This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2020.Doi Number

Optimization of Hoop Layouts for Reducing Vibration Amplitude of Pipeline System Using the Semi-analytical Model and Genetic Algorithm

Xudong Liu[1,2], Wei Sun[1,2], Zhihui Gao[1,2]
[1]School of Mechanical Engineering and Automation, Northeastern University, Shenyang 110819, China
[2]Key Laboratory of Vibration and Control of Aero-Propulsion Systems Ministry of Education of China, Northeastern University, Shenyang 110819, China

Corresponding author: Wei Sun (e-mail: weisun@mail.neu.edu.cn).

This work was supported by the Fundamental Research Funds for the Central Universities of China (Grant Nos. N180312012, N170308028).

ABSTRACT The pipeline system on the outside of aero-engine needs to work in resonance environment in some cases, therefore, it is necessary to reduce the vibration amplitude of pipeline systems to improve the reliability in the dynamic design stage. In this paper, a single pipeline system with multi-hoop supports was taken as the object and a method based on genetic algorithm to realize the layout of hoops and effectively reduce the resonance amplitude of pipeline system was proposed. Considering that the system belongs to the statically indeterminate structure, a new semi-analytical model was developed, that is, the pipeline is modeled under free boundary conditions firstly, and then the hoop is introduced into the pipeline system in the form of spring-damping structure. Meanwhile, in the process of modeling, to improve the analysis accuracy of the model, the non-uniform distribution springs were used to simulate the support stiffness of hoops, and the uniformly distributed dampers were used to simulate the support damping of hoops. Taking the position of the hoop as the design variable, the optimization model of the hoop layout with minimizing the maximum vibration amplitude of pipeline systems as the optimization objective was established, furthermore the optimization solution process of the hoop layout based on genetic algorithm was given. Finally, a case study was carried out to verify the rationality of the proposed semi-analytical model and the optimal hoop support positions were obtained by the proposed optimization model and method.

INDEX TERMS Genetic algorithm, hoop layouts, optimization of reducing the vibration amplitude, pipeline system with multi-hoop supports, semi-analytical method.

I. INTRODUCTION

The pipeline system on the outside of aero-engine is an important system which undertakes the task of transporting hydraulic medium such as fuel oil, lubricating oil, etc. It is usually fixed on the aero-engine casing by special hoops, so the vibration generated by the aero-engine rotor will be transmitted to the pipeline through the casing. When a certain order frequency of the pipeline system is consistent with the working frequency of aero-engine rotor, the resonance will occur and the vibration of the pipeline will be extremely severe and the vibration amplitude will be increased. The excessive vibration amplitude may lead to serious mechanical failures, such as pipe cracks, hoop looseness and fracture, which will seriously affect flight safety and even lead to catastrophic accidents. In order to avoid flight accidents caused by such mechanical failures, it is usually necessary to reduce the vibration amplitude of the pipeline system. Generally, the vibration amplitude of the pipeline is reduced by avoiding the frequency of the excitation source and then avoiding the resonance of the system. However, due to some reasons, the pipeline system has to work in the resonance environment sometimes, so it is necessary to reduce the vibration amplitude as much as possible in this condition. Fig.1 is a pipeline system with multi-hoop supports. It can be seen that a hoop is consists of metal belt and metal rubber and metal rubber is a kind of

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

homogeneous elastic porous material, which is made of metal wire and can provide damping for the pipeline system [1]. The hoop is mainly used to fix the pipeline and provide support stiffness and damping for the pipeline system. Therefore, the stiffness and vibration mode of the pipeline system can be changed by adjusting the position of the hoop, and then the vibration amplitude of pipeline can be changed, i.e. the vibration amplitude of the pipeline can be reduced by optimizing the position of the hoop.

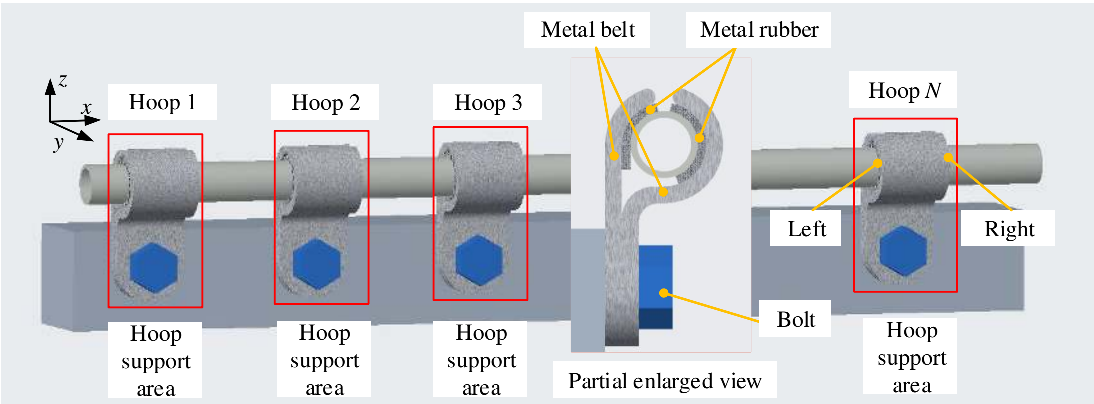


FIGURE 1. The schematic diagram of pipeline system.

In order to effectively complete the optimization research, a reasonable dynamic model needs to be created to analyze the vibration characteristics of pipeline system firstly. The aero-engine rotor will produce periodic exciting force during working, so many pipeline systems fixed on the outside of aero-engine casing are mainly forced vibration. At present, there are few dynamic models based on the pipeline system on the outside of aero-engine, but there are many researches on the dynamic modeling of pipeline system for other industrial fields. Modeling methods of pipeline system can be divided into transfer matrix method, finite element method, semi-analytical method or analytical method, etc. In order to investigate multi-span fluid-conveying pipe with multiple complex supports, a hybrid analytical method was developed in the study of Liu et al. [2], in fact, this method belongs to transfer matrix method. Liu and Li [3] also studied the dynamics of pipeline with multi-span boundary conditions. Koo and Park [4] used transfer matrix method to analyze the vibration of pipeline system with periodic supports. Li et al. [5] developed a user-defined pipe element and analyzed the vibration of the pipeline systems. Sadeghi and Karimidona [6] used an FEM-state space approach to study the dynamic behavior of the pipeline conveying fluid. Gao et al. [7-9] used spring element to simulate the boundary supports and developed the pipeline system conveying fluid by finite element method. Chai et al. [10] developed a dynamic modeling approach of the curved pipeline system with clamps and investigated the nonlinear vibration.

Because of less degrees of freedom and simple calculation formulas, the analytical or semi-analytical method has also been applied to the vibration analysis of the pipeline system. In order to study the influence of gas pressure on the pipeline, Tian et al. [11] established a pipeline model with different diameters by semi-analytical method. Païdoussis [12-17] made prominent contributions in the field of pipeline conveying fluid. In their studies, based on Euler-Bernoulli beam theory or Timoshenko beam theory, models of pipeline conveying fluid with different boundary conditions were created and the influence of fluid on the pipelines was analyzed. Jijun et al. [18] developed a model of pipeline conveying fluid with clamped-clamped boundary and analyzed the dynamic response by generalized integral transform technique. Huang et al. [19] used eliminated element-Galerkin method to study the pipeline conveying fluid with different supports. Li and Yang [20] adopted He’s variational iteration method to analyze the vibration of pipeline conveying fluid with various boundary conditions. Liang et al. [21] used Laplace transform and differential quadrature method to analyzed pipeline conveying fluid. Zhang [22] applied the method of multiple scales and the Galerkin’s procedure to analyze the dynamics of cantilevered pipe. Liang et al. [23] used the Hamilton principle to investigate the vibration of spinning pipes conveying fluid. Although a large number of scholars use semi-analytical or analytical methods to complete the modeling of pipeline system under various boundary conditions, these boundary conditions are some classical boundary conditions, such as cantilever pipeline, fixed support at both ends, etc. The pipeline system with multi-hoop supports belongs to the statically indeterminate structure, and it’s a challenge to conduct its modeling process.

In order to better simulate the real vibration characteristic of the pipeline system, it is necessary to deal with the boundary condition of the pipeline system reasonably. As mentioned above, the pipeline system is generally fixed by hoops, which is generally treated as elastic boundary conditions. Most scholars usually use spring-damping structure to simulate the elastic boundary

IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

conditions, i.e. a translational spring, a torsional spring and a damper are used to simulate the elastic support in one direction. However, it may not achieve satisfactory accuracy to simulate the hoop support with this method, which is mainly because the hoop has a certain width and the area supported by the hoop is a small section of pipeline instead of a point. The hoop is usually fixed on the aero-engine casing by bolts, so the stiffness of the hoop is affected by the bolt preload. Zhang et al. [24] found that it is unreasonable to set the spring stiffness affected by bolt preload to the same value for a specific structure. In addition, considering the influence of bolt preload, the damping effect with large bolt preload may be smaller than that with small bolt preload. Based on these situations, this paper will use non-uniform distribution spring stiffness value and uniform distribution damping value to simulate the support effect of the hoop.

In order to improve the dynamic performance of the pipeline system, some scholars studied the optimization of hoop layouts. By using the genetic algorithms to optimize the hoop locations, the noise in pipeline systems was reduced in the study of Kwong and Edge [25]. Similarly, Herrmann et al. [26] optimized the mounting position of the break pipe to reduce the sound of the pipeline systems. Tang et al. [27] adopted Sequential Quadratic Programming to optimize the hoops of the hydraulic pipeline systems and reduced the vibration of pipeline under random excitation. With smoothness and natural frequency taken into account, non-dominated sorting genetic algorithm-II was used to optimize the pipe layouts [28]. To reduce the size of optimization problem, sensitivity analysis method was used in the studies [29, 30] to find out the clamp position those which have little effect on optimization target and the result shows that the method can improve the efficiency of optimization. Li et al. [31] took the maximum impedance value as the optimization objective and obtained the optimal position of the clamp. Liu et al. [32] obtained the optimal hoop positions of pipeline system by optimization of avoiding vibration. For the optimization of hoop layouts, most scholars took reducing noise, increasing impedance and avoiding resonance frequency, etc. as optimization objectives to improve the dynamic performance of pipeline system. To the authors' knowledge, there are few relevant researches about the optimization of hoop layouts which only aims at reducing the resonance amplitude of the pipeline system.

In this paper, based on the optimization of hoop layouts for aero-engine pipeline as the background, a single pipeline system with multi-hoop supports is taken as the research object. The dynamic model and the optimization model of reducing vibration amplitude of the pipeline system are established. The hoop positions are taken as the design variable, and then the optimal hoop positions which can minimize the resonance amplitude of the pipeline system are obtained by using genetic algorithm. This paper is organized as follows. In Section 2, a new semi-analytical model was developed, that is, the pipeline is modeled under free boundary conditions firstly, and then the hoop is introduced into the pipeline system in the form of spring-damping structure. In Section 3, under the assumption that the pipeline system is in the resonance environment, an optimization model of hoop layouts with minimizing the maximum vibration amplitude of pipeline system as the optimization objective was established. In Section 4, the genetic algorithm was used to solve the optimization model. On the basis of a brief description of the genetic algorithm, the process of applying the genetic algorithm to optimize hoop layouts of pipeline system was described emphatically. In Section 5, a single pipeline system with three hoop supports was chosen as the research object, and the semi-analytical modeling method is used to model, and then the genetic algorithm is used to optimize hoop positions. The conclusions are listed in Section 6.

II. SEMI-ANALYTICAL MODELING OF THE PIPELINE SYSTEM WITH MULTI-HOOP SUPPORTS

Here, the Euler-Bernoulli beam theory and Lagrange equation are used to complete the semi-analytical modeling of the pipeline system with multi-hoop supports. Due to the pipeline system with multi-hoop supports belongs to statically indeterminate structure, as is shown in Fig. 2, the classical beam modeling method is not suitable for this study. Therefore, the pipeline and hoops are modeled separately. Firstly, the dynamic model of the pipeline under free boundary condition is carried out, and then the hoops are introduced into pipeline system in the form of spring and damper for dynamic modeling. The relevant modeling process is briefly described in the following.

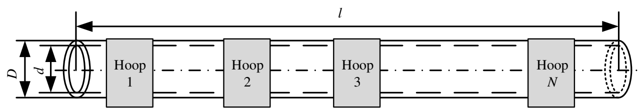


FIGURE 2. The pipeline system with multi-hoop supports.

A. THE PIPELINE MODELING UNDER FREE BOUNDARY CONDITIONS

As shown in Fig. 3, a three-dimensional rectangular coordinate system is established for the analysis of pipeline dynamics. A pipeline with free boundary conditions is considered and the length of pipe body is l, the outer diameter and inner diameter are D and d respectively. Only the transverse displacement w of the pipeline is considered here, i.e. the displacement can be along y or z direction.

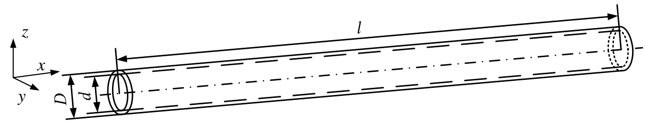


FIGURE 3. Pipeline with free boundary conditions.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

In an arbitrary straight pipeline model, the transverse displacement of pipeline at any time and position can be expressed as

$$w(x,t)=\sum_{j=1}^{n}a_j(t)\varphi_j(x) \tag{1}$$

where $a_j(t)=a_j\mathrm{e}^{\mathrm{i}\omega t}$, a_j is the corresponding coefficient, ω is the angular frequency of free vibration of the pipeline; $\varphi_j(x)$ is a series of characteristic polynomials, which can be obtained by Gram-Schmidt orthogonalization , and specific solving steps [33] are shown as follows,

$$\begin{cases}\psi_2(x)=(x-B_1)\psi_1(x)\\ \psi_{k+1}(x)=(x-B_k)\psi_k(x)-C_k\psi_{k-1}(x)\end{cases},\quad k\geq 2 \tag{2}$$

where

$$\begin{aligned}B_k&=\int_0^l x\psi_k^2(x)\mathrm{d}x\Big/\int_0^l\psi_k^2(x)\mathrm{d}x\\ C_k&=\int_0^l x\psi_k(x)\psi_{k-1}(x)\mathrm{d}x\Big/\int_0^l\psi_{k-1}^2(x)\mathrm{d}x\end{aligned} \tag{3}$$

Eq. (2) is normalized as

$$\varphi_k(x)=\frac{\psi_k(x)}{\sqrt{\int_0^l[\psi_k(x)]^2\mathrm{d}x}} \tag{4}$$

A series of characteristic orthogonal polynomials satisfying Eq. (5) can be obtained by Eqs. (2) - (4)

$$\int_0^l[\varphi_{q_1}(x)\varphi_{q_2}(x)]\mathrm{d}x=\begin{cases}0, & q_1\neq q_2\\ 1, & q_1=q_2\end{cases} \tag{5}$$

The first term $\varphi_1(x)$ of the characteristic orthogonal polynomial needs to satisfy the initial boundary conditions. For this paper, the boundary conditions that need to be satisfied are free boundary conditions at both ends.

Based on above displacement assumptions, the potential energy and kinetic energy of the pipeline under free boundary conditions can be obtained from Euler-Bernoulli beam theory, which can be described as

$$U=\frac{1}{2}\int_0^l EI[\frac{\partial^2 w(x,t)}{\partial x^2}]^2\mathrm{d}x \tag{6}$$

$$T=\frac{1}{2}\int_0^l\rho A[\frac{\partial w(x,t)}{\partial t}]^2\mathrm{d}x \tag{7}$$

where E is elastic modulus, I represents the moment of inertia of the cross-section of pipeline, ρ is the density of pipeline, and A represents the cross-sectional area of pipeline.

B. THE SUPPORT SIMULATION OF THE HOOP

It can be seen from Fig. 1 that hoops can fix pipeline by encircling, and will provide support stiffness and damping effect on it. In order to simplify the modeling, many scholars use spring and damper to simulate the support stiffness and damping.

In order to obtain the response of the pipeline system with multi-hoop supports, it is necessary to simplify the model reasonably to make the simplified result close to the real value. It can be seen from Fig. 1 that each hoop has a certain width, and it is unreasonable to simulate the support area with only one translational spring, one torsional spring and one translational damper. Therefore, the spring and damper with certain distribution forms are proposed to simulate support stiffness and damping effect of hoops. Referring to Fig. 4, a torsional spring, a translational spring and a translational damper are defined to form a spring-damping structure, and a hoop support area is simulated with m spring-damping structures. For the convenience of research, the width of m spring-damping structures is usually set to be the same as the width of hoop, i.e. the distance between the first spring-damping structure and the m-th spring-damping structure in red dotted line box. In the semi-analytical modeling, the position coordinates are established along x-axis direction shown in Fig. 1. The spring-damping structure is set at position p_ξ^ς to represent the corresponding constraint area of the hoop, where superscript (ς=1,2,..., N) of p represents the number of the hoop in pipeline system, and subscript (ξ=1,2,...,m) of p represents the number of spring-damping structures.

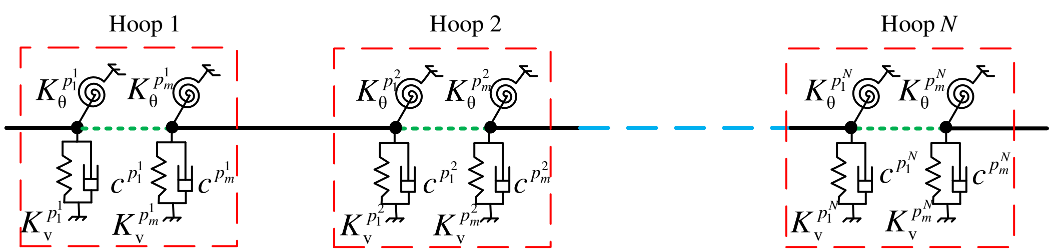
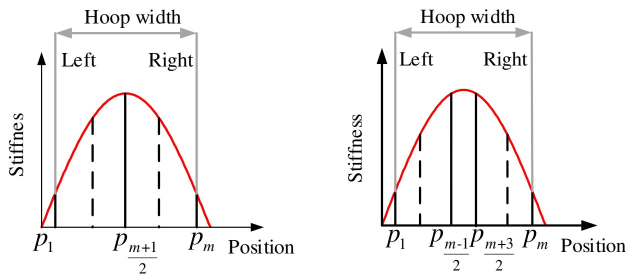


FIGURE 4. The simplified diagram of hoop-pipeline system.

As can be seen clearly from the enlarged view in Fig. 1, the hoop fixes pipeline by bolt connection, so the phenomenon of uneven bolt preload will occur. As mentioned above, m spring-damping structures are used to simulate the supporting effect of the hoop on pipeline. Considering the uneven force in the hoop support area due to uneven preload of bolts, it is unreasonable to set the stiffness value of the translational spring or the torsional spring to be same. In order to simulate the force situation of the hoop support area, it is assumed that the spring stiffness value of the hoop support area is set according to a distribution of a half-sinusoidal. Considering that the number of springs m may be odd or even, different distribution figures of the spring stiffness value are drawn for different situations. Fig. 5 (a) and (b) represent the distribution mode when m is odd and even respectively.



(a) when m is odd (b) when m is even

FIGURE 5. The distribution figure of the spring stiffness value

The damping effect in the hoop support area is produced by the metal rubber in hoops, and the damping effect of metal rubber may be relatively weak at the place where the force is large, thus it is different with the spring stiffness. For the convenience and simplification, the damping effect of the hoop is simulated by the uniformly distributed damping value.

According to the above distribution of the spring and damping, the formula of each spring stiffness and damping in the corresponding hoop support area can be written as

$$\begin{cases} K_\theta^{p_\xi^\varsigma} = K_\theta \sin(\frac{\xi\pi}{m+1}) \\ K_v^{p_\xi^\varsigma} = K_v \sin(\frac{\xi\pi}{m+1}), & \begin{cases} \varsigma = 1,2,\cdots N \\ \xi = 1,2,\cdots m \end{cases} \\ c^{p_\xi^\varsigma} = c \end{cases} \tag{8}$$

where $K_\theta^{p_\xi^\varsigma}$ is the stiffness value of ξ-th torsional spring in ς-th hoop, $K_v^{p_\xi^\varsigma}$ is the stiffness value of ξ-th translational spring in ς-th hoop, $c^{p_\xi^\varsigma}$ is the damping value of ξ-th damper in ς-th damper.

The stiffness value of the hoop is dispersive, and the stiffness value obtained by static test may be quite different from that of the hoop in real pipeline system [34]. In this study, the corresponding stiffness and damping values can be obtained by inverse identification method [35-36]. According to Eq. (8), only one translational stiffness value, one torsional stiffness value and one damping value need to be identified. The specific identification procedure is shown in Fig. 6.

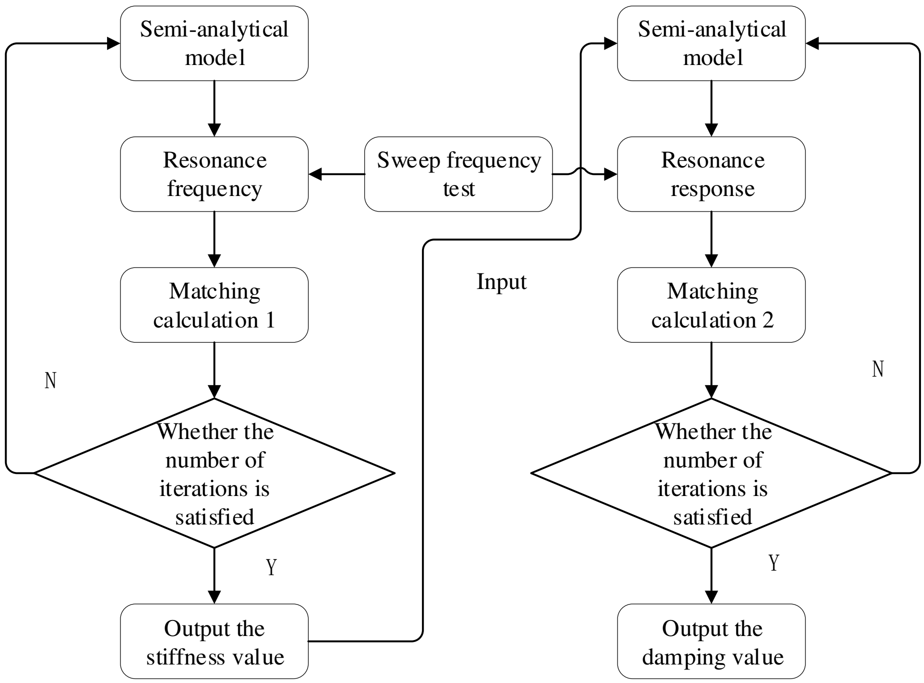


FIGURE 6. The procedure of the stiffness and the damping identification.

Firstly, the resonance frequency obtained by the semi-analytical model need be matched with that obtained by sweep frequency test (named as matching calculation 1). After the maximum number of iterations is satisfied, the obtained stiffness value is gotten and input into the semi-analytical model to calculate vibration response, and then the response value gotten by semi-analytical model is matched with that gotten by the frequency sweep test

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

(named as matching calculation 2). Based on this, the damping value is obtained after the maximum number of iterations is met. In this paper, the genetic algorithm is used to identify the stiffness and damping. In order to obtain more accurate identification value, several kinds of hoop layout schemes can be analyzed and tested at the same time. The objective function in matching calculation 1 can be described as

$$\min\ e_{\mathrm{f}}=\sqrt{[\sum_{i_1=1}^{n_1}\mu_{i_1}(e_{\mathrm{f}i_1}-\overline{e}_{\mathrm{f}})]/n_1} \tag{9}$$

where μ_{i_1} is the weight coefficient of i_1-th scheme of hoop layout, $e_{\mathrm{f}i_1}=\left|f_{i_1}^{\mathrm{A}}-f_{i_1}^{\mathrm{E}}\right|/f_{i_1}^{\mathrm{E}}$ is the absolute value of the difference between the calculated and tested first order frequencies in the i_1-th scheme of hoop layout, $f_{i_1}^{\mathrm{A}}$ is a certain order frequency of semi-analytical method in the i_1-th scheme of hoop layout, $f_{i_1}^{\mathrm{E}}$ is a certain order frequency of experiment in the i_1-th scheme of hoop layout, n_1 is the total number for the scheme of hoop layout, $\overline{e}_{\mathrm{f}}=\sum_{i_1=1}^{n_1}e_{\mathrm{f}i_1}\Big/n_1$.

The objective function in match calculation 2 can be described as

$$\min\ e_{\mathrm{R}}=\sqrt{[\sum_{i_1=1}^{n_1}\eta_{i_1}(e_{\mathrm{R}i_1}-\overline{e}_{\mathrm{R}})]/n_1} \tag{10}$$

where η_{i_1} is the weight coefficient of i_1-th scheme of hoop layout, $e_{\mathrm{R}i_1}=\left|R_{i_1}^{\mathrm{A}}-R_{i_1}^{\mathrm{E}}\right|/R_{i_1}^{\mathrm{E}}$ is the absolute value of the difference between the calculated and tested resonance responses in i_1-th scheme of hoop layout. $R_{i_1}^{\mathrm{A}}$ is a certain order response obtained by semi-analytical method in i_1-th scheme of hoop layout, $R_{i_1}^{\mathrm{E}}$ is a certain order response obtained by experiment in i_1-th scheme of hoop layout, $\overline{e}_{\mathrm{R}}=\sum_{i_1=1}^{n_1}e_{\mathrm{R}i_1}\Big/n_1$.

Parameters need to be set in the process of stiffness and damping identification based on genetic algorithm. First of all, it is necessary to set the maximum number of iterations, the population size, that is, the number of individuals in each generation where individuals refer to the translational spring stiffness, the torsional spring stiffness and the damping), binary digits, the crossover probability, and the mutation probability. Then, iterative calculation is performed. When the maximum number of iterations achieve the set value, the stiffness value of the translational spring and torsional springs and the damping value of damper can be obtained. Because the genetic algorithm is also used in the subsequent optimization of hoop layouts, the detailed description of the algorithm is shown in Section 4.

C. THE MODELING OF THE PIPELINE SYSTEM WITH MULTI HOOP SUPPORTS

The energy analysis of the pipeline with free boundary conditions has been presented in previous section, the energy generated by the spring-damping structure need to be considered further. The energy $U_{\mathrm{s}}^{p_\xi^\varsigma}$ generated by a translational spring and a torsional spring at position p_ξ^ς can be expressed as

$$U_{\mathrm{s}}^{p_\xi^\varsigma}=\frac{1}{2}K_{\mathrm{v}}^{p_\xi^\varsigma}w^2(p_\xi^\varsigma,t)+\frac{1}{2}K_{\theta}^{p_\xi^\varsigma}[\frac{\partial w(x,t)}{\partial x}|_{x=p_\xi^\varsigma}]^2 \tag{11}$$

Then, the energy U_{s} generated by translational and torsional springs at all locations can be written as

$$U_{\mathrm{s}}=\sum_{\varsigma}^{N}\sum_{\xi}^{m}U_{\mathrm{s}}^{p_\xi^\varsigma} \tag{12}$$

The energy $C^{p_\xi^\varsigma}$ produced by the damper at position p_ξ^ς can be expressed as

$$C^{p_\xi^\varsigma}=\frac{1}{2}c^{p_\xi^\varsigma}[\frac{\partial w(x,t)}{\partial t}|_{x=p_\xi^\varsigma}]^2 \tag{13}$$

Similarly, the energy C generated by dampers at all locations can be written as

$$C=\sum_{\varsigma}^{N}\sum_{\xi}^{m}C^{p_\xi^\varsigma} \tag{14}$$

In general, the pipeline system fixed on the engine casing is mainly affected by the working frequency of the aero-engine rotor, which is under base excitation and belongs to the forced vibration. In order to solve the response of the pipeline system, the work W done by the base excitation need to be calculated in addition to the above energy. The expression of work done by the base excitation is

$$W=\rho gA\int_0^l w(x,t)\mathrm{d}x \tag{15}$$

Finally, the Lagrange equation can be written based on the above equations

$$\begin{cases}\dfrac{\partial J}{\partial\dot{a}_r(t)}-\dfrac{\partial J}{\partial a_r(t)}+\dfrac{\partial C}{\partial\dot{a}_r(t)}=\dfrac{\partial W}{\partial a_r(t)}\\(r=1,2,...,n)\end{cases} \tag{16}$$

where $J=T-U-U_{\mathrm{s}}$.

Furthermore, the dynamic equation of the pipeline system can be obtained by simplifying Eq. (16)

$$(\boldsymbol{K}+\boldsymbol{K}_{\mathrm{s}}+\mathrm{i}\omega\boldsymbol{C}-\omega^2\boldsymbol{M})\boldsymbol{a}=\boldsymbol{F} \tag{17}$$

where $\boldsymbol{K}$ is the stiffness matrix of pipeline, $\boldsymbol{K}_{\mathrm{s}}$ represents the stiffness matrix superimposed by translational springs and torsional springs, $\boldsymbol{C}$ is the damping matrix of pipeline, $\boldsymbol{M}$ is the mass matrix of the pipeline, $\boldsymbol{F}=[F_1,F_2,...,F_n]^{\mathrm{T}}$ is the column vector of the exciting force, $\boldsymbol{a}$=$[a_1,a_2,...,a_n]^{\mathrm{T}}$ is the coefficient vector not including time t, and ω is the arbitrary excitation angular frequency.

The characteristic equation used to solve natural frequencies of pipeline system can be obtained by omitting damping and exciting force terms and can be expressed as

$$|\boldsymbol{K}+\boldsymbol{K}_{\mathrm{s}}-\omega_{\tau}^{2}\boldsymbol{M}|=0,\quad \tau=1,2,\ldots,n \tag{18}$$

According to Eq.(18), the τ -th order natural angular frequency of pipeline system can be obtained, and the τ -th order natural frequency of pipeline system can be written as $f_{\tau}=\dfrac{\omega_{\tau}}{2\pi}(\tau=1,2,\ldots,n)$.

According to Eq. (17), the frequency domain response of any position of the pipeline can be gotten and written as

$$X=\left|\boldsymbol{\varphi}(x_u)\frac{\boldsymbol{F}}{\boldsymbol{K}+\boldsymbol{K}_{\mathrm{s}}+\mathrm{i}\omega\boldsymbol{C}-\omega^{2}\boldsymbol{M}}\right| \tag{19}$$

where $\boldsymbol{\varphi}(x_u)=[\varphi_1(x_u),\varphi_2(x_u),\ldots,\varphi_n(x_u)]$ represents the row vector.

It should be noted that the above derivation mainly focuses on the radial movement of single pipeline, such as y or z direction in Fig. 3, and does not consider the coupling of two directions. In addition, it is assumed that the damping in pipeline system is completely provided by hoops.

III. THE OPTIMIZATION MODEL FOR THE HOOP LAYOUT OF PIPELINE SYSTEM

In order to achieve the goal of effectively reducing the vibration amplitude of pipeline system by optimizing the hoop position, and it is necessary to create a reasonable optimization model. Fig. 7 shows the essential factors to be considered in creating the optimization model, including hoop positions, response measuring points and optimization objectives. The position of hoops is the design variable, the specific value is calculated with the left end as coordinate origin, and the section line area in the figure shows the allowed movable range of the hoop. The response measuring point is on pipeline, and the response is different when different measuring points are selected for a chosen resonance state. The optimization objective describes the specific requirements for reducing the vibration amplitude of pipeline system.

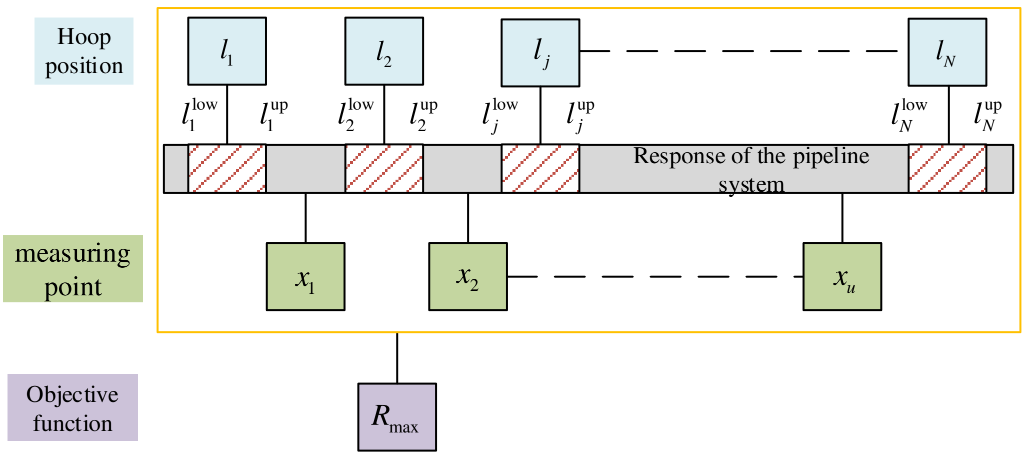


FIGURE 7. Schematic diagram of hoop position and objective function.

A. SPECIFIC OPTIMIZATION MODEL

In aero-engines, due to the limited outer space of the casing, positions and quantity of hoops that can be installed are limited, so the adjustable range for natural frequencies of pipeline system is limited. Maybe there is a specific situation, no matter how to adjust the position of the hoop, the pipeline system will be in the resonance frequency range, which will result in excessive vibration amplitude. In order to reduce the damage caused by resonance for the pipeline as much as possible, it is necessary to take the response of the pipeline system as the optimization objective and the position of the hoop as the design variable to reduce the vibration amplitude of the pipeline. Because the change of hoop position will change the measuring point position of the maximum response value, it is unreasonable to take minimizing the response value of a fixed measuring point as the optimization objective. The change of hoop position will change the position of the maximum response value point, it is unreasonable to take minimizing the response value of a fixed measuring point as the optimization objective. Based on the background that a certain order of pipeline system works in the resonance frequency range, an optimization model is established and the optimization objective is set as minimizing the maximum response of pipeline system. The detailed optimization model can be described as

$$\begin{aligned}&\min\ \ R_{\max}(l_1,l_2,\ldots.,l_j,\ldots,l_N,x_u)\\&\text{s.t.}\ \ \ l_j^{\mathrm{low}}<l_j<l_j^{\mathrm{up}}\ \ (j=1,2,\ldots,N)\end{aligned} \tag{20}$$

where $R_{\max}$ is the maximum response value of pipeline system, l_j is the position of j-th hoop, l_j^{low} is the lower limit of the position of j-th hoop, l_j^{up} is the upper limit of

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

the position of j-th hoop, x_u is the position of the measuring point for the maximum response of pipeline system.

B. SOLUTION OF THE MAXIMUM RESPONSE VALUE FOR PIPELINE SYSTEM

During the optimization calculation process, the position of the hoop is always changing. As mentioned above, because the change of hoop position will lead to the change of the maximum response position, the maximum response value and the position of measuring point should be accurately obtained in each iteration calculation to ensure reasonable optimization result. The following describes the method for obtaining the maximum response of the piping system during the optimization process.

In the optimization process, the minimum movement accuracy of the hoop is set as δ, and the movement accuracy of the measuring point is also set as δ. In the optimization process, the minimum movement accuracy of the hoop and the measuring point are both set as δ. Therefore, there are $\frac{l}{\delta}+1$ response measuring points on the pipeline, and the position of each response measuring point can be expressed as $x_u(u=0,1,...,\frac{l}{\delta})$. The process of solving the maximum response is described in Fig. 8: Firstly, set the starting frequency f_start, ending frequency f_end and frequency interval f_t; At the beginning, frequency f=f_start is input to execute the main loop and the measuring point position x_u=x_0 is input to execute the nested loop, and then the response value is calculated according to Eq. (19) and the calculation result is recorded; When $u \le \frac{l}{\delta}$, the nested loop will continue to be executed; When $u > \frac{l}{\delta}$, the nested loop will be ended, and then the maximum response value is calculated and the result is recorded at this time; When $f \le f\_\text{end}$, the main loop will continue to be executed; When $f > f\_\text{end}$, the main loop will end, and then the maximum response value of the pipeline system will be obtained.

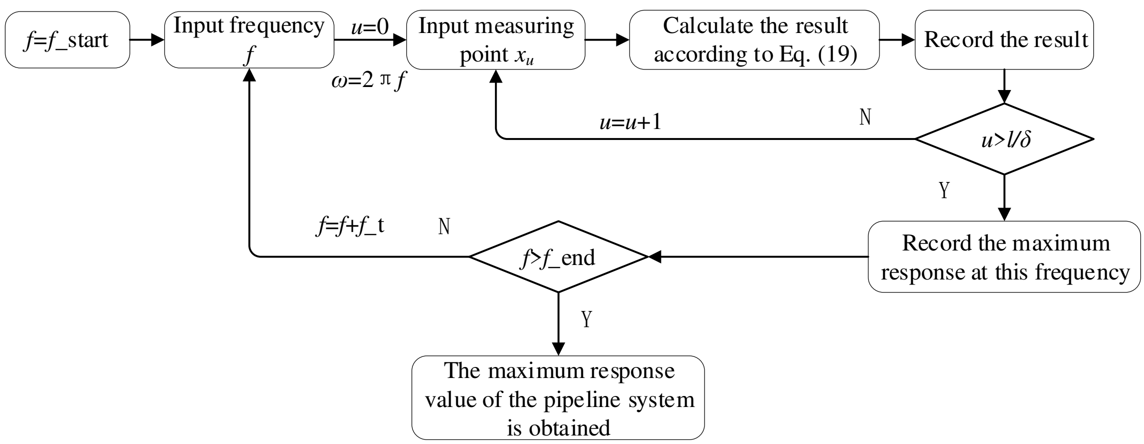


FIGURE 8. Solution flow of maximum response value for pipeline system.

IV. SOLUTION BASED ON GENETIC ALGORITHM

A. GENETIC ALGORITHM

Inspired by the biological genetic phenomenon in nature, Holland and his colleagues developed the theory and method of genetic algorithm [37]. In genetic algorithm, the binary codes are used to describe decision variable and a chromosome is composed of multiple binary codes. At the beginning, a random binary code with a certain length is used to represent the chromosome, that is, the genetic information of an individual. Then, two chromosomes and random crossover positions were selected according to the crossover probability, and the corresponding binary codes on the two chromosomes were exchanged to generate two new individuals. Similarly, the chromosome and random position are selected according to the mutation probability, and the binary code of the position is reversed to generate a new individual.

Genetic algorithm (GA) is an algorithm that simulates the genetic phenomena in nature. It has the advantages of parallel search, highly efficient search and global search. Genetic algorithm takes fitness function value transformed from objective function value as search information, which is suitable for solving objective function which cannot be derived in practical application. These advantages make genetic algorithm become a common optimization algorithm in engineering.

B. OPTIMIZATION OF HOOP LAYOUT

In Section 3, an optimization model was proposed to minimize the maximum response of pipeline system. The following will describe the optimization process of the hoop layout based on genetic algorithm.

In genetic algorithm, the population is composed of multiple individuals, and each individual is composed of e decision variables. These e decision variables can be written as $\boldsymbol{X}=[X_1,X_2,...,X_e]$ and expressed by the symbol $X_b(b=1,2,...,e)$. Decision variables can be regarded as the phenotypes of individuals, and each individual has its own genetic information (chromosomes). In the optimization of hoop layouts, the positions of N hoops represent N decision variables. These N decision variables can be written as vector $\boldsymbol{l}=[l_1,l_2,...,l_N]$ or marked as $l_j(j=1,2,...,N)$, and the N positions represent an individual. The binary codes corresponding to N hoop positions represent chromosomes. The relationship among hoop position, individual, population, chromosome and gene is illustrated in Fig. 9.

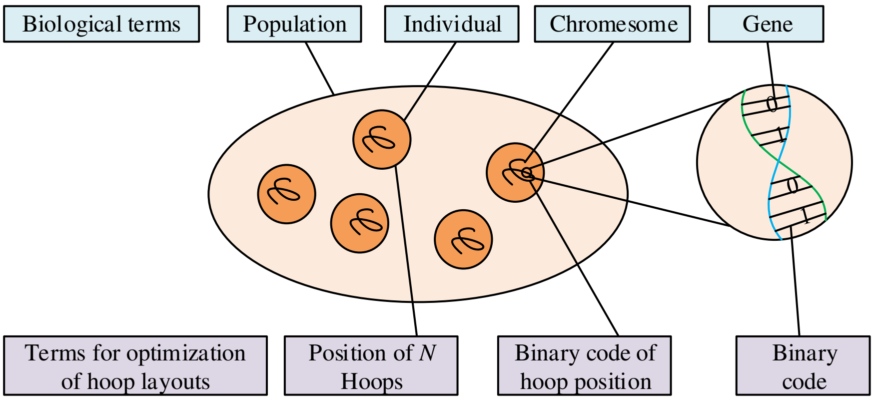


FIGURE 9. Relationship between biological terms and optimization of hoop layouts.

In genetic algorithm, the meanings of the terms of different optimization problems are diverse. Here, individual, chromosome and gene are defined as follows:
Individual: Position of N hoops.
Chromosome: A string of binary codes corresponding to the positions of N hoops.
Gene: Binary codes.

The optimization procedure of hoop layout is shown in Fig. 10

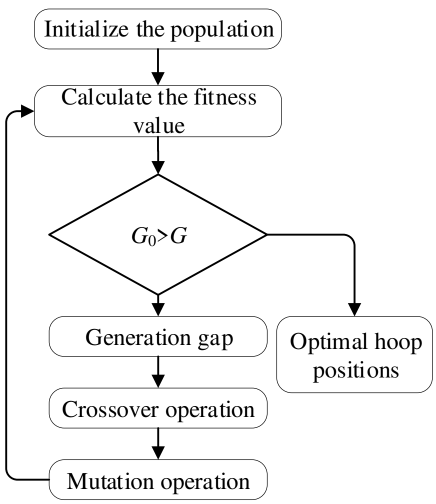


FIGURE 10. Procedure of hoop layout optimization.

1) Initialization: Initialize the population and set the generation counter G_0=0. Set the maximum number of iterations is G.
2) Individual evaluation: Calculate the fitness value according to the objective function.
3) Generation gap: According to a certain probability, select the excellent individual.
4) Crossover operation: For the selected individuals, a part of chromosomes between individuals are exchanged to generate new individuals according to a certain probability.
5) Mutation operation: For the selected individuals, some genes on the individuals are selected to change according to a certain probability to generate new individuals.
6) Termination conditions: After the above steps, if $G_0 \le G$, the program will continue to run from step 2; if $G_0>G$, the optimal individual value, that is, the optimal hoop position, will be output.

V. CASE STUDY

A. DESCRIPTION OF THE PROBLEM

In this paper, a single pipeline system supported by three hoops is chosen to display the semi-analytical method of pipeline system modeling and the optimization method of hoop layouts, and only the y-direction vibration shown in Fig. 3 is considered here. The geometric parameters and material parameters of the pipeline are shown in Tab. 1

TABLE 1. The geometric parameters and material parameters of the pipeline

Length of the pipeline *l*/mm	Outer diameter *D*/mm	Inner diameter *d*/mm	Elastic modulus *E*/Pa	Density *ρ*/kg/m^3	Poisson's ratio
500	8	6.4	1.99×10^{11}	7850	0.3

The structure and shape of these three hoops are identical, and the width of each hoop is 14mm. Therefore, the coordinate of the left side of hoops is chosen to express the position of each hoop. Firstly, four schemes of hoop layouts are selected randomly and the specific positions of the hoop are shown in Tab.2 and Fig. 11. The pipeline is fixed on the fixture by hoops and bolts, and the preload is $8\ \text{N}\cdot\text{m}$.

TABLE 2. Scheme of hoop layouts (m)

scheme	Hoop 1	Hoop 2	Hoop 3	Measuring point
1	0.025	0.150	0.400	0.325
2	0.030	0.160	0.430	0.400
3	0.049	0.251	0.426	0.325
4	0.103	0.250	0.450	0.225

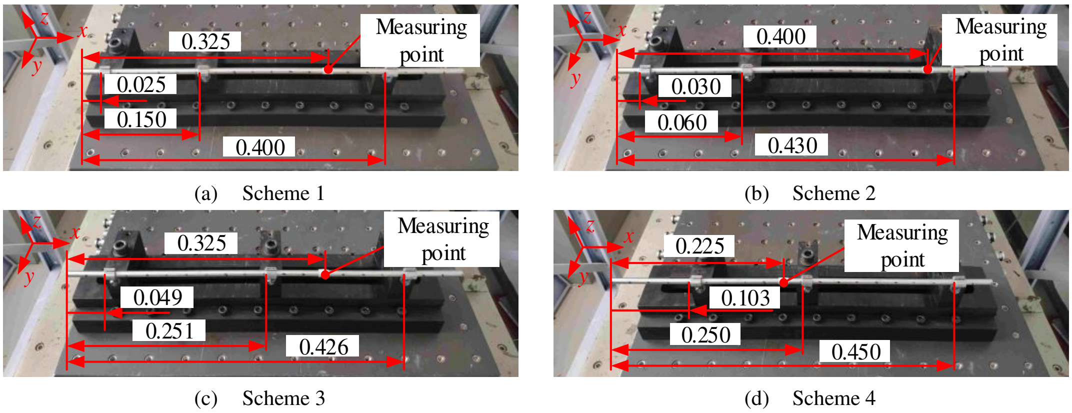


FIGURE 11. Scheme of hoop layouts

The purpose of this paper is to reduce the vibration amplitude of the pipeline system, so it is necessary to carry out sweep frequency test to obtain the response of pipeline system at the measuring point. The sweep frequency test system is shown in Fig. 12.

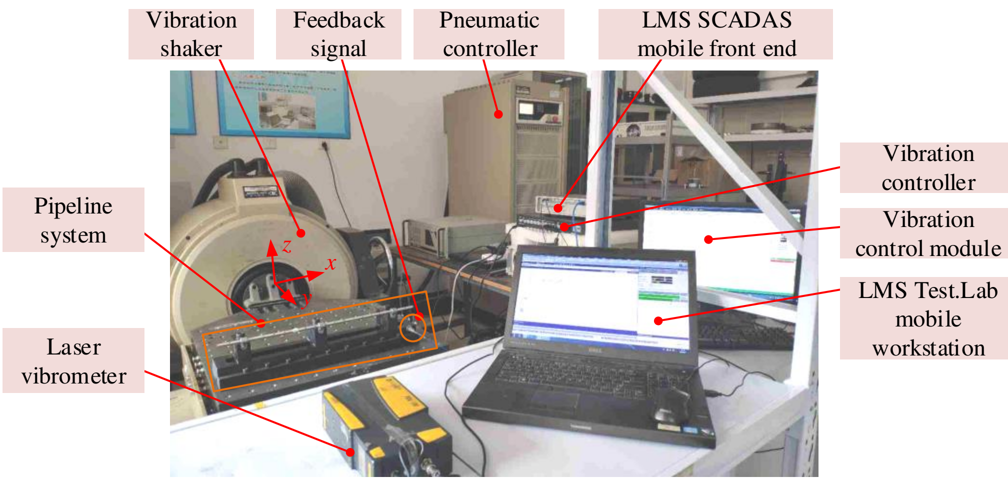


FIGURE 12. The sweep frequency test system

The yellow box in Fig. 12 shows the pipeline system. In the specific experiment, the pipeline system is placed according to the hoop layout scheme shown in Fig. 11, and the laser vibrometer is used to measure the vibration in *y* direction. The process of the experiment is described in Fig. 13.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

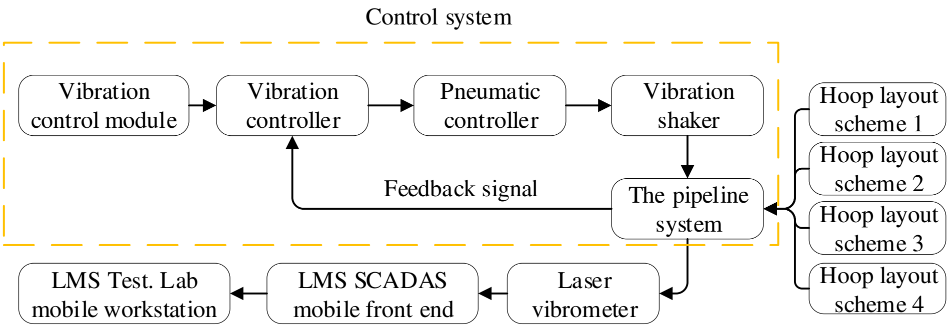


FIGURE 13. The process of the experiment

At the beginning of the experiment, the sinusoidal excitation signal is input into the vibration control module, and the excitation amplitude is set as 0.1g. The excitation signal is generated in the vibration controller and transmitted to the vibration shaker by the pneumatic controller, thus driving the pipeline system to do reciprocating motion in *y* direction, and forcing the pipeline to produce vibration in *y* direction. Then the vibration signal measured by laser in *y* direction is collected by LMS SCADAS mobile front end and transmitted to the LMS Test. Lab mobile workstation. Finally, the resonance responses of four kinds of hoop layout schemes can be obtained, and these response data are mainly used for verifying the model. The results of the first order resonance frequency and resonance response are shown in Tab. 3 and Fig. 14.

TABLE 3. Experimental results of the pipeline system

Hoop layout scheme	Scheme1	Scheme2	Scheme3	Scheme4
First order resonance frequency (Hz)	513.75	489	784	463
First order resonance response (m/s)	0.143620	0.144579	0.041368	0.012742

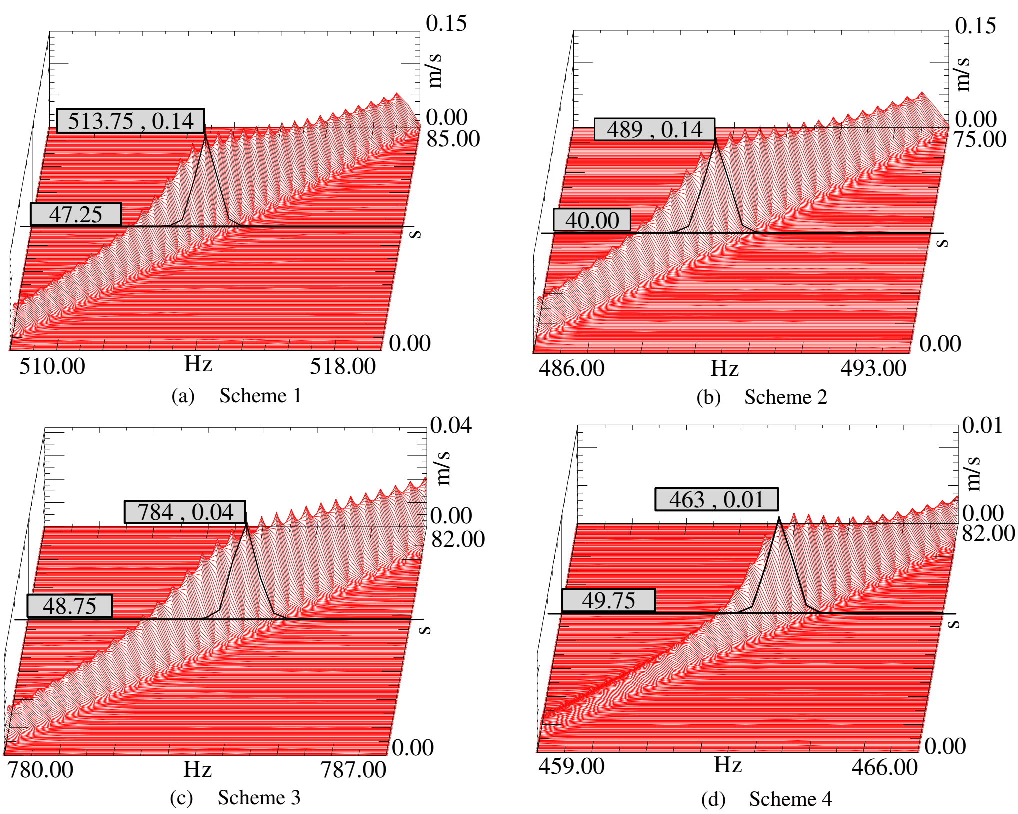


FIGURE 14. Sweep frequency test results of the pipeline system

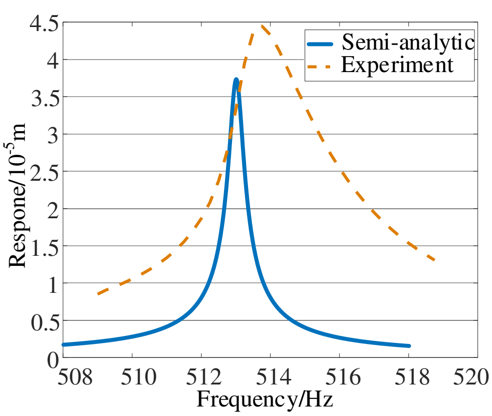
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2020.3044087, IEEE Access

B. SOLUTION FOR VIBRATION RESPONSE OF THE PIPELINE SYSTEM

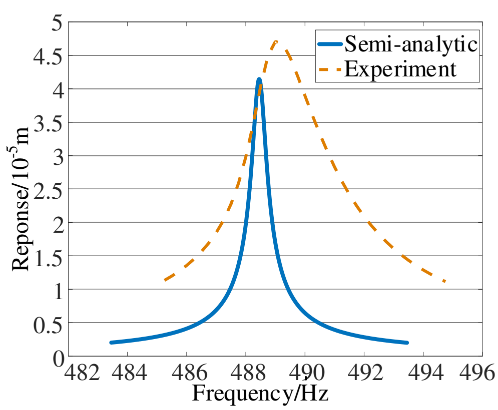
After verification, the required accuracy can be achieved when the number of polynomials is set as 20. The more spring-damping structures this system possesses, the closer to the actual constraint state of the hoop for the pipeline, but too many spring-damping structures will lead to the decrease of calculation efficiency. Therefore, this paper sets m = 15, that is, fifteen translational springs, fifteen torsional springs and fifteen dampers are used to simulate a hoop support. The distribution form shown in Fig. 5 (a) is adopted to describe the distribution of spring stiffness value here. In the process of modeling, the left end of the pipeline is taken as the coordinate origin, and the specific positions of three hoops in each scheme are determined according to the above four hoop layout schemes. When the position of the hoop is determined, the position of each spring-damping structure is also determined. The equation of kinetic energy and potential energy of pipeline in free vibration state can be obtained by Eqs. (1) - (7), and then equations of translational spring, torsional spring and damping in each hoop can be obtained according to Eq. (8). According to Eqs. (11) - (14), the energy equations of spring and damping at this position are obtained. Finally, according to Eq. (15), the equation of excitation force is obtained and Lagrange equation is constructed, thus the Eq. (19) used to solve vibration response is obtained. Although the expression of the response can be obtained by the above steps, the stiffness value and damping value need to be determined by the inverse identification method. Firstly, the parameters are set. Specifically, the maximum number of iterations is 100, the number of individuals is 80, the number of binary codes is 100, the crossover probability is 0.7, and the mutation probability is 0.01. Then, according to Eq. (9), the objective function used to identify supporting stiffness is constructed, and the weight coefficient of each item is 1. Finally, the stiffness identification is carried out several times and the result of each identification is consistent. The translational spring stiffness $K_v = 6.2133\times10^5\,\mathrm{N/m}$ and the torsional spring stiffness $K_\theta = 145.2\mathrm{N\cdot m/rad}$. The natural frequency of pipeline system can be obtained by taking the translational stiffness and torsional stiffness values obtained from the inverse identification into the model and omitting the damping term. Because the damping does not affect the resonance frequency when calculating the semi-analytical model, the natural frequency can be considered as the resonant frequency. Similarly, the objective function used to identify supporting damping is constructed according to Eq. (10), and the weight coefficients are 0.05, 0.05, 0.7 and 0.2 respectively. Then, the damping identification is also carried out several times and the result of each identification is consistent. The damping value is c=1.932$\mathrm{N\cdot m/s}$. Because of the velocity response measured in the experiment, it needs to be transformed into displacement response. The comparison between the results of semi-analytical model and experimental results are shown in Table 4 and Fig. 15.

TABLE 4. The comparison between the results of semi-analytical model and experiment

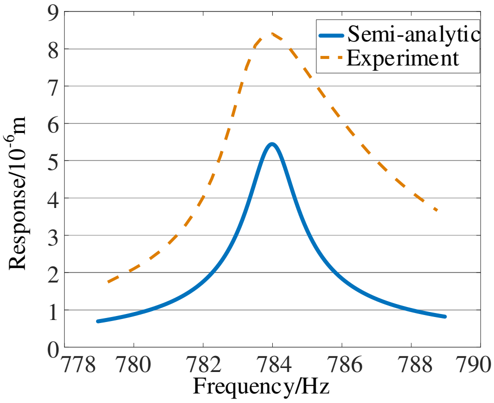
Schemes of hoop layouts	Resonance frequency			Resonance response		
	Experiment Hz	Semi-analytic Hz	Difference%	Experiment Hz	Semi-analytic Hz	Difference%
1	513.75	513.01	0.144	4.449	3.734	16.071
2	489	488.45	0.112	4.706	4.144	11.942
3	784	783.97	0.004	0.8398	0.5444	35.175
4	463	460.95	0.443	0.4380	0.5918	35.114



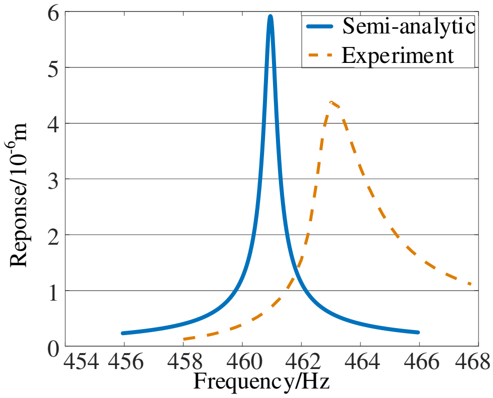
(a) Scheme 1



(b) Scheme 2



(c) Scheme 3



(d) Scheme 4

FIGURE 15. The comparison between the results of semi-analytical model and experiment

The maximum difference of resonance frequency is 0.443% and the maximum difference of resonance response is 35.175% by comparing four different hoop layout schemes, and the results objectively illustrate the rationality of semi-analytical modeling in this paper.

C. OPTIMIZATION OF HOOP LAYOUTS

In this paper, the objective is to minimize the maximum vibration response of pipeline system, the stiffness value and damping value obtained by inverse identification method in Section 4.2 are input into the semi analytical model, and the positions of the hoop are set as variables to facilitate the call of genetic algorithm, the length of binary code is 50, the generation gap is 0.95, the crossover probability is 0.7, the mutation probability is 0.01, the iteration counter is set as 0, and the objective function is $R_{\max}(l_1,l_2,l_3,x_u)$, the variable range of three hoops is shown in Tab. 5.

TABLE 5. The variable range of hoops (m)

Variable range	Hoop 1	Hoop 2	Hoop 3
Lower limit	0.020	0.120	0.420
Upper limit	0.080	0.180	0.480

The movement accuracy of the hoop and measuring point are both set as 1 mm, so there are 501 measuring points. In order to obtain the maximum response value of pipeline system, the range of first order frequency f_1 is set as 10Hz, that is, $f\_\text{start}{=}f_1-5\text{Hz}$, $f\_\text{end}{=}f_1+5\text{Hz}$, and the frequency interval is set as $f\_\text{t}{=}0.01\text{Hz}$, then, f_1 according to the method proposed in Section 3.2, the maximum vibration response value of pipeline system can be obtained.

After several optimization tests and according to the above settings, the convergence results can be obtained, and each optimization calculation time is less than 65 seconds. One of the optimization results is chosen as an example, and the optimization results are shown in Fig. 16.

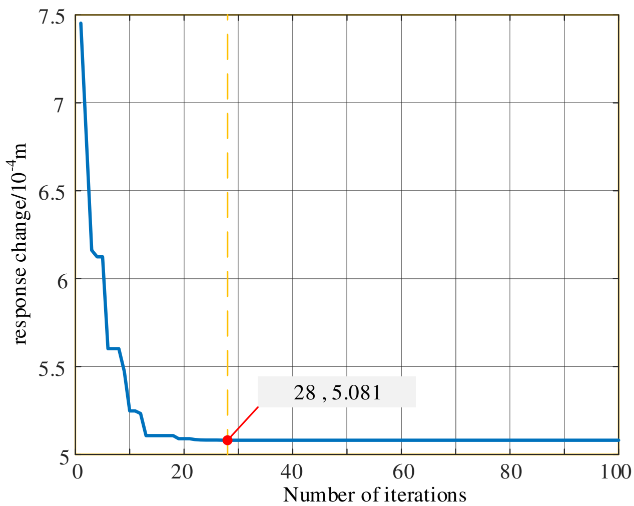
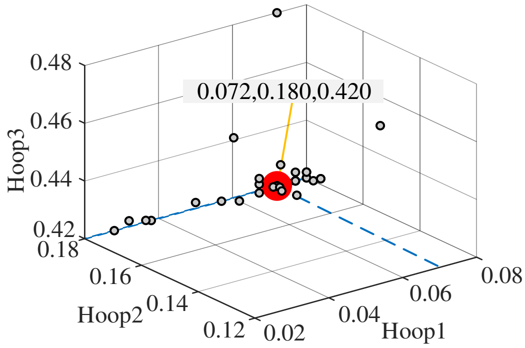
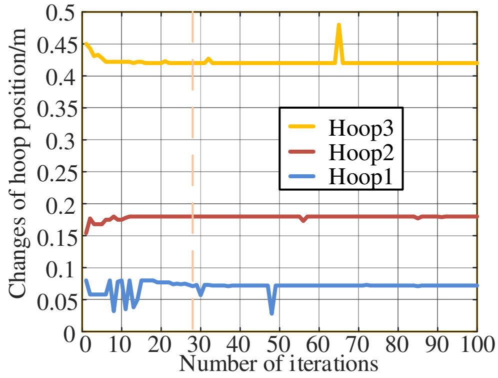


FIGURE 16. Optimization results of the pipeline system response

It can be seen from Fig. 16 that convergence results have been obtained in the 28th generation. At this time, the maximum response value of the pipeline system is 5.081×10^{-5}m , and the position of measuring point is 0.311m, and the position of the hoop is shown in Fig. 17.



(a) Individual optimization



(b) Changes of hoop position

FIGURE 17. Changes of hoop position

The grey circle in Figure 17(a) represents an individual in the genetic algorithm. It can be seen from figure 17(a) that the individual finally converges to the red circle, which is the optimal position of the individual, that is, the optimal position of three hoops, and the values are $l_1 = 0.072\text{m}$, $l_2 = 0.180\text{m}$,and $l_3 = 0.420\text{m}$. Fig. 17(b) shows the change of each hoop position. Corresponding to Fig. 16, the optimal position has been obtained in the 28th generation. In the subsequent iteration, the fluctuation of hoop position is caused by genetic algorithm itself. In order to verify the reliability of optimization results, two groups of hoop layout schemes are randomly selected to compare with the optimization results. The comparison results of the selected hoop layout scheme and the optimal position are shown in Tab. 6. It can be seen from the results in Tab. 6 that the results of schemes 1 and 2 are larger than those of the optimal position, which verifies the reliability of the optimization results in this paper.

TABLE 6. Validation of optimization results (m)

Hoop layout scheme	Hoop 1	Hoop 2	Hoop 3	Measuring point	Resonance response
Scheme 1	0.058	0.139	0.470	0.315	$4.6626\text{x}10^{-4}$
Scheme 2	0.069	0.146	0.426	0	$1.3305\ \text{x}10^{-4}$
Optimal position	0.072	0.180	0.420	0.311	$0.5081\ \text{x}10^{-4}$

VI. CONCLUSION

It is crucial to reduce the vibration amplitude of the aero-engine pipeline system in dynamic design stage. This paper takes the pipeline system with multi-hoop supports as the object, and proposes a method based on genetic algorithm to optimize the layout of hoops to reduce the vibration amplitude effectively. Some important conclusions are listed as follows:

(1) This paper proposes to model the pipeline and the hoop separately to solve the problem of statically indeterminate pipeline modeling supported by multiple hoops. The specific process of modeling is to model the pipeline under free boundary conditions first, and then introduce the hoop into pipeline system in the form of spring-damping structures. During the process of modeling, the influence of hoop width and bolt preload band on the stiffness of pipeline system are considered, a non-uniform distribution spring (specifically half period of sinusoidal function) is proposed to simulate the support stiffness of the hoop, and the uniformly distributed damper is used to simulate the damping provided by the hoop. The example shows that the difference of resonance frequency obtained by analysis is less than 0.443% compared with the experiment value, and the resonance response difference obtained by analysis is less than 35.175%, which proves that this modeling method can effectively simulate the dynamic characteristics of pipeline system.

(2) The high pressure and low pressure rotors in aero-engine are main reasons for the vibration of pipeline system attached to the casing, and in some cases, the pipeline system inevitably works in resonance environment. In this paper, minimizing the maximum response of pipeline system is taken as the optimization objective, and the hoop position is taken as the design variable. Through optimization, the vibration amplitude of the pipeline is reduced, which has a certain guiding role for the hoop layout.

(3) Genetic algorithm is a high efficient intelligent optimization algorithm, which can effectively solve the optimization problem in engineering. The optimization problem of the hoop layout described in this paper can also be well solved. The individual in the genetic algorithm corresponding to the optimization model represents the position of each hoop. The example calculation shows that this algorithm has high convergence speed and calculation efficiency.

Thus the optimal supporting position of the hoop which can effectively reduce the vibration amplitude can be quickly found.

REFERENCES

[1] D. Z. Zhang, F. Scarpa, Y. H. Ma, J. Hong and Y. Mahadik, "Dynamic mechanical behavior of nickel-based superalloy metal rubber," *Materials & Design*, vol, 56, pp. 69-77, 2014.

[2] M. Y. Liu, Z. C. Wang and Y. Z. Qu, "Vibration response of multi-span fluid-conveying pipe with multiple accessories under complex boundary conditions," *European Journal of Mechanics A-solids*, vol, 72, pp. 41-56, 2018.

[3] G. M. Liu and Y. H. Li, "Vibration analysis of liquid-filled pipelines with elastic constraints," *Journal of Sound and Vibration*, vol.330, no. PP. 3166-3181, 2011.

[4] G. H. Koo and Y. S. Park, "Vibration reduction by using periodic support in a piping system," *Journal of Sound and Vibration*, vol. 210, no. 1, pp. 53–68, 1998.

[5] Z. Y. Li, J. J. Wang and M. X. Qiu, "Dynamic Characteristics of Fluid-Conveying Pipes with Piecewise Linear Support," *International Journal of Structural Stability and Dynamics*, vol. 16, no. 4, 2016. Doi:10.1142/s021945541550025x.

[6] M. H. Sadeghi and M. H. Karimidona, "Dynamic behavior of a fluid conveying pipe subjected to a moving sprung mass — An FEM-state space approach". *International Journal of Pressure Vessels and Piping*, vol. 88, no. 4pp. 123-131, 2011.

[7] P. X. Gao, J. Y. Zhai, Y. Y. Yan., Q. K. Han, F. Z. Qu and X. H. Chen, "A model reduction approach for the vibration analysis of hydraulic pipeline system in aircraft.," *Aerospace Science and Technology*, vol. 49, pp. 144-153, 2016.

[8] P. X. Gao, J. Y. Zhai, F. Z. Qu and Q. K. Han, "Vibration and damping analysis of aerospace pipeline conveying fluid with constrained layer damping treatment," Proceed*ings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, vol. 32, no. 8, pp. 1529-1541, 2018.

[9] P. X. Gao, J. Y. Zhai and Q. K. Han, "Dynamic response analysis of aero hydraulic pipeline system under pump fluid pressure fluctuation," *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*. vol. 233, no. 5, pp. 1585-1595, 2019.

[10] Q. D. Chai, J. Zeng, H. Ma, K. Li and Q. K. Han, "A dynamic modeling approach for nonlinear vibration analysis of the L-type pipeline system with clamps," *Chinese Journal of Aeronautics*, 2020.

[11] J. L. Tian, C. F. Yuan, L. Yang, C. M. Wu, G. Liu and Z. Yang, "The Vibration Characteristics Analysis of Pipeline Under the Action of Gas Pressure Pulsation Coupling," *Journal of Failure Analysis and Prevention*, vol. 16, no. 3, pp. 499-505, 2016.

[12] M. P. Païdoussis and N. T. Issid, "Dynamic Stability of Pipes Conveying Fluid," *Journal of Sound and Vibration*, vol. 33, no. 3, pp. 267-294, 1974.

[13] M. P. Païdoussis, T. P. Luu and B. E. Laithier, "Dynamics of finite-length tubular beams conveying fluid," *Journal of Sound and Vibration*, vol, 106, no. 2, pp. 311-331, 1986.

[14] M. P. Païdoussis, "Flutter of Conservative Systems of Pipes Conveying Incompressible Fluid," *Journal of Mechanical Engineering Science*, vol. 17, no. 1, pp. 19-25, 1975.

[15] M. P. Païdoussis and B. E. Laithier, "Dynamics of Timoshenko Beams Conveying Fluid," *Journal of Mechanical Engineering Science*, vol. 18, no. 4, pp. 210-220, 1976.

[16] M. P. Païdoussis, "Dynamics of Tubular Cantilevers Conveying Fluid." *Journal of Mechanical Engineering Science*, vol. 12, no. 2, pp. 85-103, 1970.

[17] M. P. Païdoussis, C. Semler C and M. Wadham-Gagnon and S. Saaid, "Dynamics of cantilevered pipes conveying fluid. Part 2: Dynamics of the system with intermediate spring support," *Journal of Fluids and Structures*, vol. 23, no. 4, pp. 569-587, 2007.

[18] J. J. Gu, C. An, M. G. Duan, C. Levi and J. Su, "Integral transform solutions of dynamic response of a clamped–clamped pipe conveying fluid," *Nuclear Engineering and Design*, pp. 237-245, 2013.

[19] Y. M. Hang, Y. S. Liu, B. H. Li, Y. J. Li and Z. F. Yue, "Natural frequency analysis of fluid conveying pipeline with different boundary conditions" *Nuclear Engineering and Design*, vol. 240, no. 3, pp. 461-467, 2010.

[20] Y. D. Li and Y. R. Yang, "Vibration analysis of conveying fluid pipe via He's variational iteration method," *Applied Mathematical Modelling*, vol. 43, pp. 409-420, 2017.

[21] X. Liang, X. Zha, X. Jiang, L. Z. Wang, J. X. Leng and Z. Cao, "Semi-analytical solution for dynamic behavior of a fluid-conveying pipe with different boundary conditions," *Ocean Engineering*, vol. 163, pp. 183-190, 2018.

[22] Y. F. Zhang, M. H. Yao, W. Zhang and B. C. Wen, "Dynamical modeling and multi-pulse chaotic dynamics of cantilevered pipe conveying pulsating fluid in parametric resonance," *Aerospace Science and Technology*, vol. 68, pp. 441-453, 2017.

[23] F. Liang, X. D. Yang, Y. J. Qian and W. Zhang, "Transverse free vibration and stability analysis of spinning pipes conveying fluid," *International Journal of Mechanical Sciences*, vol. 137, pp. 195-204, 2018.

[24] Y. Zhang, W. Sun, J. Yang and Q. K. Han, "Nonlinear vibration analysis of a hard-coating cylindrical shell with elastic constraints by finite element method," *Nonlinear Dynamics*, vol. 90, no. 4, pp. 2879-2891, 2017.

[25] A. H. M. Kwong and K. A. Edge, "A method to reduce noise in hydraulic systems by optimizing pipe hoop locations," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol, 212, no. 4, pp. 267-280, 1998.

[26] J. Herrmann, T. Haag and L. Gaul, "Experimental and numerical investigation of the dynamics in spatial fluid - filled piping systems," *Journal of the Acoustical Society of America*, vol. 123, no. 5, pp. 3422-3422, 2008.

[27] Z. C. Tang, Z. Z. Lu, D. W. Li and F. Zhang, "Optimal design of the positions of the hoops for a hydraulic pipelines system," *Nuclear Engineering and Design*, vol. 241, no. 12, pp. 4840-4855, 2011.

[28] Q. Liu and G. S. Jiao, "A Pipe Routing Method Considering Vibration for Aero-Engine Using Kriging Model and NSGA-II," *IEEE Access*, pp. 6286-6292, 2018.

[29] W. X. Wang, C. C. Zhou, H. S. Gao and Z. Zhang, "Application of non-probabilistic sensitivity analysis in the optimization of aeronautical hydraulic pipelines," *Structural and Multidisciplinary Optimization*, vol. 57, no. 6, pp. 2177-2191, 2018.

[30] Z. Zhang, Zhou, C. C. Zhou, W. X. Wang and Z. F. Yue, "Optimization design of aeronautical hydraulic pipeline system based on non-probabilistic sensitivity analysis," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 233, no. 5, pp. 815-825, 2019.

[31] X. Li, L. J. Zhang, S. P. Wang, C. Zhang, "Impedance analysis and hoop locations optimization of hydraulic pipeline system in aircraft," *2015 International Conference on Fluid Power and Mechatronics (FPM),* pp. 1034-1039, 2015.

[32] X. D. Liu, W. Sun, Y. Gao and H. Ma, "Optimization of pipeline system with multi-hoop supports for avoiding vibration, based on particle swarm algorithm," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, Aug. 2020, DOI:10.1177/0954406220947115.

[33] R. B. Bhat, "Natural frequencies of rectangular plates using characteristic orthogonal polynomials in Rayleigh-Ritz method," *Journal of Sound and Vibration*, vol. 102, no. 4, pp. 493-499, 1985.

[34] X. M. Guo, H. Ma, X. F. Zhang, Z. Ye, Q. Fu, Z. H. Liu, and Q. K. Han, "Uncertain Frequency Responses of Clamp-Pipeline Systems Using an Interval-Based Method," *IEEE Access*, pp. 29370-29384, 2020.

[35] W. Sun, Z. Wang, X. F. Yan, and M. W. Zhu, "Inverse identification of the frequency-dependent mechanical parameters of viscoelastic materials based on the measured FRFs," *Mechanical Systems and Signal Processing*, vol. 98, no. 1, pp. 816-833, 2018.

[36] Y. Gao, W. Sun, "Inverse identification of the mechanical parameters of a pipeline hoop and analysis of the effect of preload," *Frontiers of Mechanical Engineering*, vol. 14, no. 3, pp. 358-368, 2019.

[37] J. H. Holland, "Adaptation in natural and artificial systems," Michigan Univ, Ann Arbor, MI, USA,1975.

Xudong Liu was born in Dandong, Liaoning, China in 1995. He received the B. S. degrees in school of mechanical and power engineering from Dalian Ocean University, China, in 2018. He is currently pursuing the M.S. degree in school of mechanical engineering and automation at Northeastern University, China.

His research interests optimization of hoop position and vibration reduction of aeroengine.

Wei Sun was born in Yingkou, Liaoning, China in 1975. He received the M.S. and Ph.D. degrees in school of mechanical engineering and automation from Northeastern University, China, in 2004 and 2007, respectively, where he is currently with the Northeastern University.

His research interests include advanced damping technology, test method for vibration characteristic parameters of mechanical system, modeling, analysis and optimization design of dynamic characteristics of complex mechanical systems and dynamics of machine tools. He has undertaken more than 9 projects, such as the National Natural Science Foundation Youth Foundation, the National Natural Science Foundation of China, National high technology research and development program, National major basic research project, and other projects. More than 100 papers were published, including 60 papers such as SCI/EI, and 5 invention patents.

Prof. Wei Sun's awards and honors include Liaoning province science and technology progress award, ministry of education science and technology progress award and metallurgical industry federation science and technology progress.

ZhiHui Gao was born in Yangquan, Shanxi, China in 1996. He received the B. S. degrees in school of mechanical engineering and automation from Northeastern University, China, in 2019.He is currently pursuing the M.S. degree in school of mechanical engineering and automation at Northeastern University, China.

His research interests optimization of hoop position and vibration reduction of aeroengine damping coating.