

# Optimization of extrusion force prediction model using different techniques

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## Manufacturing and processing

### ABSTRACT

**Purpose:** This research is determination of the optimal cold forward extrusion parameters with objective the minimization of tool load.

**Design/methodology/approach:** This paper deals with the different optimization approaches relating to determine optimal values of logarithmic strain, die angle and coefficient of friction with the purpose to find minimal tool loading obtained by cold forward extrusion process. To achieve this, it has been carried out two experimental plans based on factorial design of experiment and orthogonal array. By using these plans it was performed classical optimization, according to response model of extrusion forming force, and the Taguchi approach, respectively.

**Findings:** Experimental verification of optimal forming parameters with their influences on the forming forces was done. The experimental results showed an improvement in minimization of tool loading. It was compared results of optimal forming parameters obtained with different optimization approaches and based on that the analysis of the characteristics (features and limitations) of both techniques.

**Research limitations/implications:** Suggestion for future research it will be application of evolutionary algorithms namely model prediction of the process by genetic programming and optimization of extrusion parameters by genetic algorithm.

**Practical implications:** a practical (industrial) implication on the smallest energy consumption, longer tool life, better formability of the work material and the quality of the finished product.

**Originality/value:** This paper is obtained original extrusion force model for experimental domain of forming parameters and identification of parameters influence in that model.

**Keywords:** Plastic forming; Forward extrusion force model; Optimization; Taguchi approach

## 1. Introduction

The metal forming process is characterized by various process parameters including the shape of the workpiece and product, forming sequence, shapes of tools or dies, friction, forming speed, temperature and material property of the workpiece and those of the tools. Therefore, determination of the optimal forming

parameters by using optimization techniques is continuous engineering task with main aim to reduce the production cost and achieve desired product quality [1,2].

Forming technologies, that have been applied for a number of years in a definite conventional form, can be innovated by applying knowledge from the area of modelling, simulations, optimizations, theory of processes, computer technique and artificial intelligence [3]. The optimization methods have been

improved by development of applied mathematics, statistics, operational researches, design of experiment, simulation and information-computational methods. Today, there are more different optimization methods. The use of the existing methods depend on objects modelling, required degree of model accuracy, type of process and necessity of optimization.

In this research work, mathematical modelling of the extrusion force and the different optimization approaches relating to determine optimal values of logarithmic strain, die angle and coefficient of friction with the purpose to find optimal tool load obtained by cold forward extrusion process [4,5,6].

Hence, optimization i.e. minimization of the cold forward extrusion force has been carried out by two experimental plans based on factorial design of experiment and orthogonal array. By using these plans it was performed classical mathematical optimization, according to response model of extrusion forming force, and the Taguchi approach, respectively. Finally, the confirmation experiment was conducted to verify the optimal extrusion parameters with the minimal tool load and to confirm the effectiveness of these approaches. The value of presented techniques and obtained results have a practical implication on the smallest energy consumption, longer tool life, better formability of the work material and the quality of the finished product.

## 2. Experimental setup and results

The processes of cold and hot extrusion are classified depending upon the direction of material flow in relation to the tool movement direction. Another method of classifying these processes is by their geometry, namely, solid and hollow components [7]. In the solid forward extrusion process, analyzed in this paper, the flow of metal is in the same direction as the direction of action of the machine (punch), where final product is a solid workpiece with a profile determined by the shape of the die opening, shows in Fig. 1. Forward extrusion force value can be obtained both experimentally using definite measurement equipment and analytically according to well know expression for the total extrusion force [3,6,7]. Consequently, it can be concluded that forward extrusion force basically depends on material properties, logarithmic strain, die angle, coefficient of friction and initial geometry of workpiece (billet).

From that point of view, the experiment has been carried out by using central composition design with five levels of the three main independent parameters, namely, logarithmic strain ( $\varphi$ ), die angle ( $\alpha$ ) and friction coefficient ( $\mu$ ) (Table 1.) [3,4,5,6]. Overall the number of experiments which was conducted for this central composition design is  $N = 2^3 + 6 + 6 = 20$  trials. The forward extrusion operations were performed on hydraulic press with alloyed carbon steel EN 16MnCr5 (workpiece material) as rod billet. Experiments were run with different friction conditions what for used the following lubricants: MoS<sub>2</sub>, phosphate

surface&oil, grease, oil, moist oil with five coefficient of frictions according to level parameters, respectively. Initial diameter of workpiece ( $d_0$ ) and height ( $h_0$ ) for all the experiments is constant.

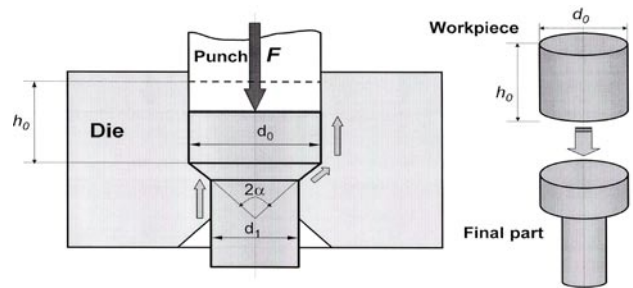


Fig. 1. Extrusion die geometry with initial and formed part

## 3. Extrusion force model prediction

Design of experiment is a powerful tool for modelling and analysing the influence of process parameters. On the basis of performed experiment can be represented the functional relationship between response of extrusion process, in this case the extrusion force, and the investigated independent parameters by the following form of mathematical model [3,4,5,6,8,9]:

$$Y = b_0 + \sum_{i=0}^k b_i X_i + \sum_{1 \leq i < m} b_{im} X_i X_m + \sum_{j=0}^k b_{ij} X_j^2 + \sum_{1 \leq i < m < k} b_{ism} X_i X_m X_k + \varepsilon \quad (1)$$

or in coding form where all the constants, including interactions, can be estimated. The obtained mathematical model has the form:

$$Y = 607,64 + 170,39 X_1 + 13,79 X_2 + 48,96 X_3 + 12,2 X_1^2 + 51,61 X_2^2 + 11,5 X_1 X_2 X_3 \quad (2)$$

or after transformation Eq. (2) extrusion force model as function of the logarithmic strain ( $\varphi$ ), die angle ( $\alpha$ ) and friction coefficient ( $\mu$ ) has the following physical form:

$$F = 52,93 + 915,33 \varphi + 147 \varphi^2 - 10,45 \alpha + 0,355 \alpha^2 + 5423,62 \mu - 16,56 \varphi \alpha - 4991 \varphi \mu - 98,325 \alpha \mu + 165,6 \varphi \alpha \mu \quad (3)$$

For the 95% confidence level the  $R^2 = 0,99$  what shows a good interdependency of the input parameters ( $\varphi$ ,  $\alpha$ ,  $\mu$ ) and response ( $F$ ). According to that, forming force model (3) describes accurately enough (model explains 99% of the variability in force  $F$ ) the experimental results within range of experiment (Table 2.).

Table 1. Levels of independent extrusion parameters

Symbol	Parameters / Levels	Lowest	Low	Centre	High	Highest
	Coding	-1,6817	-1	0	+1	+1,6817
A	Logarithmic strain $\varphi$	0,112	0,308	0,596	0,884	1,080
B	Half-die angle $\alpha$ (°)	10	18	30	42	50
C	Coefficient of friction $\mu$	0,066	0,08	0,10	0,12	0,134

Table 2. Design of experiment with experimental and model results

№ trial	$\varphi \leftrightarrow X_1$		Parameters $\alpha \leftrightarrow X_2$				Extrusion force $F$ (kN)		
	logarithmic strain	coding	half-die angle (°)	coding	friction coefficient	coding	Experiment (average $F$ )	Predicted model (3)	Analytical model (5)
1	0,308	-1	18	-1	0,08	-1	445	426,81	332,05
2	0,884	+1	18	-1	0,08	-1	790	790,59	677,40
3	0,308	-1	42	+1	0,08	-1	478	477,40	433,92
4	0,884	+1	42	+1	0,08	-1	770	795,19	762,98
5	0,308	-1	18	-1	0,12	+1	560	547,73	390,48
6	0,884	+1	18	-1	0,12	+1	860	865,51	757,39
7	0,308	-1	42	+1	0,12	+1	566	552,32	485,22
8	0,884	+1	42	+1	0,12	+1	905	916,11	819,68
9	0,596	0	30	0	0,10	0	610	607,64	564,60
10	0,596	0	30	0	0,10	0	614	607,64	564,60
11	0,596	0	30	0	0,10	0	605	607,64	564,60
12	0,596	0	30	0	0,10	0	611	607,64	564,60
13	0,596	0	30	0	0,10	0	606	607,64	564,60
14	0,596	0	30	0	0,10	0	597	607,64	564,60
15	0,112	-1.6817	30	0	0,10	0	338	355,56	304,95
16	1,080	1.6817	30	0	0,10	0	963	928,76	862,10
17	0,596	0	10	-1.6817	0,10	0	725	730,45	554,76
18	0,596	0	50	1.6817	0,10	0	799	776,87	661,95
19	0,596	0	30	0	0,066	-1.6817	556	525,29	517,39
20	0,596	0	30	0	0,134	1.6817	711	689,99	611,81

### 4. Optimization of extrusion force

For a forming process such as forward extrusion, the forming conditions play an important role in the efficient use of a machine tool. Since the cost of extrusion process is sensitive to the forming conditions optimum values have to be determined before a part is put into production. To select the forming parameters properly, there are considerable number of optimization techniques [6,10,11]. The optimum forming parameters, in this case will be determined by the two different optimization approaches, classical mathematical and Taguchi, with the objective to minimize forward extrusion force. It has already known that minimal extrusion force is possible to achieve with low strain and coefficient of friction. Hence, the minimization of extrusion force will be a function of die angle only.

#### 4.1. Classical mathematical optimization

In classical mathematical analysis the optimization of extrusion process parameters were carried out by derivation of the obtained mathematical model (3). In this particular case derivation of predicted mathematical model will be performed with the aim to find optimal die angle ( $\alpha$ ):

$$\frac{dF}{dX_i} = 0 \quad i=1,2,3, \text{ that is, for die angle} \quad (4)$$

$$\frac{dF}{d\alpha} = 0$$

Furthermore, based on literature known mathematical model for total solid forward extrusion force [7]:

$$F_U = A_0 \cdot \sigma_{f,m} \left[ \frac{2}{3} \tilde{\alpha} + \left( 1 + \frac{2\mu}{\sin 2\alpha} \right) \varphi_{\max} \right] + \pi \cdot d_0 \cdot l \cdot \mu \cdot \sigma_{f,0} \quad (5)$$

or for minimum force requirements optimal die angle is:

Table 3. Levels of independent extrusion parameters according to Taguchi approach

Symbol	Parameters	Level 1	Level 2	Level 3	Degrees of freedom
A	logarithmic strain, $\varphi$	0,308	0,596	0,884	2
B	half - die angle, $\alpha$ (°)	18	30	42	2
C	friction coefficient, $\mu$	0,08	0,10	0,12	2

$$\frac{dF_{tot}}{d\alpha} = 0 \Rightarrow \cos 2\alpha_{opt} = -3\mu \varphi_{\max} \pm \sqrt{9\mu^2 \varphi_{\max}^2 + 1} \quad (6)$$

#### 4.2. Taguchi approach

In this paper optimization based on Taguchi approach [12,13,14,15] is used to achieve the more efficiency extrusion parameters, especially for die angle, and to compare results obtained with both techniques. Table 3 shows that the experimental plan has three levels and an appropriate Taguchi orthogonal array with notation  $L_9 (3^4)$  was chosen (Table 4.). The last column of parameters notation with D (Table 4.) was used to estimate the experiment error. The right side of the table includes the average results (each trial has 3 samples) of the measured force and the calculated signal-to-noise ( $S/N$ ) ratio with associated the trial number according the classical plan. The  $S/N$  ratio, as the yardstick for analysis of experimental results, is calculated:

$$S/N = \eta = -10 \log_{10} \left( \frac{1}{n} \sum_{i=1}^n y_i^2 \right) \quad (7)$$

The above equation, which is used to calculate the  $S/N$  ratio, is in relation to the *smaller-is-better* quality characteristics, what in the particular case means minimization of extrusion force.

### 5. Results and discussion

The optimal parameter values for the different approaches are presented in Table 5. Final step is to verify the improvement using optimal parameters level (about 10%). Since, the model (3) has the interaction parts the optimal die angle depends on strain and friction, i.e. it has been established optimal die angle path (Fig. 2).

Table 4.

Three-level orthogonal array,  $L_9(3^4)$ , with experimental results (average) and calculated signal-to-noise ( $S/N$ ) ratios

Trial №	Orthogonal array				Experimental results, average $F$ (kN)	$S/N$ ratio	Trial № (Table 2.)
	A logarithmic strain	B half - die angle	C friction coefficient	D experimental error			
1	1	1	1	1	445	-52,968	1
2	1	2	2	2	450	-53,065	new exp.
3	1	3	3	3	566	-55,061	7
4	2	1	2	3	658	-56,365	new exp.
5	2	2	3	1	664	-56,445	new exp.
6	2	3	1	2	645	-56,192	new exp.
7	3	1	3	2	860	-58,691	6
8	3	2	1	3	740	-57,386	new exp.
9	3	3	2	1	835	-58,434	new exp.

Table 5.

The comparison of the optimal results and confirmation test

Level	Initial parameters	Optimal forming parameters			Confirmation test
	A1B1C1	Prediction model (3) A1C1	Taguchi approach A1C1	Analytical model (6) A1C1	A1C1
Force $F$ (kN)	445	396,41	432,11	318,01	402
Optimal half-die angle $\alpha$ (°)		$B = \alpha = 27,23^\circ$	$B2 = \alpha = 30^\circ$	$B = \alpha = 10,88^\circ$	$\alpha_{opt} = 27,23^\circ$

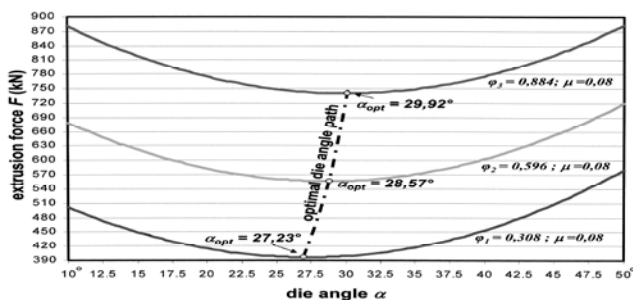


Fig. 2. The optimal die angle path

## 6. Conclusions

Classical experimental design methods are too complex and not easy to use. A large number of experiments have to be carried out especially when the number of process parameters increases. To solve this problem, the Taguchi approach uses a special design of orthogonal arrays to study the entire parameter space with a small number of experiments, what is obviously if we compare Table 2 and Table 4. Furthermore, to obtain optimal value of process parameters the classical method needs the prediction model which was used for optimization procedure, what is not necessary for orthogonal arrays design. Also, the parameters value needs to be defined strictly numerical not as description of state.

On the other hand, advantage of classical experimental design methods are possible to obtain mathematical model which is powerful tool to predict response for any of input parameters value within the experiment range, and optimal values can to be any of parameters point. This is impossible in Taguchi approach, i.e. optimal value have to be one of parameter levels, see Table 5.

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