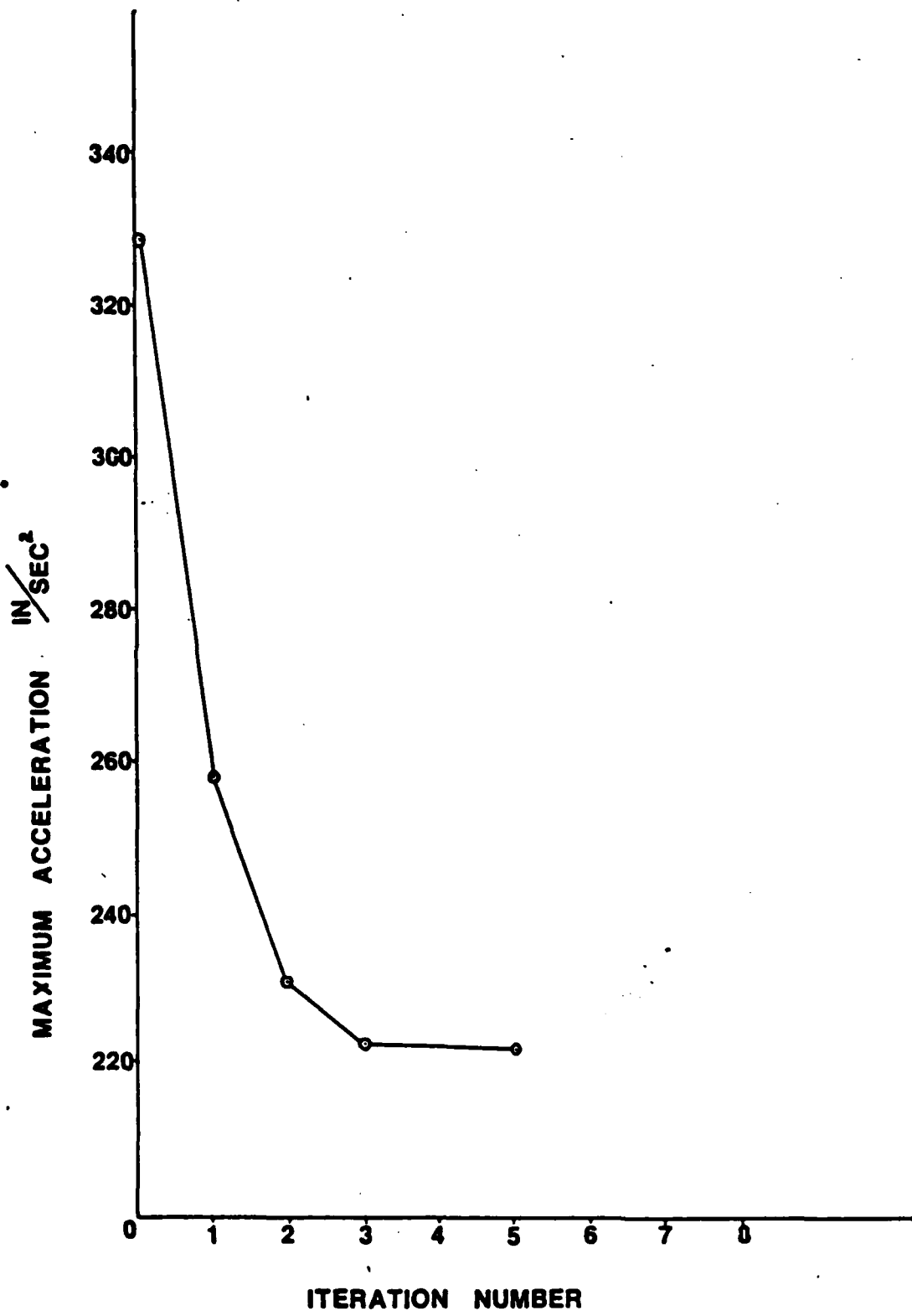


FIGURE 3: TRANSIENT RESPONSE OF FIGURE 1

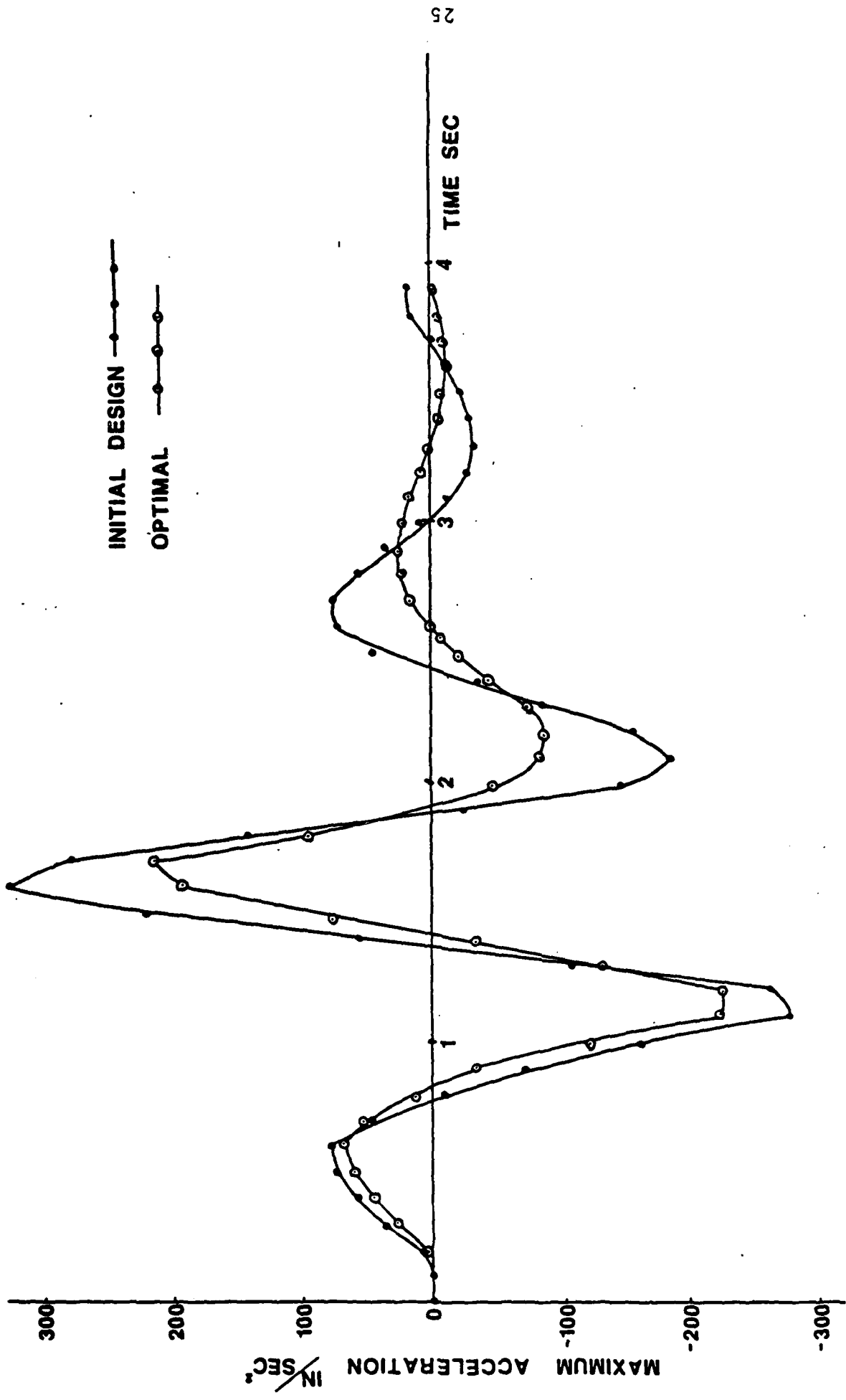


FIGURE 4: TRANSIENT RESPONSE OF FIGURE 1

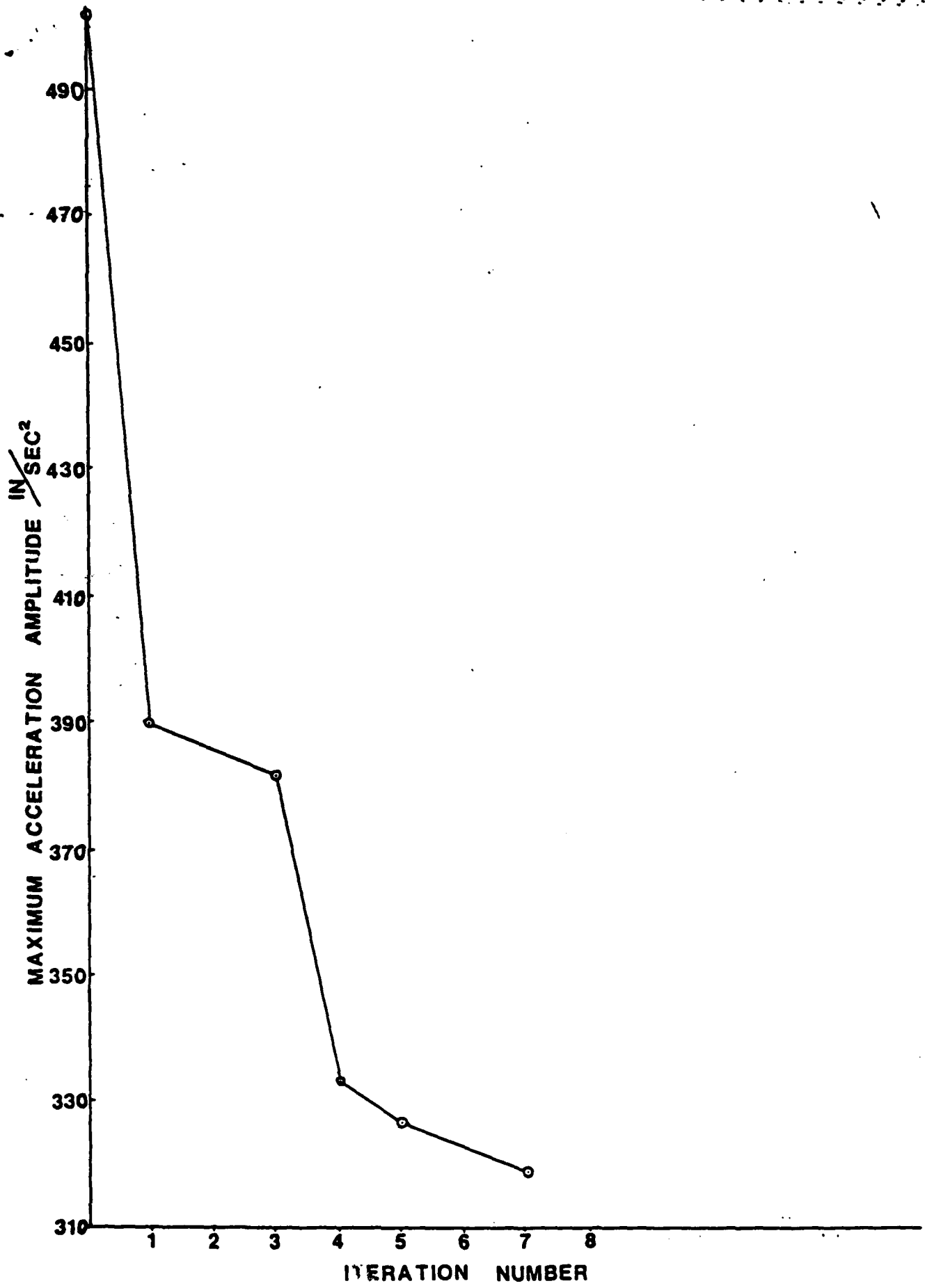


FIGURE 6: FREQUENCY RESPONSE OF FIGURE 1

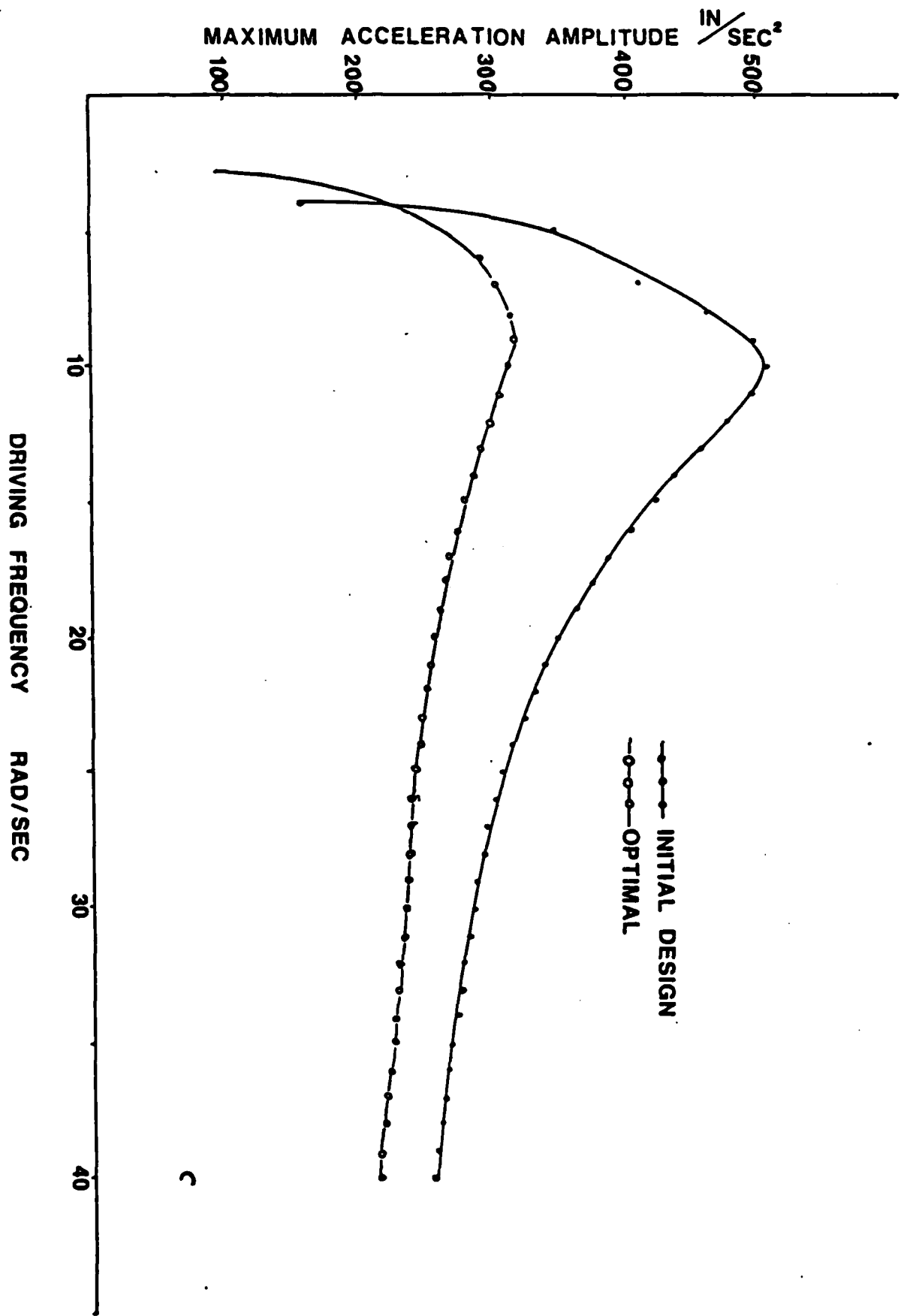


FIGURE 7 : FREQUENCY RESPONSE OF FIGURE 1

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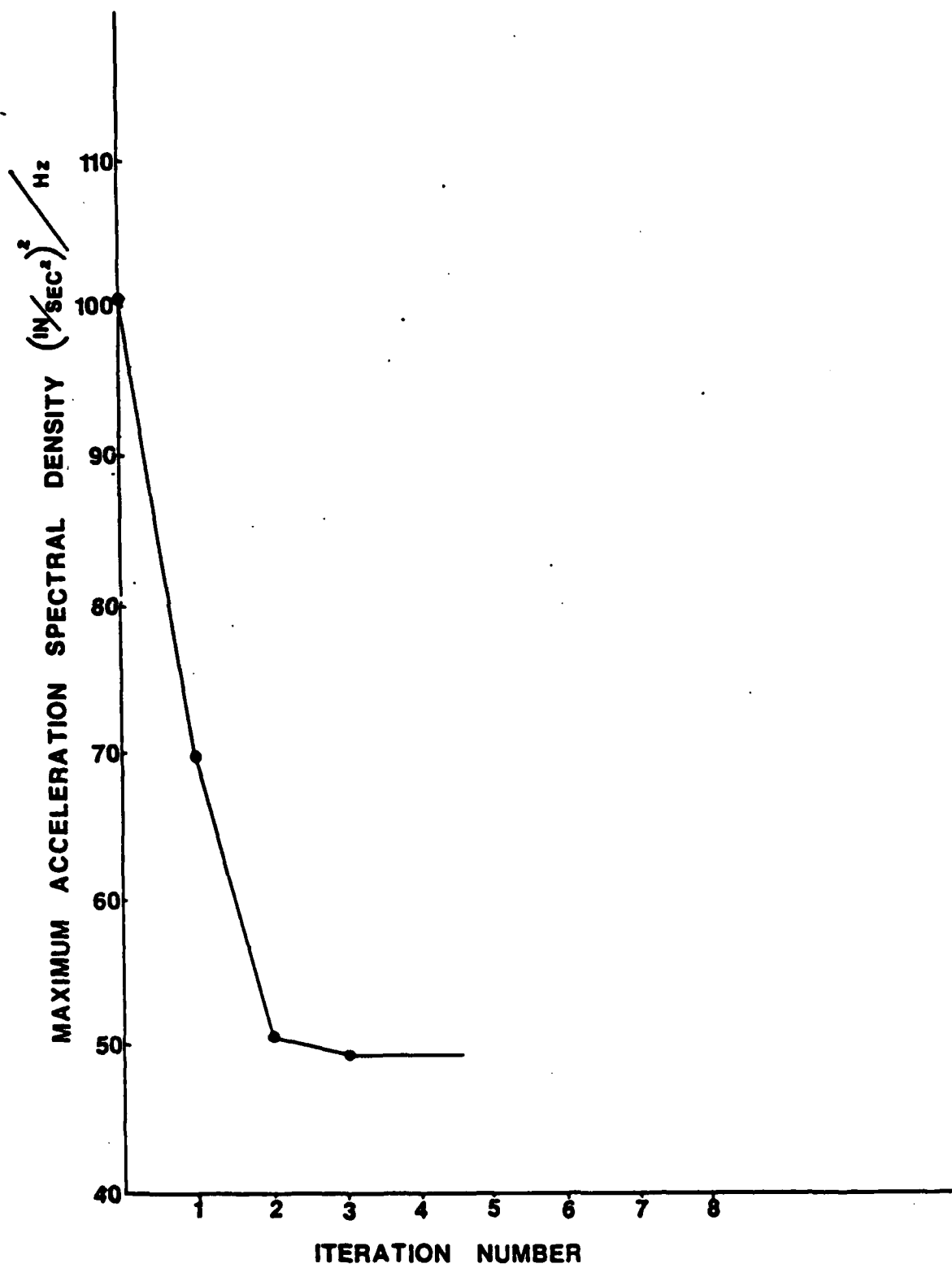


FIGURE 8: STATIONARY RANDOM RESPONSE OF FIGURE 1

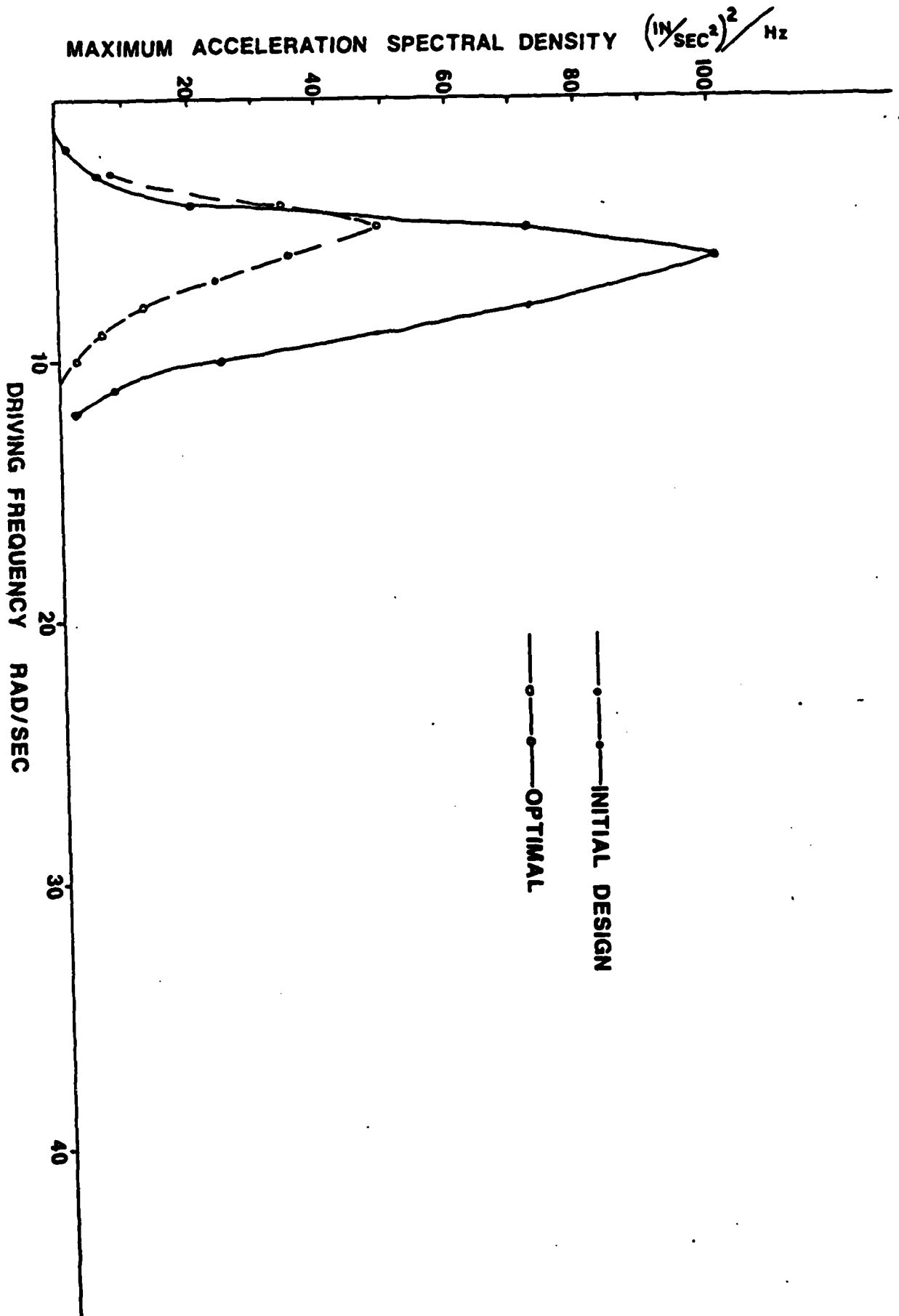


FIGURE 9: STATIONARY RANDOM RESPONSE OF FIGURE 1

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