Optimization of OFDMA-Based Cellular Cognitive Radio Networks

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***Abstract*—In this paper, we study the coexistence and optimization of a multicell cognitive radio network (CRN) which is overlaid with a multicell primary radio network (PRN). We propose a PRN-willingness-based design framework for coexistence and subchannel sharing, and a Lagrange duality based technique to optimize the weighted sum rate (WSR) of secondary users (SUs) over multiple cells. First, to avoid unacceptable SU interference to primary users (PUs), the PRN determines its interference margin based on its target performance metric and channel conditions, and broadcasts this information to the CRN. Second, each CRN cell optimizes its WSR and implements intercell iterative waterfilling (IC-IWF) to control the intercell interference. To account for the interference and transmit power limits at SUs, multilevel waterfilling (M-WF) and direct-power truncation (DPT) duality schemes are developed. Third, we develop a serial dual update technique which enables low-complexity and fast-convergence of the proposed duality schemes. Numerical results demonstrate the effects of multiple parameters, such as the number of SUs per cell, subchannel occupancy probability (SOP), and outage probability of the PUs. Our results show that the proposed duality schemes provide a large performance enhancement than the channel-greedy and access-fairness based resource allocation schemes.**

***Index Terms*—Lagrangian duality optimization, multicell overlaid CRN/PRN, OFDMA, weighted sum rate maximization.**

I. Introduction

The spectrum resource becomes scarce today and yet many available spectrum resources are not efficiently used [1]. The cognitive radio (CR) technique has been proposed as a promising solution [2], [3], where secondary users (SU) detect either white or gray space in the primary user (PU) spectrum, and implement opportunistic communication using either spectrum overlay (orthogonal channel sharing) or spectrum underlay (non-orthogonal channel sharing). To flexibly implement spectral sharing between PUs and SUs and maximize the weighted sum rate (WSR) of SUs, a dynamic spectrum allocation (DSA) design is required, using, e.g., the orthogonal frequency division multiple access (OFDMA) technique, which is a promising candidate modulation and access scheme for 4G communication systems [4]–[6]. Various DSA methods have been developed for CRN optimization coexisting with primary radio network (PRN), see e.g., [7]–[13] and references therein.

Many past results on OFDMA-based DSA methods for CRNs have assumed the single cell model [9]–[13]. A white space-access based spectrum overlay system was considered in [11] for an ad hoc any-to-any mixed CRN/PRN network. Gray space-access based spectrum underlay can be developed by imposing transmit power spectral masks (PSMs) [14], [15][1] at the SU transmitters to reduce the SU-generated interferences at PU receivers. In [10], several uplink scheduling algorithms which can reduce the SU-to-PU interference and scheduling delay were proposed. In [12], a single-cell collocated PRN/CRN downlink model was proposed and the WSR maximization of all PUs and SUs was studied using sequential quadratic programming and proportional fair scheduling (PFS) algorithms. In [13], the WSR for several SU links was optimized using the Lagrangian duality tool and both centralized and distributed algorithms were designed.

Despite the above contributions, the CRN optimization taking into account intercell interference management and the PSM constraints for multicell systems has not been well studied. Several technical difficulties are involved for CRN optimization in a multicell environment. First, the CRN/PRN coexistence involves intercell as well as intra-cell interferences. The CRN has to maximize the WSR of each cell under intercell PRN and CRN interferences, and at the same time make sure that the SU-to-PU interference upper limit at each PU receiver is not exceeded. Furthermore, this should be done using very limited intercell coordination. Second, to account for the available transmit power and PSM limits, a large number of constraints are involved in the optimization procedure (such as that used in the Lagrange duality method). The convergence of available duality methods becomes slow and unstable as the number of dual variables increases. Thus, new fast and stable dual update algorithms are needed.

For CRN/PRN coexistence, we propose a novel PRN-willingness based spectral sharing approach, in which the PUs provide a small assistance to enable CRN DSA and real-time SU-to-PU interference control. First, the PRN determines the SU interference limits (or margins) that can be accepted at each base station (BS) receiver based on its own target performance metric, such as outage probability of the signal-

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[1] Here, we borrow the name "PSM" from the ultrawideband (UWB) literature regarding signal underlay transmission. Notice, however, that in the CRN the PSM is dynamically calculated.

to-interference-plus-noise ratio (SINR). Next, each BS broadcasts to CRN pilot signals for SU-to-PU interference channel estimation, and information about its interference limit on its occupied subchannels. Finally, to avoid strong SU/PU mutual interference in the colloated cell, we propose a joint spectrum underlay/overlay method, in which the intra-cell spectrum overlay and intercell spectrum underlay is implemented. Based on these, the SUs implement DSA and WSR optimization, and real-time SU-to-PU interference control.

Besides coexistence, we propose a Lagrangian duality-based optimization framework under PSM and transmit power constraints, and develop near-optimal and low-complexity primal-dual update approaches for both uplink and downlink CRN channels. In our method, first, each CRN cell maximizes its own utility objective (such as the WSR of each cell), and also exchanges information with other CRN cells to maintain the sum SU-to-PU interference limit at each PU receiver. An intercell iterative waterfilling (IC-IWF) algorithm is developed which takes into account the intercell interference. Second, to account for both battery power and SU-to-PU interference constraints, multilevel waterfilling (M-WF) and direct power truncation (DPT) duality schemes are designed. Third, to overcome the slow convergence problem of the sub-gradient-search based duality approach, we design a new duality update technique which implements serial update of both primal and dual variables along the subchannels, in contrast to parallel dual update method in the conventional approach. This dual update technique is combined with a constrained per-link waterfilling algorithm, which solves various constraints for each link. As a result, fast and stable convergence is achieved for the WSR maximization under multiple constraints.

Furthermore, for comparison purposes, we study the performance of a greedy selective multiuser diversity (SMuD) scheme [5], [16], [17], which assigns the subchannel to the user who has the largest signal-to-noise ratio (SNR) on the considered subchannel. This method can maximize the downlink sum rate in the single-cell network. Also, we consider an access fairness SMuD scheme [17], [18], which implements the normalized SNR-ranking for channel assignment. We call these two schemes channel-greedy-SMuD scheme and access-fair-SMuD scheme, respectively. They also implement the IC-IWF in the multicell CRN, and the results are compared to the proposed duality methods.

The performance of the CRN as a function of SU transmit powers, subchannel occupancy probability (SOP), target PRN outage probability, and the number of SUs in each cell, are presented and discussed. Simulation results show that the proposed multilevel waterfilling duality scheme has a higher WSR than the direct power-truncation duality scheme, and both duality schemes provides a large performance advantage than the suboptimal SMuD schemes. Furthermore, the proposed duality schemes enjoy fast convergence and low complexity.

II. SYSTEM MODEL

Consider the system model given in Fig. 1. Each PRN cell has one BS and multiple PUs, while each CRN cell has one AP and multiple SUs, and each CRN cell is collocated with a PRN cell. To enable coexistence of CRN and PRN, we assume that (1) each CRN cell uses different subchannels from the collocated PRN cell (namely, spectrum overlay), but can use overlapping subchannels with neighboring PRN cells (spectrum underlay); (2) CRN uplink and downlink transmissions use time division duplex (TDD) mode, and they both take place in the PRN uplink transmission phase; and (3) every SU has a single transmit antenna and a single receive antenna, and exclusive subchannel assignment (ESA) is implemented inside each CRN cell.

We will design spectrum underlay/overlay coexisting techniques and Lagrangian duality-based optimization schemes to maximize the WSRs of SUs while guaranteeing the BS outage performance metrics by limiting SU-to-BS interferences.

Denote the set of SU indices in cell m as $\mathcal{C}_m$, such that $k \in \mathcal{C}_m$ when SU k is inside CRN cell m. Furthermore, we use $K_m = |\mathcal{C}_m|$ to denote the cardinality number (number of nonempty elements) of set $\mathcal{C}_m$. The total number of SUs in all $M+1$ cells is then given by $K_{\text{tot}} = \sum_{m=0}^{M} |\mathcal{C}_m| = \sum_{m=0}^{M} K_m$. In each CRN cell (say, cell m) the K_m SUs communicate with AP m, and compete for a set of N_m available frequency bands (subchannels) which are not in use by the collocated PRN cell m. To model the availability of these N_m subchannels, we define the PRN subchannel occupancy probability P_{SOP}, which is the probability that a certain subchannel is being used by the PRN cell, and it manifests the channel utilization efficiency of the PRN. The increment of P_{SOP} has three negative effects on CRN, including (1) N_m, the number of available subchannels for CRN, decreases; (2) the PSM constraint becomes more strict for SUs; and (3) intercell PU-to-SU interferences increase.

Adjacent CRN cells operate in a quasi-synchronous mode, and they are either all in uplink transmission or all in downlink transmission phases. Consider the CRN cell 0 without loss of generality. In the CRN uplink, when SU k is assigned channel n, the received signal at AP 0 on subchannel n, $y_{k,n}$, consists of the desired signal from SU k, intercell interferences from surrounding PRN/CRN, and background noise, and is given by

$$y_{k,n}^{\text{UL}} = h_{k,n} x_{k,n}^{\text{UL}} + \sum_{m=1}^{M} \left[I_{n,m}^{\text{SU,UL}} + I_{n,m}^{\text{PU,UL}} \right] + z_n, \tag{1}$$

where $h_{k,n}$ is the frequency domain channel gain from SU transmitter k to AP 0, $x_{k,n}^{\text{UL}}$ is the transmitted signal, and $I_{n,m}^{\text{SU,UL}}$ and $I_{n,m}^{\text{PU,UL}}$ are the interferences generated from SUs and PUs at neighboring cell m, $m = 1, \dots, M$, all on subchannel n. z_n is the background Gaussian noise at AP 0 on subchannel n, which has zero mean and power spectral density (PSD) $\mathcal{N}_n$. The transmit power of user k on subchannel n is given by $P_{k,n}^{\text{UL}} = |x_{k,n}^{\text{UL}}|^2$. Notice that $\sum_{n=1}^{N_c} P_{k,n}^{\text{UL}} \le P_{T,k}$ holds, where $P_{T,k}$ is the available transmit power of SU k. For conciseness, we consider only the first-tier of intercell interference in this paper, that is, $M = 6$. Nonetheless, this model can be easily extended to include more tiers of interferences, and the general approach will still apply.

The channel SINR of SU k on subchannel n is given by

$$\gamma_{k,n}^{\text{SINR,UL}} = |h_{k,n}|^2 /$$

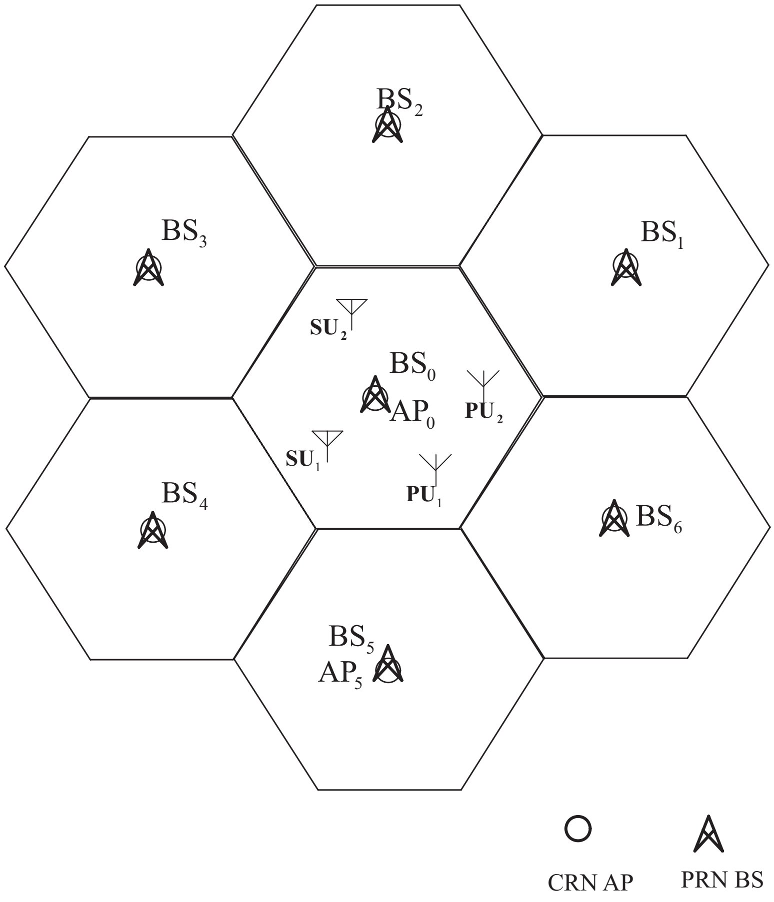

Fig. 1. Cellular system model of multicell PRN and multicell CRN. In each cell, one PRN BS and one CRN AP are collocated.

$$\left\{ \sum_{m=1}^{M} \left[E[|I^{\text{SU,UL}}_{n,m}|^2] + E[|I^{\text{PU,UL}}_{n,m}|^2] \right] + \mathcal{N}_n \right\}, \qquad (2)$$

where $E[x]$ is the expectation with respect to the fast fading part of the channel gains. We have $E[|I^{\text{SU,UL}}_{n,m}|^2] = L^{\text{SU}-\text{AP}}_{k^*,m} P^{\text{SU}}_{k^*,n,m}$, where k^* is the interfering SU who occupies channel n in cell m based on ESA, $P^{\text{SU}}_{k^*,n,m}$ is the transmit power of this user on subchannel n, and $L^{\text{SU}-\text{AP}}_{k^*,m}$ is the intercell interference channel gain (including path loss and shadowing part) between SU k^* and AP 0.

In the CRN downlink phase, the received signal at SU k on subchannel n from AP 0, $y^{\text{DL}}_{k,n}$, consists of the desired signal, CRN and PRN interferences from surrounding cells, and background noise, and is given by

$$y^{\text{DL}}_{k,n} = h_{k,n} x^{\text{DL}}_{k,n} + \sum_{m=1}^{M} \left[I^{\text{SU,DL}}_{k,n,m} + I^{\text{PU,DL}}_{k,n,m} \right] + z_{k,n}, \qquad (3)$$

where $h_{k,n}$ is the channel gain from AP 0 to SU receiver k, $x^{\text{DL}}_{k,n}$ is the transmitted signal, and $I^{\text{SU,DL}}_{n,m}$ and $I^{\text{PU,DL}}_{n,m}$ are the interferences generated from the SU and PU in neighboring cell m, $m = 1, \ldots, M$, all on subchannel n. $z_{k,n}$ is the background Gaussian noise at receiver k on subchannel n, and it has zero mean and PSD $\mathcal{N}_{k,n}$. Some frequently used symbols in this paper are listed in Table I.

III. SPECTRAL MASK FITTING APPROACHES

We will consider the optimization of cellular CRN when PRN cells are at the uplink transmission phase. This is because it is relatively easier to measure the CRN-generated interference to BSs (for PRN uplink) than to multiple PUs (for PRN downlink). Assume channel power reciprocity between CRN APs/SUs and PRN BSs. In our scheme, by measuring the pilot signals from BSs, one can determine the interference channel powers between SUs and PRN BSs. Further, the additional tolerable interference margin at each BS is broadcast to CRN APs, which can utilize such information to schedule CRN resource allocation and implement SU-to-BS interference control. On the other hand, utilizing PRN downlink transmission phase to implement CRN communication is also possible but requires more coordination between PRN and CRN and is not considered in this paper.

Our PRN assistance based design is critical for coexistence in cellular PRNs, because otherwise CRN cannot satisfy the interference constraint in either co-cell or adjacent-cell PU receivers. Here, we assume PRN victim receiver can broadcast its interference margin and subchannel availability information by either itself or a PRN scheduler (such as the BS) or spectrum broker. This framework is also related with the IEEE 802.22 TV spectrum-based CRN [19], [20]. For example, when TV tower broadcasts information regarding

TABLE I
LIST OF FREQUENTLY USED ACRONYMS AND SYMBOLS.

CRN (PRN)	Cognitive radio network (Primary radio network).
DSA	Dynamic spectrum access
IC-IWF (M-WF)	Intercell iterative waterfilling (multilevel waterfilling)
PSM	Power spectral mask
SMuD	Selective multiuser diversity
SOP	subchannel occupancy probability
SU (PU)	Secondary user (primary user).
WSR (WSE)	Weighted sum rate (weighted spectrum efficiency)
$h_{k,n}$	Complex channel gain between SU k and its AP on subchannel n.
$I^{\text{SU,UL}}_{k,n,m}$ (or $I^{\text{SU,DL}}_{k,n,m}$)	Interference at SU k in the CRN uplink (or downlink) phase from the active SU (or AP) located in cell m on subchannel n.
$I^{\text{PU,UL}}_{k,n,m}$ (or $I^{\text{PU,DL}}_{k,n,m}$)	Interference at SU k in the CRN uplink (or downlink) phase from the active PU located in cell m on subchannel n.
$\gamma^{\text{SINR,UL}}_{k,n}$ (or $\gamma^{\text{SINR,DL}}_{k,n}$)	Effective uplink (or downlink) channel SINR of SU k on subchannel n.
$g^{\text{SU}-\text{BS}}_{k,n,m'}$	Channel power between SU k and BS m' on subchannel n.
$P^{\text{SU}}_{k,n}$	Allocated power to SU k on subchannel n.
$P^{\text{PSM}}_{k,n}$	Virtual power spectral mask (PSM) of SU k on subchannel n.
$P_{T,k}$	Available transmit power of SU k.
$P^{\text{BS},\lim}_{n,m'}$	Interference margin limit at BS m' on subchannel n.
$\mathcal{C}_m$	Set of SUs in CRN cell m.
w_k	Weight factor of SU k's rate.
$\mathcal{S}_k$	Subchannel set assigned to SU k.
K_m	Number of SUs in cell m
N_m	Number of subchannels in cell m
$\text{SINR}_{n,m}$ (or $\text{SINR}^{\text{tar}}_{n,m}$)	Received PU signal SINR (or target SINR) at BS m on subchannel n.
$P^{\text{out}}_{n,m}$	Outage probability of PU signal at BS m on subchannel n.
$P^{\text{BS}}_{n,m,\text{dB}}$ (or $P^{\text{BS,CCI}}_{n,m,\text{dB}}$)	PU signal power (or PU mutual-interference power) at BS m on subchannel n (in dB).
$g^{\text{PU}-\text{BS}}_{n,m}$	Power of channel gain between the scheduled PU and BS m on subchannel n.
$P^{\text{PU}}_{n,m}$	Transmit power of the scheduled PU to BS m on subchannel n.
$L^{\text{PU}-\text{BS}}_{n,m}$	Shadowing factor for channel between the scheduled PU to BS m on subchannel n.
$\bar{L}^{\text{PU}-\text{BS}}_{n,m}$ (or $\sigma^2_{\psi,\text{PU}-\text{BS}}$)	Shadowing mean (or variance) for channel between the scheduled PU to BS m on subchannel n.
$R^{\text{SU}}_{k,n}$	Data rate of SU k on subchannel n.

its unused subchannels, the CRN can implement white-space-based access. However, in the IEEE 802.22 systems, gray-space based access appears very difficult.

A. PU/SU Subchannel-Sharing Methods

We assume that on top of the PU-to-PU-interference, the additional interference power limit (or interference margin) that can be accepted at BS m on subchannel n is given by $P^{\text{BS},\lim}_{n,m}$. Since ESA is assumed for CRN transmissions, at any time slot only one SU per CRN cell can cause interference to a neighboring PRN cell BS. To meet the interference power limit at BS m', we need the following sum SU-to-BS interference limit holds

$$\sum_{\substack{m=0\\m\neq m'}}^{M}\sum_{k\in\mathcal{C}_m} g^{\text{SU}-\text{BS}}_{k,n,m'}P^{\text{SU}}_{k,n}a_{k,n,m'} \leq P^{\text{BS},\lim}_{n,m'}, \tag{4}$$

where $P^{\text{SU}}_{k,n}$ is the transmit power of SU k on subchannel n in cell m, for $m = 0,\ldots,M$ ($m \neq m'$), and $g^{\text{SU}-\text{BS}}_{k,n,m'}$ denotes the interference channel power from the SU k to BS m' on subchannel n. $a_{k,n,m'} \in (0,1)$ is the indication factor that whether SU k generates interference to BS m', where $a_{k,n,m'} = 1$ if the interference exists and otherwise $a_{k,n,m'} = 0$. The latter case can happen when BS m' uses directional antennas to reduce (or avoid) intercell interference from certain neighboring CRN/PRN cells, or when subchannel n was not assigned to any SU in cell m. Furthermore, if BS m' does not use subchannel n, then inequality (4) can be dropped for BS m'. Notice that, due to the ESA assumption inside each CRN cell, $\sum_{k\in\mathcal{C}_m} a_{k,n,m'}$ has only one non-zero element in each cell, so (4) can be simplified to

$$\sum_{\substack{k\in\mathcal{K}_n\\k\notin\mathcal{C}_{m'}}} g^{\text{SU}-\text{BS}}_{k,n,m'}P^{\text{SU}}_{k,n} \leq P^{\text{BS},\lim}_{n,m'}, \tag{5}$$

where $\mathcal{K}_n$ is the set of indices of SUs who simultaneously access subchannel n in all CRN cells, and this poses an intercell shared subchannel assignment problem. Due to the ESA assumption we have $|\mathcal{K}_n| \leq M+1$. Second, there are multiple sum SU-to-BS interference constraints like (5) that have to be strictly satisfied by all SUs in all CRN cells. This forms a linear matrix inequality (LMI) constraint which is challenging to solve. To bypass this difficulty, we propose to dynamically split the margin $P_{n,m'}^{\text{BS},\lim}$ between the SU set $k \in \mathcal{K}_n$.

B. Acceptable Interference Power Limit at the PRN BS

Our framework is related to but different from a coexistence approach given in [21], where a single-band code-division multiple access (CDMA) multicell model was considered and the interference limits at PUs were determined based on the SU-to-PU interference violation probability and short-term fading effect. In this paper, instead, we use the PRN SINR outage probability as the target metric, and the interference limits will be determined based on the PRN channel shadowing effect.

Without loss of generality, we show how to determine $P_{n,m}^{\text{BS},\lim}$ so that the PRN SINR outage probability $P_{n,m}^{\text{out}}$ at BS m is not violated. Here, the defined SINR relates to the average signal and interference powers, and it includes the shadowing effect but does not include the fast fading part. Let $P_{n,m}^{\text{BS}}$ be the received signal power at BS m from the scheduled PU on subchannel n. Notice that here OFDMA-based ESA is assumed for channel allocation inside each PRN cell. We have $P_{n,m}^{\text{BS}} = P_{n,m}^{\text{PU}} g_{n,m}^{\text{PU}-\text{BS}} G_A$, where $P_{n,m}^{\text{PU}}$ is the transmit power of the PU who is assigned subchannel n, G_A is the receive antenna processing gain, and $g_{n,m}^{\text{PU}-\text{BS}}$ is the channel gain power between the selected PU and BS m. The SINR at BS m on subchannel n is given by $\text{SINR}_{n,m} = P_{n,m}^{\text{BS}}/P_{n,m}^{\text{BS,CCI}}$, where $P_{n,m}^{\text{BS,CCI}}$ is the total power of intercell interferences and noise. The term $P_{n,m}^{\text{BS,CCI}}$ may be decomposed to $P_{n,m}^{\text{BS,CCI}} = P_{n,m}^{\text{BS},\lim} + N'_{n,m}$, where $N'_{n,m}$ is the sum power of background noise and intercell interferences from PUs at other cells, and is assumed to be known to BS m. Notice that here the PU's SINR is defined differently from the SU's SINR given in eq. (2). To guarantee that the specified outage probability $P_{n,m}^{\text{out}}$ is not violated, we need

$$\Pr\{\text{SINR}_{n,m} < \text{SINR}_{n,m}^{\text{tar}}\} \leq P_{n,m}^{\text{out}}, \tag{6}$$

where $\text{SINR}_{n,m}^{\text{tar}}$ is the target SINR at PRN BS m for the uplink transmissions on subchannel n. It is convenient to rewrite (6) as

$$\Pr\{P_{n,m,\text{dB}}^{\text{BS}} - P_{n,m,\text{dB}}^{\text{BS,CCI}} - \text{SINR}_{n,m,\text{dB}}^{\text{tar}} < 0\} \leq P_{n,m}^{\text{out}}, \tag{7}$$

where $\Pr\{A\}$ denotes the probability of event A, $P_{n,m,\text{dB}}^{\text{BS}} = 10\log_{10} P_{n,m}^{\text{BS}}$, $P_{n,m,\text{dB}}^{\text{BS,CCI}} = 10\log_{10} P_{n,m}^{\text{BS,CCI}}$, and $\text{SINR}_{n,m,\text{dB}}^{\text{tar}} = 10\log_{10}\text{SINR}_{n,m}^{\text{tar}}$.

Due to shadowing, we assume $g_{n,m}^{\text{PU}-\text{BS}}$ has the lognormal distribution with local mean $\bar{L}_{n,m}^{\text{PU}-\text{BS}}$ and variance $\sigma_{\psi,\text{PU}-\text{BS}}^2$. Then $P_{n,m,\text{dB}}^{\text{BS}}$ has mean $\overline{P}_{\text{dB}}^{\text{BS}} = 10\log_{10}[P_{n,m}^{\text{PU}} \bar{L}_{n,m}^{\text{PU}-\text{BS}} G_A]$ and shadowing variance $\sigma_{\psi,\text{PU}-\text{BS}}^2$, that is, $P_{n,m,\text{dB}}^{\text{BS}} \sim N(\overline{P}_{\text{dB}}^{\text{BS}}, \sigma_{\psi,\text{PU}-\text{BS}}^2)$. From here on, we use $CN(\bar{x}, \sigma^2)$ and $N(\bar{x}, \sigma^2)$ to denote complex Gaussian and Gaussian distributions with mean $\bar{x}$ and variance σ^2, respectively.

We assume that each SU takes an active strategy and can accurately track the SU-to-BS interference channel power, and the outage event is related with the PU communication shadowing effect. For this case (7) leads to

$$Q\left(\frac{\overline{P}_{n,m,\text{dB}}^{\text{BS}} - P_{n,m,\text{dB}}^{\text{BS,CCI}} - \text{SINR}_{n,m,\text{dB}}^{\text{tar}}}{\sigma_{\psi,\text{PU}-\text{BS}}}\right) \leq P_{n,m}^{\text{out}}, \tag{8}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$ is the Gaussian-Q function.

The dB-valued interference limit at BS m is calculated as

$$\begin{aligned} P_{n,m,\text{dB}}^{\text{BS,CCI}} \leq\ & \overline{P}_{n,m,\text{dB}}^{\text{BS}} - \text{SINR}_{n,m,\text{dB}}^{\text{tar}} \\ & -\sigma_{\psi,\text{PU}-\text{BS}} Q^{-1}(P_{n,m}^{\text{out}}), \end{aligned} \tag{9}$$

where $Q^{-1}(\cdot)$ is the inverse-Q function. It follows that the interference margin limit at BS m is $P_{n,m}^{\text{BS,CCI}} = 10^{P_{n,m,\text{dB}}^{\text{BS,CCI}}/10}$ and

$$P_{n,m}^{\text{BS},\lim} = (P_{n,m}^{\text{BS,CCI}} - N'_{n,m})^+, \tag{10}$$

where $(x)^+ = \max(0, x)$. The interference margins $\{P_{n,m}^{\text{BS},\lim}\}_{m,n}$ on all subchannels at each BS are broadcast to the CRN. When $P_{n,m}^{\text{BS},\lim} = 0$, it means that the PRN has SINR outage even without any CRN transmission. For this case, BS m can take two different policies: In policy I, when a BS broadcasts zero margin on a subchannel, the neighboring CRNs which hear this message are prohibited from using this subchannel. In policy II, the CRN can simply drop this PSM constraint for BS m, because BS m cannot use this subchannel. In this paper we assume the more stringent policy I.

IV. WEIGHTED SUM-RATE OPTIMIZATION FOR CRN

A. WSR Optimization Model

The WSR maximization can capture the effect of different utilities (or revenue rates) of different classes of services, and is also useful to achieve the multiuser rate region. Under the sum SU-to-BS interference constraints per BS, the WSR optimization for all K SUs in the CRN uplink phase can be posed as

$$\begin{aligned} \max_{\{P_{k,n}\},\{\mathcal{S}_k\}} & \sum_{m=0}^{M} \sum_{k\in\mathcal{C}_m} w_k \sum_{n\in\mathcal{S}_k} R_{k,n} \\ \text{s.t.} & \sum_{n\in\mathcal{S}_k} P_{k,n} \leq P_{T,k},\ \forall k \\ \text{and} & \sum_{\substack{k\in\mathcal{K}_n \\ k\notin\mathcal{C}_{m'}}} g_{k,n,m'}^{\text{SU}-\text{BS}} P_{k,n}^{\text{SU}} \leq P_{n,m'}^{\text{BS},\lim}\ \ \forall m' \end{aligned} \tag{11}$$

where w_k is the utility weight factor for SU k, $\mathcal{S}_k$ is the set of subchannels assigned to user k, and $\mathcal{S}_1, \ldots, \mathcal{S}_K$ are non-overlapping sets based on ESA. $\sum_{n\in\mathcal{S}_k} P_{k,n} \leq P_{T,k}$ gives the battery power constraint for SU k, and the rate of SU k is $R_k = \sum_{n\in\mathcal{S}_k} R_{k,n}$, where $R_{k,n}$ is the throughput of user k on subchannel n. We have

$$R_{k,n} = B_n \log_2\left(1 + P_{k,n}\gamma_{k,n}^{\text{SINR}}\right), \tag{12}$$

where $\gamma_{k,n}^{\text{SINR}} = \Gamma_k \gamma_{k,n}^{\text{SINR,UL}}$, with $\gamma_{k,n}^{\text{SINR,UL}}$ being given by (2). Γ_k is an SNR gap due to modulation format and target bit-error-rate (BER) requirement. When $\Gamma_k = 1$, (12) gives the Shannon capacity, and when $\Gamma_k = -1.5/\log(5P_{e,k})$, where $P_{e,k}$ is the target BER for user k assuming continuous-rate quadrature amplitude modulation (CR-QAM) [22], (12) gives the throughput which satisfies the target BER $P_{e,k}$. To maximize WSR of SUs in all $M+1$ cells, we propose to maximize WSR in each cell while taking into account intercell interference.

B. Multi-level WF Duality Optimization

Without loss of generality, consider cell m. The WSR optimization problem for CRN cell m can be posed as

$$\begin{aligned} &\max_{\{P_{k,n}\},\{\mathcal{S}_k\}} \sum_{k\in\mathcal{C}_m} w_k R_k \\ &\text{s.t.} \quad \sum_{n\in\mathcal{S}_k} P_{k,n} \le P_{T,k}, \\ &\qquad P_{k,n} \le P_{k,n}^{\text{PSM}}, \qquad \text{and constraint} \quad (5), \end{aligned} \tag{13}$$

where $\sum_{k\in\mathcal{C}_m} w_k R_k = \sum_{k\in\mathcal{C}_m} w_k \sum_{n\in\mathcal{S}_k} R_{k,n}$ is the WSR of CRN cell m, and $P_{k,n}^{\text{PSM}}$ is a virtual PSM limit for SU k on subchannel n. To meet (5) we need

$$\sum_{\substack{k\in\mathcal{K}_n \\ k\notin\mathcal{C}_{m'}}} g_{k,n,m'}^{\text{SU}-\text{BS}} P_{k,n}^{\text{PSM}} \le P_{n,m'}^{\text{BS,lim}}. \tag{14}$$

We propose to use Lagrangian multipliers to model the virtual PSM and battery power constraints, and further use intercell coordination to ensure (14).

Define the Lagrangian

$$\begin{aligned} \mathcal{L}(\{P_{k,n}\},\{\mathcal{S}_k\}) = \sum_{k\in\mathcal{C}_m} \Bigg\{ & \sum_{n\in\mathcal{S}_k} w_k R_{k,n} \\ & -\lambda_k \left(\sum_{n\in\mathcal{S}_k} P_{k,n} - P_{T,k} \right) \\ & - \sum_{n\in\mathcal{S}_k} \mu_{k,n} \left(P_{k,n} - P_{k,n}^{\text{PSM}} \right) \Bigg\}, \end{aligned} \tag{15}$$

where $\{\lambda_k\}_{k=1,\dots,K}$ and $\{\mu_{k,n}\}_{k=1,\dots,K,n=1,\dots,N_c}$ are non-negative Lagrangian multipliers or called dual variables. Define the dual function

$$g(\{\lambda_k\},\{\mu_{k,n}\}) = \max_{\{\mathcal{S}_k\},\{P_{k,n}\}} \mathcal{L}(\{P_{k,n}\},\{\mathcal{S}_k\}). \tag{16}$$

The dual problem can be expressed as $\{\lambda_k\},\{\mu_{k,n}\} = \text{argmin}_{\{\lambda_k\},\{\mu_{k,n}\}} g(\{\lambda_k\},\{\mu_{k,n}\})$. For convenience, the dual function $g(\{\lambda_k\},\{\mu_{k,n}\})$ can be rewritten as

$$\begin{aligned} g(\{\lambda_k\},\{\mu_{k,n}\}) &= \sum_{n=1}^{N_c} g_n(\{\lambda_k\},\{\mu_{k,n}\}) \\ &+ \sum_{k\in\mathcal{C}_m} \left\{ \lambda_k P_{T,k} + \sum_{n\in\mathcal{S}_k} \mu_{k,n} P_{k,n}^{\text{PSM}} \right\}, \end{aligned} \tag{17}$$

where

$$\begin{aligned} g_n(\{\lambda_k\},\{\mu_{k,n}\}) = \max_{\{\mathcal{S}_k\},\{P_{k,n}\}} & \{ w_k \log_2(1 + P_{k,n}\gamma_{k,n}^{\text{SINR}}) \\ & -(\lambda_k + \mu_{k,n})P_{k,n} \}. \end{aligned} \tag{18}$$

Based on the Karush-Kuhn-Tucker (KKT) conditions, we obtain the following results:

Subchannel n is allocated to user k^* if

$$\begin{aligned} k^* \;=\; \text{argmax}_k & \{ w_k \log(1 + P_{k,n}\gamma_{k,n}^{\text{SINR}}) \\ & -(\lambda_k + \mu_{k,n})P_{k,n} \}. \end{aligned} \tag{19}$$

The transmit power allocated to subchannel n for user k (if $k = k^*$) is given by

$$P_{k,n} = \left(w_k/(\lambda_k + \mu_{k,n}) - 1/\gamma_{k,n}^{\text{SINR}} \right)^+, \tag{20}$$

where $P_{k,n} = 0$ if $k \neq k^*$. Here, dual variables λ_k and $\mu_{k,n}$ control the WF level of SU k on subchannel n. Since $\mu_{k,n}$ changes with n, we call this method the multilevel WF duality scheme.

Generally, to account for the transmit power and the PSM constraints, the following iterations based on sub-gradient search may be implemented:

$$\begin{aligned} \mu_{k,n}^{s+1} &= \mu_{k,n}^{s} - \beta_k (P_{k,n}^{\text{PSM}} - P_{k,n}) && (21) \\ \lambda_k^{s+1} &= \lambda_k^{s} - \beta_k (P_{T,k} - \sum_{n\in\mathcal{S}_k} P_{k,n}), && (22) \end{aligned}$$

where $\beta_k (0 < \beta_k \ll 1)$ is a gradient search step size, and λ_k^s and $\mu_{k,n}^s$ denote the values of λ_k and $\mu_{k,n}$ at stage s, respectively. Notice that generally in (21) and (22) the active-constraint conditions $P_{k,n}^{\text{PSM}} - P_{k,n} = 0$ (for all n) and $P_{T,k} - \sum_{n\in\mathcal{S}_k} P_{k,n} = 0$ (for all k) cannot be both fulfilled, and the dual iteration should stop when either of these two conditions is attained. This approach converges in theory when the dual update stepsize β_k is small enough [23]. However, gradient-based search has a very slow convergence due to the large number of dual variables involved, and this method is not suitable for either online implementation or distributed access schemes. To circumvent this problem, we design a constrained search technique to update dual variables and the proposed new duality update schemes can achieve a fast and stable convergence.

We first develop the general ***Intercell Iterative Waterfilling (IC-IWF)*** scheme (outer loop).

```
(i)  Initialize. Set outer loop stage $s_1 = 0$ and the achieved
     WSR per-cell $\sum_{k\in\mathcal{C}_m} R_k^{(s_1)} = 0$ for all $m$.
(ii) For $s_1 = 1, \dots, S_1$,
     for $m = 0, \dots, M$
       • Each CRN cell $m$ measures the interferences from
         neighboring PUs and SUs, and updates the SINRs
         for SU $k$ (for all $k \in \mathcal{C}_m$) on all subchannels.
       • Cell $m$ implements the Intra-cell duality optimization
         scheme (inner loop).
     end % m
     end % $s_1$
```

To implement the intra-cell duality optimization involved in the IC-IWF scheme, next we develop a fast-convergent *multilevel-WF duality scheme*, in which each CRN cell maximizes its own WSR in a serial way, and also exchanges information with other CRN cells to maintain the sum SU-to-BS interference limit at each BS receiver. The intra-cell update algorithm for each cell (say, cell m) is given below: (***the M-WF Duality Scheme***):

Algorithm input: The K_m SU uplinks with backlogged data, the communication and interference channel gains, and interference limits of neighboring BSs.

Algorithm Output: Subchannel and power allocation results in cell m.

```
(i)  Initialize. Set inner loop stage $s_2 = 0$ and set WSR for
     cell $m$ to zero.
(ii) For $s_2 = 1, \dots, S_2$,
       for $n = 1, \dots, N_c$,
         for $k = 1, \dots, K_m$,
```

```
      a) Find $P^{\text{PSM}}_{k,n}$ based on information from other cells regarding interference quota at BS $m'$ (for all $m'$ and $m' \neq m$).
      b) Assuming subchannel $n$ is tentatively allocated to user $k$, perform constrained per-link WF power allocation for all $k$ (as will be described next).
      c) Subchannel $n$ is assigned to user $k^*$ according to (19). Update the subchannel set $\mathcal{S}_{k^*}$ and remove subchannel $n$ from the user who originally possessed this subchannel. ■
    end % $k$
  end % $n$
end % $s_2$
```

A feasible method to handle the interference margin is to equally split the interference quota $P^{\text{BS,lim}}_{n,m'}$ among M (or less than M) CRN cells which generate interferences to BS m'. Suppose that $k \in \mathcal{C}_m$ and $k \in \mathcal{K}_n$, and $K_{n,m'}$ SUs cause interference to BS m' on subchannel n. The PSM limit imposed by BS m' to SU k is then given by $P^{\text{BS,lim}}_{n,m'}/g^{\text{SU}-\text{BS}}_{k^*,n,m'}/K_{n,m'}$. Since the PSM limit is bounded by the BS which imposes the strictest transmit power constraint, we have

$$P^{\text{PSM}}_{k,n} = \min_{m'}\{P^{\text{BS,lim}}_{n,m'}/g^{\text{SU}-\text{BS}}_{k^*,n,m'}/K_{n,m'}\}_{m'=0,\ldots,M,m'\neq m}. \quad (23)$$

When BS directional antenna is not used, we assume $K_{n,m'} = M$ for all n and m'. Notice that this quota split method maintains fairness between SUs because users with stronger interference channel gains will be more limited in transmit power. Besides this method, other adaptive margin-splitting methods can be developed.

To implement the per-link WF algorithm in Step (ii.b), a low-complexity constrained WF algorithm is designed. Assume that SU k is tentatively assigned with subchannel set $\mathcal{S}_k$ (with cardinality N_k). The transmit power constraint $\sum_{n\in\mathcal{S}_k} P_{k,n} \le P_{T,k}$ leads to

$$\sum_{n\in\mathcal{S}_k}\left(w_k/(\lambda_k+\mu_{k,n}) - 1/\gamma^{\text{SINR}}_{k,n}\right)^+ \le P_{T,k}.$$

Given w_k, if we tentatively ignore $\mu_{k,n}$, the initial value of λ_k can be derived as

$$\lambda_k = \frac{w_k N_k}{P_{T,k} + \sum_{n\in\mathcal{S}_k}(1/\gamma^{\text{SINR}}_{k,n})} \quad \text{s.t.} \quad \lambda_k \le w_k\gamma^{\text{SINR}}_{k,n}. \quad (24)$$

Based on (24), the tentative power allocation result $P^{\text{tmp}}_{k,n}$ can be obtained as

$$P^{\text{tmp}}_{k,n} = \min\{\left(w_k/\lambda_k - 1/\gamma^{\text{SINR}}_{k,n}\right)^+, P^{\text{PSM}}_{k,n}\}. \quad (25)$$

However, using $P^{\text{tmp}}_{k,n}$ as the algorithm output is suboptimal because in (25) a direct power truncation is used to include the transmission PSM constraint. Rather, we treat $P^{\text{tmp}}_{k,n}$ as a tentative value which will be used in the next step.

To include this constraint optimally, we implement two steps: first, we bound $P^{\text{tmp}}_{k,n}$ in (25) by solving $\left(w_k/(\lambda_k+\mu_{k,n}) - 1/\gamma^{\text{SINR}}_{k,n}\right)^+ \le P^{\text{PSM}}_{k,n}$, which leads to a closed-form solution $\mu_{k,n} = \left(\frac{w_k}{P^{\text{tmp}}_{k,n}+1/\gamma^{\text{SINR}}_{k,n}} - \lambda_k\right)^+$. For iterative search, we take a conservative measure that

$$\mu_{k,n} = \beta_\mu\left(\frac{w_k}{P^{\text{tmp}}_{k,n} + 1/\gamma^{\text{SINR}}_{k,n}} - \lambda_k\right)^+. \quad (26)$$

where $0 < \beta_\mu < 1$ is a control factor. In simulation, we set $\beta_\mu = 0.3 \sim 0.5$. Second, we update λ_k using a fast-convergent bi-section search to minimize the power gap given by

$$\begin{aligned}\Delta_{P,k} &= \min\{P_{T,k}, \sum_{n\in\mathcal{S}_k} P^{\text{PSM}}_{k,n}\}\\ &\quad - \sum_{n\in\mathcal{S}_k}\left(w_k/(\lambda_k+\mu_{k,n}) - 1/\gamma^{\text{SINR}}_{k,n}\right)^+. \quad (27)\end{aligned}$$

By implementing two steps (26) and (27) iteratively, the convergence is typically achieved in just a few iterations. The convergence of the inner and outer loops is demonstrated by simulation results.

Based on the analysis above, the ***Constrained Per-Link-WF*** Algorithm is summarized below, which should be implemented individually for all K_m links (in cell m).

1) Initialize. Let $N_{k,\text{eff}} = N_k$.
2) Rank $\{\gamma^{\text{SINR}}_{k,n}\}_{n\in\mathcal{S}_k}$ in a descending order resulting in $\{\gamma^{\text{SINR}}_{k,(n)}\}$, where $\gamma^{\text{SINR}}_{k,(1)} \ge \cdots \ge \gamma^{\text{SINR}}_{k,(N_k)}$ holds.
3) Tentatively ignore the PSM constraint and find λ_k using (24) with N_k being replaced by $N_{k,\text{eff}}$ therein.
4) Find $\{\mu_{k,n}\}_n$ using (26) and update $\{P_{k,n}\}_n$ using (20). Update λ_k using the bi-section search based on (27). Check if the power gap in (27) is less than a prescribed positive value. If true, go to step 5); otherwise, go to step 2).
5) Check if $\lambda_k \le w_k\gamma^{\text{SINR}}_{k,n}$ holds for all $n \in \mathcal{S}_k$. If true, go to step 6); otherwise, update set $\mathcal{S}_k$ by removing $\gamma^{\text{SINR}}_{k,(N_{k,\text{eff}})}$ from the set $\{\gamma^{\text{SINR}}_{k,(1)}, \ldots, \gamma^{\text{SINR}}_{k,(N_{k,\text{eff}})}\}$, set $N_{k,\text{eff}} = N_{k,\text{eff}} - 1$, and go to step 2).
6) The λ_k, $\{\mu_{k,n}\}_n$, $\mathcal{S}_k$, and $N_{k,\text{eff}}$ are now obtained. The allocated power to subchannel n of SU k, $P^*_{k,n}$, is given by

$$P^*_{k,n} = \min\{\left(w_k/(\lambda_k+\mu_{k,n}) - 1/\gamma^{\text{SINR}}_{k,n}\right)^+, P^{\text{PSM}}_{k,n}\}.$$

■

The constrained per-link algorithm can be implemented by each link individually without exchanging the information between the links. This is especially suitable for the distributed schemes.

C. DPT Duality Optimization

For comparison purposes, we also derive a duality approach by using direct power truncation per subchannel to fit the PSM constraint. The procedure is given below. Define the Lagrangian

$$\begin{aligned}\mathcal{L}(\{P_{k,n}\}, \{\mathcal{S}_k\}_{k\in\mathcal{C}_m}) &= \sum_{k\in\mathcal{C}_m}\sum_{n\in\mathcal{S}_k} w_k R_{k,n}\\ &\quad - \sum_{k\in\mathcal{C}_m}\lambda_k\left(\sum_{n\in\mathcal{S}_k} P_{k,n} - P_{T,k}\right). \quad (28)\end{aligned}$$

Based on the KKT conditions, we obtain the following results. Subchannel n is allocated to link k^* if

$$k^* = \text{argmax}_k\{w_k\log(1+P_{k,n}\gamma^{\text{SINR}}_{k,n}) - \lambda_k P_{k,n}\}. \quad (29)$$

The transmit power allocated to subchannel n for link k (if $k = k^*$) is given by

$$P_{k,n} = \min\{\left(w_k/\lambda_k - 1/\gamma^{\text{SINR}}_{k,n}\right)^+, P^{\text{PSM}}_{k,n}\} \quad (30)$$

and $P_{k,n} = 0$ if $k \neq k^*$. Note that in (30) DPT based on the PSM constraints is implemented and thus we call it the *DPT Duality Algorithm*. Finally, λ_k can be updated using (24).

Based on eqns. (29) – (30) the ***DPT Duality Scheme*** can be readily implemented. This algorithm fits the PSM constraints directly into the power allocation step in (30), while the WF level-related parameter λ_k is obtained by ignoring such constraints. Though this method is simpler than the *M-WF Duality Method*, its performance is inferior to the latter, as will be shown by simulation results.

D. Downlink Optimization Under PSM

The WSR optimization for downlink CRN (in cell m) can be posed as

$$\begin{aligned} &\max_{\{P_{k,n}\},\{\mathcal{S}_k\}} \sum_{k\in\mathcal{C}_m} w_k \sum_{n\in\mathcal{S}_k} R_{k,n} \\ &\text{s.t.} \quad \sum_{k\in\mathcal{C}_m}\sum_{n\in\mathcal{S}_k} P_{k,n} \le P_T, \quad \text{and} \quad P_n \le P_n^{\text{AP,PSM}}, \end{aligned} \tag{31}$$

where P_T is the available transmit power at the CRN AP, $R_{k,n}$ is given by $R_{k,n} = B_n \log_2\left(1 + P_{k,n}\gamma_{k,n}^{\text{SINR}}\right)$ and $\gamma_{k,n}^{\text{SINR}} = \Gamma_k \gamma_{k,n}^{\text{SINR,DL}}$. Define the Lagrangian

$$\begin{aligned} \mathcal{L}(\{P_{k,n}\},\{\mathcal{S}_k\}) \;=\; & \sum_{k\in\mathcal{C}_m}\left\{ w_k \sum_{n\in\mathcal{S}_k} R_{k,n} \right. \\ & -\lambda\left(\sum_{n\in\mathcal{S}_k} P_{k,n} - P_T\right) \\ & \left. -\mu_n\left(P_{k,n} - P_n^{\text{AP,PSM}}\right)\right\}, \end{aligned} \tag{32}$$

where λ and $\{\mu_n\}_{n=1}^N$ are dual variables. By deriving the KKT conditions, we have the following results: subchannel n is allocated to downlink SU k^* if

$$\begin{aligned} k^* \;=\; & \operatorname{argmax}_k\{w_k \log(1 + P_{k,n}\gamma_{k,n}^{\text{SINR}}) \\ & -(\lambda + \mu_n)P_{k,n}\}. \end{aligned} \tag{33}$$

The transmit power allocated to subchannel n for SU k (if $k = k^*$) is given by

$$P_{k,n} = \left(w_k/(\lambda + \mu_n) - 1/\gamma_{k,n}^{\text{SINR}}\right)^+, \tag{34}$$

and $P_{k,n} = 0$ if $k \neq k^*$. The *Downlink M-WF Duality scheme* can be readily designed similarly to the case of uplink, and the IC-IWF algorithm be implemented. The details are omitted here for brevity.

For comparison purposes, we also derive a duality approach by using DPT PSM fitting for downlink, as given below. The following KKT conditions are obtained. Subchannel n is allocated to SU k^* if

$$k^* = \operatorname{argmax}_k\{w_k \log(1 + P_{k,n}\gamma_{k,n}^{\text{SINR}}) - \lambda P_{k,n}\}. \tag{35}$$

The transmit power allocated to subchannel n for SU k (if $k = k^*$) is given by

$$P_{k,n} = \min\{\left(w_k/\lambda - 1/\gamma_{k,n}^{\text{SINR}}\right)^+, P_n^{\text{AP,PSM}}\}. \tag{36}$$

Finally, the WF level for all SUs is given by

$$\lambda = \frac{\sum_{k\in\mathcal{C}_m} w_k N_{k,\text{eff}}}{P_T + \sum_{k\in\mathcal{C}_m}\sum_{n\in\mathcal{S}_k}(1/\gamma_{k,n}^{\text{SINR}})} \quad \text{s.t.} \quad \lambda \le \gamma_{k,n}^{\text{SINR}} \tag{37}$$

where $N_{k,\text{eff}}$ is the number of non-zero-transmit-power subchannels in set $\mathcal{S}_k$. Based on eqns. (35) – (37), we can readily design the *Downlink DPT Duality Scheme*, and the details are omitted here for brevity.

V. NUMERICAL RESULTS

We provide simulation results for the proposed PSM fitting and rate optimization schemes in multicell overlaid CRN and PRN. The SU uplink channel gain inside cell 0, $h_{k,n}$, can be expressed as

$$h_{k,n} = \sqrt{L_k^{\text{SU-AP}}}\,\tilde{h}_{k,n}, \tag{38}$$

where $\tilde{h}_{k,n} \sim CN(0,1)$ models the normalized Rayleigh fading part, and $L_k^{\text{SU-AP}}$ models the distance-dependent channel gain (which includes the shadowing effect) between SU k and AP 0. Since we assume that the total channel bandwidth is much less than the subchannel frequency, it follows that $L_k^{\text{SU-AP}}$ is approximately independent of subchannel index n.

Popular channel models for mobile cellular communications include, for example, the IEEE 802.16 Stanford University Interim (SUI) model [24], [25], COST231 Hata model and ECC33 model [22]. Assuming the SUI model, the large-scale channel gain between SU k and AP 0 (with distance d_k) is given by (in dB)

$$\begin{aligned} L_{k,\text{dB}}^{\text{SU-AP}}(d_k) \;=\; & -20\log_{10}(4\pi d_0/\lambda) - 10\alpha_k \log_{10}(d_k/d_0) \\ & -X_f - X_h + \psi, \end{aligned} \tag{39}$$

where $\lambda = c/f_c$ is the wavelength, f_c and c are carrier frequency and speed of light, d_0 is the reference distance, and X_f and X_h are correction factors related to frequency f_c and receive antenna height, respectively. We assume $L_{k,\text{dB}}^{\text{SU-AP}}(d_k) = L_{k,\text{dB}}^{\text{SU-AP}}(d_0)$ when $d_k \le d_0$. $\psi \sim N(0, \sigma_{\psi,\text{SU-AP}}^2)$ (in dB) is a zero mean Gaussian variable which represents the shadowing effect. Consequently, $L_{k,\text{dB}}^{\text{SU-AP}} \sim N(\overline{L}_{k,\text{dB}}^{\text{SU-AP}}, \sigma_{\psi,\text{SU-AP}}^2)$ is a Gaussian variable with mean $\overline{L}_{k,\text{dB}}^{\text{SU-AP}} = -20\log_{10}(4\pi d_0/\lambda) - 10\alpha\log_{10}(d_k/d_0) - X_f - X_h$ and shadowing variance $\sigma_{\psi,\text{SU-AP}}^2$.

Both CRN and PRN have the same cell radius $R = 1000$ m, with reference distance $d_0 = 100$ m and $f_c = 1.9$ GHz. We also assume that $\alpha_k = 4$ for all SU and PU communication and interference links, and the shadowing standard deviation values are all equal and given by $\sigma_{\psi,\text{SU-AP}} = \sigma_{\psi,\text{PU-BS}} = \sigma_{\psi,\text{SU-BS}} = \sigma_{\psi,\text{PU-AP}} = 6$ dB. The noise power on each subchannel at each SU and AP is assumed to be equal to -100 dBm.

We generate 20 independent random location profiles for all PUs and SUs, and based on these we calculate the large-scale path gains for all communication and interference links. For each profile, 5 independent Rayleigh channel realizations for both uplink and downlink CRN channels are simulated.

For CRN uplink transmissions, we assume $P_{T,k} = P_T/K_m$ for all k, where P_T is the sum transmit power of all K_m SUs in cell m, for every cell. For a fair comparison between CRN downlink and uplink, we assume that the total available power at each AP is equal to P_T. Below, we assume $K_m = \overline{K}$ for all cells.

To calculate the intercell interferences from PUs to SUs, we take a pessimistic assumption that all the PUs at all

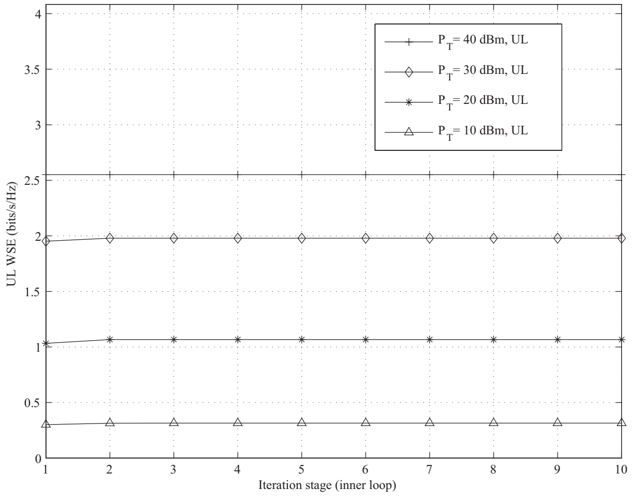

Fig. 2. Weighted spectral efficiency (WSE) per cell vs. iteration stage for the intra-cell update for the M-WF duality scheme in CRN uplink transmission, when $N_c = 16$, $\overline{K} = 8$, $P_{\text{OP}} = 0.01$, and $P_{\text{SOP}} = 0.5$.

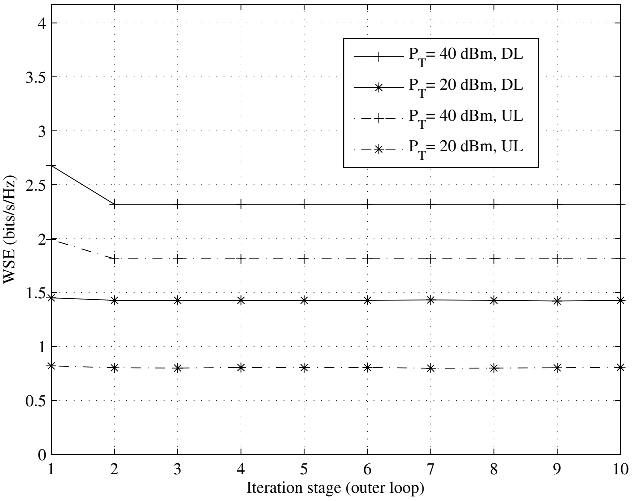

Fig. 3. WSE per cell vs. iteration stage for the intercell iterative water-filling update for the M-WF duality scheme in CRN uplink and downlink transmissions, when $N_c = 16$, $\overline{K} = 6$, $P_{\text{OP}} = 0.01$, and $P_{\text{SOP}} = 0.5$.

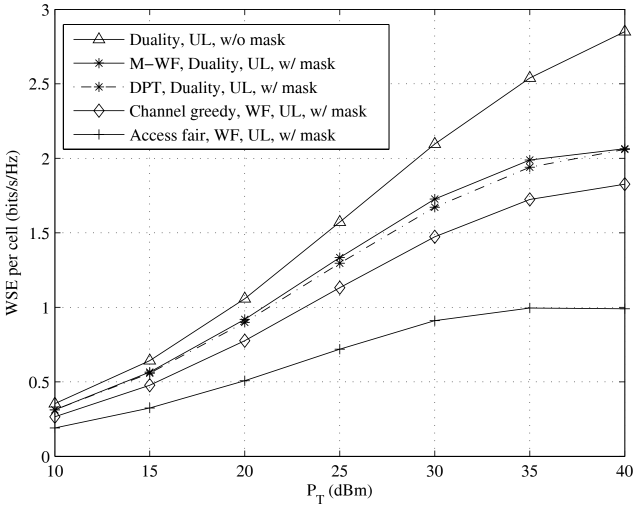

Fig. 4. WSE per cell vs. transmit power P_T for several duality and multiuser diversity schemes (channel-greedy and access-fairness) in CRN uplink transmission, when $N_c = 16$, $\overline{K} = 6$, $P_{\text{OP}} = 0.01$, and $P_{\text{SOP}} = 0.5$.

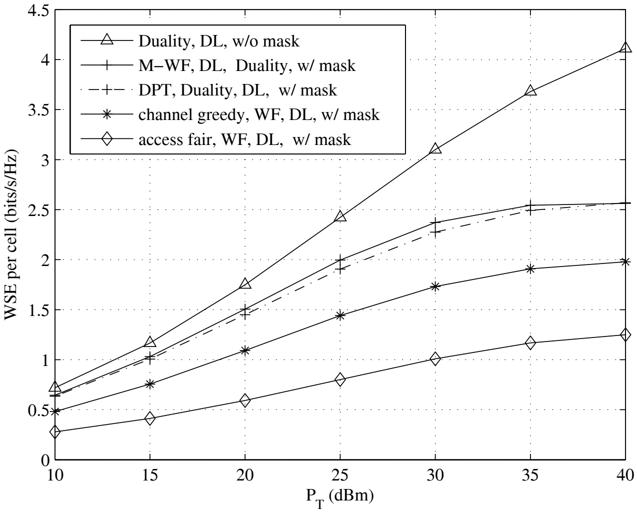

Fig. 5. WSE per cell vs. transmit power P_T for several duality and multiuser diversity schemes (channel-greedy and access-fairness) in CRN downlink transmission, when $N_c = 16$, $\overline{K} = 6$, $P_{\text{OP}} = 0.01$, and $P_{\text{SOP}} = 0.5$.

cells transmit using the maximum power $P_T^{\text{PU}} = 10$ dB on each subchannel assigned, and the adaptive modulation and coding (AMC) is used based on the channel condition. Each PU location is generated uniformly per PRN cell, but with minimum distance $R_0/4$ to its communicating BS. For the BS receiver, we assume $G_A = 0$ dB, the target SINR is uniformly distributed in [6, 15] dB to reflect different SINR requests, and PU intercell interference and noise power is $N'_{n,m} = -100$ dBm on each subchannel. The PUs at all cells are using each of the N_c subchannels independently with identical SOP $\Pr(m, n) = P_{\text{SOP}}$, and the outage probability $P_{n,m}^{\text{out}} = P_{\text{OP}}$ for all n and m.

We assume SU target BER of 10^{-5} for all K SUs, and the subchannel bandwidth is $B_n = B$ for all n. In all the figures, the WSR is normalized by N_cB, the total bandwidth of the PRN. Thus, we call the result the weighted spectrum efficiency (WSE).

To calculate the WSE in each cell, the weight vector $\mathbf{w} = [w_1, \ldots, w_{K_m}]^T$ is a normalized version of $[K_m, K_m - 1, \ldots, 1]$ such that $\sum_{k \in \mathcal{C}_m} w_k = K_m$ holds for cell m. This makes a fair comparison with unweighted sum rate where $\mathbf{w}$ is an all-one vector. Besides the duality schemes, the channel-greedy and access-fair SMuD schemes are compared assuming WF power allocation but with DPT to account for the PSM. We implement each scheme in the adjacent cells jointly and present the WSE result averaged over all 7 cells.

First, we demonstrate the “snap shot" convergence behavior of the proposed duality schemes under PSM constraint in Figs. 2 and 3, respectively. The average WSE per cell vs. the iteration stage for the intra-cell update for the M-WF duality scheme is presented in Fig. 2, when $N_c = 16$, $\overline{K} = 8$ per cell,

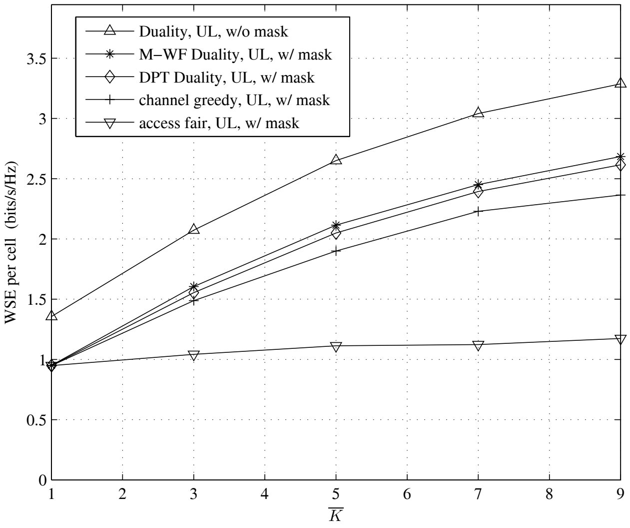

Fig. 6. WSE per cell vs. $\overline{K}$ (number of SUs per cell) for several duality and multiuser diversity schemes (channel-greedy and access-fairness) in CRN uplink transmission, when $P_T = 30$ dBm, $N_c = 16$, $P_{\text{OP}} = 0.01$ (for all n, m), and $P_{\text{SOP}} = 0.5$.

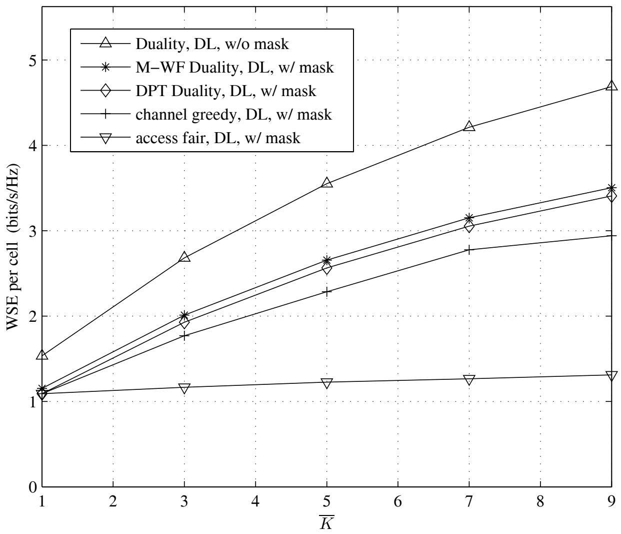

Fig. 7. WSE per cell vs. $\overline{K}$ for several duality and multiuser diversity schemes (channel-greedy and access-fairness) in CRN downlink transmission, when $P_T = 30$ dBm, $N_c = 16$, $P_{\text{OP}} = 0.01$ (for all n, m), and $P_{\text{SOP}} = 0.5$.

$P_{\text{OP}} = 0.01$, and $P_{\text{SOP}} = 0.5$. The result shows that for all the transmit power cases, the WSE converges in about two stages.

Next, in Fig. 3 we present the average WSE per cell vs. iteration stage for the intercell iterative waterfilling update for the M-WF duality scheme in CRN uplink and downlink transmissions assuming $\overline{K} = 6$. Fig. 3 shows that our IC-IWF algorithm with M-WF duality update converges very fast, in about two stages. The WSE decreases in the second stage because we initialize all the intercell interferences to be zero in the first stage. The fast convergence of our algorithms is due to two new designs: the serial dual update for subchannels, and the per-link constrained WF algorithm, presented in Section IV. For all the remaining numerical results, we use 3 stages respectively for both intra-cell and intercell update loops.

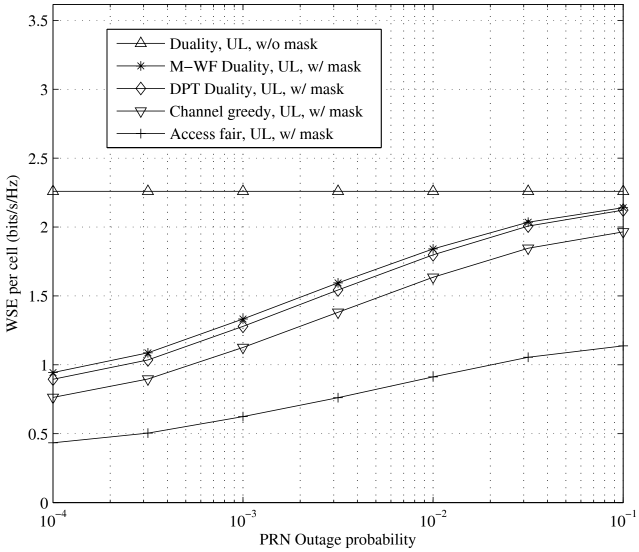

Fig. 8. WSE per cell vs. P_{OP} (target outage probability at PRN BS) for several duality and multiuser diversity schemes in CRN uplink transmission, when $\overline{K} = 6$ per cell, $P_T = 30$ dBm, $N_c = 16$, and $P_{\text{SOP}} = 0.5$.

In Figs. 4 and 5, we present the WSE per cell for the duality and SMuD schemes vs. transmit power P_T, for CRN uplink and downlink channels, respectively. Three duality schemes are considered: the M-WF and DPT duality schemes with PSM, and the duality scheme without PSM constraint imposed by adjacent-cell BSs, but it is still subject to the SOP in the colocated PRN cell.

The result shows that the WSE loss caused by the PSM constraint increases with P_T, when comparing the results with and without PSM. The M-WF duality scheme provides about 0.08, 0.25 and 0.85 bits/s/Hz WSE improvement than the DPT duality, channel-greedy and access-fair schemes, respectively, at $P_T = 30$ dBm for the CRN uplink channel. The improvement is more significant in the downlink channel.

In Figs. 6 and 7, the WSE per cell vs. $\overline{K}$ (the number of SUs per cell) is presented for the duality and SMuD schemes, for uplink and downlink channels, respectively, when $N_c = 16$, $P_T = 30$ dBm, $P_{\text{OP}} = 0.01$, and $P_{\text{SOP}} = 0.5$. When $\overline{K}$ increases, the WSE increases significantly for the duality and the channel greedy scheme. This result shows that the multiuser diversity gain is reaped for both uplink and downlink channels. On the other hand, the WSE of the access-fair scheme does not improve much with $\overline{K}$, and this shows that access fairness can cause significant performance loss. This is because the average channel SINRs differ drastically between different SU links (due to distance-based path loss and shadowing), and to provide access fairness the sum rate performance suffers significantly.

Figs. 8 and 9 present the WSE per cell in CRN uplink and downlink, respectively, vs. the PRN outage probability P_{OP} per subchannel (from 0.0001 to 0.1), when $N_c = 16$, $\overline{K} = 6$ per cell, $P_T = 30$ dBm, and $P_{\text{SOP}} = 0.5$. The PRN outage

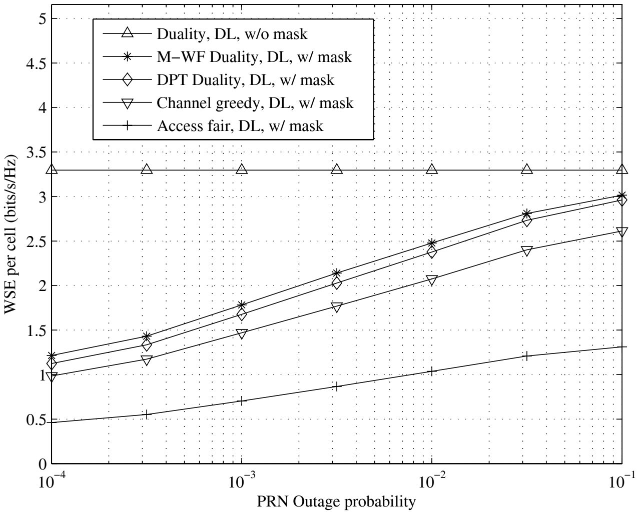

Fig. 9. WSE per cell vs. $P_{\rm OP}$ for several duality and multiuser diversity schemes in CRN downlink transmission, when $\overline{K} = 6$ per cell, $P_T = 30$ dBm, $N_c = 16$, and $P_{\rm SOP} = 0.5$.

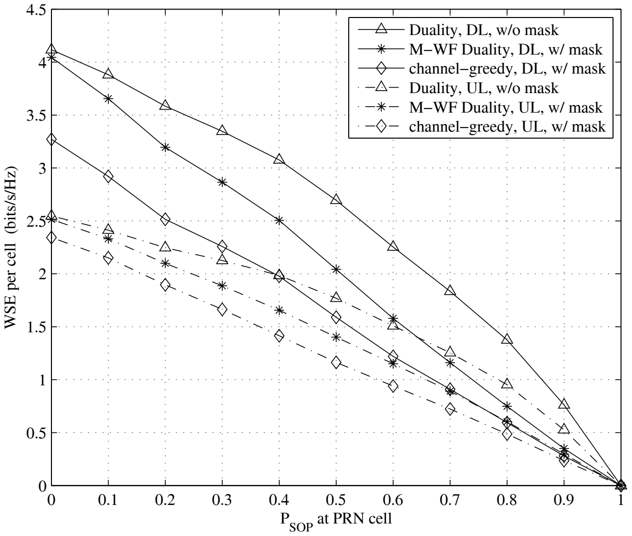

Fig. 10. WSE per cell vs. PRN subchannel occupancy probability $P_{\rm SOP}$ for several duality and multiuser diversity schemes in CRN uplink and downlink transmissions, when $\overline{K} = 6$ per cell, $P_T = 30$ dBm, $N_c = 16$, and $P_{\rm OP} = 0.01$.

probability affects the interference margins at BS receivers and the PSM at SUs. The result shows that as $P_{\rm OP}$ increases (or the PRN performance becomes worse), the WSE performance of channel-greedy, access-fair and duality schemes in uplink and downlink CRN improve significantly. This result demonstrates a tradeoff between PRN and CRN performances. A target PRN outage probability can be chosen to balance the rates between PRN and CRN. The performance of the duality scheme w/o PSM constraint is independent of the PRN outage probability, and serves as performance upperbound in this case.

Fig. 10 shows the WSE vs. the subchannel occupancy probability $P_{\rm SOP}$ for the channel-greedy and two duality optimization schemes. When $P_{\rm SOP} = 0$, all N_c channels in each PRN cell are available to all SUs, and the PSM constraint is dropped. When $P_{\rm SOP} = 1$, the PRN has used all subchannels, and there are no subchannels available to the CRN. As the $P_{\rm SOP}$ increases, the WSEs of all schemes decrease monotonically, but nonlinearly. Based on the need of each application, an optimal operating point can be chosen to balance the SOP of PRN and WSE of CRN. The WSE of the duality scheme w/o PSM also decreases to zero, because this scheme, same as other duality schemes, relies on the co-cell orthogonal PRN/CRN access.

VI. CONCLUSIONS

We have developed new coexistence and optimization techniques for a multicell CRN overlaid with a multicell PRN and proposed PRN-willingness based coexistence and near-optimal WSR maximization approaches. Simulation results have shown that the proposed M-WF duality scheme performs substantially better than the DPT duality scheme, and both duality schemes provide large performance improvement than the conventional SMuD schemes. Our results showed the critical effects of the PRN outage probability and subchannel occupancy probability on the CRN performance. Based on these results, one can readily check the effects of various system parameters on the WSR of CRN and tune some of them. The proposed framework thus provides an efficient design tool for CRN/PRN coexistence and joint optimization.

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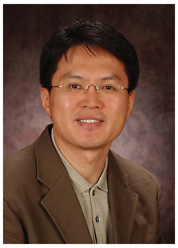

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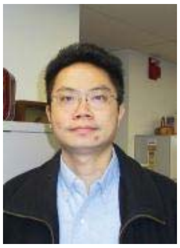

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