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Optimization of a field-widened Michelson interferometer

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This paper considers the optical design of a wide-angle fixed-path Michelson interferometer consisting of two arm glasses and an air gap. It is shown that this configuration can be optimized to give (a) extra large fringes (over 50° in diameter) over a range of wavelength, (b) a path difference nearly independent of wavelength, or (c) a path difference specified differently at two different wavelengths for observing a pair of doublets. Specific examples refer to the airglow wavelengths of 557.7, 630.0, 732.0 nm and others, and to a path difference of 4.5 cm. The properties of different glass combinations are discussed.

I. Introduction

Most practical schemes for enlarging the available solid angle of a Michelson interferometer use a refractive compensating material in the arms. The mirror positions are adjusted so a quasi-symmetry is maintained at the beam splitter, resulting in large fringes even at high resolving power.

The wide-angle Michelson Doppler imaging interferometer (WAMDII)[1] is a fixed-path field-widened device which grew out of the earlier designs of Hilliard and Shepherd[2] and Title and Ramsey.[3] It consists of a beam splitter, two plane-parallel glass arms, and an air gap in one of the arms, and its purpose is to measure the phase and visibility of fringes produced by upper atmospheric emissions, with a view of determining wind speeds and temperatures. During the design of this instrument, it was found that the optics of the interferometer could be optimized in several different ways. The optimization can be considered under four categories: (1) field wideness, (2) path difference, (3) thermal compensation, (4) achromatization of (1), (2), and (3) and the specification of different values of (2) at different wavelengths.

Ideally, one desires an instrument with maximum field wideness at a given path difference; insensitivity of these two parameters to changes of temperature; and a system which maintains this stability, provides a constant value for the path difference, and optimizes the field wideness over a selected range of wavelengths. In practice, and given the limited number of glasses available, certain compromises must be made depending on the priorities of the design.

The design by Title and Ramsey,[3] consisting of two arm glasses, was achromatically field widened and reasonably well compensated against thermal variations. The addition of an air gap in the WAMDII allows even greater scope for optimization. The combination of the above properties worked out for the WAMDII is described by Shepherd *et al.*[1] and the problem of thermal compensation is discussed by them and by Thuillier and Shepherd.[4] Here we consider other possibilities: the extremes of field wideness possible with this configuration, the achromatization of path difference, and the specification of different values of path difference at different wavelengths.

The examples given here refer specifically to the fixed-path configuration and to a path difference of 4.5 cm, appropriate for high-resolution atmospheric measurements in the visible. The same principles apply to other path differences and to scanning instruments in which refractive media are used to provide the field widening.

II. Super Field Widening

Shepherd *et al.*[1] have developed a series expansion to represent the path difference through a three-glass Michelson. Extending their expression to include one more term gives

$$\Delta = 2(n_3t_3 + n_2t_2 - n_1t_1) - \left(\frac{t_3}{n_3} + \frac{t_2}{n_2} - \frac{t_1}{n_1}\right)\sin^2\theta - \left(\frac{t_3}{n_3^3} + \frac{t_2}{n_2^3} - \frac{t_1}{n_1^3}\right)\frac{\sin^4\theta}{4} - \left(\frac{t_3}{n_3^5} + \frac{t_2}{n_2^5} - \frac{t_1}{n_1^5}\right)\frac{\sin^6\theta}{8}. \quad (1)$$

Here, θ is the incident angle in air and n and t are the various refractive indices and thicknesses, respectively, of the three arm media shown schematically in Fig. 1. This expression can be generalized to any number of glass layers by adding the appropriate terms to the parentheses. We will consider the case where the third medium is an air gap, so that $n_3 = 1.0$. In this case the path difference at normal incidence is given by

$$\Delta_0 = 2(t_3 + n_2t_2 - n_1t_1). \quad (2)$$

The useful field of view of a Michelson can be greatly increased by reducing the angular dependence of the path difference. This is generally done by using values of t that make the coefficient of $\sin^2\theta$ in Eq. (1) equal to zero:

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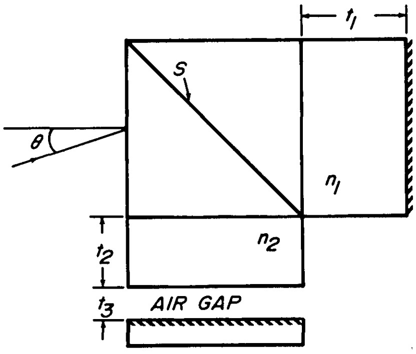

Fig. 1. Field-widened Michelson with two glasses and air gap. n_1 and n_2 are the refractive indices of the arm glasses; S is the beam splitter surface; θ is the angle of incidence in air.

$$t_3 + \frac{t_2}{n_2} - \frac{t_1}{n_1} = 0. \quad (3)$$

This technique is known as field compensation or field widening and the result is a large gain in throughput, especially at high resolving power. However, with two glasses and a vacuum gap, it is possible to enlarge the field even more by equating the coefficient of $\sin^4\theta$ to zero also, so that

$$t_3 + \frac{t_2}{n_2^3} - \frac{t_1}{n_1^3} = 0. \quad (4)$$

Equations (3) and (4) are the conditions for super field widening. Now the angular dependence of Δ involves only $\sin^6\theta$ and higher-order terms. Solving Eqs. (2)–(4) for the t terms gives

$$\left.\begin{aligned} t_1 &= \frac{\Delta_0 n_1^3}{2(n_2^2 - n_1^2)(n_1^2 - 1)}, \\ t_2 &= \frac{\Delta_0 n_2^3}{2(n_2^2 - n_1^2)(n_2^2 - 1)}, \\ t_3 &= \frac{\Delta_0}{2(n_1^2 - 1)(n_2^2 - 1)}. \end{aligned}\right\} \quad (5)$$

Because of dispersion in the glasses, the above conditions are strictly true at ony one wavelength. If it is desired to retain some degree of field widening at other wavelengths, the glasses should be chosen to make the derivative of Eq. (3) with respect to wavelength as close to zero as possible.

Differentiating the left-hand side of Eq. (3) and equating to zero give

$$\frac{t_1}{t_2} = \frac{n_1^2}{n_2^2} \cdot \frac{dn_2/d\lambda}{dn_1/d\lambda}.$$

Replacing this in Eqs. (3) and (4) yields

$$\frac{n_1^2}{n_1^2 - 1} \cdot \frac{dn_1}{d\lambda} = \frac{n_2^2}{n_2^2 - 1} \cdot \frac{dn_2}{d\lambda}.$$

Thus, to achieve an achromatically field-widened system according to condition (3), two glasses should be chosen which have

$$\frac{n^2}{n^2 - 1} \cdot \frac{dn}{d\lambda}$$

as nearly equal as possible. The superwidened condition (4) still applies only to the design wavelength, however.

Table I. Glasses Used in Search [a]

Glass	n	$\frac{dn}{d\lambda} \times 10^5$ (nm^{-1})	$\frac{dn}{d\lambda} \cdot \frac{n^2}{n^2-1} \times 10^5$ (nm^{-1})
FK5	1.486094	−3.0012	−3.691
BK7	1.515186	−3.4629	−4.049
K10	1.499607	−3.7681	−4.525
KF9	1.521409	−4.2683	−4.939
LLF1	1.545798	−4.9748	−5.534
LF5	1.580165	−5.5039	−5.810
F4	1.613340	−6.8711	−6.916
TiF2	1.530301	−4.7938	−5.467
BaK4	1.566824	−4.2802	−4.609
SK4	1.610645	−4.4314	−4.477
PSK3	1.550580	−3.7340	−4.123
BaLF4	1.577440	−4.5340	−4.805
SSK2	1.619993	−4.9140	−4.901
BaF8	1.621146	−5.5063	−5.483
SF5	1.668692	−8.4410	−7.893
SF4	1.749990	−10.9471	−9.289
SF57	1.839958	−14.0352	−10.83
BaSF10	1.646947	−6.7905	−6.531
BaSF51	1.720052	−7.7729	−6.826
LaK8	1.710372	−5.6222	−4.994
LaF21	1.784848	−6.9191	−5.650
LaF22	1.777719	−8.6052	−7.081
LaSF5	1.876499	−8.8943	−6.620

[a] Data are calculated for 630.0 nm from dispersion constants.

Other considerations may also influence the choice of glasses. When working with large incident angles, it is desirable to have rather short arms, so $n_2 - n_1$ should be large. Furthermore, thermal stability is important when accurate phase measurements are being made.

To find suitable pairs of glasses, a search was made using dispersion coefficients and thermal data from the Schott Optical Glass catalog.[5] Twenty-three preferred glasses, evenly distributed on the Schott n_d/ν_d diagram,[5,3] were selected and tested in all combinations using a computer. Combinations requiring long arms were eliminated immediately; for the remaining pairs, the path difference at normal incidence and the dependence of path difference on incident angle were calculated for various wavelengths. The thermal characteristics

$$\frac{\partial\Delta}{\partial T} \text{ and } \frac{\partial^2\Delta}{\partial\lambda\partial T}$$

were also calculated, assuming that t_3 is independent of temperature, as in the case of the WAMDII. Table I lists the glasses used and some of their properties. The list is far from complete; this study was not intended to be exhaustive but to indicate some of the possibilities.

The computer search produced 34 pairs with arm lengths under 7.5 cm. It was found that 23 of the pairs included either SF57 or LaSF5 as the high-index component. The remainder include SF4, LaF21, or LaF22.

LaSF5 contains radioactive thorium, which may exclude it from some applications.

Several of the pairs are listed in Table II, along with some of their properties. The three wavelengths in column 3 correspond to important atmospheric emission lines. The angle θ_{max} represents the approximate limit of the field of view and is somewhat arbitrarily defined as the incident angle at which $|\Delta_0 - \Delta| = \lambda/2$. The design wavelength here is 630.0 nm.

The first four pairs in Table II have the shortest arms of any pair involving their respective high-index component. FK5/LaSF5 has the shortest arms of any pair studied and K10/LaF21 has the smallest value of

$$\frac{\partial^2\Delta}{\partial\lambda\partial T}.$$

(The importance of the thermal charateristics in columns 6 and 7 in the phase stability of the Michelson is discussed by Thuillier and Shepherd.[4]) θ_{max} is remarkably constant from one pair to another at the design wavelength, where a field of view 35° in diameter is typical. At the other wavelengths, however, the size of the available field is quite variable, although even the smallest of these is much larger than the field of an uncompensated Michelson at the same path difference. Of the 34 pairs, F2/LaSF5 is the most achromatic with respect to field widening, as it has the closest match of the

$$\frac{dn}{d\lambda}\cdot\frac{n^2}{n^2-1}$$

quantities. Figure 2 gives the field wideness characteristics for this and another glass pair and shows why the useful field for F4/LaSF5 is even greater at 732.0 nm

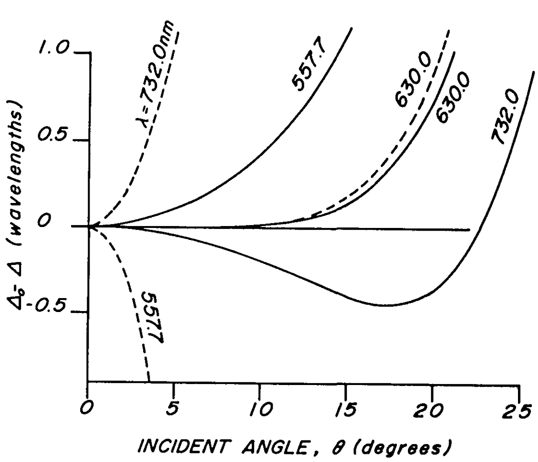

Fig. 2. Change of path difference with incident angle optimized at 630 nm for a wide field: solid line, F4/LaSF5; dashed line, FK5/SF57.

than it is at the design wavelength. This is the "defocusing" effect, similar to that described by Zwick and Shepherd.[6] Here, the small nonzero coefficients of the $\sin^2\theta$ and $\sin^4\theta$ terms in Eq. (1) partially cancel the effect of the $\sin^6\theta$ and higher terms.

Defocusing can also be designed into the system by adjusting the arm lengths to give a small nonzero value (e) to the coefficient of $\sin^4\theta$, while the $\sin^2\theta$ term is kept equal to zero. Condition (4) then becomes

$$t_3 + \frac{t_2}{n_2^3} - \frac{t_1}{n_1^3} = e, \qquad (6)$$

and when solved with Eqs. (2) and (3), the result is

Table II. Wide-Angle Combinations [a]

Glasses [b]	$t_{1,2,3}$ (cm)	λ (nm)	Δ_0 (cm)	θ_{max} (deg)	$\frac{\partial^2\Delta}{d\lambda\partial T}$ (cm nm^{-1} K^{-1})	$\frac{\partial\Delta}{\partial T}$ (cm K^{-1})
FK5/SF57	5.1918	557.7	4.5985	2.7		
	4.9920	630.0	4.5000	17.8	(No data)	
	0.7805	732.0	4.4142	3.3		
FK5/SF4	7.1554	557.7	4.5963	2.6		
	6.8462	630.0	4.5000	17.4	1.3×10^{-7}	1.2×10^{-4}
	0.9027	732.0	4.4156	3.2		
FK5/LaF22	6.4200	557.7	4.5600	3.5		
	6.1477	630.0	4.5000	17.5	7.0×10^{-8}	2.9×10^{-5}
	0.8619	732.0	4.4463	4.3		
FK5/LaSF5	4.6547	557.7	4.5459	4.5		
	4.4918	630.0	4.5000	18.0	-1.8×10^{-8}	2.3×10^{-5}
	0.7385	732.0	4.4580	5.5		
K10/LaF21	6.4854	557.7	4.5329	6.2		
	6.2478	630.0	4.5000	17.6	1.5×10^{-9}	1.0×10^{-5}
	0.8243	732.0	4.4693	7.5		
F4/LaSF5	6.4186	557.7	4.5221	10.6		
	6.4208	630.0	4.5000	18.5	-6.0×10^{-8}	-1.6×10^{-5}
	0.5568	732.0	4.4781	24.5		

[a] Data in columns 4–7 refer to the wavelengths in column 3. The design wavelength is 630 nm.
[b] The low-index member is given first and corresponds with t_1.

$$t_1' = t_1 + \frac{n_1^2 n_2^2 e}{(n_2^2 - n_1^2)(n_1^2 - 1)},$$
$$t_2' = t_2 + \frac{n_1^2 n_2^2 e}{(n_2^2 - n_1^2)(n_2^2 - 1)},$$
$$t_3' = t_3 + \frac{n_1^2 n_2^2 e}{(n_1^2 - 1)(n_2^2 - 1)}, \quad (7)$$

where the primed t terms represent the new values satisfying Eqs. (2), (3), and (6) and the unprimed t terms are those defined in Eq. (5).

Figure 3 shows the variation of path difference with angle for several values of e for the pair FK5/LaSF5 at the design wavelength. The value of $e = -0.025$ cm is produced by reducing t_1, t_2, and t_3 by 0.1821, 0.1102, and 0.638 cm, respectively. The graph shows that, in most applications where a variation in path difference of a few tenths of a wavelength across the field of view is permissible, a field diameter of over 50° is possible. To take advantage of such large angles, the Michelson and associated imaging optics have to be carefully designed.

III. Achromatic Path Difference

Dispersion in the arm glasses of a wide-angle Michelson causes the path difference to be wavelength dependent (see column 4 in Table II). In some applications this is undesirable and it is possible to remove this dependence to a large extent by requiring that $(\partial\Delta)/(\partial\lambda) = 0$, i.e., that

$$t_2 \frac{\partial n_2}{\partial \lambda} - t_1 \frac{\partial n_1}{\partial \lambda} = 0. \quad (8)$$

Solving Eq. (8) in conjunction with Eqs. (2) and (3) gives

$$t_2 = \frac{\Delta_0}{2\left[\left(n_2 - \frac{1}{n_2}\right) - \gamma\left(n_1 - \frac{1}{n_1}\right)\right]},$$
$$t_1 = \gamma t_2 \text{ and } t_3 = \frac{t_1}{n_1} - \frac{t_2}{n_2}, \quad (9)$$

where

$$\gamma = \frac{\partial n_2}{\partial \lambda} \Big/ \frac{\partial n_1}{\partial \lambda}.$$

Again a search was made for suitable combinations using the same 23 Schott glasses. The design wavelength was 630.0 nm and calculations of the path difference and θ_{max} were made from 530 to 730 nm. Thirty-three pairs have arm lengths under 5.5 cm. The path difference is shown as a function of wavelength for three of these pairs in Fig. 4. LF5/SF57 is the most achromatic of all the pairs, with a maximum variation of only 7 μm or 0.016% across the 200-nm investigated. Most of the combinations have characteristics similar to the other two curves in Fig. 4. Table III lists the same three pairs and gives their arm lengths, field size, and thermal characteristics. LaK8/SF4 is one of the best pairs for field widening and SK4/SF5 has the smallest value of

$$\frac{\partial^2\Delta}{\partial\lambda\partial T}.$$

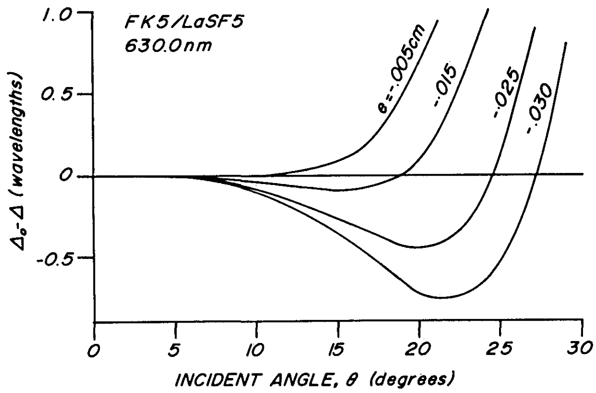

Fig. 3. Change of path difference with incident angle with different degrees of defocusing; optimized for a wide field.

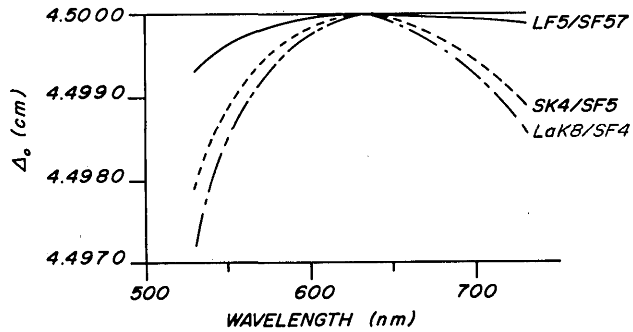

Fig. 4. Variation of path difference with wavelength; combinations optimized to minimize this variation.

Table III. Combinations for Achromatic Δ_0 [a]

Glasses	$t_{1,2,3}$ (cm)	λ (nm)	θ_{max} (deg)	$\frac{\partial^2\Delta}{\partial\lambda\partial T}$ (cm nm^{-1} K^{-1})	$\frac{\partial\Delta}{\partial T}$ (cm K^{-1})
LF5/SF57	5.1263	530.0	4.0		
	2.0103	630.0	5.8	(No data)	
	2.1516	730.0	7.2		
SK4/SF5	5.2527	530.0	5.6		
	2.7575	630.0	6.1	3.9×10^{-9}	2.3×10^{-5}
	1.6087	730.0	6.7		
LaK8/SF4	2.2204	530.0	6.1		
	4.3234	630.0	6.3	-2.0×10^{-8}	1.6×10^{-5}
	-1.2589[b]	730.0	7.0		

[a] The design wavelength is 630.0 nm.
[b] The negative sign indicates that t_3 is in the same arm as t_1.

IV. Path Difference Defined at Two Wavelengths

Phase and visibility measurements of a closely spaced doublet emission are possible with a fixed-path Michelson if the path difference is chosen so that the two sets of fringes coincide very closely in phase. This is the approach being taken with the WAMDII for observing the 0^+ doublet at 732.0 nm. It is also possible, within limits, to accommodate a second doublet in this way.

In addition to Eq. (2), a second path difference is defined:

$$\frac{\Delta_0'}{2} = t_3 + n_2't_2 - n_2't_1. \quad (10)$$

The primes refer to values at the second wavelength. The wide-angle condition is defined at an intermediate wavelength:

$$t_3 + \frac{t_2}{n_2'} - \frac{t_1}{n_1'} = 0. \quad (11)$$

When Eqs. (2), (10), and (11) are solved together, the result is

$$\left.\begin{aligned} t_1 &= \frac{d\dfrac{\Delta_0}{2} - c\dfrac{\Delta_0'}{2}}{bc - ad}, \\ t_2 &= \frac{b\dfrac{\Delta_0}{2} - a\dfrac{\Delta_0'}{2}}{bc - ad}, \\ t_3 &= \frac{t_1}{n_1'} - \frac{t_2}{n_2'}, \end{aligned}\right\} \quad (12)$$

where

$$a = n_1 - \frac{1}{n_1'}, \quad b = n_1' - \frac{1}{n_1'},$$
$$c = n_2 - \frac{1}{n_2'}, \quad d = n_2' - \frac{1}{n_2'}.$$

A numerical example was worked out using Δ_0 = 4.526 cm at 732.0 nm (the ninth beat of the 0^+ doublet) and Δ_0' = 4.512 cm at 697.8 nm [the third beat of the Λ-doubled OH (7,2) $P_1(5)$ line]. The wavelength for field widening was 714.9 nm. The same 23 glasses were used and all the pairs with acceptably short arms involved either SF4 or SF57, the glasses with the greatest dispersion. Two examples are given at the top of Table IV. LaK8/SF57 has the shortest arms of any of the pairs. In this case, the two doublets are relatively close together in wavelength and the path differences are only 0.014 cm apart. A greater difference in Δ_0 would require longer arms, which are already quite long. A larger difference in Δ_0 can be accommodated if the doublets are more widely separated in wavelength. Calculations were done for the 0^+ doublet at 732.0 nm and an imaginary doublet at 550.0 nm. The lower part of Table IV shows that, with Δ_0 fixed at 4.526 cm at 732.0 nm, a range of 4.4–4.8 cm is possible at 550.0 nm with arms under 10 cm in length. The wide-angle design wavelength is 641.0 nm.

V. Conclusions

The super field widening described in Sec. II can be put to use in a wide-field imaging interferometer or in a device with very large throughput. For a field 50° in diameter, the gain in solid angle is a factor of 10^4 over a conventional Michelson at a path difference of 4.5 cm.

The equations given here for optimizing the field size or path difference can be solved at one wavelength for any pair of glasses with different refractive indices, but other properties, such as arm length, field size at other wavelengths, or thermal properties may also be important, depending on the application. These can be manipulated to a considerable extent by selecting the glasses to be used.

The design of a Michelson interferometer with two arm glasses is rather analogous to the design of an achromatic lens, and the term defocusing was probably derived from this analogy. It is likely that, as with a lens, the inclusion of more glass elements in the design would allow a larger set of properties to be optimized.

Table IV. Combinations with Δ_0 Defined At Two Wavelengths

Glasses	$t_{1,2,3}$ (cm)	λ (nm)	Δ_0 (cm)	θ_{max} (deg)
SK4/SF57	9.2595	650.0	4.4871	6.7
	5.3467	697.8	4.5120	5.9
	2.8390	714.9	4.5193	5.6
		732.0	4.5260	5.4
		780.0	4.5416	5.1
LaK8/SF57	6.0999	650.0	4.4867	7.9
	9.0077	697.8	4.5120	6.4
	−1.9461	714.9	4.5194	6.1
		732.0	4.5260	5.7
		780.0	4.5414	5.2
SK4/SF57	9.3656	550.0	4.4000	3.9
	5.4254	641.0	4.4807	5.4
	2.8655	732.0	4.5260	4.3
LaK8/SF57	6.0592	550.0	4.4000	3.3
	8.9649	641.0	4.4815	6.9
	−1.9475	732.0	4.5260	4.2
SK4/SF57	8.0092	550.0	4.8000	2.0
	7.9074	641.0	4.6284	13.9
	0.6731	732.0	4.5260	3.0
LaK8/SF57	8.4494	550.0	4.8000	2.0
	7.6666	641.0	4.6278	8.3
	0.1119	732.0	4.5260	3.0

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