1. Introduction

Antenna array plays an important role in technology, including mobile communication, radar system, satellite, and medical treatment. For example, in [1], a concentric ring array antenna is designed for reconfigurable isoflux pattern, which uses 61 elements, and the frequency is 2.8 GHz. The antenna arrays can reduce the volume occupation and complex feeding and heat dissipation systems in a satellite antenna system. In [2], a sparse concentric rings array with isoflux coverage to earth surface is designed for low earth orbit (LEO) satellite system, which optimizes the angular separation between the antenna elements and the amplitude excitation across the array to generate the characteristics of the desired isoflux radiation. The proposed antenna arrays can generate a wide isoflux pattern with a good accuracy with respect to the isoflux mask for LEO satellites. In [3], a repeater antenna is designed for wireless body area networks (WBAN) applications, which can obtain the biological information by implanting the device to human-body for treatment, diagnosis, and potential medical applications.

The PSLL is an important criterion to evaluate the performance of the antenna array. Thus, with the multiple design constraints, including the number of elements, the aperture of array, and the minimum spacing of element, it is an important research topic to reduce the PSLL of antenna array in recent years [4]. Study on array synthesis had been carried out for many years and produced many classical methods. These methodologies include Woodward Lawson method and optimization calculation method [5, 6], which can be used to improve pattern performance, and Taylor synthesis method [7], which can be applied to obtain a given pattern. With the development of array antenna, the scale of arrays becomes larger and the structure of arrays becomes complex; the traditional method of array synthesis is no longer applicable. So a lot of intelligence methods were introduced to synthesize array antenna, including genetic algorithm (GA), simulated annealing algorithm (SAA), differential evolution algorithm (DEA), and particle swarm optimization (PSO) [8–11]. There are a few other topics about circular antenna arrays, such as the design of nonuniform circular antenna arrays for low sidelobe [12, 13], the comparison of algorithms about the multiobject design of circular antenna arrays [14], the design of electronically steerable linear arrays [15], and the synthesis of sparse circular antenna arrays for acquiring desired radiation beam pattern [16]. This article selects GA to synthesize the sparse concentric ring arrays for low PSLL. In [17], GA was proposed. In [8], GA was applied to optimize thinned array antenna and acquired efficient result. In [18], a modified real genetic algorithm (MGA) was proposed to synthesize the sparse concentric ring arrays.

Due to the difference of element sparsity degree on each auxiliary ring [19], a function model that presents the
The radiation pattern can be described as diagram of concentric ring array shown in Figure /one.fitted. In a planar array, generate a point randomly; the point is 2. The Optimization Model of Sparse Concentric Ring Array

In a planar array, generate a point randomly; the point is selected as centre point, and then generate several circles with different radius and place some elements on each ring; the resulting planar array is a concentric ring array [20]. The diagram of concentric ring array is shown in Figure 1.

Place an element in the centre of the concentric ring array; the radiation pattern can be described as

\[
F(u, v) = 1 + \sum_{n=1}^{H} \sum_{m=1}^{N_n} \omega_n \exp \left[ j k r_n \left( \cos \varphi_m u + \sin \varphi_m v \right) \right]
\]  

(1)

where

- \(N_n\) = number of elements in ring \(n\)
- \(H\) = number of rings
- \(k = 2\pi / \lambda\)
- \(\lambda\) = wavelength
- \(\omega_n\) = element weights for ring \(n\)
- \(r_n\) = radius for ring \(n\)
- \((r_n, \cos \varphi_m, r_n \sin \varphi_m)\) = element location
- \(u = \sin \theta \cos \varphi; v = \sin \theta \sin \varphi\)
- \(\varphi_m = 2\pi(m - 1)/N_n\)

For simplicity, we assume that array antenna meets the ideal conditions, including the element that is isotropic and the element that has uniform excitation amplitude and phase shift. The weight in the same ring is equal, and the elements in each ring are uniform distribution. The direction of main beam of the array points to the normal direction of the array, and the array pattern can be described as

\[
F(u, v) = 1 + \sum_{i=1}^{N-1} \omega_i \exp \left[ j k (r_i \cos \varphi_i u + r_i \sin \varphi_i v) \right]
\]  

(2)

where \(N\) is the sum of element number, \(r_i\) is the radius of \(i\)th element, and \(\varphi_i\) is the angle of \(i\)th element.

If the sparse concentric ring array has multiple constraints, including the number of rings \(H\), the maximum array aperture \(L\), the minimum element spacing \(d_m\), and the number of elements \(N_s\), the optimization goal is to obtain the minimum PSSL, and the optimal mathematical model can be described as

\[
\min \{\text{PSLL}(r_0, r_1, \ldots, r_n, \ldots, r_H)\}
\]

\[
N_n = \left[ \frac{2\pi r_n}{d_n} \right], \quad 0 < d_m < d_n, \quad n = 1, 2, \ldots, H
\]

\[
\sum_{n=0}^{H} N_n = N
\]

(3)

where

\[
\text{PSLL}(r_0, r_1, \ldots, r_n, \ldots, r_H) = \max \left\{ \left| \frac{F(u, v)}{\text{FF}_{\max}} \right| \right\}
\]

(4)

where \(\text{FF}_{\max}\) is the maximum value of array pattern; \(u, v\) are the regions excluding the main beam; \(N_n\) is the element number of \(n\)th rings. Due to the fact that the elements number of each ring must be an integer, the value of \(N_n\) must be rounded up or down to an integer. To meet the requirement of minimum element spacing \(d_m \geq d_m\), the digits to the right of the decimal point are discarded.

3. The Algorithm of Optimization

The flowchart of function model algorithm using MGA is shown in Figure 2.

3.1. Function Model. There is a conclusion in [19]: in the sparse concentric ring array, when the ring is away from the array centre, the more sparsity the element is, the lower the PSSL is. Thus we can propose a function model to present the relation between ring radius and the degree of sparsity. We select the data of /two.fitted/one.fitted as fitting data shown in Table /one.fitted.

In Table /one.fitted, \(N_n\) is the actual element number of \(n\)th rings, and \(N_{n_{\max}}\) is the maximum element number \(n\)th ring can place.

\[
N_{n_{\max}} = \left[ \frac{2\pi r_n (\lambda)}{d_m} \right]
\]

(5)

\(P_n\) is element retention rate of \(n\)th ring.

\[
P_n = \frac{N_n}{N_{n_{\max}}}
\]

(6)
Different function model is used to fit the data of Table 1, and the result is shown in Table 2.

Plot the fitting data and function model in Figure 3.

In the fitting function model, the $R^2$ is the determination coefficient of fitting degree, and the larger the $R^2$ is, the higher the fitting degree is. In Table 2, the linear model has the maximum $R^2$, so the linear model is selected as fitting function model.

$$F(X) = a \sin(X - \pi) + b \left((X - 10)^2\right) + c \quad (7)$$

$X$ is the radius of circular; the value of $F(X)$ is the retention rate of ring with radius $X$. $a$, $b$, $c$ are the coefficients of the function model, and they will be different with the variety of the number of elements and the size of the rings. The actual element number of rings with radii $r_n$ can be calculated as follows:

$$N_n = \left| F\left(r_n\right) \cdot \left(\frac{2\pi r_n}{d_c}\right)\right| \quad (8)$$
3.2. The Process of MGA. Standard genetic algorithm uses 0,1 binary code for individual, and it has a low efficiency. MGA uses real number code for individual directly, and it has a high freedom of search and efficiency. The main steps of MGA include generating initial population, calculating fitness value, selection, crossover, and mutation.

3.2.1. Set Up the Initial Population. We select ring radii \( R = [r_0, r_1, r_2, \ldots, r_n, r_{H}] \) as optimization individual. In order to meet the requirement of minimum element space \( d_c \), an increasing function is used to generate the ring radius as follows:

\[
\begin{align*}
    r_n &= r_{n-1} + d_c + a \cdot n \quad a > 0 \\
    r_0 &= 0, \\
    r_H &= \frac{L}{2}
\end{align*}
\]

Generate individual repeatedly to make up the population \( P \).

\[
P = [R_1, R_2, \ldots, R_k]
\]

Select the excellent individual by mean of truncated selection rate \( p_t \).

\[
P_E = [R_{1t}, R_{2t}, \ldots, R_{(k \cdot p_t)}]
\]

3.2.2. Calculate Fitness Value with Function Model. Bring \( R \) into equation (5), so the maximum element number of each ring can be calculated. Equation (7) can calculate the retention rate of each ring; thus the actual element number in each ring can be acquired. Because the element in each ring is uniform distribution, the angle of \( m \)th element on the \( n \)th ring can be calculated as follows:

\[
\varphi_m = \frac{2\pi (m-1)}{N_n} \quad 1 < m < N_n
\]

The polar coordinate representation of the individual is as follows:

\[
R_c = \begin{bmatrix}
    r_1 + 0j & r_1 + 2\pi j \frac{2}{N_1} & \cdots & r_1 + 2\pi j \frac{N_1 - 1}{N_1} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_n + 0j & r_n + 2\pi j \frac{2}{N_n} & \cdots & r_n + 2\pi j \frac{N_n - 1}{N_n} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_H + 0j & r_H + 2\pi j \frac{2}{N_H} & \cdots & r_H + 2\pi j \frac{N_H - 1}{N_H}
\end{bmatrix}
\]

Substitute \( R_c \) into equation (4), so the fitness value can be calculated.

3.2.3. Judgement of Termination Condition. If the termination conditions (the number of iterations reaches the limit or the PSLL meets the requirement) are met, output the optimal individual and the minimum PSLL of the current population, end. Otherwise, continue.

3.2.4. Selection Operation. Sort the individual according to fitness value.

\[
P_s = \text{sort} [R_1, R_2, \ldots, R_k] = [R_{1s}, R_{2s}, \ldots, R_{ks}]
\]

3.2.5. Crossover and Mutation. In order to meet the requirement of minimum element spacing \( d_c \), we need to set some constraints in process of crossover and mutation. Crossover. If the \( n \)th genes of \( i \)th and \( j \)th individuals are selected as crossover gene.

The precrossover genetic operator is

\[
R_i = [r_{i0}, r_{i1}, r_{i2}, \ldots, r_{in}, r_{iH}]
\]

\[
R_j = [r_{j0}, r_{j1}, r_{j2}, \ldots, r_{jn}, r_{jH}]
\]

The postcrossover genetic operator is

\[
R_i' = [r_{i0}, r_{i1}, r_{i2}, \ldots, r_{in}, r_{iH}]
\]

\[
R_j' = [r_{j0}, r_{j1}, r_{j2}, \ldots, r_{jn}, r_{jH}]
\]

Mutation. If the \( n \)th gene of \( l \)th individuals is selected as mutation gene.

The premutation genetic operator is

\[
R_i = [r_{i0}, r_{i1}, r_{i2}, \ldots, r_{ln}, r_{iH}]
\]

The postmutation genetic operator is

\[
R_i' = [r_{i0}, r_{i1}, r_{i2}, \ldots, r_{ln}, r_{iH}]
\]

4. Simulation

In order to verify the efficiency of proposed method, two simulations were performed. We set some common parameters
Table 3: Comparing the optimal result of $N = 201$ and $H = 7$ with [21, 22]. Function model algorithm using MGA (FMAMGA), element number ($N$), ring radii ($r_n(\lambda)$), and element number in the rings ($N_n$).

<table>
<thead>
<tr>
<th>method</th>
<th>PSLL (dB)</th>
<th>$N$</th>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>HGA [21]</td>
<td>-22.94</td>
<td>201</td>
<td>$r_n(\lambda)$</td>
<td>10.0</td>
<td>1.59</td>
<td>2.14</td>
<td>2.88</td>
<td>3.66</td>
<td>4.98</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_n$</td>
<td>12</td>
<td>19</td>
<td>26</td>
<td>36</td>
<td>45</td>
<td>62</td>
<td>--</td>
</tr>
<tr>
<td>Opt $r_n$ by MGA [22]</td>
<td>-25.45</td>
<td>201</td>
<td>$r_n(\lambda)$</td>
<td>0.80</td>
<td>1.38</td>
<td>1.88</td>
<td>2.43</td>
<td>3.18</td>
<td>3.91</td>
<td>4.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_n$</td>
<td>10</td>
<td>17</td>
<td>22</td>
<td>27</td>
<td>33</td>
<td>40</td>
<td>51</td>
</tr>
<tr>
<td>MGAFMA</td>
<td>-26.21</td>
<td>201</td>
<td>$r_n(\lambda)$</td>
<td>0.72</td>
<td>1.22</td>
<td>1.72</td>
<td>2.39</td>
<td>3.18</td>
<td>3.93</td>
<td>4.98</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_n$</td>
<td>9</td>
<td>15</td>
<td>21</td>
<td>28</td>
<td>35</td>
<td>41</td>
<td>51</td>
</tr>
</tbody>
</table>

Table 4: Comparing the optimal result of $N = 142$ and $H = 6$ with [21, 22]. Function model algorithm using MGA (FMAMGA), element number ($N$), ring radii ($r_n(\lambda)$), and element number in the rings ($N_n$).

<table>
<thead>
<tr>
<th>method</th>
<th>PSLL (dB)</th>
<th>$N$</th>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opt $r_n &amp; N_n$ [21]</td>
<td>-27.82</td>
<td>142</td>
<td>$r_n(\lambda)$</td>
<td>0.76</td>
<td>1.36</td>
<td>2.09</td>
<td>2.99</td>
<td>3.78</td>
<td>4.70</td>
<td>4.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_n$</td>
<td>9</td>
<td>17</td>
<td>25</td>
<td>31</td>
<td>31</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>Opt $r_n$ by MGA [22]</td>
<td>-28.07</td>
<td>142</td>
<td>$r_n(\lambda)$</td>
<td>0.76</td>
<td>1.36</td>
<td>2.09</td>
<td>2.92</td>
<td>3.77</td>
<td>4.70</td>
<td>4.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_n$</td>
<td>9</td>
<td>16</td>
<td>26</td>
<td>30</td>
<td>27</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>FMAMGA</td>
<td>-28.19</td>
<td>142</td>
<td>$r_n(\lambda)$</td>
<td>0.84</td>
<td>1.37</td>
<td>2.10</td>
<td>2.95</td>
<td>3.78</td>
<td>4.70</td>
<td>4.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$N_n$</td>
<td>10</td>
<td>17</td>
<td>25</td>
<td>29</td>
<td>29</td>
<td>31</td>
<td>31</td>
</tr>
</tbody>
</table>

4.1. Optimizing Sparse Concentric Ring Arrays with the Number of Elements $N = 201$. We set the same parameter values as [21, 22]; that is, the number of elements $N = 201$, and the aperture $L = 9.96\lambda$. If ring number $H = 6$, the arrays are almost full and the element spacing is equal to nearly $d_c$; there is no room for optimization; thus let the number of rings $H = 7$. Set the coefficient of function model $a = -0.0072$, $b = 0.0036$, and $c = 0.6973$. Five independent experiments were carried out in this simulation, and convergence curve takes the average of 5 independent experiments. The radiation pattern, the element configuration diagram, and the cut surface of $u = 0, v = 0$ are from the optimal result of 5 experiments. The element configuration of the best sparse concentric ring array is shown in Figure 4. Figure 5 shows the radiation pattern of the optimal result. The cut surface of $u = 0, v = 0$ is shown in Figure 6. The PSLL of optimal result is -26.21 dB, and it is lower than [21] and lower 0.76 dB than [22]. Figure 7 shows the convergence characteristics function model algorithm using MGA, and the proposed method has fast convergence characteristic. Contrast with that of [21, 22] is shown in Table 3.

4.2. Optimizing Sparse Concentric Ring Arrays with the Number of Elements $N = 142$. According to [21, 22], set the number of elements $N = 142$, the aperture $L = 9.4\lambda$, and the number of rings $H = 6$. Set the coefficient of function model $a = -0.1969$, $b = 0.00228$, and $c = 0.6543$. Five independent experiments were carried out in this simulation, and convergence curve takes the average of 5 independent experiments. The radiation pattern, the element configuration diagram, and the cut surface of $u = 0, v = 0$ are from the optimal result of 5 experiments. The element configuration of the best sparse concentric ring array is shown in Figure 8. Figure 9 shows the radiation pattern of the optimal result. The cut surface of radiation pattern when $u = 0$ and $v = 0$ is shown in Figure 10. The PSLL of optimal result is -28.19 dB, and it is lower 0.37 dB than [21] and lower 0.12 dB than [22]. Figure 11 shows the convergence characteristics function model algorithm using MGA, and the proposed method has fast convergence characteristic. Contrast with that of [21, 22] is shown in Table 4.
5. Conclusion

In this paper, a function model that presents the relationship between ring radius and the degree of sparsity is proposed, which is embedded into the process of MGA to synthesize the sparse concentric ring arrays for low PSLL. Comparing the optimal result with other literatures proves that the proposed method is an efficient way to reduce the PSLL and has a faster convergence rate. It is reasonable and efficient to select the element number in the rings according to the function model.

Data Availability

The data of this paper can be accessed for the public; the data have been uploaded to github repository and all the data present in the research are available. The chart, docx data used to support the findings of this study, the result of figure, and the program of this research have been deposited to the
The cut of radiation pattern \( u = 0, v = 0 \).

Figure 10: The cut of radiation pattern \( u = 0, v = 0 \).

The single trial
5 trials averaged

Figure 11: The convergence characteristics.

The research is funded by joint fund for civil aviation (U1233103).

**References**


