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Title: OPTIMIZATION OF STAGED RANKINE ENERGY CONVERSION CYCLES FOR HIGH EFFICIENCY

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Because of the fuel crisis and problems associated with thermal pollution there is new impetus and urgency for developing more efficient energy conversion systems for power generation. A preliminary analysis was undertaken to determine the potential of staged Rankine cycle systems for substantially higher efficiency. It was necessary to optimize the cycles to determine maximum potential efficiency, and the Sequential Unconstrained Minimization Technique of nonlinear programming was implemented on the Oregon State University CDC 3300 computer for this purpose. Binary, ternary, and quaternary Rankine cycle configurations were optimized for maximum efficiency under a set of realistic constraints. Liquid metal working fluids were used for the higher temperature stages with water for the low temperature stage fluid.

Maximum efficiencies are presented for the best cycle configurations with peak temperatures from 900°F to 3000°F. Sensitivity of the results to certain critical assumptions is also included. The potential efficiency gains at current peak cycle temperatures are

small, but, if high temperature expanders such as high temperature turbines, graphite helical rotor expanders, or MHD vapor expanders prove to be feasible, staged Rankine cycles can clearly provide high efficiencies with much lower temperature requirements than magnetohydrodynamic Brayton systems.

In order to determine ultimate potential of the staged cycles, conventional Rankine cycle improvements were considered for each stage also. Only extraction/regeneration was found to give any significant improvement and results are presented for a binary configuration with one to five extractions on each stage. Organic working fluids were considered as a replacement for mercury, and ammonia was considered as a low temperature stage working fluid for a stage below the steam cycle. Neither organic fluids nor ammonia proved to have any outstanding advantages for use in staged cycles. Staged cycles with a metal working fluid topping a steam cycle are probably best overall.

OPTIMIZATION OF STAGED RANKINE ENERGY CONVERSION CYCLES FOR HIGH EFFICIENCY

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CHAPTER I

INTRODUCTION

A. The Need for High Efficiency

Currently about 20% of all primary energy consumed in the United States is used by electric utilities for power generation. The National Petroleum Council's study on the U. S. Energy Outlook (reference 1) forecasts that by 1985 total energy consumption will be almost double current consumption and consumption by utilities for power generation will rise to 35.5% of that total. Several individuals who have studied the energy situation have speculated that by the end of the century 50% of all U. S. energy consumption will be for electric power generation (reference 2). Of the current consumption of energy for electric power generation, 82% is from fossil fuels, about 2% from nuclear fuel, and the balance (16%) from hydroelectric and geothermal sources. Therefore 84% of current consumption for power generation is by thermal power plants consuming a fuel. The National Petroleum Council's study predicts that by 1985 fossil fuels will account for 43%, nuclear fuel 49%, and hydroelectric and geothermal energy 8%. From these figures the forecast would indicate 92% of consumption for power generation in 1985 to be by thermal plants consuming a fuel.[1]

The average overall plant efficiency of thermal power plants in the United States is currently 33 to 34% (reference 1). The most efficient new fossil-fueled steam plants and gas-cooled nuclear power plants have an overall efficiency of 40%. Figure 1 illustrates the basic changes that would occur if plant efficiency could be increased to 50% for the same power output. It is apparent from Figure 1 that an increase in efficiency for new plants from 40% to only 50% would make possible a 20% reduction in fuel consumption and a 33% reduction in waste thermal energy. Associated with these reductions there would be reductions in the size of major system components (such as the boiler or nuclear reactor and the cooling condenser for strictly thermal plants).

The changes which would result from increasing efficiency to 50% have an important bearing on two problems which are currently considered to have significant importance: thermal pollution and the "energy crisis". All thermal power plants, since they use a thermodynamic cycle, must reject thermal energy to a low temperature "sink" (the second law of thermodynamics). This waste energy must be carried away by a coolant which generally is water from a lake or stream. Power plants using lakes or streams as sinks

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[1] In view of the current difficulties being encountered in constructing nuclear plants, the nuclear figure appears optimistic. However, the only viable option open to power companies when a nuclear plant is blocked is to construct a fossil-fueled plant, so the 92% figure for all fuel consuming plants should still be valid.

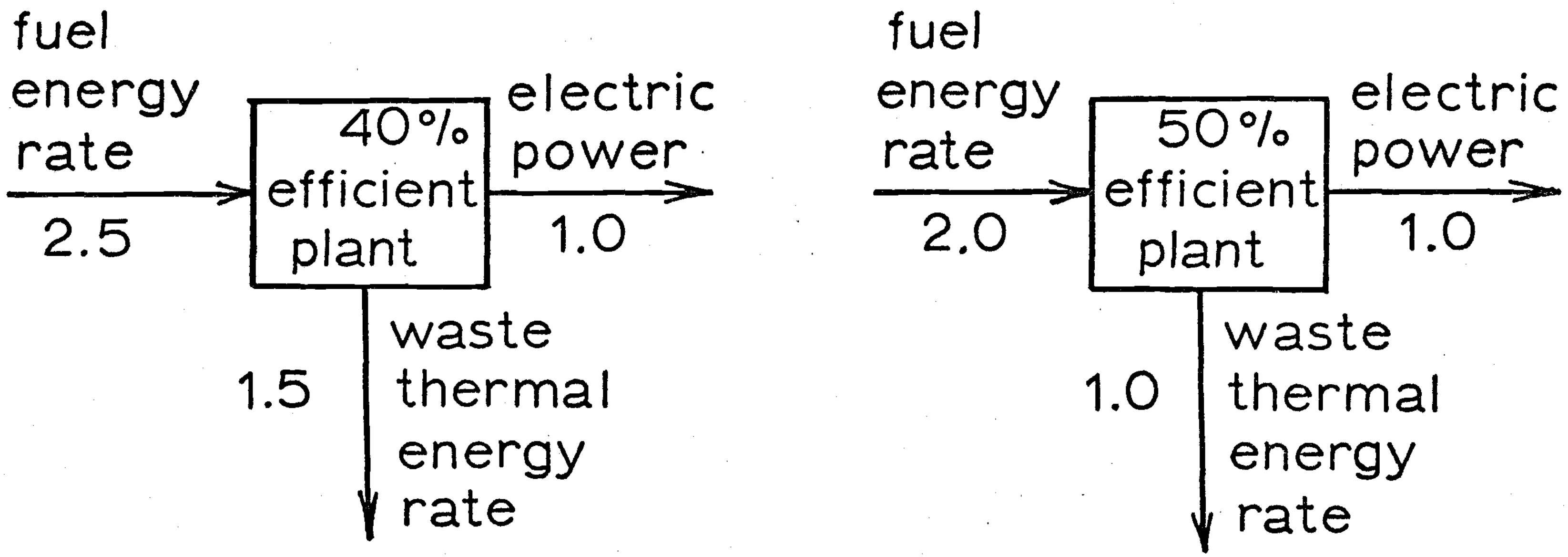

Figure 1. Effects of increasing power plant efficiency to 50%.

for the waste thermal energy from the energy conversion cycle can cause considerable biological damage because of local or widespread rises in water temperature (reference 3). The current solution to the thermal pollution problem is to provide means of cooling the water and reusing it at great expense and with currently unknown effects on the atmosphere (which generally becomes the sink for the thermal energy). If power plant efficiency could be increased to 50%, the resulting 33% drop in waste thermal energy for a given installation could bring thermal effects within acceptable bounds or at least reduce the cost of special cooling equipment. Reference 3 lists increased plant efficiency as a potential remedy to thermal pollution problems, but indicates pessimism regarding the likelihood of such an increase.

As long ago as 1967 proponents of nuclear power recognized that depletion of fuel could be a problem if water moderated thermal neutron reactors were to be the sole source of nuclear power (reference 4). The problem in the nuclear industry is expected to be alleviated substantially by the use of fast breeder reactors before fuel shortages become critical, however. The possibility of fossil fuel shortages was not widely recognized until recent years, and the discovery quickly exploded into an "energy crisis". Reference 1 shows clearly that the rapid growth in U. S. energy consumption coupled with depletion of domestic gas and oil supplies is likely to lead from a situation in 1970 with 12.4% of total requirements imported (in the form of natural gas and petroleum) to an import requirement in 1985 of 30% of total energy requirements (which are almost double the 1970 figure). The economic and political consequences are considered severe and much effort is being hastily exerted to examine alternative energy sources and means to reduce consumption (reference 2). It is widely concluded that the solution to the crisis, for the remainder of this century at least, will be the aggregate of several measures which can be taken to reduce the growth in consumption, increase the amount which can be supplied domestically, and moderate the political and economic impact of that which must be imported. With the energy for electric power generation rising from 20% to 50% of total energy consumption and with approximately 90% of that being consumed as fuels in thermal power plants, a 20% reduction in fuel consumption brought about by an increase in efficiency to 50% could be an important contribution to solution of the crisis. However, in this situation again there is a fairly common attitude that, because gains in efficiency have tapered off in the last decade, a substantial gain in efficiency is unlikely. A recent report by a representative of the President's Office of Emergency Preparedness (reference 5) concludes, "Little improvement in the efficiency of modern fossil fired steam turbine cycles can be made at the current state of metallurgy technology." Consideration of investing research and development effort into increasing efficiency is dismissed with this statement: "It is not likely that any reasonable economic incentives could bring about efficiency improvements in the electric utility sector in the near future."[2]

B. Candidates for High Efficiency

Contrary to the pessimistic conclusions mentioned in the previous section, there are several important prospects for improving thermal power plant efficiency. Obviously none of these have been developed to the point of instant availability. However, several that will be discussed are apparently technically feasible but have not been developed and implemented for economic reasons (reference 6). Thermal pollution and fuel depletion causing fuel prices to rise rapidly are new problems, and previously the cost of developing means to increase efficiency above 40% has been expected to exceed the economic gains.

Knowlton in 1960 made a survey of the important candidates for raising the efficiency of thermal power plants (reference 6); and expressed optimism that, after a brief period of leveling off at 40%, efficiency would again rise as a result of using new systems. The candidates he considered were higher temperature and pressure for Rankine steam cycles, binary mercury/steam Rankine cycles, gas turbine Brayton regenerative cycles, fuel cells, and thermoelectric and thermionic devices. Knowlton's efficiency forecasts for the first four of these are included in Figure 2 (plotted versus peak cycle temperature).

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[2] The "near future" is not defined, and no consideration is given to whether effort should be expended toward looking beyond the "near future."

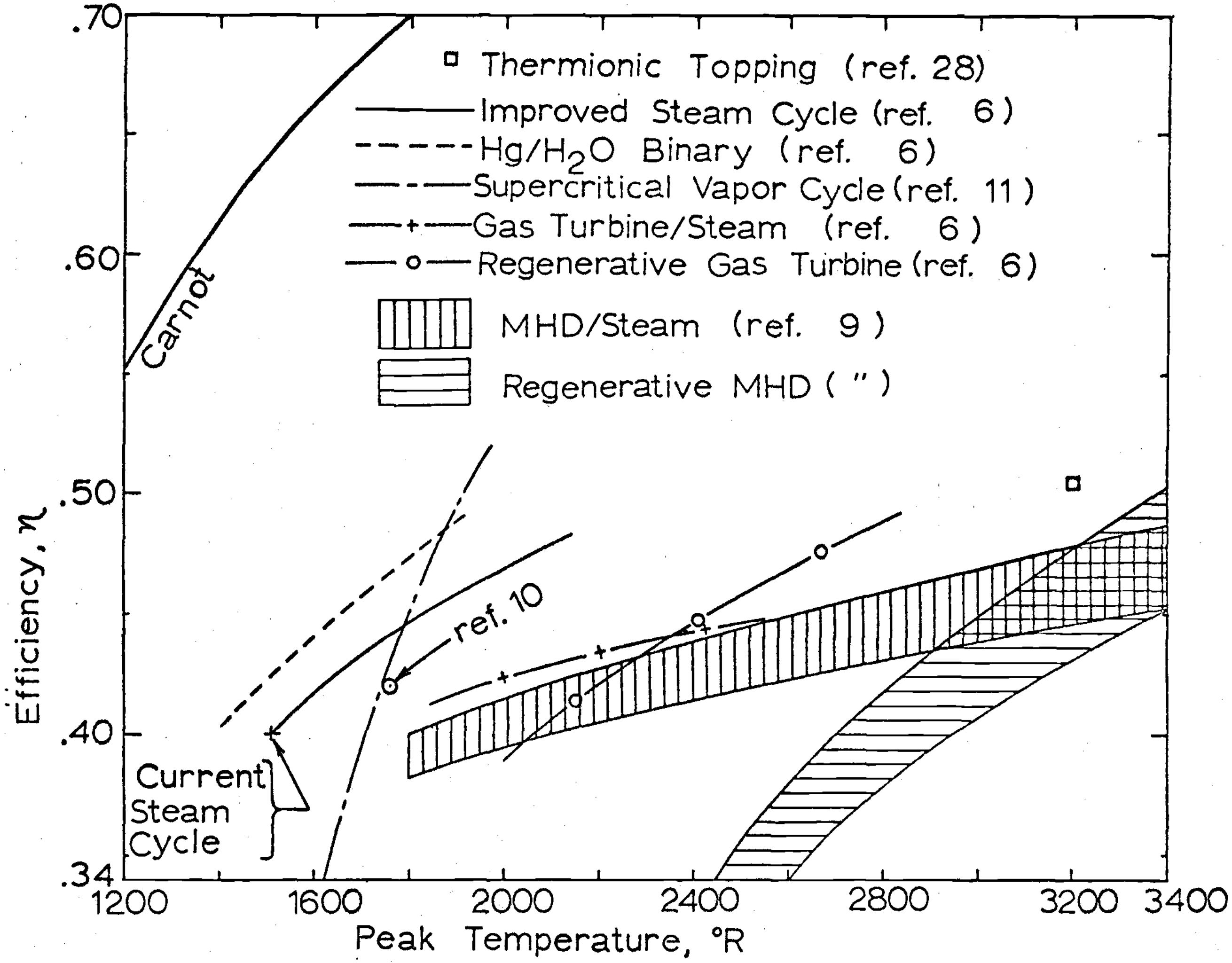

Figure 2. Efficiency potential of current candidates for providing high efficiency.

The only general surveys available since Knowlton's appear to be two surveys of theoretical possibilities made in Germany in 1969 (reference 7) and France in 1970 (reference 8). These surveys include the possibilities covered by Knowlton, and add several theoretically interesting cycles as well as two newer developments that appear feasible and are currently receiving attention: Brayton cycles using a magnetohydrodynamic (MHD) duct to generate electricity and "supercritical" vapor cycles. MHD cycles and devices are currently being studied extensively for energy conversion. General MHD efficiency capabilities from reference 9 are included in Figure 2. Feher (reference 10), Potter (reference 11) and others have considered vapor cycles in which the pressure throughout the cycle is supercritical so that a Brayton or modified Brayton cycle is used making regeneration feasible, but with pumping to raise pressure carried out in the subcooled liquid region.[3] A recent study done for the National Science Foundation's Research Applied to National Needs program strongly recommended support for research on the supercritical cycles (reference 12). Capabilities for these cycles have been extracted from references 10 and 11 and included in Figure 2.

It is apparent from Figure 2 that there is a substantial array of possible means for improving efficiency. All of the

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[3] The Brayton cycle is referred to as a "high-work cycle" because a large fraction of the turbine output work is required to compress the gas to complete the cycle. Carrying out this compression in the subcooled liquid region reduces the necessary work substantially and hence increases efficiency.

capabilitiy curves in Figure 2 are for overall plant efficiency, not just cycle efficiency. That is, each author has attempted in his analysis to account for boiler, generator, and auxiliary equipment losses as well as thermodynamic cycle losses. All of the candidate systems require higher peak cycle temperatures than are used in current plants for any substantial efficiency gain. The binary Rankine and the supercritical cycles appear to be able to achieve 50% efficiency at the lowest peak temperatures. All of these candidates should be given careful consideration, however.

C. Candidate: Improved Steam Cycles

Current large steam power plants operate at peak temperatures of about 1050°F and peak pressures around 3500 psi. Higher pressures are feasible but the costs of the higher strength of tubing, casings, etc., have been considered too high relative to the relatively small efficiency gains. As illustrated in Figure 2, the greatest potential increase in efficiency comes from increasing the peak temperature (with a corresponding small rise in the optimum operating pressure). It has been stated for a number of years that materials problems prevent higher temperatures. Strength of materials at the combined high temperatures and pressures has been inadequate. Progress has been made with development of materials with high strength at high temperatures. However, a more serious problem is likely to prevent development of high temperature steam cycles (reference 13). At temperatures above about 1100°F water begins to dissociate causing severe problems with corrosion and containment and degradation of the thermodynamic properties. As implied by the second law of thermodynamics, higher temperature is the key to higher efficiency, but some working fluid other than water will be necessary to achieve higher temperatures.

D. Candidate: Binary Rankine Cycles

The binary Rankine system consists of two Rankine cycles using two fluids so that the low temperature cycle receives the waste heat from the high temperature cycle (Figure 3). Efficiency is improved over that of a single fluid cycle with the same temperature limits because each fluid can be restricted to the temperature range where it is most effective as a working fluid, and neither fluid must be stretched into a temperature regime where its effectiveness is degraded. Also working fluids with better high temperature properties than water can be used in the top cycle. Several commercial binary cycle plants were built prior to 1950 (e.g., references 14 and 15) using mercury and steam as the working fluids. These plants had higher thermal efficiency than contemporary steam plants with similar temperature limits (reference 6).

Knowlton (reference 6) pointed out that the disadvantage of the binary system is the high cost of mercury and the mercury cycle components. Obert and Gaggioli (page 404 of reference 16) also point to the cost of mercury, special handling equipment, and special materials as the chief disadvantage to binary systems and the major reason no new binary plants have been built since 1949. Yet in

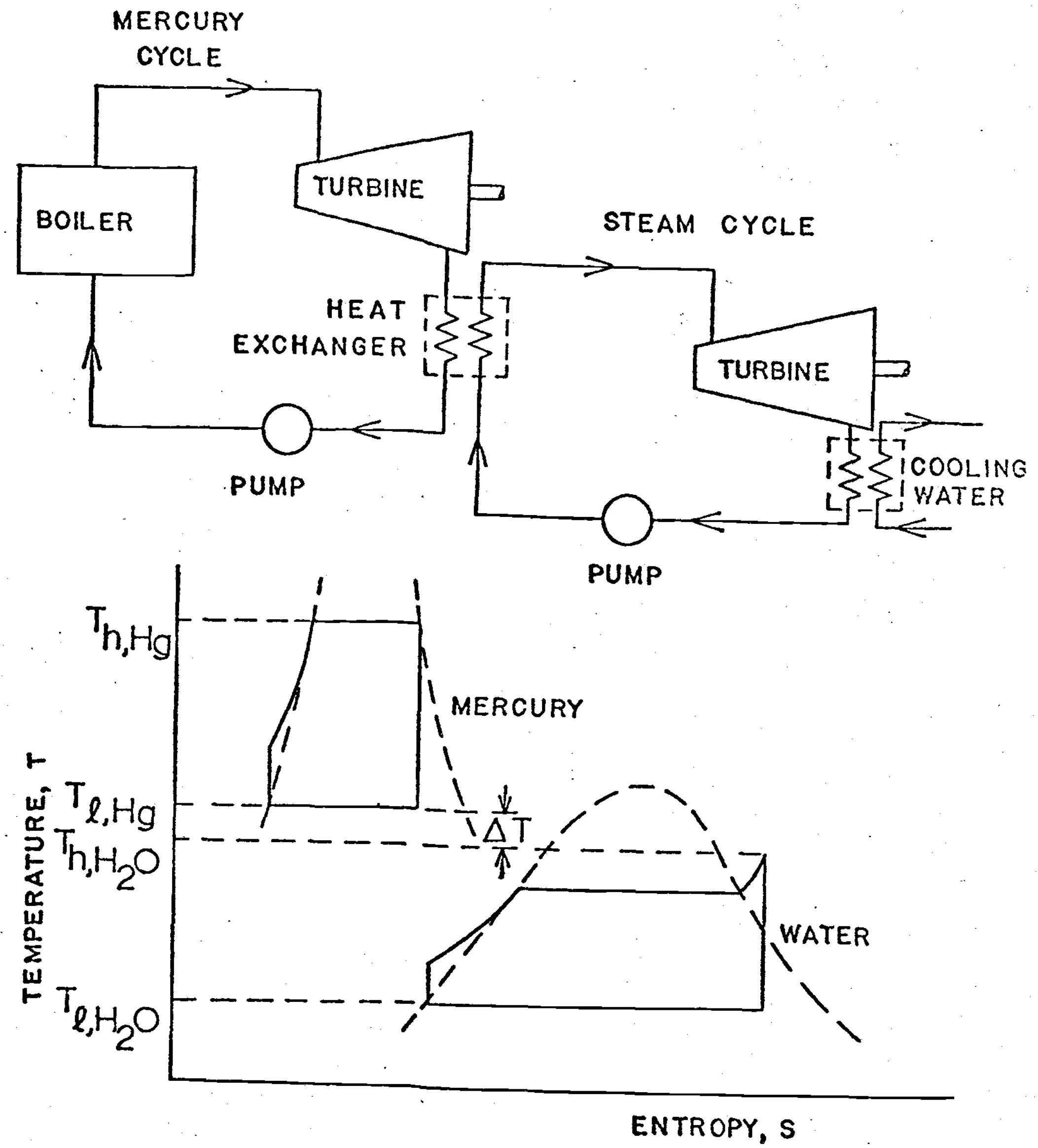

Figure 3. Simple binary Rankine cycle.

recent years considerable research has been done with liquid metal systems for cooling breeder reactors and as working fluids for special purpose power systems. The SNAP-8 power system for general space applications (e.g., references 17 and 18) uses a sodium-potassium eutectic mixture (NaK-78) as reactor coolant, mercury in the rankine turbogenerator cycle, and NaK-78 in the low temperature loop to reject waste energy to space. Numerous other projects have added working experience with materials and system component design for use with liquid metals.

In recent years there have been several proposals (e.g., references 13, 19, 20, 21) to use potassium instead of mercury with water in a binary cycle. The efficiency of such a system is less than for a mercury/steam binary cycle, but many of the problems inherent in using mercury are overcome. Considerable experience with high temperature potassium has been accumulated in the fast breeder reactor program. References 19 and 21 claim efficiencies over 50% are achievable with only some hardware development required.

E. Candidate: Gas Turbines

The regenerative gas turbine cycle can be used to solve the problem of thermal pollution of water directly because it exhausts the waste energy to the atmosphere with the exhaust gases (reference 22). The efficiency is substantially lower, however, and there are still unresolved problems regarding air pollution. Gas turbines are currently receiving much attention and are being installed in many places primarily because of an acute need for generating capacity and a fairly short installation time for gas turbine generating plants.[4]

The Brayton cycle used by gas turbines, however, is inherently less efficient than the Rankine cycle, even when regeneration and compound cycles are used to push it to its ultimate potential. The gas turbines can currently operate at about 400°F higher temperatures than steam turbines, but this difference still does not bring the gas turbine to a competitive position in terms of efficiency.

F. Candidate: Magnetohydrodynamic Conversion

Magnetohydrodynamic (MHD) energy conversion (reference 9) is one of the most prominent candidate systems for high efficiency energy conversion. MHD conversion utilizes the flow of a conducting fluid through a magnetic field to generate a current flow. In the high temperature portions of the cycle (heater and generator) no moving mechanical parts are required, so materials problems are substantially less than in conventional turbine cycles permitting higher temperatures. Liquid metals are considered as working fluids, but with relatively little enthusiasm because overall efficiencies are predicted to be relatively low. Gas (plasma) systems hold much greater current interest. However, for a gas to be electrically conducting to any substantial degree, either very high temperature or an alkali metal "seed" (e.g., cesium) is required (see conductivity

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[4] Construction time for nuclear and fossil fueled plants is typically several years, with the time being extended and some being blocked by environmental concerns.

curves in Appendix B of reference 9). Even with a seeded gas, plasma MHD systems do not have efficiency advantages at source temperature below about 2000°K (3600°R) (Chapter 8 of reference 9). Because of the high temperatures required, MHD conversion is currently being considered primarily for fossil fueled plants, especially coal fired open cycle systems.

In 1966 Booth (reference 23) could see no possibility of MHD being utilized with a nuclear energy source in the near future. Temperature limitations on current reactors place them well below the temperatures required for competitive MHD conversion, and development costs to make them compatible were considered by Booth to be unjustifiably high. By 1970, Jackson, et al. (reference 24), were more optimistic because of developments in nonequilibrium plasma and liquid metal systems. However, they still consider nuclear/MHD systems a long range prospect awaiting development of higher reactor temperatures. Kylstra (reference 25) predicts plasma core reactors will provide a thermal energy source at about 6000°R, but substantial development work must be done on such reactors so this system has long term potential only.

MHD systems produce dc power, and for central plant generation, expensive dc/ac inverters make up a large part of the plant cost. Decentralized (on-site) power generation is economically infeasible because large systems are required for MHD to be competitive.

G. Candidate: Supercritical Cycles

Feher (reference 10) proposes using CO_2 and Potter (reference 11) proposes using water in vapor cycles with the entire cycle carried out at supercritical pressures. For Feher's CO_2 cycle the required lower and upper pressures are 2000 psia and 4000 psia. For Potter's H_2O cycle they are 3300 psia and 5000 psia. In both cases the pressures are very high throughout the cycles requiring higher costs for strength of components, sealing of turbines and pumps, etc. For the working fluids mentioned there will likely be dissociation problems at the high temperatures required for an efficiency advantage. However, it may be that now the costs of development of supercritical cycle systems will be justified because of potential savings due to the higher efficiency.

H. Candidate: Direct Energy Conversion

Other candidates for high efficiency energy conversion currently receiving attention include thermoelectric and thermionic direct conversion devices and fuel cells. Thermoelectric devices are inherently low efficiency devices which produce low voltage dc power and are limited to low temperatures (reference 26). They are being used only for remote small-scale power applications where high reliability with unattended operation is more important than high efficiency. Shirazi (reference 27) studied the feasibility of using thermoelectric conversion to utilize some of the waste thermal energy from conventional power plants. He concluded that, because of very low efficiency and high materials cost, such systems are uneconomical.

Thermionic energy conversion is being considered for high temperature topping with both nuclear and chemical energy sources. By itself, thermionic conversion also has low efficiency. However, it can be used with a Rankine cycle, whose upper temperature is limited by turbine materials, to utilize higher source temperature and increase overall efficiency somewhat. Engdahl, et al., (reference 28) showed that the power output of an existing plant could be increased 25% with an overall thermal efficiency increase from 41.3% for the existing plant to 50.6% using thermionic topping. Use of in-core thermionic conversion elements for nuclear reactors is receiving considerable study, particularly for space applications where volume and mass reductions are important (reference 29). However, for large scale terrestrial power plant topping, MHD or Rankine liquid metal systems offer higher efficiency. (It is impossible to compare prospective development costs at present, but it appears that there are no more development difficulties with MHD and Rankine liquid metal systems than with thermionic systems.)

Fuel cell systems have several advantages for energy conversion. The conversion process is a chemical reaction and is not Carnot efficiency limited as are thermal conversion devices. No moving parts are required. Also large size is not required for economic systems. However, like all other systems which do not use a turbogenerator, the output power is dc, and for fuel cells only low voltages are possible. Therefore batteries of cells are required, and, if central plants are to be used, expensive dc/ac inverters are required. As a result, fuel cells are expected to be economically competitive for large-scale power generation if the power plant is located at the point of use rather than in central stations (reference 30). Currently distribution costs for fuel are lower than those for electric power and dc/ac inversion costs are avoided since many large industries can use dc as well as ac for most needs. Thus, fuel cells promise to supply part of the industrial demand for power, but are not expected to be competitive for supplying central station needs for the many users who cannot afford their own plant. Fuel cell efficiencies are expected to be about 50% with the waste being thermal energy of the products of the reaction (H_2O, CO_2, O_2, and N_2 from atmospheric air). If complete reaction can be expected at all times, fuel cells would produce essentially no unstable chemical pollution and the thermal waste can go directly to the atmosphere (water cooling of the cell itself would be required, removing something on the order of 1% of the input energy).

I. Objectives

Current fossil-fueled thermal power plants and gas-cooled nuclear power plants have a thermal efficiency of about 40%. If that efficiency could be increased to only 50%, fuel consumption and the associated products of combustion (or waste fission products for nuclear plants) would be reduced by 20% and the waste heat by 33% below a current plant of the same capacity. Potentially this could reduce costs in four areas:

(1) reduced fuel cost (and long term reduced pressure on limited fuel resources);

(2) reduced boiler or nuclear reactor size;

(3) reduced cost of waste products control or disposal;

(4) reduced thermal pollution (or reduced cost of controlling thermal poolution).

Several candidate systems appear able to provide efficiencies of 50% or more for central station power generation. Those currently showing the greatest promise are binary Rankine cycles, supercritical vapor cycles and MHD (Brayton) conversion. MHD conversion is being developed by several organizations in the U.S., Western Europe, and the U.S.S.R. (reference 9). The Oak Ridge National Laboratory has done conceptual development of a binary potassium/steam system (reference 19) and General Electric has expressed interest in hardware development for a similar system (reference 21). There is no evidence in the literature that the supercritical systems have progressed past the conceptual stage. It would be difficult at this time to choose the best of these three candidates, and all three should be developed further to determine full potential and relative cost.

Staged Rankine cycles have been chosen for further study here for several reasons:

(1) Potential for high efficiency at much lower temperatures than required for MHD conversion;

(2) Much lower pressures than required for super-critical cycles;

(3) The opportunity to exploit new developments in high temperature liquid metal technology;

(4) Staged Rankine cycles appear to be compatible with all thermal energy sources (whereas MHD is not expected to be compatible with nuclear energy, for example.

As Bidard points out (reference 8), there is no reason to restrict staged cycles to two stages (costs go up as stages are added, but, if efficiency also increases, the higher costs may be justified). As higher upper cycle temperatures are considered, the considerable range between peak and low temperatures for the system will stretch binary cycles to the point where consideration of three stages will be justified (ternary cycle). Peak temperatures from 1500 to 3400^{o}R will be considered, and binary, ternary, and quaternary systems, will be included.

Both Fraas (reference 19) and Wilson (reference 21) claim the technology is now available to develop liquid metal vapor turbine systems capable of turbine inlet temperatures up to 1540^{o}F. Further metallurgical and blade cooling developments may raise the temperature capability of turbines further but not by a very great amount. Two recent developments may make it possible to exploit much higher temperatures with staged Rankine cycles to benefit from the much higher efficiencies theoretically possible. Wells (reference 31) indicated that graphite helical rotor expanders have been operated with inert gases up to 2900°F and anticipated no difficulty with temperatures up to 4000°F. He expected expander efficiencies of 85% which compares favorably with turbines. Unfortunately there is little indication that work has been done to determine if expanders are compatible with expansion of a condensing vapor[5], and such work should be done. Rosa (reference 9) and Mokrushin, et al., in the U. S. S. R. (reference 32) indicate conceptual feasibility of using MHD expanders with a condensing vapor, but again there is no evidence of actual experimental work. The helical rotor expander and Rankine MHD expander would make feasible much higher temperatures with corresponding much higher efficiency from staged Rankine systems.

Ideally, a proposal for an advanced energy conversion system should be evaluated on an economic basis. However, costs for an undeveloped system are obscure at best, and the costs associated with environmental pollution (which could be a major cost factor)

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[5] At a conference titled "Symposium on New Sources of Energy" at University of Southern California on March 9, 1973, a presentation titled "Helical Expander as a Geothermal Prime Mover" was made by Roger Sprenkle of Hydrothermal Power Co. This may imply some work with helical expanders using condensing steam.

are not firmly established but should evolve over the next few years. Likewise it is fairly clear that fuel costs will rise and the price relationship among competing fuels will change (reference 33). However, much uncertainty exists in price projections because of supply uncertainties, market uncertainties, and the strong (and difficult to predict) effects of governmental decisions. It is hoped that an assessment concentrating on efficiency capabilities will provide a basis for economic evaluation as the cost projections are more firmly established.[6]

Since the goal is to determine efficiency capability of staged Rankine systems, a procedure is required to determine operating parameters which yield maximum efficiency. The peak temperature (Figure 3) will be treated as an independent variable (ranging from 1500°R to 3400°R). The low temperature for final waste energy rejection is determined by the temperature of available cooling water. The temperature difference required for energy exchange between stages will be assumed to be fixed by design feasibility and the requirement of reasonable heat exchanger size for reasonable cost. This leaves, then, for each stage the state of the working fluid entering the expander (turbine, helical rotor expander, or MHD expander) to be determined to maximize overall efficiency. The highest temperature stage (top stage) will have only pressure

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[6] Fraas (reference 19) attempts a rough preliminary estimate for the potassium/steam system and concludes the cost may be less than for a conventional coal-fired plant.

at expander entrance to be determined, and all other stages will have temperature and pressure at expander entrance to be determined. The maximum possible number of free variables to be optimized will range from three for two stages (binary) to seven for four stages (quaternary). It is apparent that, if a thorough study of efficiency potential is to be undertaken, a fairly sophisticated optimization procedure will be required.

Reasonable pressures will be determined and all stages restricted to conform to these. Also reasonable values will be incorporated for pump and expander efficiencies and temperature differences between stages. All working fluids will be considered which show promise of yielding high efficiency and for which sufficiently comprehensive thermodynamic data are available to allow optimization.

CHAPTER II

STAGED SIMPLE CYCLES

A. Optimization

The nonlinear optimization problem proposed in the previous section is a complex one and clearly is not one that could be solved by trial-and-error with any reasonable amount of effort. Up to seven free (independent) variables must be optimized subject to certain constraints on temperatures and pressures. Several of the relationships between variables (namely the thermodynamic properties of the working fluids as functions of temperature and pressure) are available and most conveniently used in tabular form. Appendix I presents a general discussion of the techniques available for solving nonlinear optimization problems. The Sequential Unconstrained Minimization Technique (SUMT) developed by Fiacco and McCormick (reference 34) was selected as the most appropriate technique for this problem. Appendix I includes a discussion of SUMT and the characteristics which make it appropriate.

Appendix II presents the computer programs developed to implement SUMT for staged Rankine cycles. Initially a program was developed to maximize efficiency with the expander inlet vapor being superheated. Experience with that program showed that, for maximum efficiency, the vapor at expander inlet would be saturated on all stages whenever upper pressure is not restricted. Therefore a second version of the program was written restricting the expander inlet condition to saturated vapor. This reduced the dimensionality of the problem and saved substantial computer time in subsequent work. These programs are discussed in Appendices II.A and II.B.

Implementation of SUMT, although long complex computational procedures are required, is fairly straightforward. In actual use, however, considerable experience and experimentation are required to determine the scaling constants (in the augmented objective function), search step sizes, and convergence tolerances which are appropriate for a given problem.[7] Once these are established the program can be used as a production program to generate results for many cases with relatively little effort and computer time expended.

B. Assumptions

Since it was not possible to do any pioneering work in discovering new working fluids, it was necessary to depend on the published experience of others to choose the working fluids to be considered. Pages 393 to 394 of reference 16 give a fairly comprehensive list of desirable characteristics for a Rankine cycle working fluid. Only a few of the more important characteristics contributing to feasibility and to high efficiency will be considered here.

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[7] See reference 35 for some comments in this regard.

(1) Vapor pressure should be reasonable over the range of temperatures for which the fluid is to be used.

(2) The fluid must not decompose or dissociate in the temperature range (stable fluid).

(3) Latent heat of vaporization should be large to maximize the fraction of the cycle which approximates the Carnot cycle (see page 392 of reference 16).[8]

(4) Specific heat capacity of the liquid (C_p) should be small so that constant pressure lines on the temperature-entropy diagram are steep, reducing the fraction of the cycle which does not approximate the Carnot cycle.

Figure 4 gives the vapor pressure curves for some common working fluids and for the alkali metals which are being considered for space power programs and for fast breeder reactor coolants. All of these fluids are used or considered for Rankine cycles because they meet a substantial number of the requirements for a good working fluid. (As pointed out in reference 16, all known fluids fail to have some of the desirable characteristics.) For the present, ammonia and Freon-12 will not be considered. Their properties make them most appropriate for rather low temperatures compared to those

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[8] Also for staged cycles, using a fluid with small latent heat of vaporization results in a large flow rate required for that stage to balance energy transfer to other stages. This will be seen to be the case for mercury.

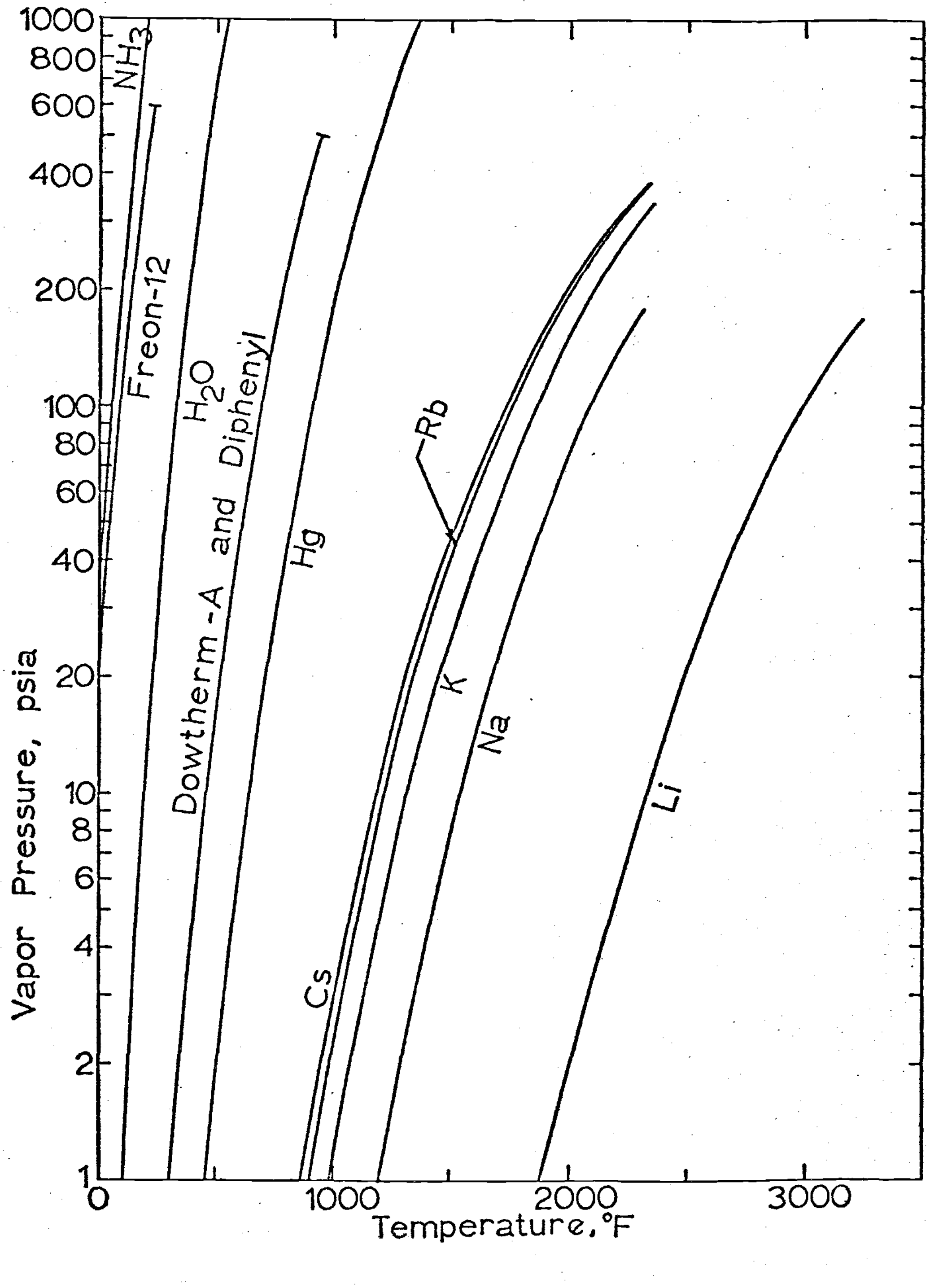

Figure 4. Vapor pressure curves for Rankine cycle working fluids

available for power generation, and they are used primarily in vapor refrigeration cycles. No fluid was found which is superior to water in the range of 80°F to about 700°F. Organic working fluids such as Dowtherm A[9] and diphenyl[10] have favorable properties for about 400°F to about 800°F but begin to decompose, seriously affecting performance, at the high end of this range. They will be considered in section III.B. Mercury appears to be favorable in the range of 500°F to 1500°F and apparently has no competitors. In the temperature range of 1000°F to 2500°F there are several candidate fluids, and for 2000°F to 3500°F lithium is the only working fluid now being considered. The Handbook of Chemistry and Physics (reference 36) gives the following approximate prices for the metals

Mercury	\$4/lb[11]
Cesium	\$100 to \$150/lb
Rubidium	\$300/lb
Potassium	\$2/lb
Sodium	15¢ to 20¢/lb
Lithium	\$8/lb

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[9] Dowtherm A is a eutectic mixture of diphenyl and diphenyl oxide developed and marketed by Dow Chemical Company (reference 49)

[10] Also called biphenyl.

[11] Until about 1969 mercury was about \$28/lb.

Cesium, rubidium, potassium, and sodium are roughly equal in corrosiveness (reference 37). For the simple cycles in this section water, mercury, potassium, sodium, and lithium will be considered as working fluids.[12] Ammonia, diphenyl, and cesium will be considered briefly in section III.

A range for the upper temperature (high temperature of the highest temperature stage) of about 1000°F to 3000°F was selected to cover the expected range for present and future fossil-fueled and gas-cooled nuclear power plants. There appear to be potential expanders to operate at some or all of this temperature range (see section I), so, since efficiency is a fairly strong function of upper temperature, a large range should be considered. The lower temperature for a power plant cycle depends on the temperature and quantity available of coolant, and it usually varies during the year. However, to provide a consistent basis for comparing the cycles, it was assumed that cooling water is available at 80°F. Since cooling water would be liquid throughout the condenser, and usually large coolant flow rates are possible, a reasonable size heat exchanger is possible with a fairly low temperature difference between the hot and cold sides. A difference of about 35°F or less (reference 40) is fairly common in power plants and this difference was used. The resulting low temperature for all cycles is 115°F or 575°R.

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[12] Thermodynamic data for water were taken from reference 38, and for the metals from reference 39.

Pressure limits were imposed to keep the complexity and cost of equipment reasonable. The limits chosen are essentially arbitrary and should be subject to reconsideration in cases where a trade-off between cost of equipment and cost savings due to higher efficiency would favor relaxing them. Very high pressures have the obvious disadvantage of requiring higher cost for strength of system components and for turbine sealing. Current plants have leveled off in peak pressure at about 3000 to 3500 psia, and very large installations are required to justify the capital cost of these installations. An upper pressure limit of 3000 psia was chosen for each stage in the staged cycles. Very low pressures in a cycle lead to sealing problems, excessive expander outlet size, and even molecular migration from component materials to the working fluid. About 0.5 psia is considered the lower limit for steam plants, but this is primarily because it is the saturation pressure for about 80°F (which is approximately a lower temperature limit for steam cycles.) Provision for such low pressures is costly, however, because of the considerations mentioned. A lower limit of 0.5 psia was chosen for each of the staged cycles. It may be necessary to reconsider this limit for certain of the working fluids.

The choice of a ΔT between stages (Figure 3) with condensing on one side of the heat exchanger and boiling on the other is quite complex. It was desired to choose one value to be used in all cases and to use a reasonably feasible value. Increasing ΔT decreases overall thermodynamic efficiency, so ΔT should not be too large. On the other hand as ΔT approaches zero the heat exchanger area grows to infinity (and cost with it). In general, condensing takes place at approximately constant temperature, and fairly reasonable size is obtained with a fluid-to-wall temperature difference of 20° to 40°F (reference 40). The energy transfer in boiling depends strongly on the fluid-to-wall temperature difference, and the transfer rate (per unit area) reaches a peak just before transition from nucleate to film boiling (see page 72 of reference 40). Just below this peak is the optimum design point because it gives minimum area (hence cost) for a given required energy transfer rate. The temperature difference at the peak varies with pressure (and with surface roughness) so that choice of ΔT for even one combination of fluids would vary over the temperature range. This combined with the economic trade-off of cost (area) versus ΔT makes it impossible to choose a ΔT appropiate for all fluids and all temperatures. The following boiling fluid-to-wall data for peak energy transfer rate were used as a guide:

Fluid	Boiling Pressure	Boiling Temperature	Fluid-to-wall Temp. Diff.*	Reference
Water	100 psig	338°F	4°F	40
Mercury	30 psig	800°F	30°F	41
Sodium	400 mm Hg	1500°F	25°F	42

*In general these are for smooth wall surfaces, and they decrease as pressure increases.

According to reference 40 the fluid-to-wall temperature difference at peak energy transfer rate would be lower for potassium and lithium than that for sodium. In the boiler three energy transfer regimes will exist: heating the subcooled liquid, boiling, and superheating the vapor. The boiling process dominates the design because most of the energy is transferred in that process (latent heat of vaporization substantially exceeds specific heats of liquid and vapor). Combining 20° to 40°F for condensing with 4° to 30°F for boiling leads to a choice of 100°F for the ΔT between stages whenever the lower stage is not superheated. If the lower stage is superheated 50°F or more,then $\Delta T=50°F$ is considered justified because boiling would occur with an overall difference of 100° or more. It is clear that these choices must be considered preliminary and approximate. When a staged Rankine system is finally designed, the parameters of heat exchangers must be included in the design optimization.

The turbines in the largest modern steam power plants have an efficiency[13] of about 85 to 87%. For staged cycles efficiencies were assumed to be as follows:

All pumps	50%
Steam turbine	80%
Potassium, sodium or mercury expander	75%
Lithium expander	70%

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[13] For a turbine, efficiency is actual work divided by ideal (isentropic) work. For a pump it is the reciprocal of that.

Turbine and expander efficiencies were selected to approximately account for the expected lower efficiency when exotic working fluids are used in new systems at higher temperatures, and to account for the lower efficiencies which would be expected for staged systems wherein each stage expands over a lower pressure ratio than would normally be the case. It is hoped the values chosen are conservative, but all such values are conjectural at this point.

It is traditional in steam power plants to be concerned about the quality of the steam in the last stages of the turbine. Too much liquid results in blade erosion so that frequent blade replacement is necessary. It was originally planned to include a lower limit on the expander exit quality for each of the staged cycles. However, a survey of the current situation regarding turbine erosion (reference 43) makes it clear such a limit would be an unnecessary constraint on a preliminary staged system study. Several means have been developed to avoid erosion of expanders by the liquid drops in the working fluid. Probably the most important example is extraction of the liquid between turbine stages. For current nuclear power plants, superheating is complicated and so is not done. Low quality would be a problem except that liquid extraction has been developed to handle the problem very well. Typically quality is increased from about .76 to about .86 by liquid extraction and no blade erosion problems are encountered. When removal of liquid is necessary, cost is increased and efficiency decreased slightly. However, it is better to analyze this trade-off in the final design optimization rather than arbitrarily restrict expander exit quality in preliminary design studies.

All losses (or cycle irreversibilities) not already mentioned are considered negligible. These include pressure losses in connecting lines and heat exchangers, and heat losses in connecting lines. The major irreversibilities have been accounted for and those neglected are usually small, certainly less important than the uncertainty in such assumptions as expander efficiencies.

C. Results

In order to test the program and determine preliminary characteristics of optimum cycles, ideal (fully reversible) cycles were considered first. This involved neglecting all heat losses and assuming turbine and pump efficiencies of 100% and no temperature difference required for heat transfer in heat exchangers. Figure 5 gives the maximum efficiency versus peak (upper cycle) temperature for a number of configurations of unconstrained ideal cycles. Ideally at least, staged Rankine cycles can approach Carnot efficiency very closely. In all cases it reduces efficiency to superheat any of the stages. A large part of the total temperature range at any particular peak temperature is taken up by the liquid metal cycles with the consequence that the optimum condensing pressure of the liquid metal cycle just above the steam cycle is extremely low. For the optimum ideal cycles these pressures go as low as 10^{-2} psia for mercury, 10^{-6} psia for potassium, 10^{-8} psia for sodium, and 10^{-4}

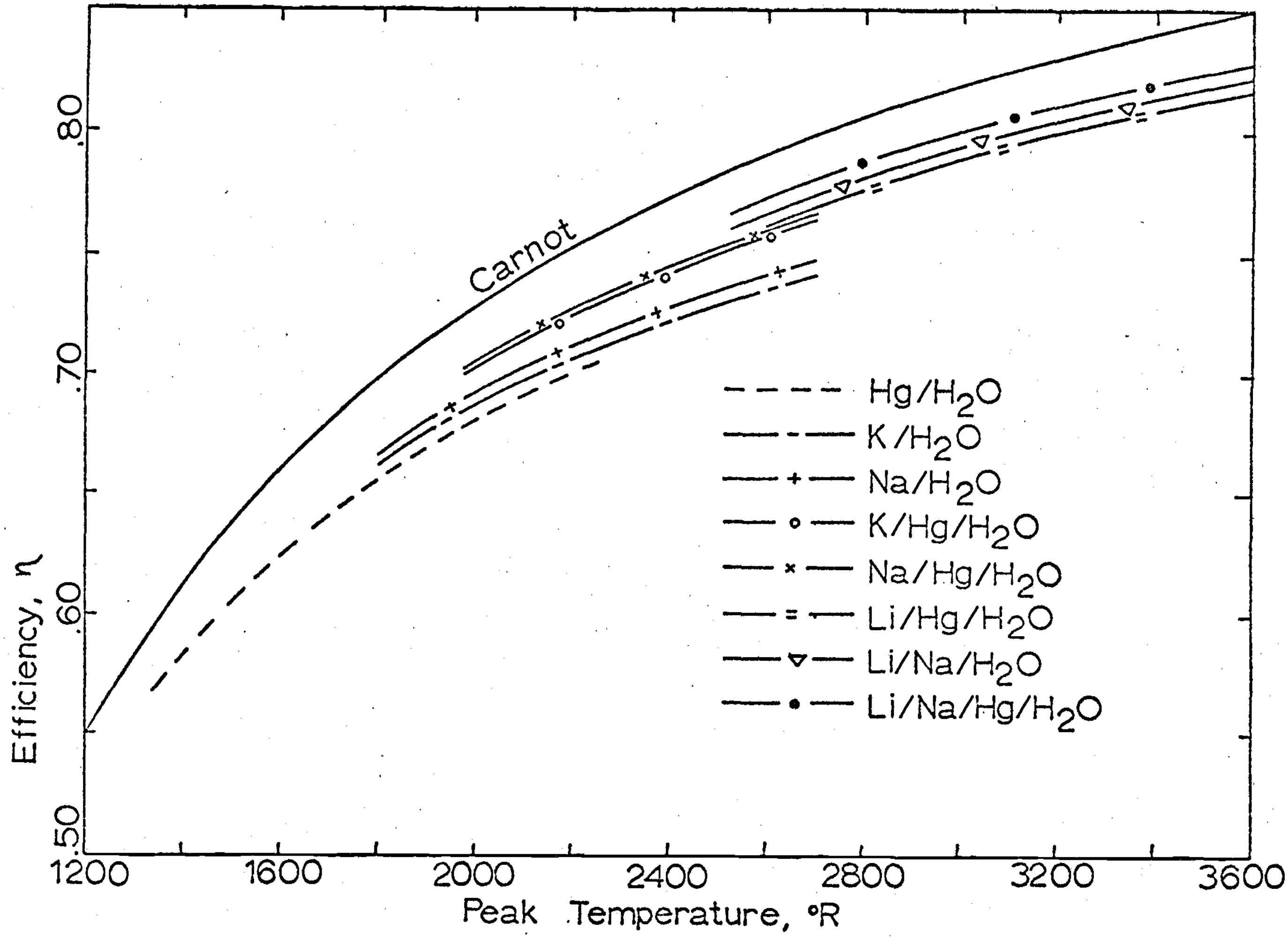

Figure 5. Efficiency of unconstrained ideal (reversible) staged Rankine cycles.

psia for lithium. Such low pressures would lead to turbine sealing problems, extremely large turbine or expander exit area, and molecular migration from materials into the working fluid. Hence, the ideal cycles were re-optimized with a minimum limit of 0.5 psia on the low pressure of all cycles and a maximum limit of 3000 psia on the high pressure. The resulting efficiencies are given in Figure 6. The potassium/steam and sodium/steam binary cycles are reduced substantially in efficiency by imposition of the 0.5 psia constraint.

Figure 7 gives the results for optimized cycles in which the major irreversibilities are included. Results for K/H_2O, Na/H_2O, and $Li/Hg/H_2O$ have been omitted since efficiencies for these fell substantially below those presented. At about 2194^oR the saturation pressure of mercury reaches the 3000 psia bound. This is the only instance where an upper pressure reaches the upper bound on pressure. Many of the stages have the lower pressure on the 0.5 psia bound however. Tables 1A, 2, 3, and 4 in Appendix III give the optimum pressures and temperatures as well as turbine exit quality, power, and flow rate for each stage. For all maximum-efficiency configurations saturated vapor enters the turbine; superheating in any stage would reduce efficiency. Table 1C illustrates this for the Hg/H_2O case where the upper temperature of the H_2O stage remains fixed, but various pressures below the saturation pressure are used. Tables 1B, 1D, and 1E give the sensitivities of the efficiency, turbine exit qualities, power distribution, and mass flow rates for the Hg/H_2O configuration to interstage temperature and turbine efficiencies.

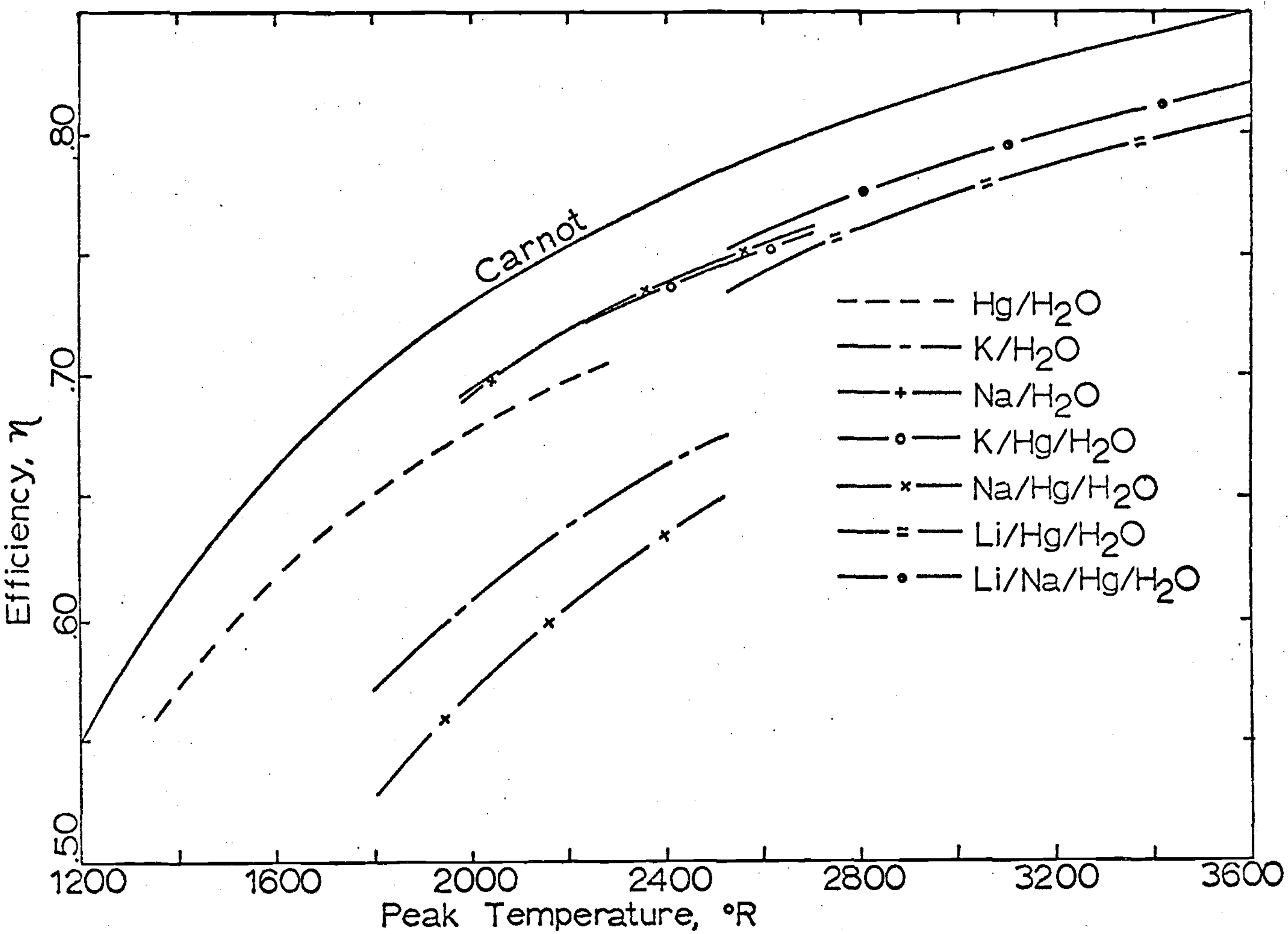

Figure 6. Efficiency of ideal (reversible) staged Rankine cycles with pressure constrained to $0.5 \leq p \leq 3000$ psia

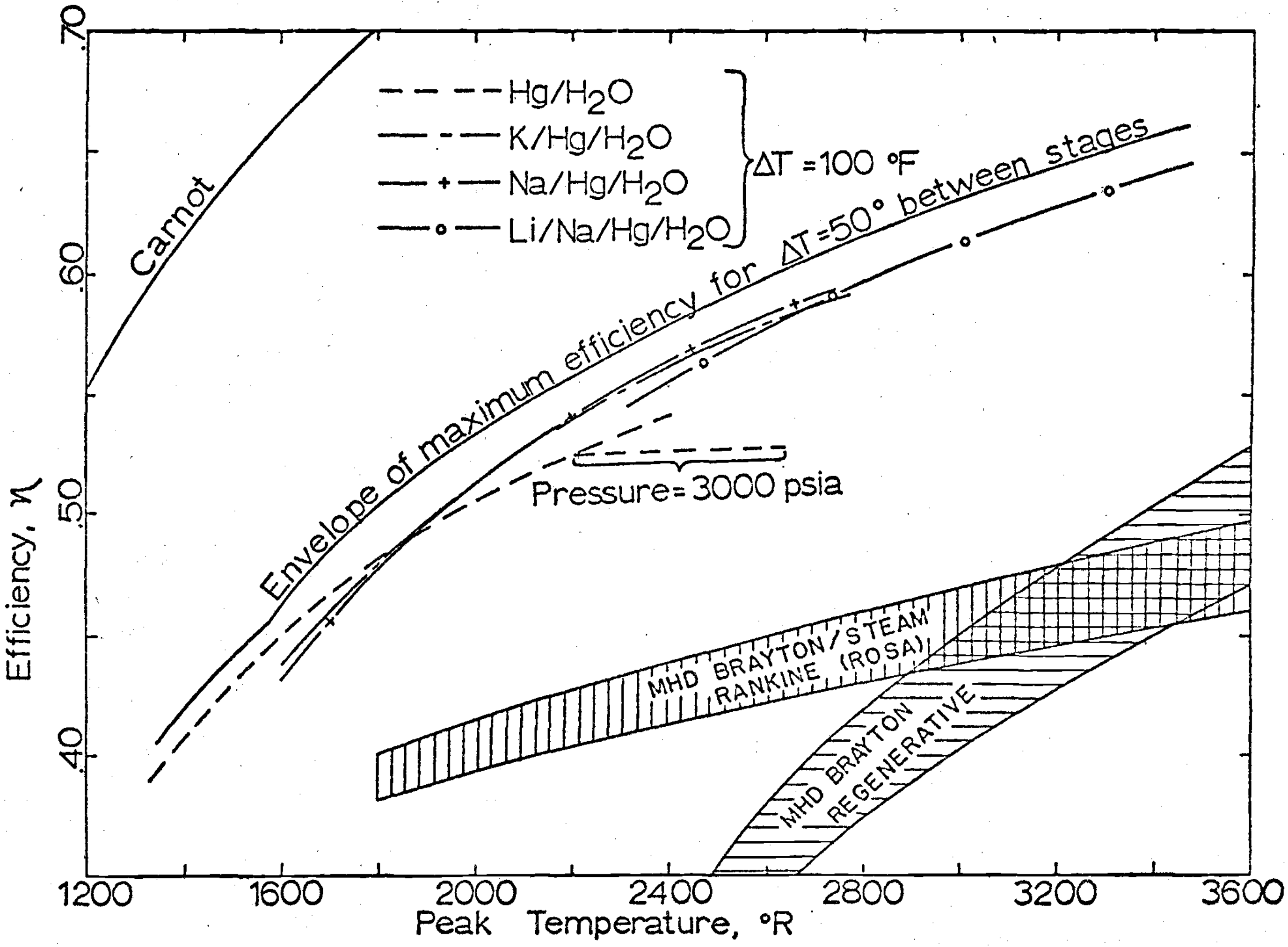

Figure 7. Efficiency of constrained irreversible real cycles.

In order to get an idea of the sensitivity of the results to ΔT, the staged cycles were re-optimized for $\Delta T=50^{\circ}F$ between stages. Just the envelope of these results is included in Figure 7 for comparison to the other curves. It appears likely that in many cases a ΔT of $50^{\circ}F$ would require an uneconomically large heat exchanger, so the envelope for $\Delta T=50^{\circ}F$ should be used only to give an indication of the relative effect of changes in ΔT.

Also included in Figure 7 are the approximate capabilities of MHD energy conversion from reference 9. Comparing the maximum efficiency curves in Figure 7 to those for MHD Brayton cycles makes it clear that staged Rankine cycles can provide higher efficiencies or competitive efficiencies at much lower temperatures. Of course, feasibility of those staged Rankine cycles at the higher temperatures requires availability of a high temperature turbine or some other high temperature expander, such as the graphite helical rotor expander or Rankine cycle MHD expander. It is seen from the tables giving optimum pressures and temperatures (Tables 1A, 2, 3, and 4) that for binary and ternary cycles only the highest temperature cycle would ever require an exotic expander. The other stages have peak temperatures within the range of turbines. Quaternary cycles would probably require exotic expanders for the top two stages. Pressures are very nominal on all stages for all cases.

Because of its very low latent heat of vaporization compared to the other working fluids, flow rates of mercury must be very large to get the required energy exchange between cycles at any particular overall power level. Because of this, its toxicity, and high temperature corrosion problems, there has been a long-standing search for a suitable replacement for mercury to be used in staged cycles. Several organic working fluids have been developed, but decomposition at high temperatures limits them to operation at temperatures below 1000°F leaving them non-competitive. No completely satisfactory replacement for mercury has been found. From Table 1A it is clear that operating at higher temperatures reduces the required flow rate substantially because of higher efficiency. Higher temperatures do increase corrosion problems, however, and would require special materials. From tables 2, 3, and 4 it is seen that adding more stages reduces the power burden on the mercury cycle, hence somewhat further reducing flow rate. In all cases, however, the mercury flow rate is an order of magnitude greater than for the other stages. The most satisfactory solution would still be to find a replacement for mercury.

Two general areas of investigation remain to give a complete picture of the efficiency potential of staged Rankine cycles:

(1) consideration of additional working fluids;

(2) optimization of cycles including cycle improvements such as extraction/regeneration, superheat with reheat, and condenser subcooling to increase mean interstage temperature difference for heat exchange.

CHAPTER III

AUGMENTED STAGED CYCLES

A. Conventional Rankine Cycle Improvements

Other than higher temperatures and pressures (which are already accounted for in the optimization), the usual means of improving Rankine cycles are (references 16 and 44) superheating, reheating, regenerative feedwater heating,[14] and improved efficiency of system components. Component efficiency improvement will not be considered here because that is a hardware design problem rather than a system design problem. Superheating improves efficiency only if a cycle has reached an upper bound on pressure, and higher temperatures are still possible. If it is possible to use the saturation pressure corresponding to the peak cycle temperature (so that the fluid is saturated vapor at turbine inlet), the efficiency will always be higher using this pressure than using a lower pressure with superheating. Therefore superheating will be advantageous when upper pressure is on a bound, and this has already been incorporated into the analyses in section II (it was necessary only for Hg/H_2O cycles).

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[14] To make clear the distinction between Brayton cycle regeneration, which is an energy exchange in a heat exchanger, and Rankine cycle regeneration, which requires extraction of hot fluid from the expander, this will be referred to as extraction/regeneration.

Adding an infinite number of reheats would clearly increase the efficiency of a Rankine cycle with superheat because it amounts to adding a Carnot cycle at the highest cycle temperature (see page 20 of reference 44). Reheaters are very costly so this is obviously impractical. Adding one reheat to a cycle with superheat can raise the efficiency slightly if it begins at a high enough temperature (page 395 of reference 16). In general, however, reheating is done primarily to increase the quality at the final expander outlet to reduce blade erosion problems. For a cycle with no superheat it appears that reheating would offer little, if any, efficiency improvement since it is marginal in a cycle with superheat. A check of the effect of adding reheat to a non-superheat steam cycle with upper temperature of 700°R (25.0 psia) and lower temperature of 575°R (1.47 psia) was made. Without reheat the ideal cycle efficiency is 16.9% and with a single reheat starting at 636.8°R (7.0 psia) the ideal efficiency is 16.4%. While reheating offers no efficiency advantages it appears the reduction in efficiency may be small enough to consider reheating as a means to obtain increased quality at final expander exit. If it is necessary to reduce the quantity of liquid in the expander, the overall cost of using superheating should be compared to that for liquid extraction.

The only feasible way to accomplish regeneration in most Rankine cycles is to extract hot fluid from the turbine and use its energy to preheat the fluid entering the boiler. Two types of heating are possible:

using an open feedwater heater (mixing the hot and cold fluids - also called contact or mixing regeneration) or a closed feedwater heater (exchange energy in a heat exchanger prior to mixing). Mixing regeneration is more efficient but requires an added pump for each feedwater heater. Closed feedwater heaters are preferred because, when several extractions are used, the fluid extracted from the expander can be flashed to low pressure (after giving up thermal energy in the heat exchanger) and then ultimately mixed with the cold liquid leaving the condenser. The closed feedwater heating system can be designed to use only one additional pump regardless of the number of extractions, but flashing to lower pressure wastes a small amount of the energy. Mixing regeneration was selected for consideration here because it will give an indication of the maximum potential gain from extraction/regeneration. Ultimate selection of closed feedwater heating would reduce the overall efficiency gains slightly.

Figure 8 illustrates the cycle arrangement for a stage having a single extraction with mixing regeneration. A fraction of the fluid passing through the turbine is extracted and mixed with fluid which has been pumped to the same pressure after leaving the condenser. Thermal energy (represented by e'f on the T-s diagram) is given to the liquid to raise its temperature (d'f on the diagram). The resulting mixture at f is then pumped to boiler pressure (g'). The mass fraction to be extracted is fixed by the requirement that the energy given up in condensing must just equal the energy

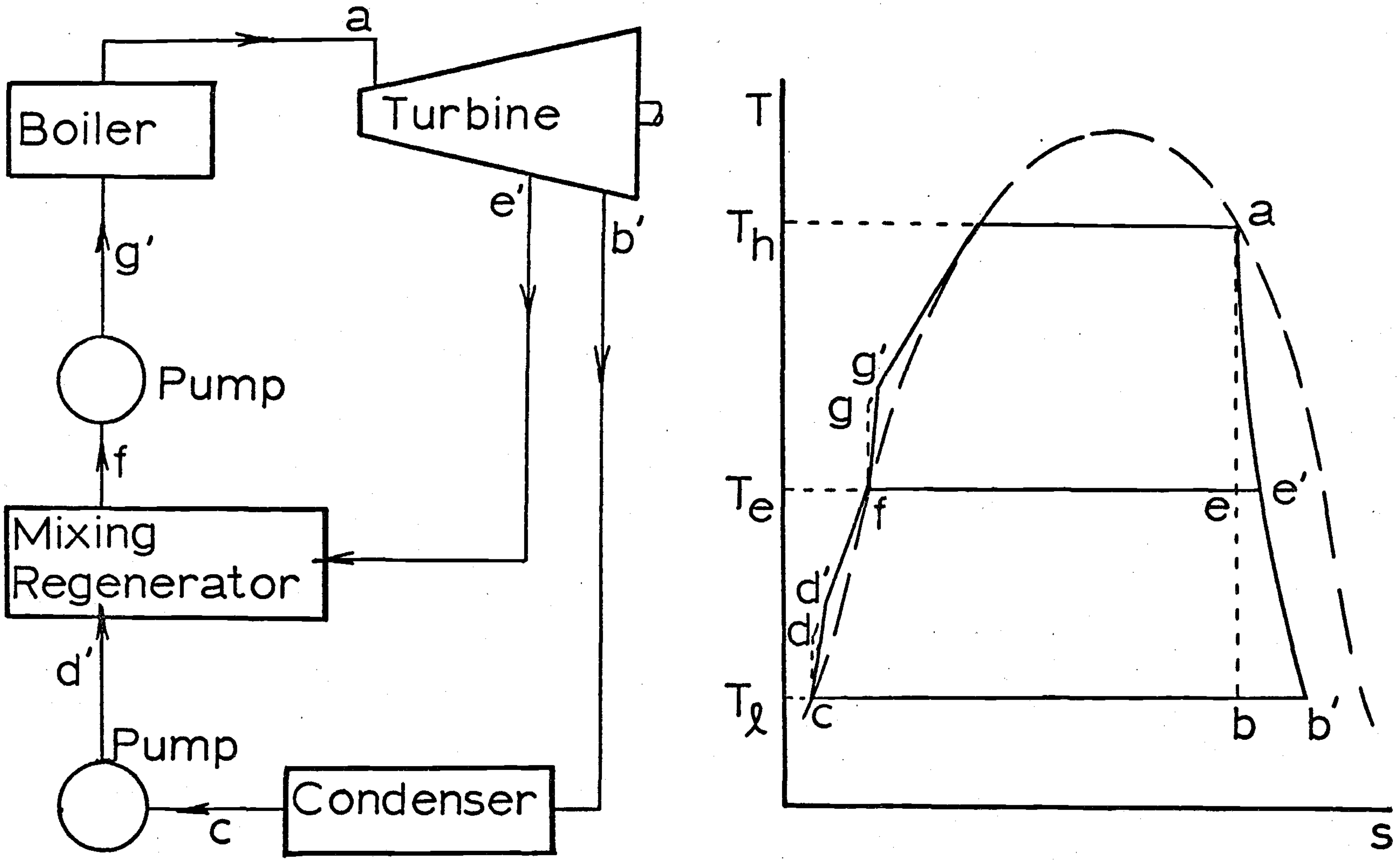

Figure 8. Rankine cycle with a single extraction and mixing regeneration.

required to raise the temperature of the remainder of the mass from d' to f. Therefore, the only free (independent) variable added by the extraction is the extraction temperature, T_e. Initially a subroutine was written which would find the optimum extraction temperature, T_e (i.e., to maximize stage efficiency which in turn gives maximum overall efficiency). This subroutine is given in Appendix II.C. Details of optimized cycles for a Hg/H_2O binary system with one optimized extraction/ regeneration in each stage are given in Table 6 of Appendix III. It is worth noting from a comparison of Table 1.A and Table 6 that the optimum temperatures for the heat exchange between stages are quite different for staged simple cycles and staged cycles with one extraction/regeneration.

It was also noted in Table 6 that the optimum extraction temperature, T_e, for each stage is very close to being midway between the high and low temperatures of the stage. Runs were made re-optimizing the Hg/H_2O cycles but with the extraction in each stage midway between the high and low temperatures. Results (Table 7) showed negligible change in optimum Hg-H_2O interface temperature and in efficiency. As more extractions are added, narrowing the temperature range between extractions, the optimum extraction points should be even more evenly spaced in temperature. Therefore, to reduce required computer time, multiple extraction cases were optimized with extraction points equally spaced in temperature in each stage. Tables 7, 8, 9, 10, and 11 give detailed results for one through five extractions on each stage. Figure 9 also presents

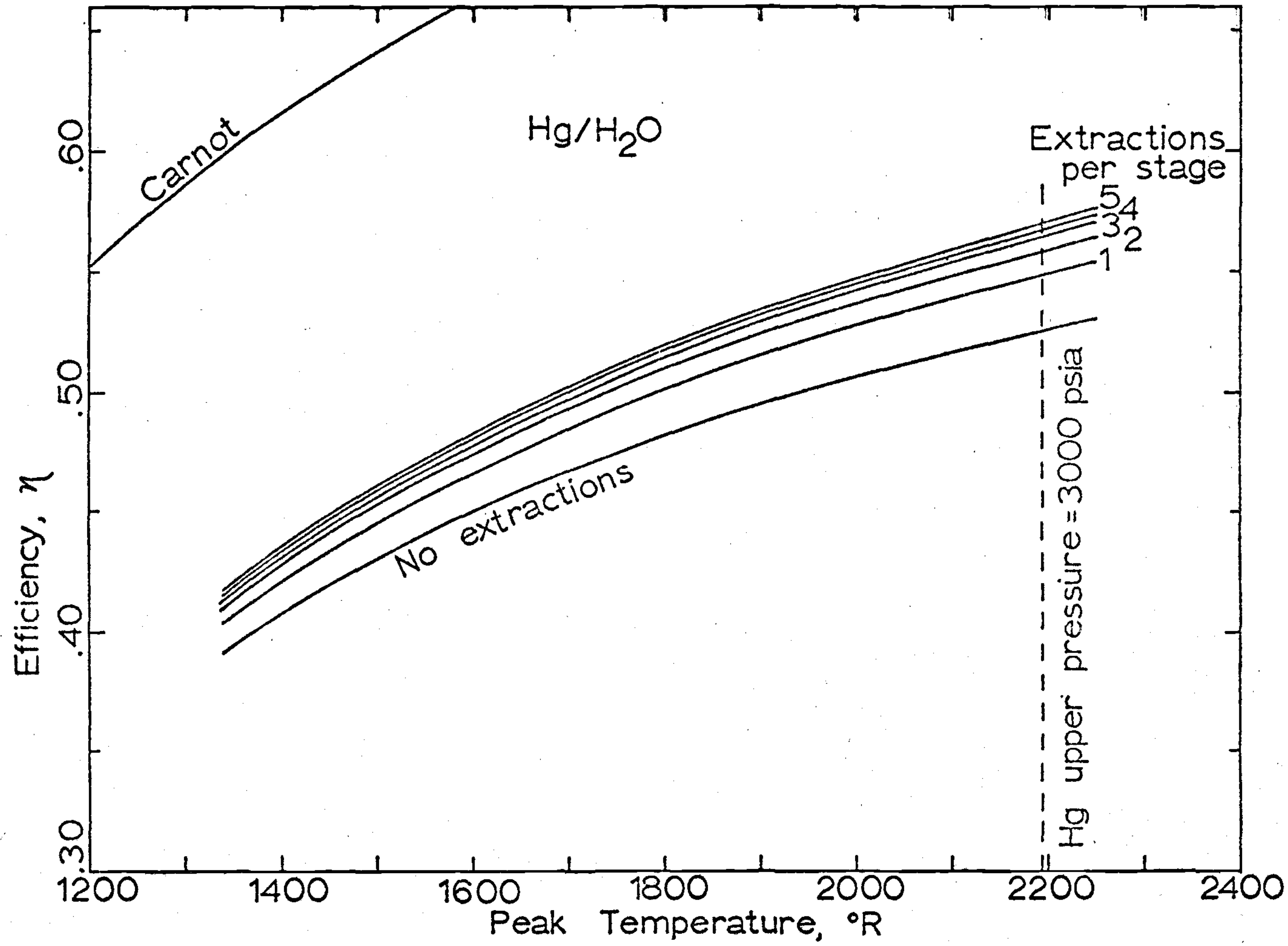

Figure 9. Efficiency of mercury/steam binary cycles with extraction/regeneration.

the efficiency potential of the binary cycles with extraction/regeneration. It is clear from Figure 9 that one extraction/regeneration per stage gives a fairly substantial efficiency gain, one to two percentage points. As more are added the gain diminishes rapidly. As usual, the decision of how many extractions to use would be based on an overall cost analysis, trading cost of extra hardware against cost advantages of increased efficiency.

With Rankine steam cycles having an upper limit on pressure, and a wide temperature range between cycle peak temperature and condenser temperature, cycle improvements such as extraction, regeneration and superheat with reheat can add substantially to efficiency. However, because upper pressure constraints do not, in general, constrict the cycles, and because the temperature range for each working fluid is much less, the efficiency improvement for staged cycles is somewhat less than for single-stage cycles.

B. Organic Working Fluids

It was concluded in section II.C that a satisfactory fluid to replace mercury is badly needed. Mercury proves to be the most advantageous fluid to use above water for all staged cycles in terms of providing greatest efficiency. However, it is costly, toxic, highly corrosive at high temperatures, and requires extremely high flow rates. All of these disadvantages were clear when mercury/steam binary plants were developed, and early proposals were made to substitute an organic working fluid, specifically diphenyl $(C_6H_5)_2$, for mercury in binary cycles (references 45 and 46).

Diphenyl (as well as several other organic fluids) has fairly good thermodynamic characteristics, is non-toxic and non-corrosive, and has the peculiar characteristic of having its saturated vapor line on a T-s diagram with a positive slope. This latter characteristic results in a superheated vapor at expander exit so that moisture problems are eliminated and regeneration similar to that used in Brayton Cycles is possible.

Organic fluids did not replace mercury as proposed because increased thermal efficiency was achieved through advances which permitted higher pressures and temperatures in steam cycles, so that binary cycles were abandoned (reference 46). However, more recently, organic fluids have been used for purposes other than for central station power generation (references 47 and 48). Initial applications were in the area of power for space missions using a nuclear source. Both isotope heat sources and nuclear reactors developed for space applications provide thermal energy at fairly low temperatures (700°F), and organic fluids have been found to yield relatively high efficiencies at these temperatures (about 20%). Other specialized applications, where a low temperature heat source is available, are being considered.

Organic fluids have one rather serious disadvantage which is important in considering them to help achieve high efficiency in staged cycles. The upper temperature of all the fluids presently available is limited to about 700° to 1000°F because of decomposition. The limit is not clearly defined for a given fluid because the rate of decomposition increases with temperature so that the limiting temperature depends on operating life of the system and the degree of decomposition which can be tolerated. For Dowtherm A, for example, the marketing data (reference 49) give experimental results for decomposition rate versus temperature. In addition, the manufacturer recommends an upper limit of $750^{\circ}F$ where the rate is quite low. The relatively low limit on the upper temperature of the fluid makes organic fluids totally noncompetitive for binary cycles. At such low temperatures the efficiency gain using binary cycles would certainly be too low to offset the added cost. Organic working fluids may be useful to replace mercury in ternary cycles where an upper bound can be placed on the upper temperature of the organic cycle.

Figure 10 illustrates a simple Rankine cycle using an organic working fluid. (In staged cycles the "boiler" and "condenser" would be heat exchangers exchanging energy with the next higher temperature stage and the next lower temperature stage.) Because the saturated vapor line has a positive slope, the state at expander exit (point C') is superheated vapor with a temperature substantially higher than the low temperature for the cycle. A' is actually only a few degrees higher in temperature than D, so C' is generally also substantially higher in temperature than A'. As a result it is possible to transfer energy in a heat exchanger from the vapor leaving the expander to the liquid leaving the pump (regeneration). The regenerative cycle using an organic fluid is shown in Figure 11.

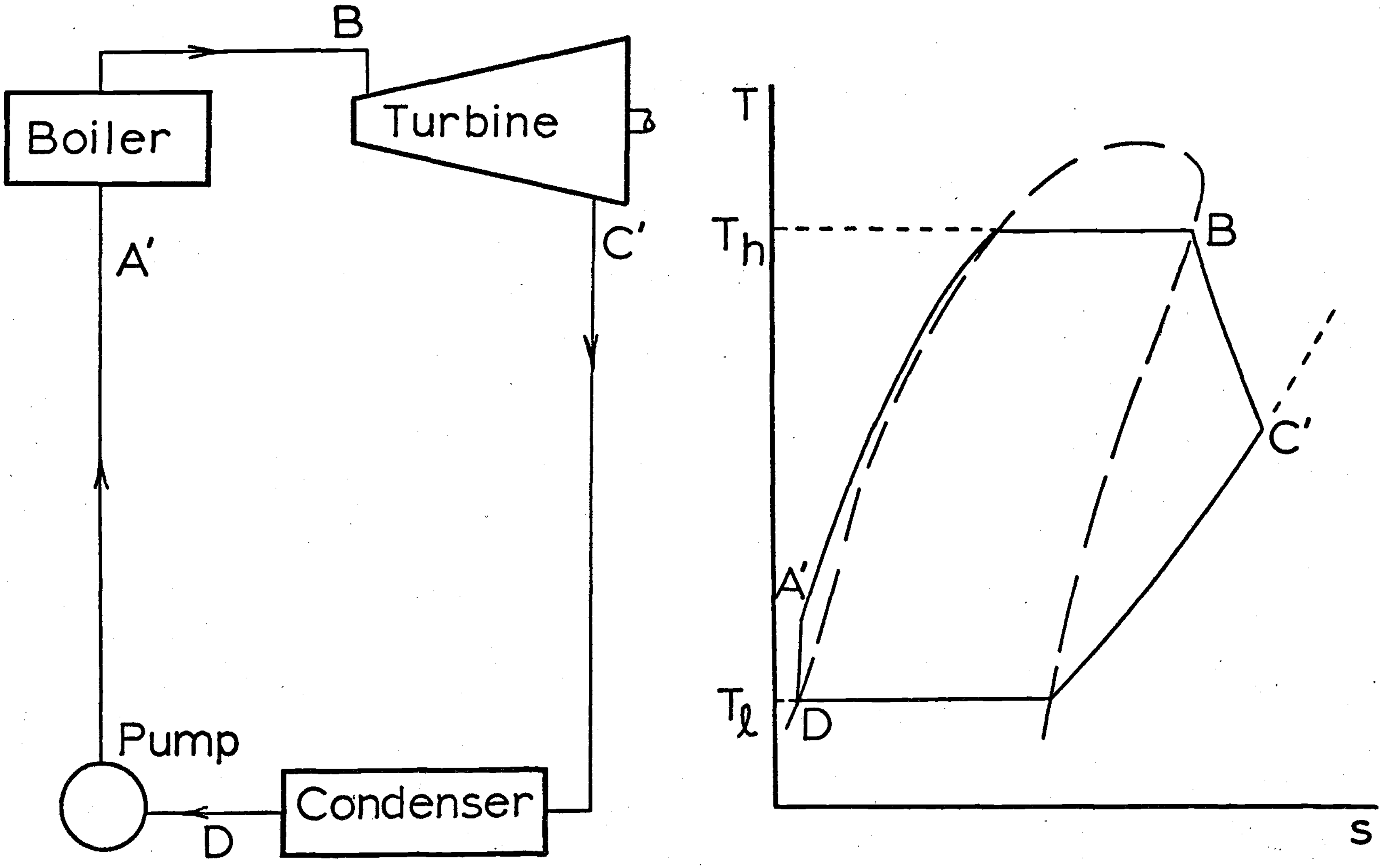

Figure 10. Simple Rankine cycle with an organic working fluid (no regeneration)

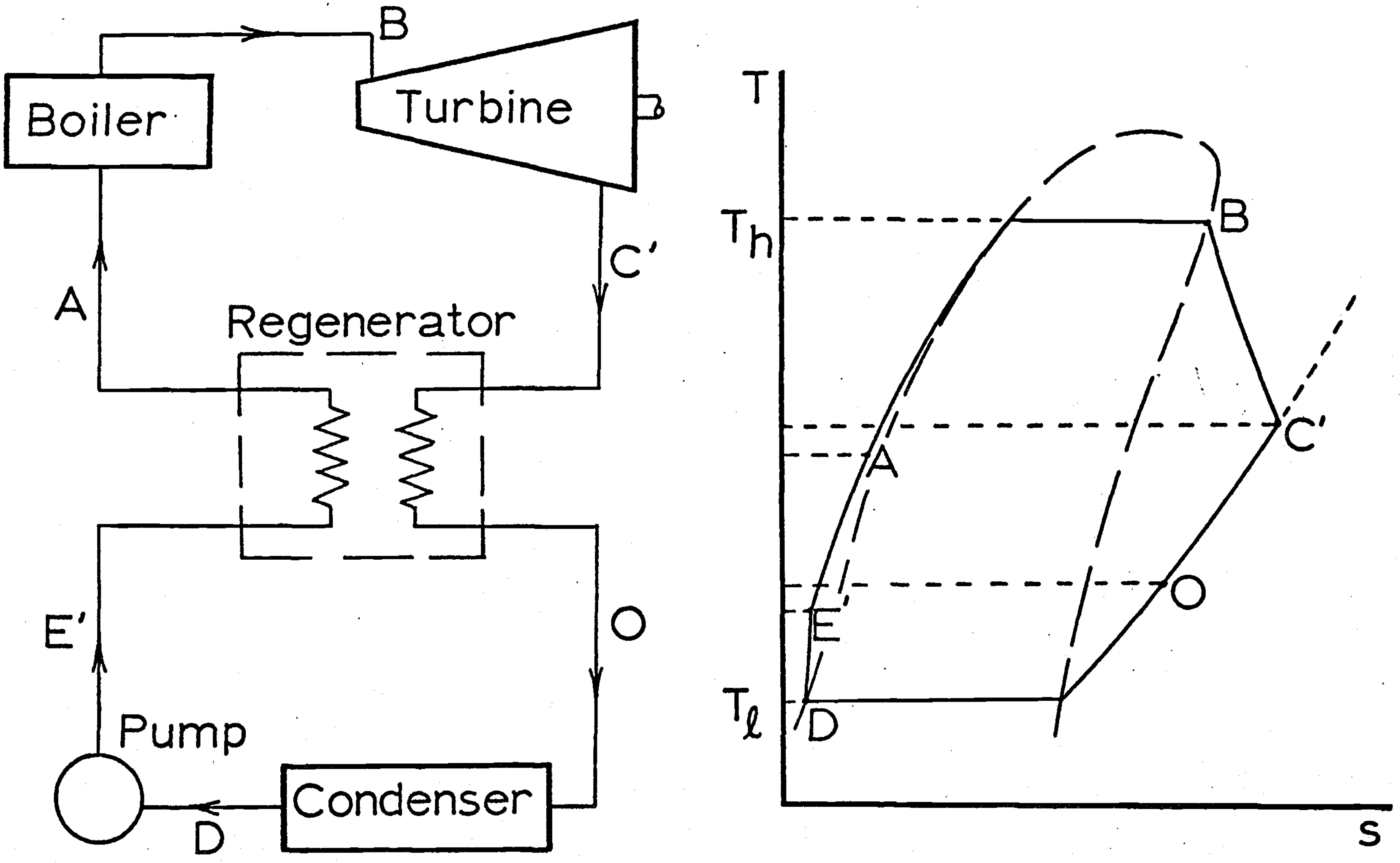

Figure 11. Rankine cycle with an organic working fluid using heat-exchange regeneration.

Two additional possibilities exist when an organic fluid stage is used in staged cycles. The higher temperature at C' could be used to reduce the temperature difference (ΔT) between the organic stage and the next lower stage because the average temperature difference would be higher than the specified ΔT. Also the higher temperature at C' could be used to superheat and raise the peak temperature of the next lower stage above T_D.

Diphenyl was chosen as a typical organic working fluid because its properties are quite typical, and because thermodynamic data for superheated vapor were readily available in usable form (reference 45). A turbine designed for diphenyl should have an efficiency advantage because the fluid throughout is dry vapor (reference 46). Therefore a turbine efficiency of 80% was used in the diphenyl stage. Binary diphenyl/steam cycles were considered first. The three possible uses for the elevated temperature at expander outlet were considered. A simple nonregenerative Rankine cycle using diphenyl was used above a non-superheat steam cycle with ΔT reduced to 50^oF. Also a regenerative diphenyl cycle was used above a non-superheat steam cycle with $\Delta T=100^oF$ (the usual assumption in section II). The subroutines for organic stages are given in Appendix II.D. The results for a peak temperature of 800^oF (region of the upper limit) are given in Table 12 (Appendix III). The efficiencies are nearly equal at about 35.5% and are too low for binary diphenyl/steam cycles to be useful. The required flow rate for diphenyl is about six times as great as for the water in the lower stage.

For mercury the required flow rate was about ten times as great as for the water stage, so diphenyl does give some improvement. The third possibility, that of using the high turbine outlet temperature to superheat the next lower stage, was also considered with a temperature difference of 100°F at both ends of the heat exchanger. The result was a 2% lower overall efficiency, so this alternative was not considered further.

Organic working fluids do not offer any advantage in binary cycles, but in ternary cycles the upper temperature of the organic stage can be limited and a higher temperature liquid metal stage used to exploit higher temperatures. Potassium/diphenyl/steam cycles were considered. Meeting the low pressure constraint of 0.5 psia on the potassium stage required a low temperature of 914°F on that stage. This in turn required a high temperature of 814°F on the diphenyl stage (which is probably very marginal in terms of maintaining the purity of the diphenyl). Results are given in Tables 13 and 14 for regenerative and nonregenerative diphenyl stages respectively. The resulting efficiencies are very nearly identical. These can be compared to Table 2 which gives results for potassium/mercury/steam cycles. Comparison of efficiencies is given in Figure 12. The ternary cycles using diphenyl in place of mercury have lower efficiencies by about one to one and one-half percentage points. The gain in using diphenyl would be lower toxicity and corrosiveness, lower cost, and somewhat lower flow rates than required when mercury is used.

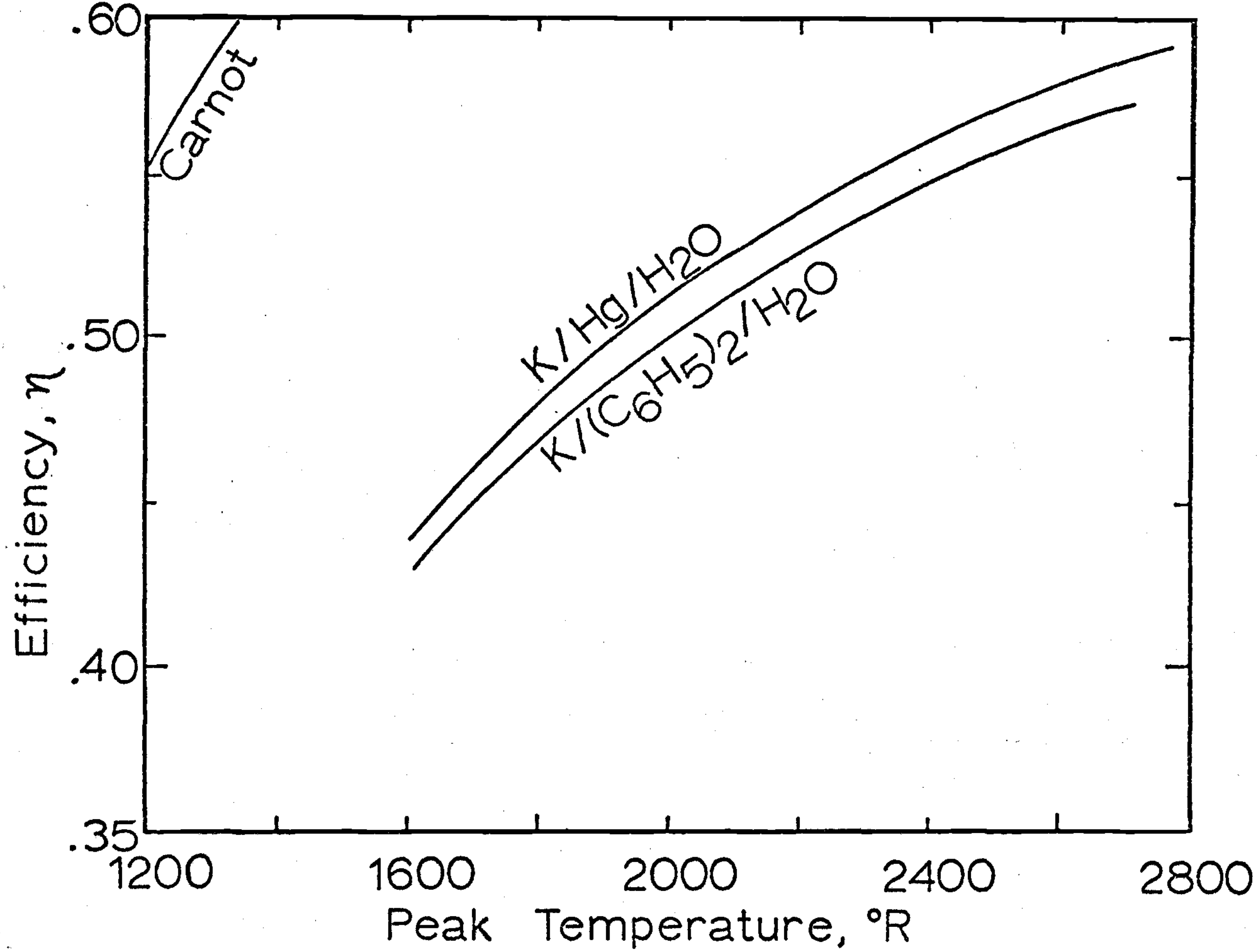

Figure 12. Comparison of $K/(C_6H_5)_2/H_2O$ ternary cycles (with and without regeneration in the diphenyl stage) to $K/Hg/H_2O$ cycles.

C. Other Fluids and Configurations

One possible version of staged Rankine cycles that has not been considered in previous sections would be using a low temperature stage below the steam cycle. This could possibly improve efficiency and would have the added benefit of increasing the low pressure in the lowest stage. It is considered desirable in Rankine cycles to operate with a condenser pressure near or above atmospheric pressure to reduce costly sealing and working fluid contamination. At the low temperature used in this study, 575°R (115°F), the saturation pressure of water is 1.47 psia (pressures down as low as 0.5 psia are common in steam plants). Adding a lower stage with a fluid having higher saturation pressures could alleviate this situation. From Figure 4 it is clear that only the fluids commonly used as refrigerants are feasible to use for a stage below the steam cycle. Ammonia was selected as a typical refrigerant and because it is more likely to be stable at high temperatures than Freon-12.[15] Since tabulated thermodynamic data are available only up to about 130°F, it was necessary to use the experimental correlation equations given in reference 50 to extrapolate data to about 180°F. The extrapolated data probably are in error by as much as 10 to 15%, but will still be adequate to indicate the suitability of ammonia for a low temperature Rankine stage. The results for binary steam/ammonia cycles showed the

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[15] Information regarding characteristics of these fluids at high temperatures is difficult to find because they are commonly used only as refrigerants.

efficiency to be lower than for steam cycles alone. The thermodynamic properties of ammonia are not superior to those of water in the temperature range considered, and the sacrifice of the 100°F of temperature range required for thermal energy exchange between stages reduced efficiency substantially. Pressures were improved with the low pressures of the steam and ammonia cycles being on the order of 40 psia and 270 psia respectively. The necessary sacrifice in efficiency to achieve this is unacceptable, however. Ternary mercury/steam/ammonia cycles were also considered and these also had lower efficiency by 3 to 4 percentage points than binary mercury/steam cycles.

In many of the optimized staged cycles using potassium or sodium stages the low pressure of the potassium or sodium stage was on the 0.5 psia bound (see Tables 2, 3, 4, 5, 13, 14). As pointed out earlier, imposition of the low pressure bound had a substantial reducing effect on efficiency (compare Figures 5 and 6). From Figure 4 it is seen that cesium has vapor pressure characteristics in the same range as potassium and sodium but has somewhat higher vapor pressures. It may be that using cesium in place of potassium or sodium could yield higher efficiencies if the 0.5 psia bound proved to be not restraining for cesium stages. Cesium stages were tried in binary and ternary configurations and it did prove true that the low pressure of cesium stages was less frequently on the 0.5 psia bound. Efficiencies, however, were not improved, and in all cases cesium stages yielded about the same efficiencies overall as did potassium stages. Only the results for ternary cesium/diphenyl/steam staged cycles are presented in Table 15 of Appendix III to illustrate the comparison of a potassium stage (Table 14) with a cesium stage (Table 15). The potassium stages in all optimum cases in Table 14 had the lower pressure on the 0.5 psia bound. Substituting cesium with its higher vapor pressures resulted in the low pressure in most cases in Table 16 being above the 0.5 psia bound. Efficiencies are not improved, however, and, since the cost of cesium is about 50 times that of potassium, it is unlikely cesium will ever be considered seriously as a working fluid.

References 19, 20 and 21 argue in favor of adding a topping cycle to present steam cycle configurations. There is a good economic argument for this since it would make maximum use of existing components. However, the current high efficiency plants are already very costly because of extremely high boiler and turbine inlet pressures, and because of the equipment required for several extraction/regeneration stages and reheats. It may be less expensive to use optimized simple cycles which achieve high efficiencies and operate at very nominal pressures (the steam cycle would never have to exceed 500 psia). Also, table 1B indicates that interstage temperature can be adjusted over a fairly wide range, if optimum pressures or temperatures are unsatisfactory, with little penalty in efficiency.

CHAPTER IV

CONCLUSIONS

A. Optimization Technique

The Sequential Unconstrained Minimization Technique proved very satisfactory for optimizing staged cycles including any variations and modifications of the cycles that were tried. In a number of cases where several of the free variables optimized on the bounds (became fixed), SUMT was probably more sophisticated than necessary However, this could not be anticipated in advance.

The program would easily handle additional free variables and could be extended to do more detailed analysis once general configuration and working fluids were selected based on the preliminary studies. For example, once working fluids were chosen, it would be possible to include calculation of optimum ΔT between stages based on heat transfer characteristics of the fluids. Also incorporating variation of expander efficiency with pressure ratio and exit quality could be easily incorporated when fluid and peak temperatures are chosen and expander operating characteristics are determined. In final detailed design optimization, minimum lifetime cost could become the objective function. Then trade-offs between capital costs and savings in boiler size and pollution control could be incorporated as well as between operating costs and fuel savings so that the true cost effects of higher efficiency are incorporated. Cost trade-offs on heat exchanger size and extractions per stage could then also be incorporated. Such an optimization program would represent the ultimate in system design optimization, and could probably save large amounts in capital and operating costs for the extremely large power plants now being built. A design optimization of a new system such as the staged Rankine cycles would require a great deal of a priori information regarding design and construction costs. However, experience in operations research and systems engineering has shown that design optimization based even on order of magnitude estimates of system parameters is better than design based on intuition with no attempt at an overall optimization.

B. Results

The efficiencies calculated for staged Rankine cycles and presented in Figure 7 clearly show that staged cycles offer the potential for either higher efficiencies than MHD power (using Brayton cycles) or similar high efficiencies at much lower peak temperatures. Utilizing this potential requires availability of an expander capable of the required high temperatures, but turbines using liquid metal working fluids have already been developed for temperatures of about 2000°R, and there is some evidence that helical rotor expanders and MHD expanders for a vapor can be developed to extend to much higher temperatures. Development of MHD conversion has been very costly and is not completed yet. On the other hand a preliminary cost analysis by Fraas (reference 19) has shown that, because of cost savings attributable to higher efficiency, a binary

Rankine plant may well cost less than a conventional coal-fired steam power plant.

This study has considered the efficiency potential for binary, ternary, and quaternary staged Rankine cycles. It is clear that for smaller total temperature ranges (i.e., for lower peak temperatures) one stage is adequate. As peak temperature increases, higher efficiency is achieved by adding stages rather than stretching one working fluid. In terms of temperature capability, the current situation indicates it is time for the transition from one to two stages, and for the next several years only binary systems are likely to be utilized. As capability for higher temperatures develops, the same arguments leading to adding a high temperature stage to current steam plants will lead to adding additional stages. Efficiency continues to rise and will offset the cost of added stages.

Fraas at Oak Ridge National Laboratory (reference 19) and Wilson at General Electric (reference 21) have shown that potassium is an acceptable compromise substitute for mercury in binary systems. Using potassium gives lower efficiency, but still high enough efficiency so that binary cycles are still attractive. A better replacement for mercury is still needed, however, and it may be that one can be found. Historically, the development of high temperature working fluids has been by way of the work in nuclear power for space applications and fast breeder reactor development. Therefore, a prime consideration has been that the fluid have a low neutron absorption cross-section. Because of this consideration, fluids may have been overlooked that would offer advantages in operation in fossil-fueled plants. Sulfur, for example, has a saturation temperature of 718°K (830°F, reference 36) at atmospheric pressure placing it fairly close to mercury (673°F) in terms of vapor pressure characteristics (Figure.4). Thermodynamic characteristics of sulfur liquid-vapor systems may or may not be suitable, but sulfur should be investigated to determine suitability. Similarly zinc should have characteristics similar to potassium and sodium but has not been investigated as a thermodynamic working fluid, probably because it has a high neutron absorption cross-section. A thorough investigation could probably yield several other working fluid candidates once a commitment is made to develop staged Rankine cycles.

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APPENDICES

APPENDIX I

NONLINEAR PROGRAMMING AND THE SEQUENTIAL UNCONSTRAINED MINIMIZATION TECHNIQUE

A. The Design Problem

Engineering design problems are frequently problems in optimizing the design of a system or component so that some characteristic is minimized or maximized. For example, in design of commercial products it is desirable to find the design which minimizes the cost of the product subject to constraints on performance. In designing systems for space applications or for aircraft or ship installations, manufacturing cost may be less important, but it may be very important to minimize weight or volume (again subject to specifications on performance). System design problems are usually quite complex, involving the interaction of a large number of independent and interdependent system variables. The procedure for finding an optimum design has frequently been trial-and-error with the intuitive application of what the designer calls his "art." All too often the optimum solution was the "best one achieved when the money runs out" (Hyde in reference 35, p. 2). Automatic digital computers have made it possible to systematize the solution of design problems. Unfortunatley, the earlier efforts consisted merely of mechanizing the trial-and-error procedures used in the past. In recent years, however, systematic and reliable procedures have been developed for solving design problems, and these are now receiving considerable attention.

The problem consists formally of the following:

Objective function: $f(\underline{x})$ (1)

(continuous, but derivatives need not be continuous)

where $\underline{x} \equiv (x_1, x_2, \ldots, x_n) = \langle x_i \rangle$, the vector of n independent variables. The objective function is to be minimized[16] subject to

Equality constraints: $g_k(\underline{x})=0;\ k=1,2,\ldots,K<n$ (2)

Inequality constraints: $h_\ell(\underline{x})\leq 0;\ \ell=1,2,\ldots,L$ (3)[17]

The objective function may be the total of initial cost and present worth of operating costs, or it may be system weight or volume, etc. The equality constraints, (2), are relations among the system variables and imply that not all of the x_i are independent, i.e., one could be eliminated for each equation (2). However, it is often impossible or inconvenient to use the equalities to eliminate variables, so allowance is made for retaining them as constraints. The inequality constraints impose performance standards, size limitations, etc. Discrete design variables such as materials to be used, cycle to be used, types of components (e.g. turbine vs. reciprocating engine), etc. are best handled by trial-and-error unless they can be made continuous (e.g. composition of an alloy.).

B. Solution of the Problem

There are two general approaches to the solution of optimization

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[16] A minimization problem will be outlined. However, note that this is general since max $f(\underline{x})=-\min[-f(\underline{x})]$ and a maximization problem can be converted to a minimization problem.

[17] A constraint $g(\underline{x})\geq 0$ can be converted to (3) by multiplying by (-1). Note that frequently independent variables must be non-negative. In such cases inequality constraints must be included to guarantee this.

problems: "direct methods" which are numerical search techniques giving a numerical answer; and "indirect methods" which are analytical techniques yielding necessary conditions for a minimum in the form of equations. Direct methods are the most general and widely applicable, because they can be applied to problems where the objective function is tabular or otherwise not easily described analytically. Also indirect methods are usually difficult for very large problems, even when a solution is theoretically possible.

The general solution procedure for direct methods is as follows:

(1) Choose a starting vector $\underline{x}_1$ (for most methods this must be feasible, i.e., satisfy the inequality constraints).

(2) Iterate according to the vector sum $\underline{x}_{j+1} = \underline{x}_j + k\,\underline{d}$ (4) where k and $\underline{d}$ are a scalar accelerator and a direction chosen at each step by some procedure to move toward the optimum with the least possible computational effort.

(3) Stop when the improvement becomes negligible, usually $|\underline{x}_{j+1} - \underline{x}_j| \leq \delta \cdot |\underline{x}_{j+1}|$ where δ is small and specified (e.g. $\delta = .01$ for $\underline{x}$ to be within about 1% of the optimum point).

The minimum found by this procedure is a local minimum in the vicinity of the starting point. However, design problems are usually unimodal (having one minimum). If the objective is not unimodal, the existence of multiple solutions will not be detected by indirect methods and the choice of $\underline{x}_1$ can then be very important.

The procedure makes use of techniques developed for unconstrained optimization problems with special modifications made for handling constraints where appropriate. Therefore, unconstrained optimization techniques will be outlined first, followed by discussion of the handling of constraints.

C. Solution of Unconstrained Problems

Unconstrained problems are solved by the iteration procedure outlined in the previous section. The key to solution of a problem is the choice of a technique for finding k and $\underline{d}$.

Three general procedures are available for determining the optimum distance, k, once the optimum direction, $\underline{d}$, has been determined.

(1) If $f(\underline{x})$ is differentiable, use differential calculus to find min f along $\underline{d}$, i.e.

$$\frac{df(\underline{x}_j+k\underline{d})}{dk} = \frac{df(k)}{dk} = 0 \quad \text{(k is the only variable)}$$

A Newton-Raphson iteration, e.g., may be required to solve this for k.

(2) Use an approximation for f(k) (e.g. quadratic), calculate points and/or derivatives as required to find the coefficients, find min analytically, and iterate until convergence criteria on k are satisfied.

(3) Use any of several types of single variable search (see reference 51, pp. 104-109). For example, choose $k=k_1$

If $f(\underline{x}_j+k_1\underline{d})>f(\underline{x}_j)$ narrow search to $0<k<k_1$

If $f(\underline{x}_j+k_1\underline{d})<f(\underline{x}_j)$ choose $k_2>k_1$ and proceed.

A procedure is developed whereby the interval containing the minimum is reduced until it is as small as desired.

The accelerator k will usually be positive, but some methods for finding $\underline{d}$ require that it be indefinite in sign (e.g. the first method given below).

A large number of methods have been developed for determining the optimum direction of descent toward the minimum. Only selected examples, starting with the simplest possible, will be given here.

(1) Univariate search: Take $\underline{d}_1 = \hat{x}_1$ (unit vector) for the first step, then $\underline{d}_2 = \hat{x}_2, \ldots, \underline{d}_n = \hat{x}_n$, then recycle through the variables until convergence criteria are met. This method is illustrated in Figure I.1. The contours are hypothetical loci of constant f. For illustration it is necessary to limit $\underline{x}$ to two dimensions (x_1, x_2). However most problems will have n considerably greater than two, and it is this multidimensionality which requires use of special techniques for solution. Most problems could be solved easily by other methods if n were restricted to two.

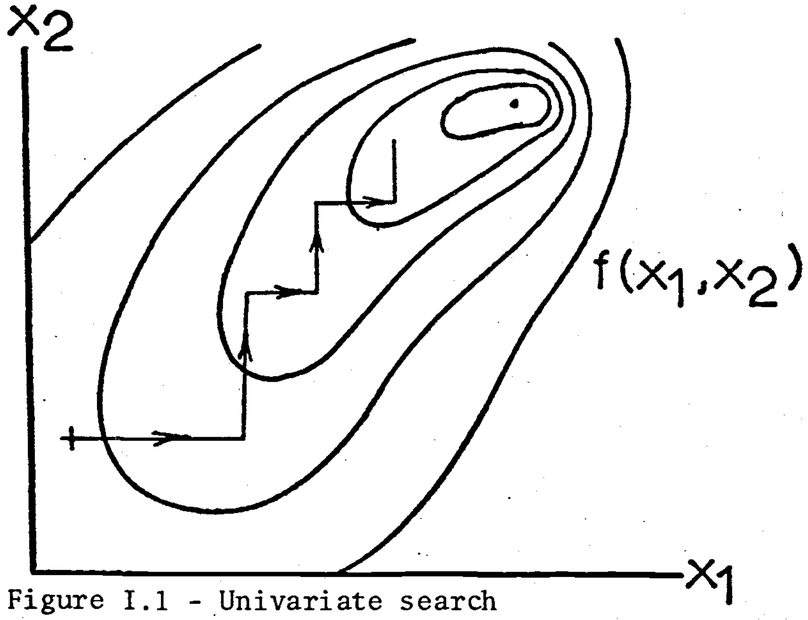

Figure I.1 - Univariate search

(2) Gradient techniques: Take $\underline{d}=-\nabla f$ (grad f) so that the move will be in the local direction of steepest descent. If $f(\underline{x})$ is not differentiable the gradient vector can be approximated numerically by

$$\nabla f = \left\langle \frac{\Delta f}{\Delta x_i} \right\rangle .$$

where the Δx_i are as small as practical (i.e. Δf must be somewhat larger than the expected roundoff errors of computation). This method is illustrated in Figure I.2.

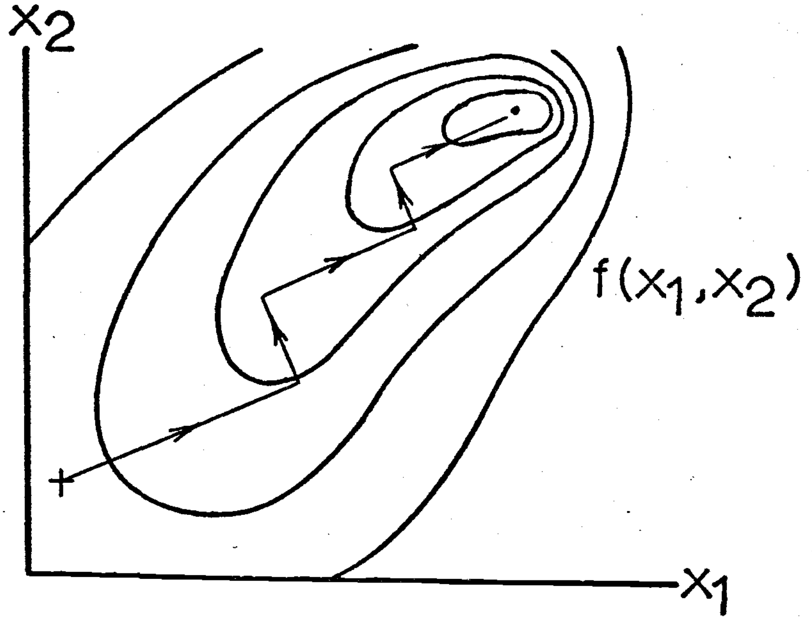

Figure I.2 - Gradient technique

(3) Add "pattern moves" (Schinzinger, reference 35, p. 170) to either of the above to accelerate the descent. That is, after n steps take the vector sum

$$\underline{d}_{n+1}=k_1\,\underline{d}_1+k_2\,\underline{d}_2+\ldots.+k_n\underline{d}_n$$

This is illustrated in Figure I.3.

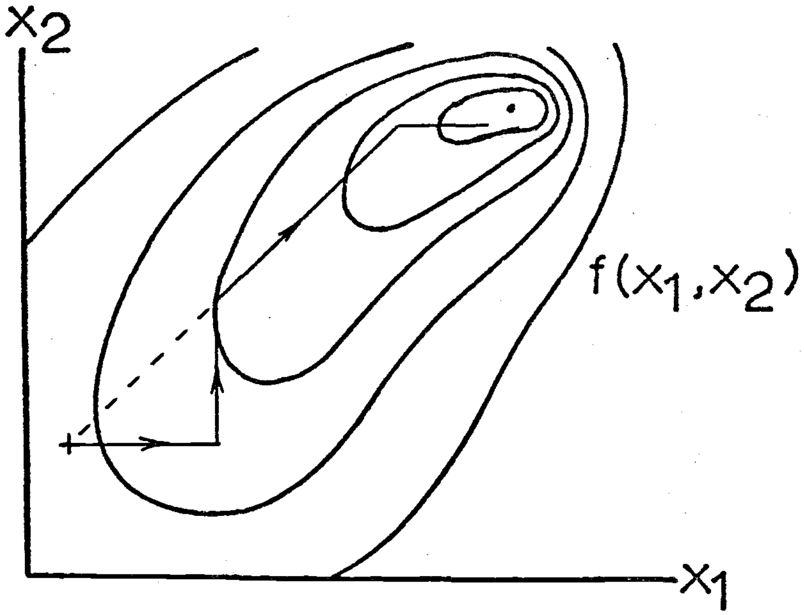

Figure I.3 - Pattern move (with univariate search)

(4) Conjugate gradient method (reference 52): Take

$$\underline{d}_1 = -\nabla f(\underline{x}_1) = -\nabla f_1$$

Then for subsequent steps

$$\underline{d}_{i+1} = -\nabla f_{i+1} + \frac{|\nabla f_{i+1}|^2}{|\nabla f_i|^2} \underline{d}_i$$

where the second term coefficient is the ratio of squared vector magnitudes. This method deflects the path $\underline{d}_{i+1}$ toward the path of the previous step $\underline{d}_i$ and hence toward the minimum. The net result in certain cases is convergence in fewer steps than required for simple gradient methods. This is illustrated in Figure I.4.

(5) Methods using the Hessian or an approximation to the Hessian: The Hessian is the n x n matrix of second partial derivatives of the objective function

$$H \equiv [h_{ij}] \ , \ h_{ij} \equiv \frac{\partial^2 f}{\partial x_i \partial x_j} \ , \ \begin{array}{l} i = 1,2,\ldots,n \\ j = 1,2,\ldots,n \end{array}$$

If H is available and easily inverted use

$$k\underline{d} = -H^{-1} \ \nabla f$$

or, in the more general case when H^{-1} is not easily available, it can be approximated numerically (see reference 53, p. 331 ff. or reference 51, p. 117 ff.). This method is also illustrated approximately by Figure I.4.

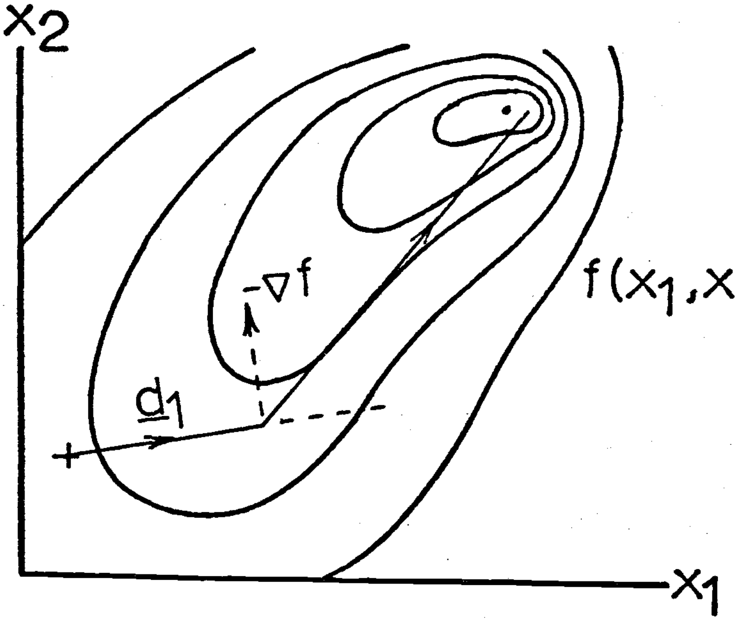

Figure I.4 - Conjugate gradient method or methods using the Hessian

Of the methods given, (4) appears to be the best compromise between simplicity and rapid convergence. If narrow ridges or valleys occur in $f(\underline{x})$, convergence may be slow because many small steps will be taken along the ridge or valley. Methods have been developed to accelerate convergence in these cases (e.g. see reference 53, p. 304 ff.).

D. Handling Constraints

A problem with constraints is solved using the same methods as those for unconstrained problems, but with special provision made for handling constraints.

(1) Augment the objective function with penalty functions.

or (2) Use special tactics in choosing $\underline{d}$ whenever a constraint is encountered.

The most widely used version of method (1) is the Sequential Unconstrained Minimization Technique (SUMT references 34 and 54). The basic idea involved is to augment the objective function so that inequality constraints (which are not to be crossed) are converted to high ridges in the f hypersurface, and equality constraints become valleys. The solution can be on one or more of the inequality constraint bounds. To allow the solution to approach the bound the steepness of the ridge must be sequentially increased. Similarly, to guarantee that the solution will lie on the equality constraints, the steepness of the valleys is sequentially increased. The augmented objective function f_a becomes, e. g.

$$f_a(\underline{x}) = f(\underline{x}) - \sum_{\ell} \frac{L_\ell}{h_\ell(\underline{x})} + \sum_{k} K_k \quad [g_k(\underline{x})]^2$$

where the h_ℓ are from the inequality constraints (3) and are negative in the feasible region. The g_k are from the equality constraints (2). Squaring g_k guarantees non-negativity and the minimum value of these terms is zero from (2). The L_ℓ and K_k are positive constants. L_ℓ is decreased and K_k increased sequentially each time convergence is achieved. That is, particular L_ℓ and K_k are set and the search proceeds until a local minimum is found. The L_ℓ and K_k are then adjusted to steepen the ridges and valleys and the search proceeds to find a new local minimum. This procedure is followed until an overall convergence criterion on $f(\underline{x})$ or $\underline{x}$ is satisfied at the latest local minimum. Schinzinger (reference 35, pp. 176-179) discusses briefly some experience with this procedure. Choosing and progressively altering the L_ℓ and K_k require some foresight and experience and a number of techniques have been tried. In using this method for handling constraints it is probably wise to monitor the constraints at each step during the single-variable search for k. The ridges may be narrow enough so that a ridge may be crossed in a single step with no indication that this has occurred.

There are a number of tactics which can be used to select $\underline{d}$ which avoid altering the objective function. Reference 53 (Chapters 2, 3, and 7) discusses several of these. A simple procedure as an example (reference 53, p. 338) would be to monitor the constraints at each step in determining k, and when a bound is crossed interpolation is used to stop right on the bound. The $\underline{d}$ for the next step is then chosen to again proceed downhill but to also avoid the possibility of hitting the same bound again immediately. A possible choice would be to make d the vector sum of

(a) The negative unit (normalized) gradient vector and

(b) The normal to the constraint pointing to the feasible side.

That is

$$\underline{d} = \frac{-\nabla f}{|\nabla f|} - \sum_{\ell} \frac{\nabla h_{\ell}}{|\nabla h_{\ell}|}$$

where the sum includes only those constraints which are currently "active", i.e. ℓ is determined by $h_{\ell}=0$. Figure I.5 illustrates this method.

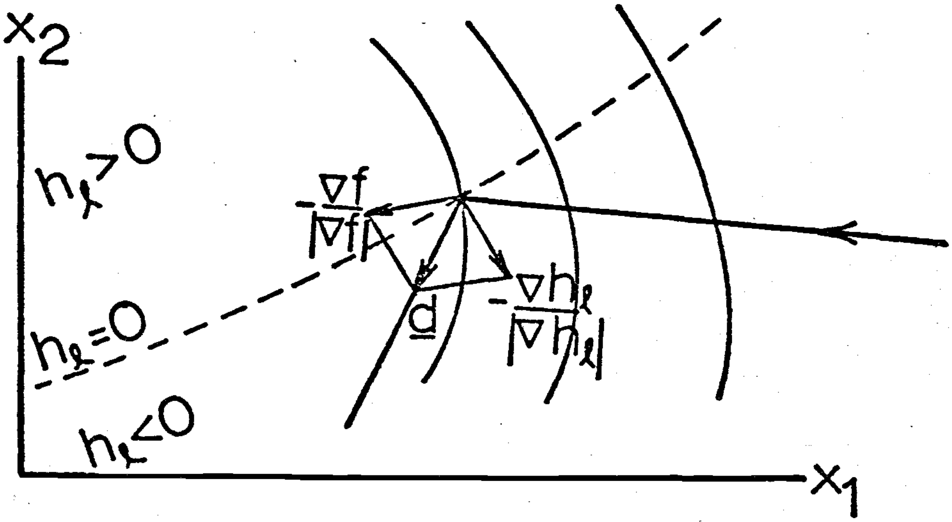

Figure I.5 - Choosing d at a bound

If for a particular problem it is desirable to handle only one type of constraint, inequality constraints can be converted to equality constraints by using "slack variables."

$$h_\ell(\underline{x})\leqslant 0$$

becomes

$$h_\ell(x_1,\ldots,x_n) + x_{n+\ell}=0 \ ; \quad x_{n+\ell}\geqslant 0;\ \ell=1,\ldots,L$$

One slack variable is added for each inequality constraint converted to an equality (but note that a non-negativity constraint on the slack variable is added also). In special cases it may be possible to change equality constraints to inequality constraints. For example a specification that power output of a generating device must be equal to the rated value

$$P(\underline{x}) = P_o \text{ or } P(\underline{x}) - P_o=0$$

can be replaced by

$$P(\underline{x}) \geqslant P_o \text{ or } P_o - P(\underline{x})\leqslant 0$$

If the solution gives a $P>P_o$ then it is learned from this that a higher power output than specified can be achieved with no penalty to the objective function (on physical grounds this is hardly to be expected--the solution would most likely be on the bound). An equality constraint can always be replaced by two inequality constraints.

$$P(\underline{x}) = P_o \rightarrow \begin{cases} P(\underline{x})\leq P_o \\ P(\underline{x})\geq P_o \end{cases}$$

E. SUMT for Staged Rankine Cycles

Because of the possibility of as many as seven independent variables to be optimized and because some of the functional relationships are available most accurately in tabular form[18], indirect approaches such as the method of Lagrange multipliers would not be feasible to optimize staged Rankine cycles. Therefore direct search methods were investigated for this purpose.

The conjugate gradient approach appears very attractive as a search method because it combines a simple method for determining direction $\underline{d}$ with maximum use of past and current available information regarding the gradient. For strictly quadratic objective functions it gives quickest convergence (reference 52). However some experimenting with a non-quadratic objective function revealed a definite problem of slow convergence resulting from a solution path which spirals around the minimum. In the example in figure I.6, the solution starting at (0,1) reaches the vicinity of the minimum rapidly. However, starting at (1,1.25) results in a path taking many iterations in spiral fashion about the minimum. Fletcher and Reeves in the original article describing this method (reference 52) also noted this difficulty. They solved the problem by arbitrarily making a gradient move (moving in the direction of the negative gradient vector) at intervals during the search. This improves

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[18] Examples for any particular working fluid: saturation pressure, enthalpy of vaporization and entropy of vaporization as functions of temperature and other thermodynamic data.

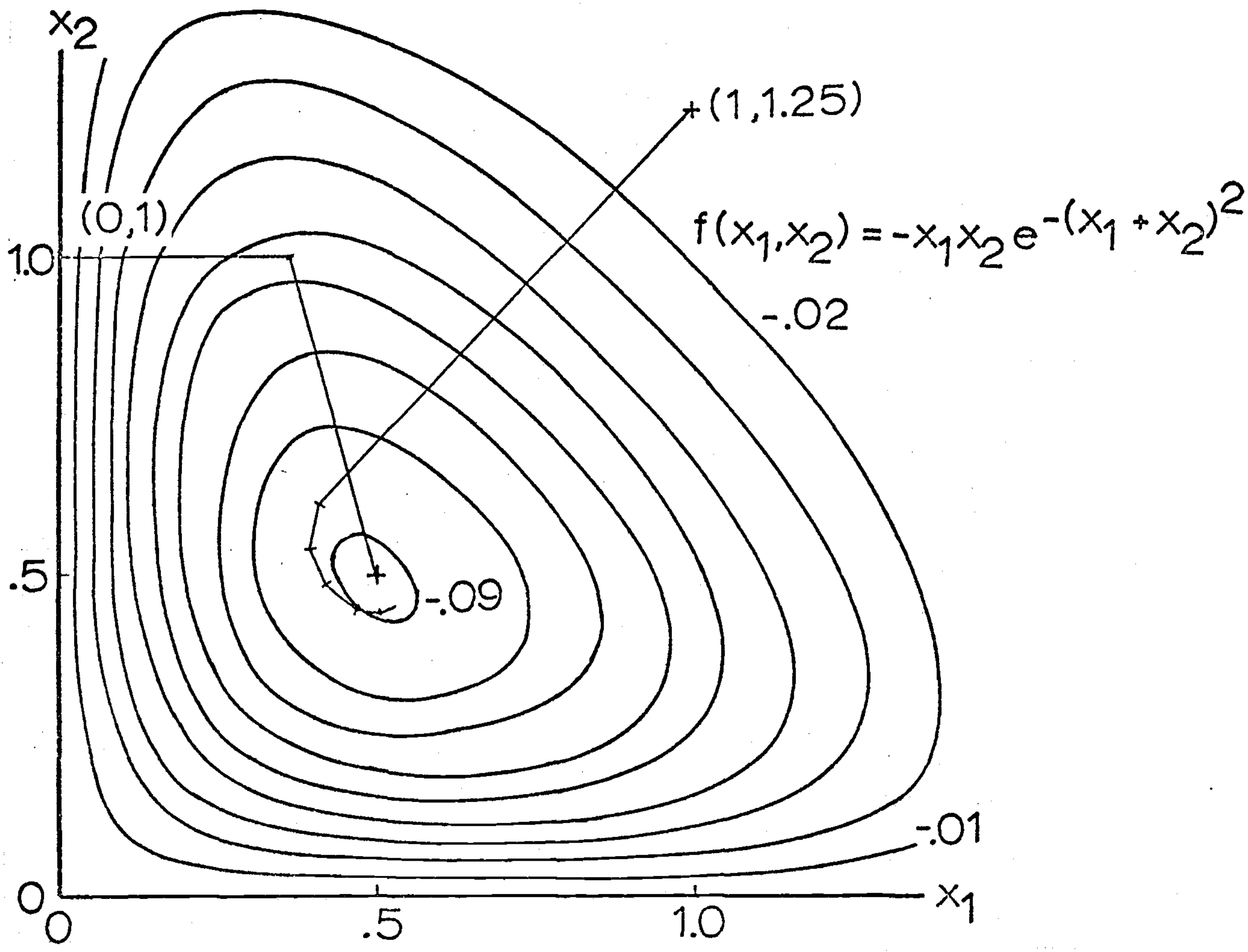


Figure I.6. Unconstrained minimization using the conjugate gradient method.

convergence, but does not eliminate the basic problem of slow convergence in some cases.

Because of the difficulty encountered with the conjugate gradient method, the gradient, or steepest descent, method was chosen for extension to the fully constrained problem. Two general techniques for handling constraints were previously discussed,

(1) use of penalty functions to augment the objective function (SUMT)

(2) use of special tactics in choosing the direction for the next step when a bound is encountered (barrier techniques)

The second of these methods provides only for inequality constraints, so each equality constraint must be replaced by two inequality constraints

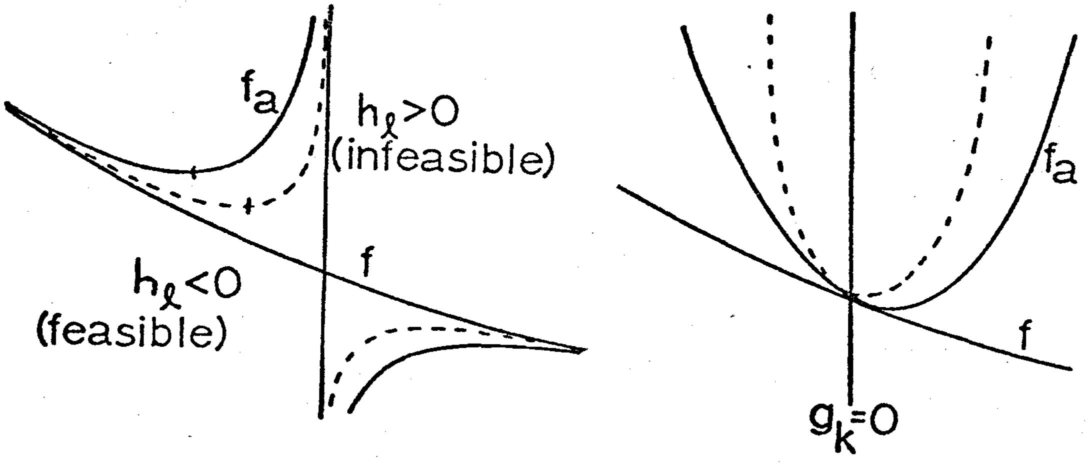
$$g_k(\underline{x})=0 \quad \rightarrow \quad -g_k(\underline{x})\leq 0 \quad \text{and} \quad g_k(\underline{x})\leq 0$$

In actual use, a small distance ϵ must be allowed between the bounds to permit the search to move along the constraint. To avoid this complication, the penalty function method or sequential unconstrained minimization technique was adopted.

The augmented objective function chosen was

$$f_a(\underline{x}) = f(\underline{x}) - \sum_{\ell} \frac{K_1}{h_\ell(\underline{x})} + \sum_{k} K_2[g_k(\underline{x})]^2$$

K_1 and K_2 are positive constants which are decreased and increased respectively at each phase during the search. The inequality constraints become infinite ridges and the equality constraints become deep valleys. The penalty functions in cross-section are shown in Figure I.7.



Penalty function for
an inequality constraint

Penalty function for
an equality constraint

Figure I.7 - Cross section of penalty functions

The dotted lines show f_a for decreased K_1 and increased K_2. Since the equality constraints become long narrow valleys, convergence would be extremely slow without the use of some acceleration technique. The simplest available acceleration technique, that of pattern moves, was used. Figure I.8 illustrates the search using the gradient technique with pattern moves for the unconstrained example.

Appendix II gives the programs developed to optimize staged Rankine cycles. In general, each program is divided into three basic parts:

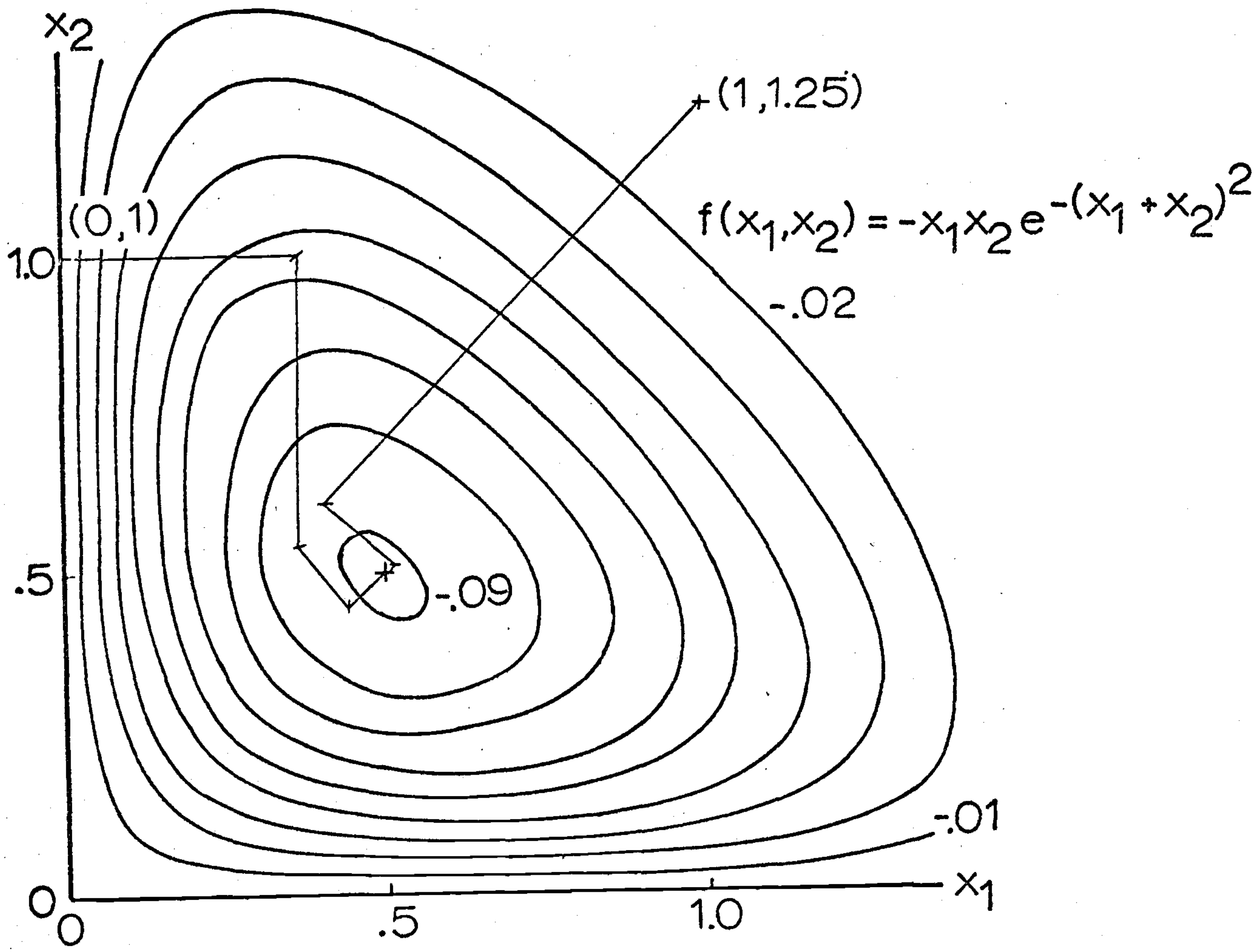


Figure I.8. Unconstrained minimization using the gradient technique with pattern moves

Main Program (SUMT)-Controls input and output, and chooses the direction for each step in the search (steps are numbered: $J = 1,2,\ldots.$).

Subroutine GRAD-Computes the constraint functions, g_k and h_ℓ; the augmented objective function, f_a; and the components of the gradient vector, ∇f_a.

Subroutine SEARCH-Conducts the search along the direction chosen by the main program for the minimum of f_a. A constant step of $|\underline{x}|/cJ$ is used until f_a increases. Quadratic interpolation is then used iteratively until the minimum is found within the desired tolerance.

APPENDIX II

COMPUTER PROGRAMS

Listings are given here of the various computer programs required to optimize the staged Rankine cycles. Comments are included in each program to make it as self-explanatory as possible. Additional description and explanation of input variables is given at the beginning of each program.

A. SUMT Optimization of Staged Rankine Cycles Allowing Superheated Vapor at Expander Entrance

This program is designed to find the maximum efficiency of a staged Rankine cycle system of up to four stages using the Sequential Unconstrained Minimization Technique. Superheated vapor at expander entrance is permitted, but this can be constrained to be on the saturation line using the constraint multiplier CON in SUBROUTINE GRAD. (When saturated vapor is desired at expander entrance for all stages it is preferable to use the program version described in Appendix II.B.)

The objective is to maximize the overall thermodynamic efficiency, η, of the n_s stages.

Maximize

$$\eta = 1 - \prod_{i=1}^{n_s} (1-\eta_i) \tag{1}$$

For each stage, i, the efficiency is

$$\eta_i = \left\{ \frac{\eta_{turb}[h_b(T_h,p_h)-h_c(T_\ell,T_h,p_h)] - \Delta h_{da}(T_\ell,p_h)/\eta_{comp}}{h_b(T_h,p_h) - h_a(T_\ell,p_h)} \right\}_i \tag{2}$$

where the points in the cycle are defined in the temperature-entropy diagram of the Rankine cycle shown in Figure II.1.

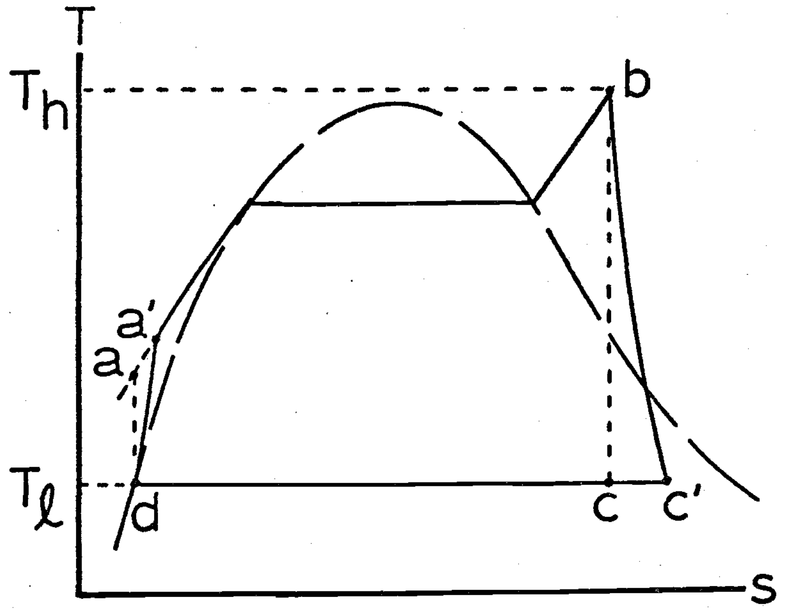


Figure II.1 Simple Rankine cycle nomenclature

T_h, T_ℓ = high and low temperatures of stage

p_h, p_ℓ = high and low pressures of stage

h = enthalpy per unit mass, Btu/lbm

η_{turb} = expander efficiency

η_{comp} = compressor (pump) efficiency

The maximization is subject to the following constraints for each stage, i.

$$T_{\ell_i} \geqslant T_{min_i} \quad (T_{min_i} \text{ specified}) \tag{3}$$

$$p_{h_i} \leqslant p_{max_i} \quad (p_{max_i} \text{ specified}) \tag{4}$$

$$T_{h_i} \geqslant T_{\ell_i} \qquad (5)$$

$$T_{h_i} \geqslant T_{sat_i}(p_{h_i}) \qquad (6)$$

$$T_{\ell_i} = T_{h_{i+1}} + \Delta T \; ; \; i=1,..,n_s-1 \qquad (7)$$

$(T_{\ell_{n_s}}$ = specified low temperature)

where ΔT is the specified temperature difference between stages (see Figure 3). Additionally (6) can be converted to an equality constraint

$$T_{h_i} = T_{sat_i}(p_{h_i})$$

by setting the constraint multiplier, CON, for the stage to one on input. The independent variables are

$$p_{h_1}$$

$$T_{h_i} \; , \; p_{h_i} \; ; \; i=2,.....,n_s$$

Note p_{ℓ_i} is the saturation pressured fixed by T_{ℓ_i} which in turn is a function of $T_{h_{i+1}}$ from (7). Therefore p_{ℓ_i} and T_{ℓ_i} are not independent variables.

Since the program is written for minimization, the objective function internal to the program is the negative of (1). Positive values of efficiency are printed, however. Constraints (3) and (4) permit placing upper and lower limits on the stage if the working fluid properties require such limits. Constraint (5) is required because mathematically the temperature range of a stage could be negative, but physically it cannot. Since thermodynamic data for the expander inlet state is superheated vapor data, a constraint must be applied to prevent the computer from extrapolating that data into the liquid-vapor mixture region, constraint (6).

The general procedure followed by the program is as follows. SUMT reads the set-up data required and then calls GRAD which computes the objective function, and the gradient vector at the starting point. GRAD makes use of R1, R2, R3, and RSTEAM as required (one for each stage) to calculate the efficiency of each stage from thermodynamic data for the fluid used by that stage. R1, R2, R3, and RSTEAM in turn use LOOK whenever it is necessary to interpolate in a table of thermodynamic data. When GRAD returns the state at the starting point to SUMT, SUMT chooses the direction $\underline{d}$ and calls SEARCH to find the minimum of the augmented objective function in that direction. SEARCH calls GRAD whenever the value of the augmented objective function or the gradient is required during the search. When the minimum along $\underline{d}$ is found by SEARCH, SUMT chooses a new $\underline{d}$ and again calls SEARCH to find the minimum. This continues iteratively until a stage tolerance is satisfied or until the number of steps (new directions, d) exceeds a specified maximum. That completes phase 1 and the minimum of the augmented objective function is found. The second phase is then started by adjusting the scaling in the augmented objective function to considerably steepen the ridges and valleys and again find the minimum iteratively. Phase 2 is then completed and phase 3 begins with new scaling, etc. This procedure continues until a specified number of phases have been completed. The optimum point is then assumed to be found with sufficient accuracy, and OUTPUT is called to print the results.

Input Required:

Program	Variable	Description	Format No.
SUMT	N	see program SUMT	10
	MAXIT	maximum number of iterations in each phase	10
	ERR	convergence tolerance on objective function in each phase	10
	AJ	scaling factor used in augmented objective function in GRAD	10
	AJM	scaling factor multiplier to increase scaling factor in each phase	10
	ITMAX	maximum number of phases	10
	IPRINT	print flag - if IPRINT is not zero then details of every step in search will be printed	10
	X(I),I=1,N	starting values of independent variables (see program SUMT)	10

GRAD	NS	see SUBROUTINE GRAD	5
	K(I),I=1,4	" " "	5
	PMAX(I)	" " "	5
	TMIN(I)	" " "	5
	DT(I)	" " "	5
	DFM(I)	" " "	6
	CON(I)	" " "	7
R1,R2, or R3 in order called by GRAD	NTS	number of saturation temperatures in table	10
	TS1	first saturation temperature (°R)	10
	DTS	saturation temperature interval (°R)	10
	DT	superheat temperature interval (°R)	10
	ETURB	expander efficiency	10
	ECOMP	compressor (pump) efficiency	10
	NT(I)	number of superheat temperatures for current sat. temp.	12
	PSAT(I)	saturation pressure for current saturation temperature (psia)	12
	F1(I)	enthalpy of saturated liquid for current sat. temp. (Btu/lbm)	12
	F2(I)	entropy of saturated liquid at current sat. temp. (Btu/lbm-°F)	12

	F3(I)	enthalpy of vaporization at current sat. temp. (Btu/lbm)	12
	F4(I)	entropy of vaporization at current sat. temp. (Btu/lbm-°F)	12
	F5(I)	specific volume of saturated liquid at current sat. temp. (ft^3/lbm)	12
	H(I,J1), J1=1,NT(I)	list of enthalpies at the NT(I) superheat points for current sat. temp. (Btu/lbm) (pseudonym D1(L) in program)	17
	S(I,J1), J1=1,NT(I)	list of entropies at the NT(I) superheat points for current sat. temp. (Btu/lbm-°F) (pseudonym D2(L) in program)	17
RSTEAM (usually called last by GRAD)	NPS	Number of saturation pressures in table	10
	PS1	first saturation pressure (psia)	10
	DPS	interval of saturation pressures (psia)	10
	DT	superheat temperature interval (°R)	10
	ETURB	expander efficiency	10
	ECOMP	compressor (pump) efficiency	10
	NT2	number of saturation temperatures in low temperature data (for condenser)	10
	T21	first temperature in low temperature data (°R)	10

DT2	saturation temperature interval in low temperature data (°R)	10
NT(I)	number of superheat temperatures for current sat. press.	12
TSAT(I)	saturation temperature for current sat. press. (°R)	12
TI(I)	first superheat temperature for current sat. press. (°R)	12
H(I,J1), J1=1,NT(I)	list of enthalpies at the NT(I) superheat points for current sat. press. (Btu/lbm) (pseudonym D1(L) in program)	17
S(I,J1), J1=1,NT(I)	list of entropies at the NT(I) superheat points for current sat. press. (Btu/lbm-°F) (pseudonymn D2(L) in program)	17
F1(I)	enthalpy of saturated liquid at the current saturation temp. [low temp. (condenser) data] (Btu/lbm)	35
F2(I)	entropy of saturated liquid at the current sat. temp. (Btu/lbm - °F)	35
F3(I)	enthalpy of vaporization at current sat. temp. (Btu/lbm)	35
F4(I)	entropy of vaporization at current sat. temp. (Btu/lbm-°F)	35
F5(I)	specific volume of saturated liquid at current sat. temp. (ft^3/lbm)	35
F6(I)	saturation pressure at current sat. temp. (ft^3/lbm)	35

In order to permit several cases (usually varying peak temperature) for the same stage configuration, the program repeats at the conclusion of each case. New data for SUMT must be provided but not for any of the subroutines. A value of zero for N in SUMT causes the program to halt. The stages are handled by the program from the highest temperature stage down to the lowest. Therefore X(1) is the upper temperature of stage 1 which is the highest temperature stage; X(2) is the upper pressure of stage 1; X(3) is the upper temperature of stage 2; etc. K(1) is the number of the Rankine cycle subroutine to use for stage 1. A value of K(1) of one causes the program to use R1 for stage 1, two causes use of R2, three causes use of R3, and four causes use of RSTEAM. R1, R2, and R3 are designed to use liquid metal working fluids. RSTEAM is designed to use steam. The thermodynamic data is read by the program starting with the highest temperature stage and proceeding to the lowest.

```
      PROGRAM SUMT
C     THIS PROGRAM FINDS THE MINIMUM OF THE NEGATIVE OVERALL EFFICIENCY
C     FOR STAGED RANKINE CYCLES USING THE METHOD OF STEEPEST DESCENT
C     WITH PATTERN MOVES.  THIS VERSION PERMITS SUPERHEATED VAPOR AT
C     TURBINE INLET ON ALL STAGES.
C         NS=NUMBER OF STAGES (1, 2, 3, OR 4 PERMITTED)
C         N=NUMBER OF INDEPENDENT VARIABLES=2*NS+1
C         X(I)=INDEPENDENT VARIABLES=UPPER TEMPERATURE OF STAGE (I+1)/2
C                                    IF I IS ODD (AND NOT EQUAL NS)
C                                   =UPPER PRESSURE OF STAGE I/2 IF
C                                    I IS EVEN
C                                   =LOWER TEMPERATURE OF LAST STAGE
C                                    WHEN I=NS
C         GK(K)=0 -- EQUALITY CONSTRAINTS (THERE CAN BE AS MANY AS
C                    NS OF THESE)
C         GL(L).LE.0 -- INEQUALITY CONSTRAINTS (THERE ARE 4*NS OF THESE)
C         J=COUNTER FOR NUMBER OF STEPS IN CURRENT PHASE OF OPTIMIZATION
C         K=COUNTER FOR NUMBER OF STEPS SINCE LAST PATTERN MOVE
C         IT=PHASE COUNTER (SCALING IN THE AUGMENTED OBJECTIVE FUNCTION
C             IS CHANGED IN EACH PHASE).
C
C
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION X( 9),DF( 9),S( 9),GK(4),GL(16),X0( 9)
    5 READ(60,10)N,MAXIT,ERR,AJ,AJM,ITMAX,IPRINT,(X(I),I=1,N)
   10 FORMAT(2I3,E13.6,     2F3.0,2I3,2(/5E13.6))
      IF(N.EQ.0) GO TO 999
      J=0
      NK=(N-1)/2
      NL=4*NK
      IT=1
    7 K=0
      DO 11 I=1,N
   11 X0(I)=X(I)
      F1=1.E20
      DO 12 I=1,N
   12 S(I)=0.
      CALL GRAD(1,X,DF,F,FA,J,GK,GL)
C     PRINT OUT DETAILS OF EACH STEP ONLY IF IPRINT=1
   15 IF(IPRINT.EQ.0)GO TO 22
      WRITE(61,18)(I,X(I),I=1,N)
   18 FORMAT(///(3H X(I2,4H) = E13.6))
      WRITE(61,19)FA,F,(I,DF(I),I=1,N)
   19 FORMAT(5H0F = E13.6//14H0AUGMENTED F =E13.6//17H GRADIENT VECTOR!/
     1    (8H      DF(I2,4H) = E13.6))
      IF(NK.EQ.0) GO TO 20
      WRITE(61,200)(I,GK(I),I=1,NK)
  200 FORMAT(42H0EQUALITY CONSTRAINTS (ALL SHOULD BE ZERO)/
     1  (8H      GK(I1,4H) = E13.6))
   20 IF(NL.EQ.0) GO TO 22
      WRITE(61,300)(I,GL(I),I=1,NL)
  300 FORMAT(56H0INEQUALITY CONSTRAINTS (ALL SHOULD BE NEGATIVE OR ZERO)
     1  /(8H      GL(I2,4H) = E13.6))
   22 J=J+1
      K=K+1
      IF(J.GT.MAXIT) GO TO 100
      DELTF=ABS(F-F1)
      F1=F
      IF(DELTF-ERR)90,25,25
   25 KN=K/(N-1)
      IF(N.LE.3)KN=K/4
```

```
      IF(KN.LT.1) GO TO 28
C     PATTERN MOVE EVERY (N-1) STEPS TO ACCELERATE CONVERGENCE
      DO 26 I=1,N
   26 S(I)=X(I)-X0(I)
      K=0
      GO TO 40
C     NORMAL GRADIENT MOVE
   28 DO 30 I=1,N
   30 S(I)=-DF(I)
   40 CALL SEARCH(N,X,DF,S,F,FA,J,NL,GK,GL)
      IF(K.GT.0) GO TO 15
      DO 50 I=1,N
   50 X0(I)=X(I)
      GO TO 15
   90 WRITE(61,95)
   95 FORMAT(31H0CONVERGENCE CRITERIA SATISFIED)
      GO TO 111
  100 WRITE(61,110)
  110 FORMAT(19H0MAXIMUM ITERATIONS)
  111 IT=IT+1
      AJ=AJ*AJM
      J=0
      IF(IT-ITMAX)7,7,120
C     WHEN OPTIMIZATION IS COMPLETED, CALL OUTPUT TO PRINT DETAILS OF
C     OPTIMUM CONFIGURATION.
  120 CALL OUTPUT(N,X)
      GO TO 5
  999 END
C
```

```
      SUBROUTINE LOOK(N,T1,DT,F,U,FT,DFT,IFLAG)
C     TABLE LOOKUP USING QUADRATIC INTERPOLATION.
C     (IF VALUE OUTSIDE TABLE, QUADRATIC EXTRAPOLATION WITH MESSAGE)
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION F(23)
      TN=T1+FLOAT(N-1)*DT
      I=(1.5+(U-T1)/DT)
      IF(I.LT.2) I=2
      IF(I.GT.N-1) I=N-1
      F1=F(I-1)
      F2=F(I)
      F3=F(I+1)
C     IF IFLAG=1, INTERPOLATE USING LOGS OF DEPENDANT VARIABLE VALUES
      IF(IFLAG.NE.1)GO TO 20
      F1=ALOG(F1)
      F2=ALCG(F2)
      F3=ALOG(F3)
   20 X1=FLOAT(I-2)*DT+T1
C     NORMALIZE BOTH COORDINATES BEFORE INTERPOLATION
       Y2=(F2-F1)/(F3-F1)
      X=(U-X1)/(2.*DT)
      A=2.-4.*Y2
      B=1.-A
      Y=A*X**2+B*X
      FT=Y*(F3-F1)+F1
      DFT=(F3-F1)*(2.*A*X+B)/(2.*DT)
      IF(IFLAG.NE.1)GO TO 30
      FT=EXP(FT)
      DFT=FT*DFT
   30 IF(U.LT.T1.OR.U.GT.TN) WRITE(61,10) U,FT
   10 FORMAT(32H0TABLE LOOKUP OUTSIDE TABLE * F(E13.6,2H)=E13.6)
      RETURN
      END
C
```

```
      SUBROUTINE SEARCH(N,X,DF,S,F,FA,J,NL,GK,GL)
C     THIS SUBPROGRAM COMPUTES THE GRADIENT AND CONDUCTS A UNIVARIATE
C     SEARCH ALONG THE DIRECTION OF THE NEGATIVE GRADIENT FOR THE
C     MINIMUM OF THE AUGMENTED OBJECTIVE FUNCTION, F.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION X( 9),DF( 9),S( 9),GK(4),GL(16)
      K=0
      DU=0.
      SMAG=0.
      DO 1 I=1,N
      DU=DU+X(I)**2
    1 SMAG=SMAG+S(I)**2
C     DU=STEP SIZE IN SEARCH ALONG DIRECTION OF GRADIENT VECTOR
C             (NOTE SCALING BY STEP COUNTER, J).
      DU=SQRT(DU)/(100.*FLOAT(J))
C     SMAG=MAGNITUDE OF GRADIENT VECTOR, S
      SMAG=SQRT(SMAG)
      F1=F
      U=DU
    2 K=K+1
C     K=SEARCH STEP COUNTER FOR SEARCH ALONG DIRECTION OF GRADIENT
C         VECTOR (LIMITED TO 20) AT EACH VALUE OF J
      IF(K.GT.20.AND.F1.NE.1.E20) GO TO 39
      DO 3 I=1,N
    3 X(I)=X(I)+U*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GK,GL)
      DO 100 I=1,NL
      IF(GL(I).GT.0.)GO TO 110
  100 CONTINUE
      GO TO 200
C     IF A BOUND IS CROSSED AT ANY STEP, TURN AROUND.
  110 DU=-DU
      F1=1.E20
      U=DU
C     IF A BOUND IS CROSSED ON FIRST STEP, TURN AROUND AND USE
C         STEPS 1/10 AS LARGE.
      IF(K.LE.1)DU=DU/10.
      GO TO 2
  200 F2=F
      G=F2-F1
      IF(G.GE.0.) GO TO 4
      F1=F2
      U=DU
      GO TO 2
C     WHEN INTERVAL OF MINIMUM IS LOCATED BY SEARCH, USE QUADRATIC
C     INTERPOLATION TO GET LOCATION OF ACTUAL MINIMUM (PROCESS REPEATED
C     UNTIL CONVERGED).
    4 DO 5 I=1,N
    5 X(I)=X(I)-.75*U*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GK,GL)
      DU=DU/4.
    8 U=0.
      FM1=F
      K=K+1
      IF(K.GT.20) GO TO 39
      DO 9 I=1,N
    9 X(I)=X(I)+DU*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GK,GL)
      FM2=F
      DO 10 I=1,N
   10 X(I)=X(I)+DU*S(I)/SMAG
```

```
      CALL GRAD(0,X,DF,F,FA,J,GK,GL)
      FM3=F
      B=(4.*FM2-FM3-3.*FM1)/(2.*DU)
      A=(FM2-B*DU-FM1)/DU**2
      U2=-B/(2*A)
C     FOR INSURANCE
      U2=-B*ABS(U2/B)
      H1=2.*ABS(DU)
      H2=ABS(U2)
      IF(H2.GT.H1) U2=H1*U2/H2
      DO 30 I=1,N
   30 X(I)=X(I)+(U2-2.*DU)*S(I)/SMAG
      CALL GRAD(1,X,DF,F,FA,J,GK,GL)
      DO 31 I=1,NL
      IF(GL(I).LE.0.) GO TO 31
      U=U2
      DU=U2/2.
      GO TO 4
   31 CONTINUE
      IF(ABS(B*DU/F)-1.E-6)40,35,35
   35 DU=U2/4.
      GO TO 8
   39 CALL GRAD(1,X,DF,F,FA,J,GK,GL)
   40 RETURN
      END
C
```

```
      SUBROUTINE GRAD(M,X,DF,F,FA,J,GK,GL)
C     THIS SUBPROGRAM COMPUTES THE OBJECTIVE FUNCTION, AUGMENTED
C     OBJECTIVE FUNCTION, AND COMPONENTS OF THE GRADIENT VECTOR AT ANY
C     GIVEN POINT.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION X( 9),DF( 9),GL(16),K(4),QI(4),DET(4,3),TMIN(4),DT(4),
     1   DFM(9),GK(4),QO(4),ROM(3),ROP(3),PMAX(4),TSAT(4),DTSAT(4),
     2   CON(16)
      IF(NS.GT.0)GO TO 10
C     NS=NUMBER OF STAGES
C     K(I)=NUMBER OF SUBROUTINE TO USE FOR STAGE I
C     PMAX(I)=MAXIMUM ALLOWED PRESSURE FOR STAGE I
C     TMIN(I)=MINIMUM ALLOWED TEMPERATURE FOR STAGE I
C     DT(I)=DIFFERENCE BETWEEN LOWER TEMPERATURE OF STAGE I AND UPPER
C          TEMPERATURE OF STAGE I+1 (OR OF COOLING WATER FOR LAST STAGE
      READ(60,5)NS,(K(I),I=1,4),(PMAX(I),TMIN(I),DT(I),I=1,4)
    5 FORMAT(5I3/12F6.0)
      NV=2*NS+1
C     DFM(I)=PARTIAL DERIVATIVE MULTIPLIER (NORMALLY 1).  TO FIX
C           VARIABLE  I, SET DFM(I)=0.
      READ(60,6)(DFM(I),I=1,NV)
    6 FORMAT(9F1.0)
      N3=3*NS
      IN3=1+N3
      IN4=NS+N3
C     CON(I)=CONSTRAINT MULTIPLIER (NORMALLY 0). WHEN
C           CON(I)=1 THE I-TH CONSTRAINT BECOMES AN EQUALITY.
C           (CURRENTLY PERMITTED ONLY FOR I=3*NS+1 TO 4*NS -- I.E., FOR
C           STAGE (I-3*NS), THE UPPER TEMPERATURE WILL BE ON THE
C           SATURATION LINE).
      READ(60,7)(CON(I),I=IN3,IN4)
    7 FORMAT(4F1.0)
   10 DO 100 I=1,NS
C     INEQUALITY CONSTRAINT -- LOWER TEMPERATURE OF STAGE I GREATER
C     THAN OR EQUAL TMIN(I)
      GL(I)=TMIN(I)-X(2*I+1) -DT(I)
      N=NS+I
C     INEQUALITY CONSTRAINT -- UPPER PRESSURE OF STAGE I LESS THAN OR
C     EQUAL PMAX(I)
      GL(N)=X(2*I)-PMAX(I)
      L=K(I)
C     T1=UPPER TEMPERATURE AND T2=LOWER TEMPERATURE OF STAGE
      T1=X(2*I-1)
      T2=X(2*I+1)+DT(I)
      I1=2*I-1
      I2=I1+1
      T(I1)=T1
      T(I2)=T2
      PH=X(2*I)
      P(I1)=PH
C     CALL PROPER SUBROUTINE TO GET EFFICIENCY AND PARTIALS OF
C     EFFICIENCY WITH RESPECT TO T1 AND T2.
      GO TO (11,12,13,14),L
   11 CALL R1(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,I)
      GO TO 20
   12 CALL R2(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,I)
      GO TO 20
   13 CALL R3(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,I)
      GO TO 20
   14 CALL RSTEAM(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,I)
   20 ET(I)=E
```

```
      QI(I)=QIOM
      QO(I)=(1.-E)*QIOM
      DET(I,1)=DE1
      DET(I,2)=DE2
      DET(I,3)=DE3
      TSAT(I)=TSATI
      DTSAT(I)=DTSATI
      N=2*NS+I
C     INEQUALITY CONSTRAINT -- T1 GREATER THAN OR EQUAL T2
      GL(N)=T2-T1
      N=3*NS+I
C     INEQUALITY CONSTRAINT -- UPPER TEMPERATURE OF STAGE GREATER THAN
C     OR EQUAL SATURATION TEMPERATURE FOR GIVEN UPPER PRESSURE
C     (NULLIFIED LATER IF CON(3*NS+I)=1)
      GL(N)=TSAT(I)-X(2*I-1)
C     EQUALITY CONSTRAINT -- UPPER TEMPERATURE OF STAGE EQUAL
C     SATURATION TEMPERATURE (NULLIFIED IF CON(3*NS+I)=0)
      GK(I)=(TSAT(I)-X(2*I-1))*CON(N)
  100 CONTINUE
      Q=1.
      DO 110 I=1,NS
  110 Q=Q*(1.-ET(I))
C     FA=OBJECTIVE FUNCTION (NEGATIVE OVERALL EFFICIENCY)
      FA=Q-1.
      SUM=0.
      SUMEQ=0.
      DO 112 I=1,NS
  112 SUMEQ=SUMEQ+GK(I)**2
      N=4*NS
      DO 115 I=1,N
  115 SUM=SUM-(1./GL(I))*(1.-CON(I))
C     F=AUGMENTED OBJECTIVE FUNCTION
      F=FA+SUM/AJ**4+AJ*SUMEQ
C     M.LT.1 IMPLIES ONLY F AND FA ARE REQUIRED, NOT THE GRADIENT.
      IF(M.LT.1)GO TO 1000
      DF(1)=0.
      N=2*NS
      DO 120 I=2,N
      A=FLOAT(I/2)-FLOAT(I)/2.
      IF(A.LT.-0.1)GO TO 118
      I2=I/2
      I3=NS+I2
      I4=3*NS+I2
C     COMPUTATION OF PARTIALS OF AUGMENTED OBJECTIVE FUNCTION WITH
C     RESPECT TO UPPER TEMPERATURE
      DF(I)=-Q*DET(I2,2)/(1.-ET(I2))+(1./GL(I3)**2+(1.-CON(I4))*
     1    DTSAT(I2)/GL(I4)**2)/AJ**4+2.*AJ*GK(I2)*DTSAT(I2)
      DF(I)=DF(I)*DFM(I)
      GO TO 120
  118 I2=(I-1)/2
      I3=I2+1
      I4=2*NS+I3
      I5=2*NS+I2
      I6=3*NS+I3
C     COMPUTATION OF PARTIALS OF AUGMENTED OBJECTIVE FUNCTION WITH
C     RESPECT TO UPPER PRESSURE
      DF(I)=-Q*DET(I2,3)/(1.-ET(I2))-Q*DET(I3,1)/(1.-ET(I3))-(1./GL(I2)
     1   **2+1./GL(I4)**2-1./GL(I5)**2+(1.-CON(I6))/GL(I6)**2)/AJ**4
     2   -2.*AJ*GK(I3)
      DF(I)=DF(I)*DFM(I)
  120 CONTINUE
      DF(2*NS+1)=0.
```

```
      N=4*NS
 1000 DO 150 I=1,N
  150 GL(I)=GL(I)*(1.-CON(I))
C     M.GT.-1 IMPLIES F, FA, AND THE GRADIENT ARE REQUIRED, NOT THE
C     CYCLE DETAILS (POWER, FLOW RATES, ETC.).
      IF(M.GT.-1)GO TO 2000
      NS1=NS-1
      DO 116 I=1,NS1
      ROM(I)=QI(I+1)/QO(I)
  116 ROP(I)=ET(I)/(ET(I+1)*(1.-ET(I)))
      SUMPR=0.
      DO 240 I=1,NS1
      FLOP(I)=1.
      POP(I)=1.
      OO 230 J=I,NS1
      FLOP(I)=FLOP(I)*ROM(J)
  230 POP(I)=POP(I)*ROP(J)
  240 SUMPR=SUMPR+POP(I)
      DEN=SUMPR+1.
      FRNS=3413./(ET(NS)*QI(NS)*DEN)
      DC 260 I=1,NS1
      POP(I)=POP(I)/DEN
  260 FLOP(I)=FLOP(I)*FRNS
      POP(NS)=1./DEN
      FLOP(NS)=FRNS
 2000 RETURN
      END
C
```

```
      SUBROUTINE R1(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR A SIMPLE RANKINE CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION PSAT(19),NT(19),H(19,21),S(19,21),F1(19),F2(19),F3(19),
     1          F4(19),F5(19),F(21),D1(4),D2(4)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY
      IF(NTS.GT.1) GO TO 100
      READ(60,10)NTS,TS1,DTS,DT,ETURB,ECOMP
   10 FORMAT(I3,3F6.0,2F6.3)
      DO 20 I=1,NTS
C     READ SATURATION AND SUPERHEAT DATA
      READ(60,12)NT(I),PSAT(I),F1(I),F2(I),F3(I),F4(I),F5(I)
   12 FORMAT(I3,E13.6,F10.3,F8.5,F10.3,F8.5,F8.5)
      NTI=NT(I)
      N=0.76+FLOAT(NTI)/4.
      DO 15 J=1,N
      READ(60,17)(D1(L),D2(L),L=1,4)
   17 FORMAT(4(F10.3,F8.5))
      DO 13 L=1,4
      J1=4*(J-1)+L
      IF(J1.GT.NTI) GO TO 15
      H(I,J1)=D1(L)
   13 S(I,J1)=D2(L)
   15 CONTINUE
   20 CONTINUE
  100 DO 110 I=1,NTS
      IF(I-NTS+1)105,125,125
  105 IF(PH-PSAT(I)) 120,120,110
  110 CONTINUE
C     LOOK UP  SATURATION PRESSURE FOR CURRENT UPPER TEMPERATURE
  120 IF(PSAT(I)-PH.GT.PH-PSAT(I-1))I=I-1
  125 X1=TS1+FLOAT(I-2)*DTS
      X2=X1+DTS
      X3=X2+DTS
      X12=X1**2
      X22=X2**2
      A1=ALOG(PSAT(I-1))
      A2=ALOG(PSAT(I))
      A3=ALOG(PSAT(I+1))
      PA=ALOG(PH)
      A=(2.*A2      -A3        -A1        )/(2.*X22-X3**2-X12)
      B=(A2      -A1        -A*(X22-X12))/DTS
      C=A2     -A*X22-B*X2
      X=(-B+SQRT(B**2-4.*A*(C-PA)))/(2.*A)
      Y=ABS(X-X2)
      IF(Y.GT.DTS.AND.I.LT.NTS-1)  X=(-B-SQRT(B**2-4.*A*(C-PA)))/(2.*A)
      TSATI=X
      PSATI=EXP(A*X**2+B*X+C)
      DTSATI=1./((2.*A*X+B)*PSATI)
      N=NT(I-1)
C     LOOK UP H AND S AT TURBINE INLET
      DO 130 J=1,N
  130 F(J)=H(I-1,J)
      CALL LOOK(N,X1,DT,F,T1,H1,DH1,0)
      DO 140 J=1,N
  140 F(J)=S(I-1,J)
      CALL LOOK(N,X1,DT,F,T1,S1,DS1,0)
      N=NT(I)
      DO 150 J=1,N
  150 F(J)=H(I,J)
      CALL LOOK(N,X2,DT,F,T1,H2,DH2,0)
```

```
      DO 160 J=1,N
  160 F(J)=S(I,J)
      CALL LOOK(N,X2,DT,F,T1,S2,DS2,0)
      N=NT(I+1)
      DO 170 J=1,N
  170 F(J)=H(I+1,J)
      CALL LOOK(N,X3,DT,F,T1,H3,DH3,0)
      DO 180 J=1,N
  180 F(J)=S(I+1,J)
      CALL LOOK(N,X3,DT,F,T1,S3,DS3,0)
      F(1)=H1
      F(2)=H2
      F(3)=H3
      CALL LOOK(3,X1,DTS,F,X,HB,DHB,0)
      F(1)=S1
      F(2)=S2
      F(3)=S3
      CALL LOOK(3,X1,DTS,F,X,SB,DSB,0)
      F(1)=DH1
      F(2)=DH2
      F(3)=DH3
      CALL LOOK(3,X1,DTS,F,X,DHDT,D,0)
      F(1)=DS1
      F(2)=DS2
      F(3)=DS3
      CALL LOOK(3,X1,DTS,F,X,DSDT,D,0)
      DHDP=DHB*DTSATI
      DSDP=DSB*DTSATI
C     LOOK UP OTHER THERMODYNAMIC PROPERTIES
      CALL LOOK(NTS,TS1,DTS,F1,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,DH,DDH,0)
      CALL LOOK(NTS,TS1,DTS,F4,T2,DS,DDS,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,VD,DVD,0)
      CALL LOOK(NTS,TS1,DTS,PSAT,T2,PL,DPL,1)
      I2=2*IST
      P(I2)=PL
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=STAGE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
C     DE1=PARTIAL OF EFFICIENCY WITH RESPECT TO T1 (UPPER TEMPERATURE)
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT)/(HB-HAPRI)
      DHCDP=DH*DSDP/DS
      DHDADP=VD*144./778.
C     DE2=PARTIAL OF EFFICIENCY WITH RESPECT TO PH (UPPER PRESSURE)
      DE2=(ETURB*(DHDP-DHCDP)-DHDADP/ECOMP-E*(DHDP-DHDADP/ECOMP))/
     1   (HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=-144.*(VD*DPL+PL*DVD)/778.+PH*DVD*144./778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE3=PARTIAL OF EFFICIENCY WITH RESPECT TO T2 (LOWER TEMPERATURE)
      DE3=(E*DHAPDT-ETURB*DHCDT-DHDADT/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HC)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE R2(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR A SIMPLE RANKINE CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION PSAT(19),NT(19),H(19,21),S(19,21),F1(19),F2(19),F3(19),
     1         F4(19),F5(19),F(21),D1(4),D2(4)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY
      IF(NTS.GT.1) GO TO 100
      READ(60,10)NTS,TS1,DTS,DT,ETURB,ECOMP
   10 FORMAT(I3,3F6.0,2F6.3)
      DO 20 I=1,NTS
C     READ SATURATION AND SUPERHEAT DATA
      READ(60,12)NT(I),PSAT(I),F1(I),F2(I),F3(I),F4(I),F5(I)
   12 FORMAT(I3,E13.6,F10.3,F8.5,F10.3,F8.5,F8.5)
      NTI=NT(I)
      N=0.76+FLOAT(NTI)/4.
      DO 15 J=1,N
      READ(60,17)(D1(L),D2(L),L=1,4)
   17 FORMAT(4(F10.3,F8.5))
      DO 13 L=1,4
      J1=4*(J-1)+L
      IF(J1.GT.NTI) GO TO 15
      H(I,J1)=D1(L)
   13 S(I,J1)=D2(L)
   15 CONTINUE
   20 CONTINUE
  100 DO 110 I=1,NTS
      IF(I-NTS+1)105,125,125
  105 IF(PH-PSAT(I)) 120,120,110
  110 CONTINUE
C     LOOK UP  SATURATION PRESSURE FOR CURRENT UPPER TEMPERATURE
  120 IF(PSAT(I)-PH.GT.PH-PSAT(I-1))I=I-1
  125 X1=TS1+FLOAT(I-2)*DTS
      X2=X1+DTS
      X3=X2+DTS
      X12=X1**2
      X22=X2**2
      A1=ALOG(PSAT(I-1))
      A2=ALOG(PSAT(I))
      A3=ALOG(PSAT(I+1))
      PA=ALOG(PH)
      A=(2.*A2      -A3        -A1        )/(2.*X22-X3**2-X12)
      B=(A2      -A1        -A*(X22-X12))/DTS
      C=A2      -A*X22-B*X2
      X=(-B+SQRT(B**2-4.*A*(C-PA)))/(2.*A)
      Y=ABS(X-X2)
      IF(Y.GT.DTS)X=(-B-SQRT(B**2-4.*A*(C-PA)))/(2.*A)
      TSATI=X
      PSATI=EXP(A*X**2+B*X+C)
      DTSATI=1./((2.*A*X+B)*PSATI)
      N=NT(I-1)
C     LOOK UP H AND S AT TURBINE INLET
      DO 130 J=1,N
  130 F(J)=H(I-1,J)
      CALL LOOK(N,X1,DT,F,T1,H1,DH1,0)
      DO 140 J=1,N
  140 F(J)=S(I-1,J)
      CALL LOOK(N,X1,DT,F,T1,S1,DS1,0)
      N=NT(I)
      DO 150 J=1,N
  150 F(J)=H(I,J)
      CALL LOOK(N,X2,DT,F,T1,H2,DH2,0)
```

```
      DO 160 J=1,N
  160 F(J)=S(I,J)
      CALL LOOK(N,X2,DT,F,T1,S2,DS2,0)
      N=NT(I+1)
      DO 170 J=1,N
  170 F(J)=H(I+1,J)
      CALL LOOK(N,X3,DT,F,T1,H3,DH3,0)
      DO 180 J=1,N
  180 F(J)=S(I+1,J)
      CALL LOOK(N,X3,DT,F,T1,S3,DS3,0)
      F(1)=H1
      F(2)=H2
      F(3)=H3
      CALL LOOK(3,X1,DTS,F,X,HB,DHB,0)
      F(1)=S1
      F(2)=S2
      F(3)=S3
      CALL LOOK(3,X1,DTS,F,X,SB,DSB,0)
      F(1)=DH1
      F(2)=DH2
      F(3)=DH3
      CALL LOOK(3,X1,DTS,F,X,DHDT,D,0)
      F(1)=DS1
      F(2)=DS2
      F(3)=DS3
      CALL LOOK(3,X1,DTS,F,X,DSDT,D,0)
      DHDP=DHB*DTSATI
      DSDP=DSB*DTSATI
C     LOOK UP OTHER THERMODYNAMIC PROPERTIES
      CALL LOOK(NTS,TS1,DTS,F1,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,DH,DDH,0)
      CALL LOOK(NTS,TS1,DTS,F4,T2,DS,DDS,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,VD,DVD,0)
      CALL LOOK(NTS,TS1,DTS,PSAT,T2,PL,DPL,1)
      I2=2*IST
      P(I2)=PL
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=STAGE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
C     DE1=PARTIAL OF EFFICIENCY WITH RESPECT TO T1 (UPPER TEMPERATURE)
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT)/(HB-HAPRI)
      DHCDP=DH*DSDP/DS
      DHDADP=VD*144./778.
C     DE2=PARTIAL OF EFFICIENCY WITH RESPECT TO PH (UPPER PRESSURE)
      DE2=(ETURB*(DHDP-DHCDP)-DHDADP/ECOMP-E*(DHDP-DHDADP/ECOMP))/
     1   (HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=-144.*(VD*DPL+PL*DVD)/778.+PH*DVD*144./778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE3=PARTIAL OF EFFICIENCY WITH RESPECT TO T2 (LOWER TEMPERATURE)
      DE3=(E*DHAPDT-ETURB*DHCDT-DHDADT/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE R3(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR A SIMPLE RANKINE CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION PSAT(19),NT(19),H(19,21),S(19,21),F1(19),F2(19),F3(19),
     1          F4(19),F5(19),F(21),D1(4),D2(4)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY
      IF(NTS.GT.1) GO TO 100
      READ(60,10)NTS,TS1,DTS,DT,ETURB,ECOMP
   10 FORMAT(I3,3F6.3,2F6.3)
      DO 20 I=1,NTS
C     READ SATURATION AND SUPERHEAT DATA
      READ(60,12)NT(I),PSAT(I),F1(I),F2(I),F3(I),F4(I),F5(I)
   12 FORMAT(I3,E13.6,F10.3,F8.5,F10.3,F8.5,F8.5)
      NTI=NT(I)
      N=0.76+FLOAT(NTI)/4.
      DO 15 J=1,N
      READ(60,17)(D1(L),D2(L),L=1,4)
   17 FORMAT(4(F10.3,F8.5))
      DO 13 L=1,4
      J1=4*(J-1)+L
      IF(J1.GT.NTI) GO TO 15
      H(I,J1)=D1(L)
   13 S(I,J1)=D2(L)
   15 CONTINUE
   20 CONTINUE
  100 DO 110 I=1,NTS
      IF(I-NTS+1)105,125,125
  105 IF(PH-PSAT(I)) 120,120,110
  110 CONTINUE
C     LOOK UP  SATURATION PRESSURE FOR CURRENT UPPER TEMPERATURE
  120 IF(PSAT(I)-PH.GT.PH-PSAT(I-1))I=I-1
  125 X1=TS1+FLOAT(I-2)*DTS
      X2=X1+DTS
      X3=X2+DTS
      X12=X1**2
      X22=X2**2
      A1=ALOG(PSAT(I-1))
      A2=ALOG(PSAT(I))
      A3=ALOG(PSAT(I+1))
      PA=ALOG(PH)
      A=(2.*A2     -A3        -A1        )/(2.*X22-X3**2-X12)
      B=(A2     -A1       -A*(X22-X12))/DTS
      C=A2     -A*X22-B*X2
      X=(-B+SQRT(B**2-4.*A*(C-PA)))/(2.*A)
      Y=ABS(X-X2)
      IF(Y.GT.DTS)X=(-B-SQRT(B**2-4.*A*(C-PA)))/(2.*A)
      TSATI=X
      PSATI=EXP(A*X**2+B*X+C)
      DTSATI=1./((2.*A*X+B)*PSATI)
      N=NT(I-1)
C     LOOK UP H AND S AT TURBINE INLET
      DO 130 J=1,N
  130 F(J)=H(I-1,J)
      CALL LOOK(N,X1,DT,F,T1,H1,DH1,0)
      DO 140 J=1,N
  140 F(J)=S(I-1,J)
      CALL LOOK(N,X1,DT,F,T1,S1,DS1,0)
      N=NT(I)
      DO 150 J=1,N
  150 F(J)=H(I,J)
      CALL LOOK(N,X2,DT,F,T1,H2,DH2,0)
```

```
      DO 160 J=1,N
  160 F(J)=S(I,J)
      CALL LOOK(N,X2,DT,F,T1,S2,DS2,0)
      N=NT(I+1)
      DO 170 J=1,N
  170 F(J)=H(I+1,J)
      CALL LOOK(N,X3,DT,F,T1,H3,DH3,0)
      DO 180 J=1,N
  180 F(J)=S(I+1,J)
      CALL LOOK(N,X3,DT,F,T1,S3,DS3,0)
      F(1)=H1
      F(2)=H2
      F(3)=H3
      CALL LOOK(3,X1,DTS,F,X,HB,DHB,0)
      F(1)=S1
      F(2)=S2
      F(3)=S3
      CALL LOOK(3,X1,DTS,F,X,SB,DSB,0)
      F(1)=DH1
      F(2)=DH2
      F(3)=DH3
      CALL LOOK(3,X1,DTS,F,X,DHDT,0,0)
      F(1)=DS1
      F(2)=DS2
      F(3)=DS3
      DHDP=DHB*DTSATI
      DSDP=DSB*DTSATI
C     LOOK UP OTHER THERMODYNAMIC PROPERTIES
      CALL LOOK(3,X1,DTS,F,X,DSDT,0,0)
      CALL LOOK(NTS,TS1,DTS,F1,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,DH,DDH,0)
      CALL LOOK(NTS,TS1,DTS,F4,T2,DS,DDS,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,VD,DVD,0)
      CALL LOOK(NTS,TS1,DTS,PSAT,T2,PL,DPL,1)
      I2=2*IST
      P(I2)=PL
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=STAGE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
C     DE1=PARTIAL OF EFFICIENCY WITH RESPECT TO T1 (UPPER TEMPERATURE)
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT)/(HB-HAPRI)
      DHCDP=DH*DSDP/DS
      DHDADP=VD*144./778.
C     DE2=PARTIAL OF EFFICIENCY WITH RESPECT TO PH (UPPER PRESSURE)
      DE2=(ETURB*(DHDP-DHCDP)-DHDADP/ECOMP-E*(DHDP-DHDADP/ECOMP))/
     1   (HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=-144.*(VD*DPL+PL*DVD)/778.+PH*DVD*144./778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE3=PARTIAL OF EFFICIENCY WITH RESPECT TO T2 (LOWER TEMPERATURE)
      DE3=(E*DHAPDT-ETURB*DHCDT-DHDADT/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE RSTEAM(T1,PH,T2,E,DE1,DE2,DE3,TSATI,DTSATI,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR A SIMPLE RANKINE CYCLE.
C     THERMODYNAMIC DATA FORMAT FOR THIS SUBPROGRAM IS DIFFERENT THAN
C     FOR R1, R2, AND R3.  THIS SUBPROGRAM IS NORMALLY USED FOR
C     THE STEAM CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),OIOM
      DIMENSION TSAT(23),NT(23),H(23,13),S(23,13),F1(13),F2(13),F3(13),
     1      F4(13),F5(13),F6(13),F(13),TI(23),D1(5),D2(5)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY
      IF(NPS.GT.1) GO TO 100
      READ(60,10)NPS,PS1,DPS,DT,ETURB,ECOMP,NT2,T21,DT2
   10 FORMAT(I3,3F6.0,2F6.3,I3,2F6.0)
C     READ HIGH TEMPERATURE SATURATION AND SUPERHEAT DATA
      DO 20 I=1,NPS
      READ(60,12)NT(I),TSAT(I),TI(I)
   12 FORMAT(I3,F8.2,F6.0)
      NTI=NT(I)
      N=0.76+FLOAT(NTI)/5.
      DO 15 J=1,N
      READ(60,17)(D1(L),D2(L),L=1,5)
   17 FORMAT(5(F7.1,F7.4))
      DO 13 L=1,5
      J1=5*(J-1)+L
      IF(J1.GT.NTI) GO TO 15
      H(I,J1)=D1(L)
   13 S(I,J1)=D2(L)
   15 CONTINUE
   20 CONTINUE
C     READ LOW TEMPERATURE SATURATION DATA
      DO 30 I=1,NT2
   30 READ(60,35)F1(I),F2(I),F3(I),F4(I),F5(I),F6(I)
   35 FORMAT(F7.2,F7.4,F7.1,F7.4,F8.5,F8.4)
C     LOOK UP  SATURATION PRESSURE FOR CURRENT UPPER TEMPERATURE
  100 CALL LOOK(NPS,PS1,DPS,TSAT,PH,TSATI,DTSATI,0)
C     LOOK UP H AND S AT TURBINE INLET
      I=1.5+(PH-PS1)/DPS
      IF(I.LT.2)I=2
      IF(I.GT.NPS-1)I=NPS-1
      N=NT(I-1)
      TI1=TI(I-1)
      DO 110 J=1,N
  110 F(J)=H(I-1,J)
      CALL LOOK(N,TI1,DT,F,T1,H1,DH1,0)
      DO 120 J=1,N
  120 F(J)=S(I-1,J)
      CALL LOOK(N,TI1,DT,F,T1,S1,DS1,0)
      N=NT(I)
      TI2=TI(I)
      DO 130 J=1,N
  130 F(J)=H(I,J)
      CALL LOOK(N,TI2,DT,F,T1,H2,DH2,0)
      DO 140 J=1,N
  140 F(J)=S(I,J)
      CALL LOOK(N,TI2,DT,F,T1,S2,DS2,0)
      N=NT(I+1)
      TI3=TI(I+1)
      DO 150 J=1,N
  150 F(J)=H(I+1,J)
      CALL LOOK(N,TI3,DT,F,T1,H3,DH3,0)
      DO 160 J=1,N
  160 F(J)=S(I+1,J)
```

```
      CALL LOOK(N,TI3,DT,F,T1,S3,DS3,0)
      F(1)=H1
      F(2)=H2
      F(3)=H3
      P1=PS1+FLOAT(I-2)*DPS
      CALL LOOK(3,P1,DPS,F,PH,HB,DHDP,0)
      F(1)=S1
      F(2)=S2
      F(3)=S3
      CALL LOOK(3,P1,DPS,F,PH,SB,DSDP,0)
      F(1)=DH1
      F(2)=DH2
      F(3)=DH3
      CALL LOOK(3,P1,DPS,F,PH,DHDT,D,0)
      F(1)=DS1
      F(2)=DS2
      F(3)=DS3
      CALL LOOK(3,P1,DPS,F,PH,DSDT,D,0)
C     LOOK UP OTHER THERMODYNAMIC PROPERTIES
      CALL LOOK(NT2,T21,DT2,F1,T2,HD,DHD,0)
      CALL LOOK(NT2,T21,DT2,F2,T2,SD,DSD,0)
      CALL LOOK(NT2,T21,DT2,F3,T2,DH,DDH,0)
      CALL LOOK(NT2,T21,DT2,F4,T2,DS,DDS,0)
      CALL LOOK(NT2,T21,DT2,F5,T2,VD,DVD,0)
      CALL LOOK(NT2,T21,DT2,F6,T2,PL,DPL,1)
      I2=2*IST
      P(I2)=PL
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=STAGE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
C     DE1=PARTIAL OF EFFICIENCY WITH RESPECT TO T1 (UPPER TEMPERATURE)
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT)/(HB-HAPRI)
      DHCDP=DH*DSDP/DS
      DHDADP=VD*144./778.
C     DE2=PARTIAL OF EFFICIENCY WITH RESPECT TO PH (UPPER PRESSURE)
      DE2=(ETURB*(DHDP-DHCDP)-DHDADP/ECOMP-E*(DHDP-DHDADP/ECOMP))/
     1   (HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=-144.*(VD*DPL+PL*DVD)/778.+PH*DVD*144./778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE3=PARTIAL OF EFFICIENCY WITH RESPECT TO T2 (LOWER TEMPERATURE)
      DE3=(E*DHAPDT-ETURB*DHCDT-DHDADT/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
C
```

```
      SUBROUTINE OUTPUT(N,X)
C     THIS SUBPROGRAM PRINTS OUT THE DETAILS OF THE FINAL OPTIMUM CYCLE
C     CONFIGURATION.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION X( 9),DF( 9),GK(4),GL(16)
      CALL GRAD(-1,X,DF,F,FA,1,GK,GL)
      EFF=-FA
      NS=(N-1)/2
      WRITE(61,10)
   10 FORMAT(≠3STAGE≠,5X,≠T-HIGH≠,5X,≠P-HIGH≠,5X,≠T-LOW≠,6X,≠P-LOW≠,6X,
     1   ≠STAGE≠,10X,≠TURBINE EXIT≠,5X,≠P/P-TOTAL≠,5X,≠M-DOT/P-TOTAL≠/
     2   ≠ ≠,10X,≠(DEG R)≠,4X,≠(PSIA)≠,5X,≠(DEG R)≠,4X,≠(PSIA)≠,5X,
     3   ≠EFFICIENCY≠,5X,≠QUALITY≠,24X,≠(LBM/HR PER KW)≠/)
      DO 20 I=1,NS
      I1=2*I-1
      I2=I1+1
   20 WRITE(61,30)I,T(I1),P(I1),T(I2),P(I2),ET(I),QUAL(I),POP(I),FLOP(I)
   30 FORMAT(≠ ≠,I3,7X,F6.1,5X,F6.1,5X,F6.1,5X,F6.2,5X,F7.5,8X,F6.4,11X,
     1   F6.4,8X,F6.3)
      WRITE(61,40)EFF
   40 FORMAT(≠0≠,33X,≠OVERALL EFFICIENCY =≠,F8.5)
      RETURN
      END
```

B. SUMT Optimization of Staged Rankine Cycles with Saturated Vapor at Expander Entrance

Experience with the program in Appendix II.A showed that, in general, saturated vapor at expander entrance always yields maximum efficiency. The time required by that program to search along the saturation line for each stage (which is a constraint line) is fairly great. Therefore a version of the program was written which only allows saturated vapor at expander entrance and which is much faster, hence saving computer time. This version is described here.

Now, since T_{h_i} is constrained to be on the saturation line, p_{h_i} and T_{h_i} are no longer independent of each other. The independent variables are

$$T_{h_i} \; ; \; i=2,\ldots,n_s$$

and the dimensionality of the optimization problem is considerably reduced. The problem now becomes

maximize

$$\eta = 1 - \prod_{i=1}^{n_s} (1-\eta_i)$$

where

$$\eta_i = \left\{ \frac{\eta_{turb}[h_b(T_h)-h_c(T_\ell,T_h)] - \dfrac{\Delta h_{da}(T_\ell,T_h)}{\eta_{comp}}}{h_b(T_h) - h_{a'}(T_\ell,T_h)} \right\}_i$$

subject to (for each stage)

$$T_{\ell_i} \geq T_{\min_i}$$

$$T_{h_i} \geq T_{\ell_i}$$

$$T_{\ell_i} = T_{h_{i+1}} + \Delta T \;;\; i=1,\ldots,n_s-1$$

$(T_{\ell_{n_s}}$ specified)

The procedure followed by this program is generally the same as that described in II.A.

Input Required:

Program	Variable	Description	Format No.
SUMT	All as described in II.A		
GRAD	NS K(I) TMIN(I) DT(I) DFM(I)	as described in II.A	
R1,R2,R3, or RSTEAM in order called by GRAD	NTS TS1 DTS ETURB ECOMP	as described in II.A	
	F1(I)	enthalpy of saturated vapor for current saturation temperature (Btu/lbm)	12
	F2(I)	entropy of saturated vapor for current sat. temp. (Btu/lbm-°F)	14
	F3(I)	saturation pressure at current sat. temp. (psia)	16

F4(I)	enthalpy of saturated liquid for current sat. temp. (Btu/lbm)	12
F5(I)	entropy of saturated liquid for current sat. temp. (Btu/lbm-°F)	14
F6(I)	specific volume of saturated liquid for current sat. temp. (ft^3/lbm)	14

Stages are always handled from highest temperature stage to lowest as described in II.A.

```
      PROGRAM SUMT
C     THIS PROGRAM FINDS THE MINIMUM OF THE NEGATIVE OVERALL EFFICIENCY
C     FOR STAGED RANKINE CYCLES USING THE METHOD OF STEEPEST DESCENT
C     WITH PATTERN MOVES.  THIS VERSION ASSUMES THE WORKING FLUID IS
C     SATURATED VAPOR AT THE TURBINE INLET ON ALL STAGES.
C         NS=NUMBER OF STAGES (1,2,3, OR 4 PERMITTED)
C         N=NUMBER OF INDEPENDENT VARIABLES=NS+1
C         X(I)=INDEPENDENT VARIABLES=TOP TEMPERATURE OF EACH STAGE PLUS
C               BOTTOM TEMPERATURE OF LAST (LOWEST TEMPERATURE) STAGE
C         GL(L).LE.0 -- INEQUALITY CONSTRAINTS (THERE ARE 2*NS OF THESE)
C         J=COUNTER FOR NUMBER OF STEPS IN CURRENT PHASE OF OPTIMIZATION
C         K=COUNTER FOR NUMBER OF STEPS SINCE LAST PATTERN MOVE
C         IT=PHASE COUNTER (SCALING IN THE AUGMENTED OBJECTIVE FUNCTION
C               IS CHANGED AT EACH PHASE).
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION X( 5),DF( 5),S( 5),GL(8),X0(5)
    5 READ(60,10)N,MAXIT,ERR,AJ,AJM,ITMAX,IPRINT,(X(I),I=1,N)
   10 FORMAT(2I3,E13.6,2F3.0,2I3/5E13.6)
      IF(N.EQ.0) GO TO 999
      J=0
      NL=2*(N-1)
      IT=1
      IOFLAG=0
    7 K=0
      DO 11 I=1,N
   11 X0(I)=X(I)
      F1=1.E20
      DO 12 I=1,N
   12 S(I)=0.
      CALL GRAD(1,X,DF,F,FA,J,GL)
C     PRINT OUT DETAILS OF EACH STEP ONLY IF IPRINT=1
   15 IF(IPRINT.EQ.0)GO TO 22
      WRITE(61,18)(I,X(I),I=1,N)
   18 FORMAT(///(3H X(I2,4H) = E13.6))
      WRITE(61,19)FA,F,(I,DF(I),I=1,N)
   19 FORMAT(5H0F = E13.6//14H0AUGMENTED F =E13.6//17H GRADIENT VECTOR!/
     1   (8H     DF(I2,4H) = E13.6))
      WRITE(61,300)(I,GL(I),I=1,NL)
  300 FORMAT(56H0INEQUALITY CONSTRAINTS (ALL SHOULD BE NEGATIVE OR ZERO)
     1  /(8H      GL(I2,4H) = E13.6))
   22 J=J+1
      K=K+1
      IF(J.GT.MAXIT) GO TO 100
      DELTF=ABS(F-F1)
      F1=F
      IF(DELTF-ERR)90,25,25
   25 KN=K/(N-1)
      IF(N.LE.3)KN=K/4
      IF(KN.LT.1) GO TO 28
C     PATTERN MOVE EVERY (N-1) STEPS TO ACCELERATE CONVERGENCE
      DO 26 I=1,N
   26 S(I)=X(I)-X0(I)
      K=0
      GO TO 40
C     NORMAL GRADIENT MOVE
   28 DO 30 I=1,N
   30 S(I)=-DF(I)
   40 CALL SEARCH(N,X,DF,S,F,FA,J,NL,GL)
      IF(K.GT.0) GO TO 15
      DO 50 I=1,N
   50 X0(I)=X(I)
```

```
      GO TO 15
   90 WRITE(61,95)
   95 FORMAT(31H0CONVERGENCE CRITERIA SATISFIED)
      GO TO 111
  100 WRITE(61,110)
  110 FORMAT(19H0MAXIMUM ITERATIONS)
  111 IT=IT+1
      J=0
      AJ=AJ*AJM
      IF(IT-ITMAX)7,7,120
C     WHEN OPTIMIZATION IS COMPLETED, CALL OUTPUT FOR DETAILED OUTPUT
  120 CALL OUTPUT(N,X)
      GO TO 5
  999 END
```

```
      SUBROUTINE LOOK(N,T1,DT,F,U,FT,DFT,IFLAG)
C     TABLE LOOKUP USING QUADRATIC INTERPOLATION.
C     (IF VALUE OUTSIDE TABLE, QUADRATIC EXTRAPOLATION WITH MESSAGE)
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION F(30)
      TN=T1+FLOAT(N-1)*DT
      I=(1.5+(U-T1)/DT)
      IF(I.LT.2) I=2
      IF(I.GT.N-1) I=N-1
      F1=F(I-1)
      F2=F(I)
      F3=F(I+1)
C     IF IFLAG=1, INTERPOLATE USING LOGS OF DEPENDANT VARIABLE VALUES
      IF(IFLAG.NE.1)GO TO 20
      F1=ALOG(F1)
      F2=ALOG(F2)
      F3=ALOG(F3)
   20 X1=FLOAT(I-2)*DT+T1
C     NORMALIZE BOTH COORDINATES BEFORE INTERPOLATION
       Y2=(F2-F1)/(F3-F1)
      X=(U-X1)/(2.*DT)
      A=2.-4.*Y2
      B=1.-A
      Y=A*X**2+B*X
      FT=Y*(F3-F1)+F1
      DFT=(F3-F1)*(2.*A*X+B)/(2.*DT)
      IF(IFLAG.NE.1)GO TO 30
      FT=EXP(FT)
      DFT=FT*DFT
   30 IF(U.LT.T1.OR.U.GT.TN) WRITE(61,10) U,FT
   10 FORMAT(32H0TABLE LOOKUP OUTSIDE TABLE * F(E13.6,2H)=E13.6)
      RETURN
      END
```

```
      SUBROUTINE SEARCH(N,X,DF,S,F,FA,J,NL,GL)
C     THIS SUBPROGRAM COMPUTES THE GRADIENT AND CONDUCTS A UNIVARIATE
C     SEARCH ALONG THE DIRECTION OF THE NEGATIVE GRADIENT FOR THE
C     MINIMUM OF THE AUGMENTED OBJECTIVE FUNCTION, F.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION X(5),DF(5),S(5),GL(8)
      K=0
      DU=0.
      SMAG=0.
      DO 1 I=1,N
      DU=DU+X(I)**2
    1 SMAG=SMAG+S(I)**2
C     DU=STEP SIZE IN SEARCH ALONG DIRECTION OF GRADIENT VECTOR
C          (NOTE SCALING BY STEP COUNTER, J).
      DU=SQRT(DU)/(100.*FLOAT(J))
C     SMAG=MAGNITUDE OF GRADIENT VECTOR, S.
      SMAG=SQRT(SMAG)
      F1=F
      U=DU
    2 K=K+1
C     K=SEARCH STEP COUNTER FOR SEARCH ALONG DIRECTION OF GRADIENT
C     VECTOR AT EACH VALUE OF J (K LIMITED TO 20).
      IF(K.GT.20.AND.F1.NE.1.E20) GO TO 39
      DO 3 I=1,N
    3 X(I)=X(I)+U*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GL)
      DO 100 I=1,NL
      IF(GL(I).GT.0.)GO TO 110
  100 CONTINUE
      GO TO 200
C     IF A BOUND IS CROSSED AT ANY STEP, TURN AROUND.
  110 DU=-DU
      F1=1.E20
      U=DU
C     IF BOUND IS CROSSED ON FIRST STEP, TURN AROUND AND USE STEPS
C     1/10 AS LARGE.
      IF(K.LE.1)DU=DU/10.
      GO TO 2
  200 F2=F
      G=F2-F1
      IF(G.GE.0.) GO TO 4
      F1=F2
      U=DU
      GO TO 2
C     WHEN INTERVAL OF MINIMUM IS LOCATED BY SEARCH, USE QUADRATIC
C     INTERPOLATION TO GET LOCATION OF ACTUAL MINIMUM.
    4 DO 5 I=1,N
    5 X(I)=X(I)-.75*U*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GL)
      DU=DU/4.
    8 U=0.
      FM1=F
      K=K+1
      IF(K.GT.20) GO TO 39
      DO 9 I=1,N
    9 X(I)=X(I)+DU*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GL)
      FM2=F
      DO 10 I=1,N
   10 X(I)=X(I)+DU*S(I)/SMAG
      CALL GRAD(0,X,DF,F,FA,J,GL)
```

```
      FM3=F
      B=(4.*FM2-FM3-3.*FM1)/(2.*DU)
      A=(FM2-B*DU-FM1)/DU**2
      U2=-B/(2*A)
C     FOR INSURANCE
      U2=-B*ABS(U2/B)
      H1=2.*ABS(DU)
      H2=ABS(U2)
      IF(H2.GT.H1) U2=H1*U2/H2
      DO 30 I=1,N
   30 X(I)=X(I)+(U2-2.*DU)*S(I)/SMAG
      CALL GRAD(1,X,DF,F,FA,J,GL)
      DO 31 I=1,NL
      IF(GL(I).LE.0.) GO TO 31
      U=U2
      DU=U2/2.
      GO TO 4
   31 CONTINUE
      IF(ABS(B*DU/F)-1.E-6)40,35,35
   35 DU=U2/4.
      GO TO 8
   39 CALL GRAD(1,X,DF,F,FA,J,GL)
   40 RETURN
      END
```

```
      SUBROUTINE GRAD(M,X,DF,F,FA,J,GL)
C     THIS SUBPROGRAM COMPUTES THE OBJECTIVE FUNCTION, AUGMENTED
C     OBJECTIVE FUNCTION, AND COMPONENTS OF THE GRADIENT VECTOR AT ANY
C     GIVEN POINT.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION X( 5),DF( 5),GL(8),K(4),QI(4),DET(4,2),TMIN(4),DT(4),
     1     DFM(9),       QO(4),ROM(3),ROP(3)
      IF(NS.GT.0)GO TO 10
C     NS=NUMBER OF STAGES
C     K(I)=NUMBER OF SUBROUTINE TO USE FOR STAGE I
C     TMIN(I)=MINIMUM ALLOWED TEMPERATURE FOR STAGE I
C     DT(I)=DIFFERENCE BETWEEN LOWER TEMPERATURE OF STAGE I AND UPPER
C     TEMPERATURE OF STAGE I+1 (OR OF COOLING WATER FOR LAST STAGE).
      READ(60,5)NS,(K(I),I=1,4),(        TMIN(I),DT(I),I=1,4)
    5 FORMAT(5I3/ 8F6.0)
      NV=NS+1
C     DFM(I)=PARTIAL DERIVATIVE MULTIPLIER -- NORMALLY 1
C     TO FIX VARIABLE I, SET DFM(I) TO 0.
      READ(60,6)(DFM(I),I=1,NV)
    6 FORMAT(9F1.0)
   10 DO 100 I=1,NS
C     INEQUALITY CONSTRAINT -- LOWER TEMPERATURE OF STAGE I GREATER
C     THAN OR EQUAL TO TMIN(I)
      GL(I)=TMIN(I)-X(   I+1) -DT(I)
      L=K(I)
C     T1=UPPER TEMPERATURE AND T2=LOWER TEMPERATURE OF STAGE
      T1=X(I)
      T2=X(   I+1)+DT(I)
      I1=2*I-1
      I2=I1+1
      T(I1)=T1
      T(I2)=T2
C     CALL PROPER SUBROUTINE TO GET EFFICIENCY AND PARTIALS OF
C     EFFICIENCY WITH RESPECT TO T1 AND T2 FOR CURRENT STAGE
      GO TO (11,12,13,14),L
   11 CALL R1(T1,T2,E,DE1,DE2,I)
      GO TO 20
   12 CALL R2(T1,T2,E,DE1,DE2,I)
      GO TO 20
   13 CALL R3(T1,T2,E,DE1,DE2,I)
      GO TO 20
   14 CALL RSTEAM(T1,T2,E,DE1,DE2,I)
   20 ET(I)=E
      QI(I)=QIOM
      QO(I)=(1-E)*QIOM
      DET(I,1)=DE1
      DET(I,2)=DE2
      N=NS+I
C     INEQUALITY CONSTRAINT -- T1 GREATER THAN OR EQUAL TO T2
      GL(N)=T2-T1
  100 CONTINUE
      Q=1.
      DO 110 I=1,NS
  110 Q=Q*(1.-ET(I))
C     FA=OBJECTIVE FUNCTION (NEGATIVE OF OVERALL EFFICIENCY)
      FA=Q-1.
      SUM=0.
      N=2*NS
      DO 115 I=1,N
  115 SUM=SUM-(1./GL(I))
C     F=AUGMENTED OBJECTIVE FUNCTION
```

```
      F=FA+SUM/AJ**4
C     M.LT.1 IMPLIES ONLY F AND FA ARE REQUIRED, NOT THE GRADIENT.
      IF(M.LT.1)GO TO 1000
      DF(1)=0.
      DO 120 I=2,NS
      I2=I-1
      I4=NS+I
      I5=I4-1
C     COMPUTATION OF COMPONENTS OF THE GRADIENT VECTOR
      DF(I)=-Q*DET(I2,2)/(1.-ET(I2))-Q*DET(I ,1)/(1.-ET(I ))-(1./GL(I2)
     1     **2+1./GL(I4)**2-1./GL(I5)**2                            )/AJ**4
      DF(I)=DF(I)*DFM(I)
  120 CONTINUE
      DF(  NS+1)=0.
C     M.GT.-1 IMPLIES ONLY F,FA, AND THE GRADIENT ARE REQUIRED, NOT THE
C     CYCLE DETAILS (FLOW RATES, ETC.)
 1000 IF(M.GT.-1)GO TO 2000
      NS1=NS-1
      DO 116 I=1,NS1
      ROM(I)=QI(I+1)/QO(I)
  116 ROP(I)=ET(I)/(ET(I+1)*(1.-ET(I)))
      SUMPR=0.
      DO 240 I=1,NS1
      FLOP(I)=1.
      POP(I)=1.
      DO 230 J=I,NS1
      FLOP(I)=FLOP(I)*ROM(J)
  230 POP(I)=POP(I)*ROP(J)
  240 SUMPR=SUMPR+POP(I)
      DEN=SUMPR+1.
      FRNS=3413./(ET(NS)*QI(NS)*DEN)
      DO 260 I=1,NS1
      POP(I)=POP(I)/DEN
  260 FLOP(I)=FLOP(I)*FRNS
      POP(NS)=1./DEN
      FLOP(NS)=FRNS
 2000 RETURN
      END
```

```
      SUBROUTINE R1(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR SIMPLE RANKINE CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION F1(21),F2(21),F3(21),F4(21),F5(21),F6(21)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1) GO TO 100
      READ(60,10)NTS,TS1,DTS,   ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
      READ(60,12)(F1(I),I=1,NTS)
   12 FORMAT(8F10.3)
      READ(60,14)(F2(I),I=1,NTS)
   14 FORMAT(8F10.5)
      READ(60,16)(F3(I),I=1,NTS)
   16 FORMAT(8F10.4)
      READ(60,12)(F4(I),I=1,NTS)
      READ(60,14)(F5(I),I=1,NTS)
      READ(60,14)(F6(I),I=1,NTS)
C     LOOK UP THERMODYNAMIC PROPERTIES.
  100 CALL LOOK(NTS,TS1,DTS,F1,T1,HB,DHDT,0)
      CALL LOOK(NTS,TS1,DTS,F2,T1,SB,DSDT,0)
      CALL LOOK(NTS,TS1,DTS,F3,T1,PH,DPH,1)
      CALL LOOK(NTS,TS1,DTS,F4,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,PL,DPL,1)
      CALL LOOK(NTS,TS1,DTS,F1,T2,HV,DHV,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SV,DSV,0)
      CALL LOOK(NTS,TS1,DTS,F6,T2,VD,DVD,0)
      I1=2*IST-1
      I2=I1+1
      P(I1)=PH
      P(I2)=PL
      DH=HV-HD
      DDH=DHV-DHD
      DS=SV-SD
      DDS=DSV-DSD
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=CYCLE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
      DHADT1=VD*DPH*144./778.
      DHAPDT=DHADT1/ECOMP
C     DE1=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T1
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT-DHADT1/ECOMP+DHAPDT*E)/(HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=144.*(DVD*(PH-PL)-VD*DPL)/778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE2=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T2
      DE2=(E*DHAPDT-ETURB*DHCDT-(DHADT-DHD)/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE R2(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR SIMPLE RANKINE CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION F1(21),F2(21),F3(21),F4(21),F5(21),F6(21)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1) GO TO 100
      READ(60,10)NTS,TS1,DTS,   ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
      READ(60,12)(F1(I),I=1,NTS)
   12 FORMAT(8F10.3)
      READ(60,14)(F2(I),I=1,NTS)
   14 FORMAT(8F10.5)
      READ(60,16)(F3(I),I=1,NTS)
   16 FORMAT(8F10.4)
      READ(60,12)(F4(I),I=1,NTS)
      READ(60,14)(F5(I),I=1,NTS)
      READ(60,14)(F6(I),I=1,NTS)
C     LOOK UP THERMODYNAMIC PROPERTIES.
  100 CALL LOOK(NTS,TS1,DTS,F1,T1,HB,DHDT,0)
      CALL LOOK(NTS,TS1,DTS,F2,T1,SB,DSDT,0)
      CALL LOOK(NTS,TS1,DTS,F3,T1,PH,DPH,1)
      CALL LOOK(NTS,TS1,DTS,F4,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,PL,DPL,1)
      CALL LOOK(NTS,TS1,DTS,F1,T2,HV,DHV,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SV,DSV,0)
      CALL LOOK(NTS,TS1,DTS,F6,T2,VD,DVD,0)
      I1=2*IST-1
      I2=I1+1
      P(I1)=PH
      P(I2)=PL
      DH=HV-HD
      DDH=DHV-DHD
      DS=SV-SD
      DDS=DSV-DSD
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=CYCLE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
      DHADT1=VD*DPH*144./778.
      DHAPDT=DHADT1/ECOMP
C     DE1=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T1
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT-DHADT1/ECOMP+DHAPDT*E)/(HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=144.*(DVD*(PH-PL)-VD*DPL)/778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE2=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T2
      DE2=(E*DHAPDT-ETURB*DHCDT-(DHADT-DHD)/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE R3(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR SIMPLE RANKINE CYCLE.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION F1(21),F2(21),F3(21),F4(21),F5(21),F6(21)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1) GO TO 100
      READ(60,10)NTS,TS1,DTS,   ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
      READ(60,12)(F1(I),I=1,NTS)
   12 FORMAT(8F10.3)
      READ(60,14)(F2(I),I=1,NTS)
   14 FORMAT(8F10.5)
      READ(60,16)(F3(I),I=1,NTS)
   16 FORMAT(8F10.4)
      READ(60,12)(F4(I),I=1,NTS)
      READ(60,14)(F5(I),I=1,NTS)
      READ(60,14)(F6(I),I=1,NTS)
C     LOOK UP THERMODYNAMIC PROPERTIES.
  100 CALL LOOK(NTS,TS1,DTS,F1,T1,HB,DHDT,0)
      CALL LOOK(NTS,TS1,DTS,F2,T1,SB,DSDT,0)
      CALL LOOK(NTS,TS1,DTS,F3,T1,PH,DPH,1)
      CALL LOOK(NTS,TS1,DTS,F4,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,PL,DPL,1)
      CALL LOOK(NTS,TS1,DTS,F1,T2,HV,DHV,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SV,DSV,0)
      CALL LOOK(NTS,TS1,DTS,F6,T2,VD,DVD,0)
      I1=2*IST-1
      I2=I1+1
      P(I1)=PH
      P(I2)=PL
      DH=HV-HD
      DDH=DHV-DHD
      DS=SV-SD
      DDS=DSV-DSD
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=CYCLE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
      DHADT1=VD*DPH*144./778.
      DHAPDT=DHADT1/ECOMP
C     DE1=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T1
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT-DHADT1/ECOMP+DHAPDT*E)/(HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=144.*(DVD*(PH-PL)-VD*DPL)/778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE2=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T2
      DE2=(E*DHAPDT-ETURB*DHCDT-(DHADT-DHD)/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE RSTEAM(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR SIMPLE RANKINE CYCLE.
C     (RSTEAM IS NORMALLY USED FOR THE STEAM CYCLE).
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION F1(30),F2(30),F3(30),F4(30),F5(30),F6(30)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1)GO TO 100
      READ(60,10)NTS,TS1,DTS,   ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
      READ(60,12)(F1(I),I=1,NTS)
   12 FORMAT(8F10.3)
      READ(60,14)(F2(I),I=1,NTS)
   14 FORMAT(8F10.5)
      READ(60,16)(F3(I),I=1,NTS)
   16 FORMAT(8F10.4)
      READ(60,12)(F4(I),I=1,NTS)
      READ(60,14)(F5(I),I=1,NTS)
      READ(60,14)(F6(I),I=1,NTS)
C     LOOK UP THERMODYNAMIC PROPERTIES.
  100 CALL LOOK(NTS,TS1,DTS,F1,T1,HB,DHDT,0)
      CALL LOOK(NTS,TS1,DTS,F2,T1,SB,DSDT,0)
      CALL LOOK(NTS,TS1,DTS,F3,T1,PH,DPH,1)
      CALL LOOK(NTS,TS1,DTS,F4,T2,HD,DHD,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,SD,DSD,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,PL,DPL,1)
      CALL LOOK(NTS,TS1,DTS,F1,T2,HV,DHV,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SV,DSV,0)
      CALL LOOK(NTS,TS1,DTS,F6,T2,VD,DVD,0)
      I1=2*IST-1
      I2=I1+1
      P(I1)=PH
      P(I2)=PL
      DH=HV-HD
      DDH=DHV-DHD
      DS=SV-SD
      DDS=DSV-DSD
      HC=HD+(SB-SD)*DH/DS
      HDA=VD*(PH-PL)*144./778.
      HA=HD+HDA
      HAPRI=HD+(HA-HD)/ECOMP
C     E=CYCLE EFFICIENCY
      E=(ETURB*(HB-HC)-HDA/ECOMP)/(HB-HAPRI)
      DHCDT=DH*DSDT/DS
      DHADT1=VD*DPH*144./778.
      DHAPDT=DHADT1/ECOMP
C     DE1=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T1
      DE1=(DHDT*(ETURB-E)-ETURB*DHCDT-DHADT1/ECOMP+DHAPDT*E)/(HB-HAPRI)
      DHCDT=DHD-(DH*DSD+SD*DDH)/DS-(SB-SD)*DH*DDS/DS**2+SB*DDH/DS
      DHDADT=144.*(DVD*(PH-PL)-VD*DPL)/778.
      DHADT=DHD+DHDADT
      DHAPDT=DHD+(DHADT-DHD)/ECOMP
C     DE2=PARTIAL DERIVATIVE OF EFFICIENCY WITH RESPECT TO T2
      DE2=(E*DHAPDT-ETURB*DHCDT-(DHADT-DHD)/ECOMP)/(HB-HAPRI)
      HCPRI=HB-ETURB*(HB-HC)
      QUAL(IST)=(HCPRI-HD)/DH
      QIOM=HB-HAPRI
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(N,X)
C     THIS SUBPROGRAM PRINTS OUT THE DETAILS OF THE FINAL OPTIMUM
C     CYCLE CONFIGURATION.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION X(5),DF(5),GL(8)
      IOFLAG=1
      CALL GRAD(-1,X,DF,F,FA,1,GL)
      EFF=-FA
      NS=N-1
      WRITE(61,10)
   10 FORMAT(≠3STAGE≠,5X,≠T-HIGH≠,5X,≠P-HIGH≠,5X,≠T-LOW≠,6X,≠P-LOW≠,6X,
     1   ≠STAGE≠,10X,≠TURBINE EXIT≠,5X,≠P/P-TOTAL≠,5X,≠M-DOT/P-TOTAL≠/
     2   ≠ ≠,10X,≠(DEG R)≠,4X,≠(PSIA)≠,5X,≠(DEG R)≠,4X,≠(PSIA)≠,5X,
     3   ≠EFFICIENCY≠,5X,≠QUALITY≠,24X,≠(LBM/HR PER KW)≠/)
      DO 20 I=1,NS
      I1=2*I-1
      I2=I1+1
   20 WRITE(61,30)I,T(I1),P(I1),T(I2),P(I2),ET(I),QUAL(I),POP(I),FLCP(I)
   30 FORMAT(≠ ≠,I3,7X,F6.1,5X,F6.1,5X,F6.1,5X,F6.2,5X,F7.5,8X,F6.4,11X,
     1   F5.4,8X,F6.3)
      WRITE(61,40)EFF
   40 FORMAT(≠0≠,33X,≠OVERALL EFFICIENCY =≠,F8.5)
      RETURN
      END
```

C. Rankine Cycle Stage with Extraction/Regeneration

Subroutines were written to replace R1, R2, R3 and RSTEAM when extractions with mixing regeneration were to be used. The first of these has one extraction and the optimum temperature (maximizing stage efficiency) is determined by the subroutine. When using the subroutine with one optimized extraction, no input in addition to the usual thermodynamic data is required. The extraction/regeneration subroutine simply replaces the simple Rankine cycle routine for those stages where extraction/regeneration is required. (Only R1 is given here; there are corresponding versions of R2, R3, and RSTEAM.)

The efficiency for this stage is (see Figure 8 for nomenclature):

$$\eta_i = \left\{ \frac{(h_a - h_{e'}) + (1-m)(h_{e'} - h_{b'}) - (h_{g'} - h_f) - (1-m)(h_{d'} - h_c)}{h_a - h_{g'}} \right\}_i$$

where

$$m = \text{extraction fraction} = \frac{h_f - h_d'}{h_e' - h_d'}$$

and the points in the cycle are from Figure 8.

For given high and low temperatures, a single-variable search is used to determine the T_e which yields highest efficiency.

```
      SUBROUTINE R1(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM  COMPUTES EFFICIENCY FOR RANKINE CYCLES WITH ONE
C     OPTIMIZED EXTRACTION AND MIXING REGENERATION.
C
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION F1(30),F2(30),F3(30),F4(30),F5(30),F6(30)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1)GO TO 25
      READ(60,10)NTS,TS1,DTS,   ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
      READ(60,12)(F1(I),I=1,NTS)
   12 FORMAT(8F10.3)
      READ(60,14)(F2(I),I=1,NTS)
   14 FORMAT(8F10.5)
      READ(60,16)(F3(I),I=1,NTS)
   16 FORMAT(8F10.4)
      READ(60,12)(F4(I),I=1,NTS)
      READ(60,14)(F5(I),I=1,NTS)
      READ(60,14)(F6(I),I=1,NTS)
C     NP IS POINT NUMBER.  FIVE POINTS ARE USED TO GET PARTIALS OF
C     EFFICIENCY WITH RESPECT TO T1 AND T2 USING CENTRAL DIFFERENCE
C     APPROXIMATION.
   25 NP=1
      TE=T1-5.
      EFF1=0.
      DTE=50.
C     LOOK UP THERMODYNAMIC PROPERTIES.
   30 CALL LOOK(NTS,TS1,DTS,F1,T2,HVAPB,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SVAPB,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,PL,DUM,1)
      CALL LOOK(NTS,TS1,DTS,F4,T2,HC,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,SC,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F6,T2,VC,DUM,0)
   40 CALL LOOK(NTS,TS1,DTS,F1,T1,HA,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F2,T1,SA,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F3,T1,PH,DUM,1)
      HB=HC+(SA-SC)*(HVAPB-HC)/(SVAPB-SC)
      HBP=HA-ETURB*(HA-HB)
C     EXTRACTION IS MADE AT TEMPERATURE TE, AND UNIVARIATE SEARCH IS
C     USED TO FIND OPTIMUM TE.
   60 CALL LOOK(NTS,TS1,DTS,F1,TE,HVAPE,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F2,TE,SVAPE,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F3,TE,PE,DUM,1)
      CALL LOOK(NTS,TS1,DTS,F4,TE,HF,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F5,TE,SF,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F6,TE,VF,DUM,0)
      HE=HF+(SA-SF)*(HVAPE-HF)/(SVAPE-SF)
      HEP=HA-ETURB*(HA-HE)
      HDP=HC+VC*(PE-PL)*144./(778.*ECOMP)
      HGP=HF+VF*(PH-PE)*144./(778.*ECOMP)
C     XM=MASS FRACTION OF EXTRACTION AND EFF=CYCLE EFFICIENCY
      XM=(HF-HDP)/(HEP-HDP)
      EFF=(HA-HEP+(1.-XM)*(HEP-HBP)-HGP+HF-(1.-XM)*(HDP-HC))/(HA-HGP)
      EFF2=EFF
C     CONVERGENCE CHECK IN UNIVARIATE SEARCH FOR MAXIMUM EFFICIENCY
      IF(ABS(EFF1-EFF2).LT..5E-6)GO TO 100
      ISGN=(EFF2-EFF1)/ABS(EFF2-EFF1)
      IF(ISGN)80,80,70
   70 EFF1=EFF2
      TE=TE-DTE
      GO TO 60
```

```
   80 DTE=-DTE/10.
      EFF1=EFF2
      TE=TE-DTE
      GO TO 60
C     WHEN UNIVARIATE SEARCH IS CONVERGED, GO ON TO GET EFFICIENCY AT
C     OTHER POINTS, (T1,T2).
  100 GO TO(130,150,170,190,210),NP
  130 NP=2
      QIOM=HA-HGP
      E=E1=EFF
      T1=T1-2.
      IF(IOFLAG)40,40,140
  140 WRITE(61,220)IST,XM,TE
  220 FORMAT(≠0STAGE≠,I2,≠ HAS ONE EXTRACTION WITH≠/
     1   ≠ MASS FRACTION=≠,F6.4,≠ AND EXTRACTION TEMPERATURE=≠,F8.2)
      T1=T1+2.
      I11=2*IST-1
      I22=I11+1
      P(I11)=PH
      P(I22)=PL
      QUAL(IST)=(HBP-HC)/(HVAPB-HC)
      GO TO 1000
  150 NP=3
      E2=EFF
      T1=T1+4.
      GO TO 40
  170 NP=4
      E3=EFF
      T1=T1-2.
      T2=T2-2.
      GO TO 30
  190 NP=5
      E4=EFF
      T2=T2+4.
      GO TO 30
  210 E5=EFF
      T2=T2-2.
C     CENTRAL DIFFERENCE APPROXIMATION FOR PARTIALS OF EFFICIENCY WITH
C     RESPECT TO T1 AND T2.
      DE1=(E3-E2)/4.
      DE2=(E5-E4)/4.
 1000 RETURN
      END
```

It was found that when the extractions are optimized, they divide the temperature range of the stage very nearly into equal parts. Comparing efficiency for optimized extractions with efficiency when the extractions were equally spaced in temperature showed a negligible difference. Therefore, to save the computer time that would be required to optimize several extractions on several stages, the subroutine for multiple extractions places them at uniform temperature intervals rather than optimizing.

This subroutine allows from one to five extractions with regeneration. The number of extractions, NE, must be input following the thermodynamic data for the stage (FORMAT statement number 20).

The efficiency has already been given for one extraction. For two or more extractions, the T-s diagram is given in Figure II.2, and the efficiency is given by:

$$\eta_i = \Bigg[\Bigg\{(h_a - h_{e'_1}) - (h_{g'_1} - h_{f_1}) + \sum_{k=1}^{n-1} [1 - \sum_{j=1}^{k} m_j][(h_{e'_k} - h_{e'_{k+1}})$$

$$-(h_{g'_{k+1}} - h_{f_{k+1}})]$$

$$+ [1 - \sum_{j=1}^{n} m_j][(h_{e'_n} - h_{b'}) - (h_{d'} - h_c)]\Bigg\} / (h_a - h_{g'_1})\Bigg]_i$$

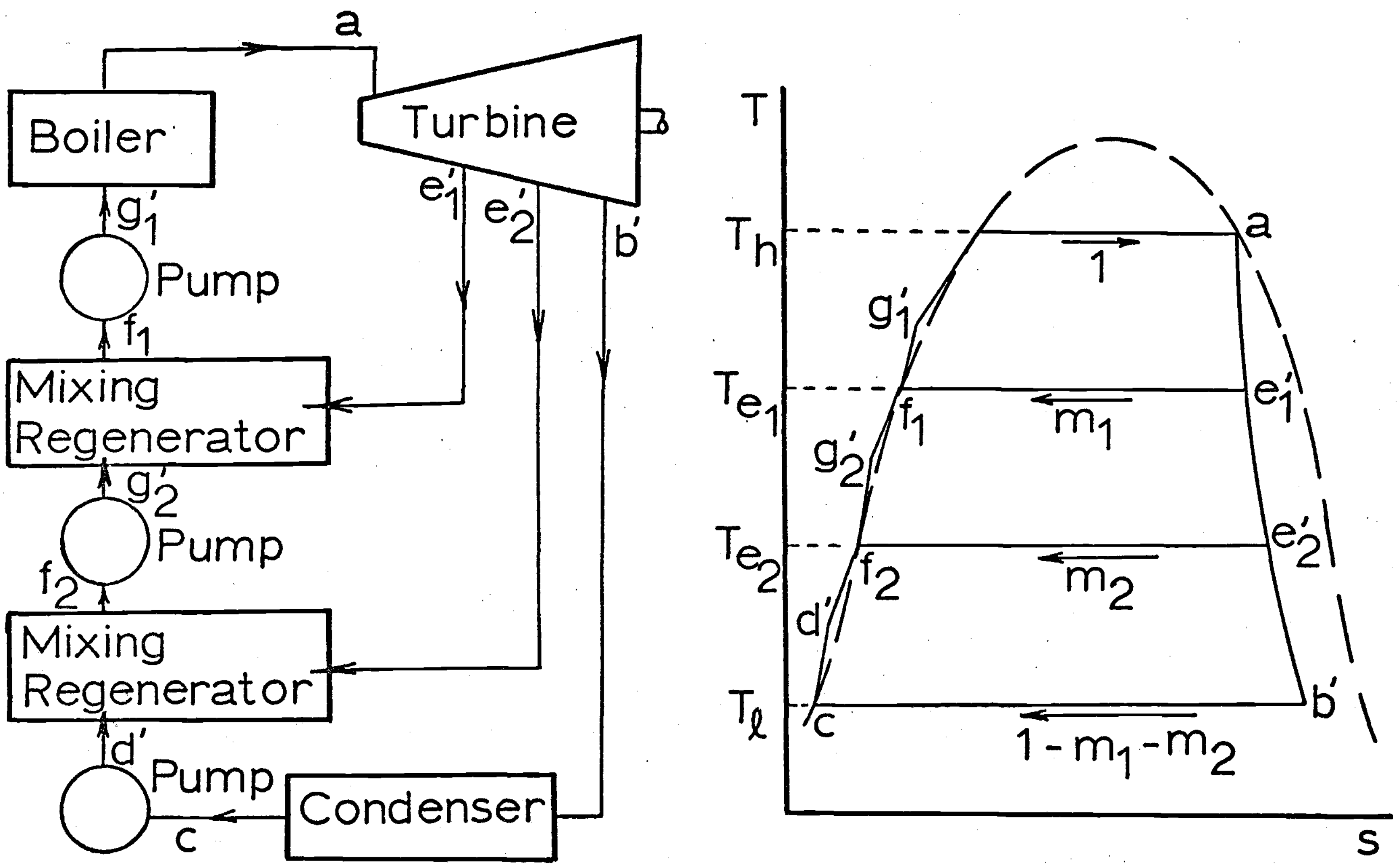


Figure II.2 Rankine cycle with two extractions and mixing regeneration where

n = number of extractions

m_1 = mass fraction of first extraction = $\frac{h_{f_1} - h_{g'_2}}{h_{e'_1} - h_{g'_2}}$

for k = 2,.....,n-1 (i.e., extractions 2 through n-1)

$$m_k = \left[1 - \sum_{j=1}^{k-1} m_j\right] \frac{(h_{f_k} - h_{g'_{k+1}})}{(h_{e'_k} - h_{g'_{k+1}})}$$

and for the last extraction

$$m_n = \left[1 - \sum_{j=1}^{n-1} m_j\right] \frac{(h_{f_n} - h_{d'})}{(h_{e'_n} - h_{d'})}$$

The extraction temperatures, T_{e_k}, are chosen so that the range, $(T_h - T_\ell)$, is divided into n+1 equal parts.

```
      SUBROUTINE R1(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR RANKINE CYCLES WITH
C     MULTIPLE EXTRACTIONS AND MIXING REGENERATION.  EXTRACTIONS (1,2,3,
C     4, OR 5) ARE EQUALLY SPACED WITH RESPECT TO TEMPERATURE AND ARE
C     NOT OPTIMIZED.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      REAL M
      DIMENSION F1(30),F2(30),F3(30),F4(30),F5(30),F6(30),TE(5),PE(5),
     1   HF(5),VF(5),HEP(5),HGP(5),M(5)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1) GO TO 25
      READ(60,10)NTS,TS1,DTS,    ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
      READ(60,12)(F1(I),I=1,NTS)
   12 FORMAT(8F10.3)
      READ(60,14)(F2(I),I=1,NTS)
   14 FORMAT(8F10.5)
      READ(60,16)(F3(I),I=1,NTS)
   16 FORMAT(8F10.4)
      READ(60,12)(F4(I),I=1,NTS)
      READ(60,14)(F5(I),I=1,NTS)
      READ(60,14)(F6(I),I=1,NTS)
C     NE=NUMBER OF EXTRACTIONS (1,2,3,4, OR 5)
      READ(60,20)NE
   20 FORMAT(I3)
      NE1=NE-1
C     NP IS POINT NUMBER.  FIVE POINTS ARE USED TO GET PARTIALS OF
C     EFFICIENCY WITH RESPECT TO T1 AND T2 USING CENTRAL DIFFERENCE
C     APPROXIMATION.
   25 NP=1
C     TE(I)=TEMPERATURE OF THE I-TH EXTRACTION
      TE(1)=T1-(T1-T2)/(NE+1.)
      DO 50 I=2,NE
   50 TE(I)=TE(I-1)-(T1-T2)/(NE+1.)
C     LOOK UP THERMODYNAMIC PROPERTIES.
   30 CALL LOOK(NTS,TS1,DTS,F1,T2,HVAPB,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F2,T2,SVAPB,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F3,T2,PL,DUM,1)
      CALL LOOK(NTS,TS1,DTS,F4,T2,HC,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F5,T2,SC,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F6,T2,VC,DUM,0)
   40 CALL LOOK(NTS,TS1,DTS,F1,T1,HA,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F2,T1,SA,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F3,T1,PH,DUM,1)
      HB=HC+(SA-SC)*(HVAPB-HC)/(SVAPB-SC)
      HBP=HA-ETURB*(HA-HB)
   60 DO 70 I=1,NE
      CALL LOOK(NTS,TS1,DTS,F1,TE(I),HVAPE,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F2,TE(I),SVAPE,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F3,TE(I),PE(I),DUM,1)
      CALL LOOK(NTS,TS1,DTS,F4,TE(I),HF(I),DUM,0)
      CALL LOOK(NTS,TS1,DTS,F5,TE(I),SF,DUM,0)
      CALL LOOK(NTS,TS1,DTS,F6,TE(I),VF(I),DUM,0)
      HE=HF(I)+(SA-SF)*(HVAPE-HF(I))/(SVAPE-SF)
   70 HEP(I)=HA-ETURB*(HA-HE)
      HDP=HC+VC*(PE(NE)-PL)*144./(778.*ECOMP)
      HGP(1)=HF(1)+VF(1)*(PH-PE(1))*144./(778.*ECOMP)
      DO 80 I=2,NE
   80 HGP(I)=HF(I)+VF(I)*(PE(I-1)-PE(I))*144./(778.*ECOMP)
C     M(I)=MASS FRACTION OF THE I-TH EXTRACTION
      M(1)=(HF(1)-HGP(2))/(HEP(1)-HGP(2))
```

```
      IF(NE.EQ.1)M(1)=(HF(1)-HDP)/(HEP(1)-HDP)
      DO 90 I=2,NE1
      I1=I-1
      SUM=0.
      DO 85 J=1,I1
   85 SUM=SUM+M(J)
   90 M(I)=(1.-SUM)*(HF(I)-HGP(I+1))/(HEP(I)-HGP(I+1))
      IF(NE.EQ.1) GO TO 100
      SUM=0.
      DO 92 J=1,NE1
   92 SUM=SUM+M(J)
      M(NE)=(1.-SUM)*(HF(NE)-HDP)/(HEP(NE)-HDP)
      SUM2=SUM+M(NE)
      SUM1=0.
      DO 96 I=1,NE1
      SUM=0.
      DO 94 J=1,I
   94 SUM=SUM+M(J)
   96 SUM1=SUM1+(1.-SUM)*(HEP(I)-HEP(I+1)-HGP(I+1)+HF(I+1))
  100 IF(NE.GT.1) GO TO 110
      SUM1=0.
      SUM2=M(1)
C     EFF=CYCLE EFFICIENCY
  110 EFF=(HA-HEP(1)-HGP(1)+HF(1)+SUM1+(1.-SUM2)*(HEP(NE)-HBP-HDP+HC))/
     1    (HA-HGP(1))
C     GO ON TO GET EFFICIENCY AT OTHER POINTS, (T1,T2).
      GO TO (130,150,170,190,210),NP
  130 NP=2
      QIOM=HA-HGP(1)
      E=E1=EFF
      T1=T1-2.
      IF(IOFLAG)40,40,140
  140 WRITE(61,220)IST,NE,(I,M(I),TE(I),I=1,NE)
      T1=T1+2.
      I11=2*IST-1
      I22=I11+1
      P(I11)=PH
      P(I22)=PL
      QUAL(IST)=(HBP-HC)/(HVAPB-HC)
      GO TO 1000
  150 NP=3
      E2=EFF
      T1=T1+4.
      GO TO 40
  170 NP=4
      E3=EFF
      T1=T1-2.
      T2=T2-2.
      GO TO 30
  190 NP=5
      E4=EFF
      T2=T2+4.
      GO TO 30
  210 E5=EFF
      T2=T2-2.
C     CENTRAL DIFFERENCE APPROXIMATION FOR PARTIALS OF EFFICIENCY WITH
C     RESPECT TO T1 AND T2.
      DE1=(E3-E2)/4.
      DE2=(E5-E4)/4.
  220 FORMAT(≠0STAGE≠,I2,≠ HAS≠,I2,≠ EXTRACTIONS AS FOLLOWS≠/
     1  ≠                    EXTRACTION      MASS FRACTION      TEMPERAT
     2RE≠/5(≠ ≠,22X,I2,13X,F6.4,13X,F8.2/))
```

```
1000 RETURN
     END
```

D. Rankine Cycle Stage for Organic Working Fluid

Organic working fluids all have a characteristic which requires special subroutines for stages using them. The saturated vapor line on a temperature-entropy diagram has a positive slope rather than a negative slope as for water and liquid metals. As a result the expansion in the turbine or other expander results in superheated vapor at the exit rather than a mixture of saturated liquid and vapor.

The first subroutine presented here is for simple Rankine cycles using an organic working fluid. When an organic working fluid is to be used this routine would replace the R3 in the program and would be used for the stage which is to have an organic working fluid.

The efficiency for a Rankine cycle stage with an organic working fluid is (see Figure 10)

$$\eta_i = \left\{ \frac{(h_B - h_{C'}) - (h_{A'} - h_D)}{(h_B - h_{A'})} \right\}_i$$

Input Required:

Variable	Description	Format No.
NTS	number of saturation temperatures	10
TS1	starting saturation temperature (°R)	10
DTS	saturation temperature interval (°R)	10
ETURB	expander efficiency	10

ECOMP	pump efficiency	10
HG(I)	enthalpy of saturated vapor for each saturation temperature (Btu/lbm)	12
PSAT(I)	saturation pressure for each saturation temperature (psia)	14
HF(I)	enthalpy of saturated liquid for each saturation temperature (Btu/lbm)	12
VF(I)	specific volume of saturated liquid for each sat. temp. (ft^3/lbm)	16
NPSH	number of pressures for which superheat data are given	25
DTSH	superheat temperature interval (°R)	25
PSH(I)	pressure for which superheat data are given (psia)	17
NTSH(I)	number of superheat temperatures for given pressure	17
H(I,J)	enthalpy at $I^{\underline{th}}$ pressure and $J^{\underline{th}}$ superheat temp. (Btu/lbm)	18
S(I,J)	entropy at $I^{\underline{th}}$ pressure and $J^{\underline{th}}$ superheat temp. (Btu/lbm-°R)	19

Since non-uniform intervals are permitted on the pressures for which superheat data are given, a special table lookup subroutine, LOOK2, is required which allows non-uniform intervals on the independent variable.

```
      SUBROUTINE R3(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES EFFICIENCY FOR RANKINE CYCLES USING AN
C     ORGANIC WORKING FLUID WITHOUT REGENERATION.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION HG(29),PSAT(29),HF(29),VF(29),PSH(7),H(7,16),S(7,16),
     1   EFF(5),SG(7),F(17),NTSH(7),X(17),G1(17),G2(17),G3(17)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1)GO TO 100
      READ(60,10)NTS,TS1,DTS,ETURB,ECOMP
   10 FORMAT(I3,2F6.0,2F6.3)
C     SATURATED VAPOR
      READ(60,12)(HG(I),I=1,NTS)
   12 FORMAT(8F10.1)
      READ(60,14)(PSAT(I),I=1,NTS)
   14 FORMAT(8F10.3)
C     SATURATED LIQUID
      READ(60,12)(HF(I),I=1,NTS)
      READ(60,16)(VF(I),I=1,NTS)
   16 FORMAT(8F10.5)
      READ(60,25)NPSH,DTSH
   25 FORMAT(I3,F10.3)
      DO 20 I=1,NPSH
      READ(60,17)PSH(I),NTSH(I)
      NT=NTSH(I)
   17 FORMAT(F10.2,I3)
C     SUPERHEATED VAPOR (PRESSURE AND SUPERHEAT ARE THE PARAMETERS)
      READ(60,18)(H(I,J),J=1,NT)
   18 FORMAT(8F10.2)
      READ(60,19)(S(I,J),J=1,NT)
   19 FORMAT(8F10.4)
   20 CONTINUE
C     NP IS POINT NUMBER.  FIVE POINTS ARE USED TO GET PARTIALS OF
C     EFFICIENCY WITH RESPECT TO T1 AND T2 USING CENTRAL DIFFERENCE
C     APPROXIMATION.
  100 NP=1
C     LOOK UP THERMODYNAMIC PROPERTIES.
  110 CALL LOOK(NTS,TS1,DTS,HG,T1,HB,DUM,0)
      CALL LOOK(NTS,TS1,DTS,PSAT,T1,PH,DUM,1)
      CALL LOOK(NTS,TS1,DTS,PSAT,T2,PL,DUM,1)
      CALL LOOK(NTS,TS1,DTS,VF,T2,VD,DUM,0)
      CALL LOOK(NTS,TS1,DTS,HF,T2,HD,DUM,0)
      HAD=VD*(PH-PL)*144./778.
      HAP=HD+HAD/ECOMP
      DO 120 I=1,NPSH
  120 SG(I)=S(I,1)
      CALL LOOK2(NPSH,PSH,SG,PH,SB,IDUM,1)
      CALL LOOK2(NPSH,PSH,SG,PL,DUM,I1,0)
      I2=I1+1
      I3=I2+1
      NUM1=NTSH(I1)
      X(1)=0.
      DO 130 J=1,NUM1
      F(J)=S(I1,J)
      G1(J)=H(I1,J)
  130 X(J+1)=X(J)+DTSH
      CALL LOOK2(NUM1,F,X,SB,TC1,IDUM,0)
      CALL LOOK2(NUM1,X,G1,TC1,H1,IDUM,0)
      NUM2=NTSH(I2)
      DO 140 J=1,NUM2
      F(J)=S(I2,J)
      G2(J)=H(I2,J)
```

```
  140 X(J+1)=X(J)+DTSH
      CALL LOOK2(NUM2,F,X,SB,TC2,IDUM,0)
      CALL LOOK2(NUM2,X,G2,TC2,H2,IDUM,0)
      NUM3=NTSH(I3)
      DO 150 J=1,NUM3
      F(J)=S(I3,J)
      G3(J)=H(I3,J)
  150 X(J+1)=X(J)+DTSH
      CALL LOOK2(NUM3,F,X,SB,TC3,IDUM,0)
      CALL LOOK2(NUM3,X,G3,TC3,H3,IDUM,0)
      X(1)=PSH(I1)
      F(1)=TC1
      X(2)=PSH(I2)
      F(2)=TC2
      X(3)=PSH(I3)
      F(3)=TC3
      CALL LOOK2(3,X,F,PL,TCSH,IDUM,1)
      X(1)=H1
      X(2)=H2
      X(3)=H3
      CALL LOOK2(3,F,X,TCSH,HC,IDUM,0)
      HCP=HB-ETURB*(HB-HC)
C     EFF=CYCLE EFFICIENCY
      EFF(NP)=(HB-HCP-HAD/ECOMP)/(HB-HAP)
C     GO ON TO GET EFFICIENCY AT OTHER POINTS, (T1,T2).
      GO TO(170,180,190,200,210),NP
  170 NP=2
      I11=2*IST-1
      I22=I11+1
      P(I11)=PH
      P(I22)=PL
      DIOM=HB-HAP
      T1=T1-5.
      GO TO 110
  180 NP=3
      T1=T1+10.
      GO TO 110
  190 NP=4
      T1=T1-5.
      T2=T2-5.
      GO TO 110
  200 NP=5
      T2=T2+10.
      GO TO 110
  210 T2=T2-5.
      E=EFF(1)
C     CENTRAL DIFFERENCE APPROXIMATION FOR PARTIALS OF EFFICIENCY WITH
C     RESPECT TO T1 AND T2.
      DE1=(EFF(3)-EFF(2))/10.
      DE2=(EFF(5)-EFF(4))/10.
      QUAL(IST)=1.
      RETURN
      END
C
```

```
      SUBROUTINE LOOK2(NX,X,F,XP,FP,I1,M)
C     THIS SUBPROGRAM LOOKS UP THE VALUE OF THE DEPENDENT VARIABLE,FP,
C     FROM A LIST,F, FOR A GIVEN VALUE OF INDEPENDENT VARIABLE,XP, IN A
C     LIST, X, USING QUADRATIC INTERPOLATION.  VARIABLE INTERVALS ARE
C     PERMITTED IN THE INDEPENDENT VARIABLE LIST.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM
      DIMENSION X(17),F(17)
      NX2=NX-2
      DO 10 I=1,NX2
      IF(X(I).GT.XP)GO TO 20
   10 CONTINUE
      I1=NX2
      I2=NX-1
      I3=NX
      GO TO 50
   20 IF(I-3)30,40,40
   30 I1=1
      I2=2
      I3=3
      GO TO 50
   40 I1=I-1
      I2=I
      I3=I+1
   50 X1=X(I1)
      X2=X(I2)
      X3=X(I3)
      XL=XP
C     IF M=0, DO NOT USE LOGS OF INDEPENDENT VARIABLE -- IF M=1, DO USE
C     LOGS TO GET CURVE WITH LESS RAPIDLY VARYING SLOPE.
      IF(M.EQ.0)GO TO 60
      X1=ALOG(X1)
      X2=ALOG(X2)
      X3=ALOG(X3)
      XL=ALOG(XL)
   60 X2=(X2-X1)/(X3-X1)
      Y2=(F(I2)-F(I1))/(F(I3)-F(I1))
      XI=(XL-X1)/(X3-X1)
      A=(X2-Y2)/(X2-X2**2)
      B=1.-A
      YI=A*XI**2+B*XI
      FP=F(I1)+YI*(F(I3)-F(I1))
      RETURN
      END
```

The fluid at expander exit can have a higher temperature than the fluid at pump exit when an organic working fluid is used. This makes heat-exchange regeneration like that used in Brayton cycles possible. This subroutine is for a Rankine cycle using an organic fluid with regeneration. The efficiency for a Rankine cycle stage using an organic working fluid and heat-exchange regeneration is (see Figure 11)

$$\eta_i = \left\{ \frac{(h_B - h_{C'}) - (h_{E'} - h_D)}{(h_B - h_A)} \right\}_i$$

The temperature difference at the low temperature end of the regeneration heat exchanger, $T_0 - T_{E'}$, is the minimum temperature difference and is fixed on input as DTREG. The temperature difference at the high temperature end, $T_{C'} - T_A$, is included in the output. Regenerator effectiveness, defined as follows, is also printed out.[19]

$$\eta_{Reg} = \frac{T_{C'} - T_0}{T_{C'} - T_{E'}}$$

(Regenerator energy balance requires that $h_A - h_{E'} = h_{C'} - h_0$.)

The thermodynamic data are the same as described earlier in this section, except that at the end of the first card (following ECOMP) add

DTREG temperature difference at low temperature end of regenerator (FORMAT no. 10)

---

[19] Note the definition given here is for "cooling effectiveness". Heating effectiveness, $(T_A - T_{E'})/(T_{C'} - T_{E'})$, is also commonly used, and care must be used in making comparisons.

```
      SUBROUTINE R3(T1,T2,E,DE1,DE2,IST)
C     THIS SUBPROGRAM COMPUTES THE EFFICIENCY FOR RANKINE CYCLES USING
C     AN ORGANIC WORKING FLUID AND HEAT-EXCHANGE REGENERATION.
      COMMON AJ,T(8),P(8),ET(4),QUAL(4),POP(4),FLOP(4),QIOM,IOFLAG
      DIMENSION HG(29),PSAT(29),HF(29),VF(29),PSH(7),H(7,16),S(7,16),
     1   EFF(5),SG(7),F(29),NTSH(7),X(17),G1(17),G2(17),G3(17)
C     READ THERMODYNAMIC DATA FIRST TIME CALLED ONLY.
      IF(NTS.GT.1)GO TO 100
C     DTREG=TEMPERATURE DIFFERENCE AT LOW TEMPERATURE END OF REGENERATOR
C                (USUALLY 50 DEGREES).
      READ(60,10)NTS,TS1,DTS,ETURB,ECOMP,DTREG
   10 FORMAT(I3,2F6.0,3F6.3)
C     SATURATED VAPOR
      READ(60,12)(HG(I),I=1,NTS)
   12 FORMAT(8F10.1)
      READ(60,14)(PSAT(I),I=1,NTS)
   14 FORMAT(8F10.3)
C     SATURATED LIQUID
      READ(60,12)(HF(I),I=1,NTS)
      READ(60,16)(VF(I),I=1,NTS)
   16 FORMAT(8F10.5)
      READ(60,25)NPSH,DTSH
   25 FORMAT(I3,F10.3)
      DO 20 I=1,NPSH
      READ(60,17)PSH(I),NTSH(I)
      NT=NTSH(I)
   17 FORMAT(F10.2,I3)
C     SUPERHEATED VAPOR (PRESSURE AND SUPERHEAT ARE THE PARAMETERS)
      READ(60,18)(H(I,J),J=1,NT)
   18 FORMAT(8F10.2)
      READ(60,19)(S(I,J),J=1,NT)
   19 FORMAT(8F10.4)
   20 CONTINUE
C     NP IS POINT NUMBER.  FIVE POINTS ARE USED TO GET PARTIALS OF
C     EFFICIENCY WITH RESPECT TO T1 AND T2 USING CENTRAL DIFFERENCE
C     APPROXIMATION.
  100 NP=1
C     LOOK UP THERMODYNAMIC PROPERTIES.
  110 CALL LOOK(NTS,TS1,DTS,HG,T1,HB,DUM,0)
      CALL LOOK(NTS,TS1,DTS,PSAT,T1,PH,DUM,1)
      CALL LOOK(NTS,TS1,DTS,PSAT,T2,PL,DUM,1)
      CALL LOOK(NTS,TS1,DTS,VF,T2,VD,DUM,0)
      CALL LOOK(NTS,TS1,DTS,HF,T2,HD,DUM,0)
      HED=VD*(PH-PL)*144./778.
      HEP=HD+HED/ECOMP
      F(1)=TS1
      DO 115 I=2,NTS
  115 F(I)=F(I-1)+DTS
      CALL LOOK2(NTS,HF,F,HEP,TEP,IDUM,0)
      TO=TEP+DTREG
      TOSH=TO-T2
      DO 120 I=1,NPSH
  120 SG(I)=S(I,1)
      CALL LOOK2(NPSH,PSH,SG,PH,SB,IDUM,1)
      CALL LOOK2(NPSH,PSH,SG,PL,DUM,I1,0)
      I2=I1+1
      I3=I2+1
      NUM1=NTSH(I1)
      X(1)=0.
      DO 130 J=1,NUM1
      F(J)=S(I1,J)
```

```
      G1(J)=H(I1,J)
  130 X(J+1)=X(J)+DTSH
      CALL LOOK2(NUM1,F,X,SB,TC1,IDUM,0)
      CALL LOOK2(NUM1,X,G1,TC1,H1,IDUM,0)
      CALL LOOK2(NUM1,X,G1,TOSH,HO1,IDUM,0)
      NUM2=NTSH(I2)
      DO 140 J=1,NUM2
      F(J)=S(I2,J)
      G2(J)=H(I2,J)
  140 X(J+1)=X(J)+DTSH
      CALL LOOK2(NUM2,F,X,SB,TC2,IDUM,0)
      CALL LOOK2(NUM2,X,G2,TC2,H2,IDUM,0)
      CALL LOOK2(NUM2,X,G2,TOSH,HO2,IDUM,0)
      NUM3=NTSH(I3)
      DO 150 J=1,NUM3
      F(J)=S(I3,J)
      G3(J)=H(I3,J)
  150 X(J+1)=X(J)+DTSH
      CALL LOOK2(NUM3,F,X,SB,TC3,IDUM,0)
      CALL LOOK2(NUM3,X,G3,TC3,H3,IDUM,0)
      CALL LOOK2(NUM3,X,G3,TOSH,HO3,IDUM,0)
      X(1)=PSH(I1)
      F(1)=TC1
      X(2)=PSH(I2)
      F(2)=TC2
      X(3)=PSH(I3)
      F(3)=TC3
      CALL LOOK2(3,X,F,PL,TCSH,IDUM,1)
      X(1)=H1
      X(2)=H2
      X(3)=H3
      CALL LOOK2(3,F,X,TCSH,HC,IDUM,0)
      HCP=HB-ETURB*(HB-HC)
      X(1)=PSH(I1)
      X(2)=PSH(I2)
      X(3)=PSH(I3)
      F(1)=HO1
      F(2)=HO2
      F(3)=HO3
      CALL LOOK2(3,X,F,PL,HO,IDUM,1)
      HA=HEP+HCP-HO
C     EFF=CYCLE EFFICIENCY
      EFF(NP)=(HB-HCP-HEO/ECOMP)/(HB-HA)
C     GO ON TO GET EFFICIENCY AT OTHER POINTS, (T1,T2).
      GO TO(170,180,190,200,210),NP
  170 NP=2
      I11=2*IST-1
      I22=I11+1
      P(I11)=PH
      P(I22)=PL
      QIOM=HB-HA
      X(1)=0.
      DO 160 I=1,15
  160 X(I+1)=X(I)+DTSH
      CALL LOOK2(NUM1,G1,X,HCP,TCP1,IDUM,0)
      CALL LOOK2(NUM2,G2,X,HCP,TCP2,IDUM,0)
      CALL LOOK2(NUM3,G3,X,HCP,TCP3,IDUM,0)
      X(1)=PSH(I1)
      F(1)=TCP1
      X(2)=PSH(I2)
      F(2)=TCP2
      X(3)=PSH(I3)
```

```
      F(3)=TCP3
      CALL LOOK2(3,X,F,PL,TCP,IOUM,1)
      F(1)=TS1
      DO 175 I=2,NTS
  175 F(I)=F(I-1)+DTS
      CALL LOOK2(NTS,HF,F,HA,TA,IOUM,0)
C     TCPA=TEMPERATURE DIFFERENCE AT HIGH-TEMPERATURE END OF REGENERATOR
      TCPA=TCP-TA+T2
C     EFFREG=REGENERATOR EFFECTIVENESS
      EFFREG=(TCP-TOSH)/(TCP+T2-TEP)
      IF(IOFLAG.EQ.1)WRITE(61,177)TCPA,EFFREG
  177 FORMAT(≠0T(C-PRIME) - T(A) = ≠,F6.1,10X,
     1   ≠REGENERATOR EFFECTIVENESS = ≠,F6.3)
      T1=T1-5.
      GO TO 110
  180 NP=3
      T1=T1+10.
      GO TO 110
  190 NP=4
      T1=T1-5.
      T2=T2-5.
      GO TO 110
  200 NP=5
      T2=T2+10.
      GO TO 110
  210 T2=T2-5.
      E=EFF(1)
C     CENTRAL DIFFERENCE APPROXIMATION FOR PARTIALS OF EFFICIENCY WITH
C     RESPECT TO T1 AND T2.
      DE1=(EFF(3)-EFF(2))/10.
      DE2=(EFF(5)-EFF(4))/10.
      QUAL(IST)=1.
      RETURN
      END
```

APPENDIX III

TABULATED RESULTS

Detailed specifications for selected optimized staged Rankine configurations are presented here. Most of the tables have been typed from the computer output to reduce the space required. However, to illustrate the format of the output, the computer output for binary cycles with extraction/regeneration (tables 6 through 11) and for staged cycles using an organic working fluid stage (tables 13, 14, and 15) is given directly.

Nomenclature

P_h, P_ℓ = high and low pressures for a particular stage, psia

T_h, T_ℓ = high and low temperatures for a particular stage, °R

ΔT = temperature difference between stages for heat exchange, °R

η = overall thermodynamic efficiency

η_{Turb} = turbine efficiency

χ = turbine exit quality

P/P_{Tot} = ratio of power output for stage to total power output for all stages

$\dot{m}/P_{Tot}$ = mass flow rate of stage per kw of total power output for all stages, lbm/hr per kw

$T_{C'}$ = expander outlet temperature for organic fluid stage (temperature of vapor entering regenerator), °R

T_A = temperature of liquid leaving regenerator for organic fluid stage, °R

$T_C - T_A$ = temperature difference at high temperature end of regeneration heat exchanger, °R

T_0 = temperature of vapor leaving regenerator for organic fluid stage, °R

$T_{E'}$ = pump outlet temperature for organic fluid stage (temperature of liquid entering regenerator), °R

$T_0 - T_{E'}$ = temperature difference at low temperature end of regeneration heat exchanger, °R

Table 1A - Optimum Cases for Hg/H_2O Binary Cycles (ΔT = 100°F)

Mercury				Water			Quality, χ		P/P_{Tot}		$\dot{m}/P_{Tot}$	
T_h	P_h	T_ℓ	P_ℓ	T_h	P_h	η	Hg	H_2O	Hg	H_2O	Hg	H_2O
1350	89	925	1.14	825	163	.394	.847	.845	.572	.428	62.51	6.03
1500	226	950	1.65	850	220	.430	.830	.836	.604	.396	55.97	5.27
1710	624	985	2.65	885	326	.468	.817	.822	.631	.369	49.70	4.59
1890	1237	1005	3.45	905	402	.494	.812	.813	.651	.349	45.73	4.19
2250	3000*	1020	4.17	920	467	.525	.816	.806	.683	.317	40.38	3.72
2610	3000*	1020	4.17	920	467	.528	.854	.806	.686	.314	38.13	3.68
*On bound				T_ℓ = 575								
				P_ℓ = 1.47								

Table 1B - Sensitivity of Hg/H_2O Results to the High Temperature of the Steam Cycle

Mercury		Water			Quality, χ		P/P_{Tot}		$\dot{m}/P_{Tot}$	
T_ℓ	P_ℓ	T_h	P_h	η	Hg	H_2O	Hg	H_2O	Hg	H_2O
875	.5*	775	83	.4274	.803	.868	.687	.313	55.34	5.12
900	.77	800	118	.4289	.812	.856	.658	.342	55.46	5.16
925	1.14	825	163	.4297	.821	.845	.630	.370	55.66	5.21
950	1.65	850	220	.4298	.830	.836	.604	.396	55.97	5.27
1000	3.23	900	382	.4288	.847	.815	.552	.448	56.75	5.42
1050	6.00	950	622	.4256	.864	.794	.503	.497	57.83	5.64
T_h = 1500		T_ℓ = 575								
P_h = 226		P_ℓ = 1.47								

*On bound

Table 1C - Effect of Superheating the Steam with Fixed High Temperature (Hg/H_2O)

Water		Quality, χ		P/P_{Tot}		$\dot{m}/P_{Tot}$	
p_h	η	Hg	H_2O	Hg	H_2O	Hg	H_2O
220	.4298	.830	.836	.604	.396	55.97	5.27
200	.4274	.830	.842	.608	.392	56.28	5.28
180	.4250	.830	.849	.611	.389	56.60	5.29
160	.4225	.830	.856	.615	.385	56.94	5.30

Mercury: $T_h = 1500$, $p_h = 226$, $T_\ell = 950$, $p_\ell = 1.65$

Water: $T_h = 850$, $T_\ell = 575$, $p_\ell = 1.47$

Table 1D - Sensitivity of Hg/H_2O Results to Mercury Turbine Efficiency (each case optimized)

Hg	Mercury		Water			Quality, χ		P/P_{Tot}		$\dot{m}/P_{Tot}$	
η_{Turb}	T_ℓ	p_ℓ	T_h	p_h	η	Hg	H_2O	Hg	H_2O	Hg	H_2O
.80	910	0.90	810	135	.4439	.795	.852	.667	.333	53.70	4.88
.75	950	1.65	850	220	.4298	.830	.836	.604	.396	55.97	5.27
.70	990	2.83	890	344	.4167	.862	.819	.540	.460	58.25	5.67
	$T_h = 1500$		$T_\ell = 575$								
	$p_h = 226$		$p_\ell = 1.47$								

Table 1E - Sensitivity of Hg/H_2O Results to Steam Turbine Efficiency (each case optimized)

H_2O	Mercury		Water			Quality, χ		P/P_{Tot}		$\dot{m}/P_{Tot}$	
η_{Turb}	T_ℓ	P_ℓ	T_h	P_h	η	Hg	H_2O	Hg	H_2O	Hg	H_2O
.85	988	2.76	888	336	.4407	.843	.803	.550	.450	55.06	5.24
.80	950	1.65	850	220	.4298	.830	.836	.604	.396	55.97	5.27
.75	913	0.94	813	140	.4196	.817	.865	.658	.342	56.85	5.31
	T_h =	1500	T_ℓ =	575							
	P_h =	226	P_ℓ =	1.47							

Table 2 - Optimum Cases for $K/Hg/H_2O$ Ternary Cycles (ΔT = 100°F)

Potassium				Mercury				Water			Quality, χ			P/P_{Tot}			$\dot{m}/P_{Tot}$		
T_h	P_h	T_ℓ	P_ℓ	T_h	P_h	T_ℓ	P_ℓ	T_h	P_h	η	K	Hg	H_2O	K	Hg	H_2O	K	Hg	H_2O
1620	3.64	1374	0.50*	1274	50.7	912	.93	812	138	.442	.909	.859	.851	.252	.411	.337	8.39	50.16	4.92
1800	10.9	1428	.82	1328	76.2	918	1.02	818	149	.480	.880	.849	.849	.312	.393	.296	7.69	43.82	4.24
1980	26.6	1477	1.24	1377	107.	926	1.16	826	165	.510	.855	.842	.845	.357	.375	.268	7.14	39.32	3.77
2160	55.6	1527	1.86	1427	147.	934	1.30	834	182	.535	.838	.836	.842	.387	.365	.248	6.83	36.05	3.43
2340	103.	1576	2.67	1476	197.	942	1.47	842	200	.556	.825	.831	.839	.410	.356	.234	6.53	33.51	3.16
2520	174.	1626	3.79	1526	260.	950	1.65	850	220	.573	.817	.826	.836	.426	.352	.222	6.29	31.55	2.96
2700	272.	1676	5.25	1576	337.	958	1.83	858	242	.588	.811	.823	.833	.439	.348	.213	6.10	29.97	2.79
								T_ℓ =	575										
								P_ℓ =	1.47										

*On bound

Table 3 - Optimum Cases for Na/Hg/H_2O Ternary Cycles (ΔT = 100°F)

Sodium			Quality, χ	P/P_{Tot}			$\dot{m}/P_{Tot}$		
T_h	p_h	η	Na	Na	Hg	H_2O	Na	Hg	H_2O
1620	.69	.436	.984	.040	.580	.380	4.38	54.30	5.12
1800	2.72	.477	.917	.187	.491	.321	3.98	45.99	4.34
1980	8.20	.510	.865	.287	.431	.282	3.70	40.36	3.81
2160	20.5	.536	.825	.358	.388	.254	3.49	36.32	3.42
2340	44.7	.558	.794	.412	.356	.233	3.32	33.28	3.14
2520	85.8	.576	.770	.453	.331	.216	3.19	30.95	2.92
2700	150.5	.590	.749	.485	.311	.204	3.08	29.13	2.75

Sodium: T_ℓ = 1582, p_ℓ = .5*

Mercury: T_h = 1482, p_h = 204, T_ℓ = 943, p_ℓ = 1.49, χ = .831

Water: T_h = 843, p_h = 203, T_ℓ = 575, p_ℓ = 1.47, χ = .838

*On bound

Table 4 - Optimum Cases for Li/Na/Hg/H_2O Quaternary Cycles (ΔT = 100°F)

Lithium				Sodium			Quality, χ		P/P_{Tot}				$\dot{m}/P_{Tot}$			
T_h	p_h	T_ℓ	p_ℓ	T_h	p_h	η	Li	Na	Li	Na	Hg	H_2O	Li	Na	Hg	H_2O
2340	1.02	2227	.50*	2127	17.6	.548	.969	.832	.063	.325	.370	.242	.70	3.30	34.63	3.27
2520	2.80	2227	.50*	2127	17.6	.569	.923	.832	.141	.298	.339	.222	.67	3.03	31.76	3.00
2700	6.64	2288	.74	2188	23.4	.588	.901	.820	.178	.302	.314	.206	.65	2.85	29.44	2.78
2880	14.3	2350	1.08	2250	30.8	.604	.882	.809	.208	.306	.294	.192	.64	2.70	27.50	2.59
3060	27.9	2414	1.57	2314	40.2	.618	.867	.798	.231	.311	.277	.181	.62	2.57	25.90	2.44
3240	50.5	2478	2.24	2378	51.7	.631	.856	.789	.250	.316	.262	.172	.61	2.47	24.54	2.31
3420	86.5	2542	3.13	2442	65.6	.642	.846	.780	.267	.320	.250	.163	.60	2.38	23.36	2.20

Sodium: T_ℓ = 1582, p_ℓ = .5*

Mercury: T_h = 1482, p_h = 204, T_ℓ = 943, p_ℓ = 1.49, χ = .831

Water: T_h = 843, p_h = 203, T_ℓ = 575, p_ℓ = 1.47, χ = .838

Table 5. Optimum Cases for Li/K/Hg/H_2O Quaternary Cycles (ΔT = 100°F)

Lithium				Potassium				Mercury				Water			Quality, X			
T_h	p_h	T_ℓ	p_ℓ	T_h	p_h	T_ℓ	p_ℓ	T_h	p_h	T_ℓ	p_ℓ	T_h	p_h	η	Li	K	Hg	H_2O
2340	1.0	2223	.5*	2123	48.3	1538	2.0	1438	157	940	1.4	840	196	.547	.967	.846	.836	.839
2520	2.8	2223	.5*	2123	48.3	1538	2.0	1438	157	940	1.4	840	196	.569	.922	.846	.836	.839
2700	6.6	2223	.5*	2123	48.3	1538	2.0	1438	157	940	1.4	840	196	.587	.886	.846	.836	.839
2880	14.3	2293	.76	2193	62.7	1553	2.3	1453	172	942	1.5	842	200	.603	.869	.840	.835	.839
3060	27.9	2358	1.13	2258	78.9	1580	2.7	1480	201	947	1.6	847	212	.617	.855	.837	.832	.837
3240	50.5	2401	1.46	2301	91.2	1588	2.9	1488	211	951	1.7	851	224	.630	.840	.833	.832	.835
3420	86.5	2500	2.52	2400	124	1643	4.2	1543	284	954	1.7	854	232	.641	.838	.834	.826	.834
3600	138	2581	3.80	2481	157	1670	5.1	1570	327	958	1.8	858	241	.651	.834	.831	.823	.832
												T_ℓ = 575						
												p_ℓ = 1.47						

*On bound

P/P_{Tot}				$\dot{m}/P_{Tot}$			
Li	K	Hg	H_2O	Li	K	Hg	H_2O
.063	.346	.350	.242	.70	6.48	34.5	3.28
.144	.315	.320	.221	.67	5.93	31.6	3.00
.205	.293	.298	.204	.65	5.50	29.3	2.78
.230	.294	.283	.193	.63	5.23	27.5	2.60
.251	.289	.276	.184	.62	5.01	26.1	2.46
.276	.285	.263	.177	.61	4.79	24.8	2.34
.280	.282	.269	.170	.60	4.68	23.8	2.23
.289	.283	.265	.164	.60	4.55	22.9	2.14

Table 6. Hg (Stage 1)/H_2O (Stage 2) with One Optimized Extraction with Regeneration on Each Stage

```
STAGE 1 HAS ONE EXTRACTION WITH
MASS FRACTION= .0482 AND EXTRACTION TEMPERATURE= 1154.50

STAGE 2 HAS ONE EXTRACTION WITH
MASS FRACTION= .1442 AND EXTRACTION TEMPERATURE=  717.61


STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                       (LBM/HR PER KW)

  1       1350.0      88.8      970.6       2.18      .20635         .8643            .5060         63.839
  2        870.6     278.4      575.0       1.47      .25388         .8268            .4940          6.810

                                OVERALL EFFICIENCY =  .40784


STAGE 1 HAS ONE EXTRACTION WITH
MASS FRACTION= .0630 AND EXTRACTION TEMPERATURE= 1234.50

STAGE 2 HAS ONE EXTRACTION WITH
MASS FRACTION= .1551 AND EXTRACTION TEMPERATURE=  727.83

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                       (LBM/HR PER KW)

  1       1500.0     225.7      993.3       2.96      .24599         .8451            .5518         57.695
  2        893.3     355.6      575.0       1.47      .26496         .8174            .4482          5.975

                                OVERALL EFFICIENCY =  .44577
```

Table 6 (Continued)

```
STAGE 1 HAS ONE EXTRACTION WITH
MASS FRACTION= .0841 AND EXTRACTION TEMPERATURE= 1344.50

STAGE 2 HAS ONE EXTRACTION WITH
MASS FRACTION= .1677 AND EXTRACTION TEMPERATURE=  739.36

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)    EFFICIENCY    QUALITY                       (LBM/HR PER KW)

  1       1710.0      624.5     1018.9      4.11     .29153        .8276           .5982          51.843
  2        918.9      461.1      575.0      1.47     .27643        .8068           .4018           5.198

                                OVERALL EFFICIENCY =  .48738


STAGE 1 HAS ONE EXTRACTION WITH
MASS FRACTION= .1001 AND EXTRACTION TEMPERATURE= 1434.50

STAGE 2 HAS ONE EXTRACTION WITH
MASS FRACTION= .1800 AND EXTRACTION TEMPERATURE=  750.60

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)    EFFICIENCY    QUALITY                       (LBM/HR PER KW)

  1       1890.0     1236.7     1041.6      5.43     .32042        .8219           .6226          48.303
  2        941.6      574.0      575.0      1.47     .28579        .7970           .3774           4.789

                                OVERALL EFFICIENCY =  .51464


STAGE 1 HAS ONE EXTRACTION WITH
MASS FRACTION= .1297 AND EXTRACTION TEMPERATURE= 1595.00

STAGE 2 HAS ONE EXTRACTION WITH
MASS FRACTION= .1938 AND EXTRACTION TEMPERATURE=  762.69

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)    EFFICIENCY    QUALITY                       (LBM/HR PER KW)

  1       2250.0     3433.6     1068.2      7.34     .36779        .8217           .6629          43.148
  2        968.2      731.7      575.0      1.47     .29585        .7851           .3371           4.207

                                OVERALL EFFICIENCY =  .55483
```

Table 7. Hg (Stage 1)/H_2O (Stage 2) with One Extraction with Regeneration on Each Stage

```
    STAGE 1 HAS 1 EXTRACTIONS AS FOLLOWS
                                EXTRACTION      MASS FRACTION       TEMPERATURE
                                    1                .0496               1159.94

    STAGE 2 HAS 1 EXTRACTIONS AS FOLLOWS
                                EXTRACTION      MASS FRACTION       TEMPERATURE
                                    1                .1484                722.44


    STAGE       T-HIGH      P-HIGH      T-LOW      P-LOW       STAGE          TURBINE EXIT     P/P-TOTAL      M-DOT/P-TOTAL
                (DEG R)     (PSIA)      (DEG R)    (PSIA)      EFFICIENCY     QUALITY                         (LBM/HR PER KW)

      1         1350.0       88.8       969.9       2.16       .20674          .8640           .5069          63.928
      2          869.9      276.1       575.0       1.47       .25349          .8271           .4931           6.842

                                        OVERALL EFFICIENCY =  .40782


    STAGE 1 HAS 1 EXTRACTIONS AS FOLLOWS
                                EXTRACTION      MASS FRACTION       TEMPERATURE
                                    1                .0658               1246.73

    STAGE 2 HAS 1 EXTRACTIONS AS FOLLOWS
                                EXTRACTION      MASS FRACTION       TEMPERATURE
                                    1                .1608                734.23


    STAGE       T-HIGH      P-HIGH      T-LOW      P-LOW       STAGE          TURBINE EXIT     P/P-TOTAL      M-DOT/P-TOTAL
                (DEG R)     (PSIA)      (DEG R)    (PSIA)      EFFICIENCY     QUALITY                         (LBM/HR PER KW)

      1         1500.0      225.7       993.5       2.97       .24592          .8451           .5517          57.872
      2          893.5      356.0       575.0       1.47       .26499          .8174           .4483           6.016

                                        OVERALL EFFICIENCY =  .44574
```

Table 7 (Continued)

```
STAGE 1 HAS 1 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1              .0885            1364.26

STAGE 2 HAS 1 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1              .1742             746.76

STAGE    T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY    QUALITY                      (LBM/HR PER KW)

  1      1710.0     624.5    1018.5     4.10     .29163        .8275           .5984        52.095
  2       918.5     459.6     575.0     1.47     .27626        .8069           .4016         5.239
                         OVERALL EFFICIENCY =  .48733

STAGE 1 HAS 1 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1              .1059            1469.32

STAGE 2 HAS 1 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1              .1907             761.82

STAGE    T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY    QUALITY                      (LBM/HR PER KW)

  1      1890.0    1236.7    1048.6     5.90     .31772        .8239           .6175        48.713
  2       948.6     612.9     575.0     1.47     .28851        .7939           .3825         4.873
                         OVERALL EFFICIENCY =  .51457
```

Table 7. (Continued)

```
STAGE 1 HAS 1 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1             .1422             1659.32

STAGE 2 HAS 1 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1             .2020              771.82


STAGE    T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY    QUALITY                      (LBM/HR PER KW)

  1      2250.0    3433.6    1068.6     7.37     .36744        .8218           .6625        43.812
  2       968.6     734.6     575.0     1.47     .29597        .7849           .3375         4.254

                           OVERALL EFFICIENCY =  .55466
```

Table 8. Hg (Stage 1)/H_2O (Stage 2) with Two Extractions with Regeneration on Each Stage

```
 STAGE 1 HAS 2 EXTRACTIONS AS FOLLOWS
                     EXTRACTION      MASS FRACTION      TEMPERATURE
                         1                .0305            1232.77
                         2                .0307            1115.54

 STAGE 2 HAS 2 EXTRACTIONS AS FOLLOWS
                     EXTRACTION      MASS FRACTION      TEMPERATURE
                         1                .1179             790.54
                         2                .1006             682.77

 STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE         TURBINE EXIT      P/P-TOTAL      M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY    QUALITY                         (LBM/HR PER KW)

   1       1350.0       88.8      998.3       3.16     .19269         .8750             .4660          64.198
   2        898.3      374.5      575.0       1.47     .27356         .8154             .5340           7.386

                                  OVERALL EFFICIENCY =  .41354



 STAGE 1 HAS 2 EXTRACTIONS AS FOLLOWS
                     EXTRACTION      MASS FRACTION      TEMPERATURE
                         1                .0417            1339.42
                         2                .0417            1178.85

 STAGE 2 HAS 2 EXTRACTIONS AS FOLLOWS
                     EXTRACTION      MASS FRACTION      TEMPERATURE
                         1                .1266             803.85
                         2                .1063             689.42

 STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE         TURBINE EXIT      P/P-TOTAL      M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY    QUALITY                         (LBM/HR PER KW)

   1       1500.0      225.7     1018.3       4.08     .23602         .8537             .5218          58.358
   2        918.3      458.5      575.0       1.47     .28308         .8070             .4782           6.492

                                  OVERALL EFFICIENCY =  .45228
```

Table 8. (Continued)

```
STAGE 1 HAS 2 EXTRACTIONS AS FOLLOWS
                       EXTRACTION     MASS FRACTION     TEMPERATURE
                           1              .0561            1493.34
                           2              .0552            1276.69

STAGE 2 HAS 2 EXTRACTIONS AS FOLLOWS
                       EXTRACTION     MASS FRACTION     TEMPERATURE


                           1              .1460             831.69
                           2              .1179             703.34

STAGE     T-HIGH     P-HIGH     T-LOW     P-LOW      STAGE         TURBINE EXIT      P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)   (PSIA)     EFFICIENCY    QUALITY                          (LBM/HR PER KW)

  1       1710.0      624.5     1060.0     6.68      .27772         .8397             .5608         52.922
  2        960.0      680.1      575.0     1.47      .30108         .7888             .4392          5.827

                                 OVERALL EFFICIENCY =  .49518


STAGE 1 HAS 2 EXTRACTIONS AS FOLLOWS
                       EXTRACTION     MASS FRACTION     TEMPERATURE
                           1              .0706            1615.25
                           2              .0683            1340.51

STAGE 2 HAS 2 EXTRACTIONS AS FOLLOWS
                       EXTRACTION     MASS FRACTION     TEMPERATURE
                           1              .1488             835.51
                           2              .1195             705.25

STAGE     T-HIGH     P-HIGH     T-LOW     P-LOW      STAGE         TURBINE EXIT      P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)   (PSIA)     EFFICIENCY    QUALITY                          (LBM/HR PER KW)

  1       1890.0     1236.7     1065.8     7.14      .31579         .8282             .6034         49.634
  2        965.8      716.0      575.0     1.47      .30336         .7862             .3966          5.254

                                 OVERALL EFFICIENCY =  .52335
```

Table 8. (Continued)

```
STAGE 1 HAS 2 EXTRACTIONS AS FOLLOWS
                        EXTRACTION      MASS FRACTION      TEMPERATURE
                            1                .0971            1862.77
                            2                .0934            1475.53

STAGE 2 HAS 2 EXTRACTIONS AS FOLLOWS
                        EXTRACTION      MASS FRACTION      TEMPERATURE
                            1                .1603             850.53
                            2                .1257             712.77

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE         TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY    QUALITY                        (LBM/HR PER KW)

  1       2250.0     3433.6     1088.3       9.18     .36749          .8266            .6508        45.198
  2        988.3      871.3      575.0       1.47     .31179          .7757            .3492         4.619

                                 OVERALL EFFICIENCY =  .56470
```

Table 9. Hg (Stage 1)/H_2O (Stage 2) with Three Extractions with Regeneration on Each Stage

```
 STAGE 1 HAS 3 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1              .0218           1266.22
                        2              .0219           1182.44
                        3              .0220           1098.65

 STAGE 2 HAS 3 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1              .0988            829.90
                        2              .0854            744.94
                        3              .0757            659.97

 STAGE    T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
          (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY    QUALITY                      (LBM/HR PER KW)

   1      1350.0      88.8    1014.9      3.91    .18424         .8814          .4419       64.225
   2       914.9     443.2     575.0      1.47    .28518         .8085          .5581        7.758

                          OVERALL EFFICIENCY =  .41687
```

Table 9. (Continued)

```
STAGE 1 HAS 3 EXTRACTIONS AS FOLLOWS
                 EXTRACTION     MASS FRACTION     TEMPERATURE
                     1              .0299           1385.15
                     2              .0298           1270.31
                     3              .0299           1155.46

STAGE 2 HAS 3 EXTRACTIONS AS FOLLOWS
                 EXTRACTION     MASS FRACTION     TEMPERATURE
                     1              .1088            849.21
                     2              .0921            757.81
                     3              .0804            666.40


STAGE    T-HIGH     P-HIGH    T-LOW     P-LOW     STAGE        TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)    (PSIA)    (DEG R)   (PSIA)    EFFICIENCY   QUALITY                      (LBM/HR PER KW)

  1      1500.0     225.7     1040.6     5.37     .22607        .8614          .4958        58.552
  2       940.6     568.8      575.0     1.47     .29711        .7974          .5042         6.907
                           OVERALL EFFICIENCY =  .45601

STAGE 1 HAS 3 EXTRACTIONS AS FOLLOWS
                 EXTRACTION     MASS FRACTION     TEMPERATURE
                     1              .0420           1548.93
                     2              .0413           1387.86
                     3              .0410           1226.78

STAGE 2 HAS 3 EXTRACTIONS AS FOLLOWS .
                 EXTRACTION     MASS FRACTION     TEMPERATURE
                     1              .1192            868.03
                     2              .0987            770.36
                     3              .0848            672.68


STAGE    T-HIGH     P-HIGH    T-LOW     P-LOW     STAGE        TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)    (PSIA)    (DEG R)   (PSIA)    EFFICIENCY   QUALITY                      (LBM/HR PER KW)

  1      1710.0     624.5     1065.7     7.13     .27690        .8413          .5543        53.200
  2       965.7     715.7      575.0     1.47     .30793        .7863          .4457         6.064
                           OVERALL EFFICIENCY =  .49956
```

Table 9. (Continued)

```
STAGE 1 HAS 3 EXTRACTIONS AS FOLLOWS
                          EXTRACTION      MASS FRACTION      TEMPERATURE
                              1              .0527             1687.10
                              2              .0512             1484.20
                              3              .0502             1281.29

STAGE 2 HAS 3 EXTRACTIONS AS FOLLOWS
                          EXTRACTION      MASS FRACTION      TEMPERATURE
                              1              .1248              877.54
                              2              .1021              776.70
                              3              .0870              675.85

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE        TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY   QUALITY                        (LBM/HR PER KW)

  1       1890.0     1236.7     1078.4      8.23      .31311        .8317            .5929         50.085
  2        978.4      800.2      575.0      1.47      .31302        .7804            .4071          5.538
                                 OVERALL EFFICIENCY =  .52812


STAGE 1 HAS 3 EXTRACTIONS AS FOLLOWS
                          EXTRACTION      MASS FRACTION      TEMPERATURE
                              1              .0738             1962.94
                              2              .0692             1675.88
                              3              .0661             1388.82

STAGE 2 HAS 3 EXTRACTIONS AS FOLLOWS
                          EXTRACTION      MASS FRACTION      TEMPERATURE
                              1              .1358              895.07
                              2              .1084              788.38
                              3              .0908              681.69

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE        TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY   QUALITY                        (LBM/HR PER KW)

  1       2250.0     3433.6     1101.8     10.61      .36641        .8298            .6425         45.903
  2       1001.8      975.5      575.0      1.47      .32173        .7691            .3575          4.890
                                 OVERALL EFFICIENCY =  .57025
```

Table 10. Hg (Stage 1)/H_2O (Stage 2) with Four Extractions with Regeneration on Each Stage

```
    STAGE 1 HAS 4 EXTRACTIONS AS FOLLOWS
                       EXTRACTION     MASS FRACTION     TEMPERATURE
                           1               .0166            1285.95
                           2               .0167            1221.90
                           3               .0167            1157.85
                           4               .0168            1093.80

    STAGE 2 HAS 4 EXTRACTIONS AS FOLLOWS
                       EXTRACTION     MASS FRACTION     TEMPERATURE
                           1               .0867             858.80
                           2               .0756             787.85
                           3               .0673             716.90
                           4               .0610             645.95

    STAGE     T-HIGH     P-HIGH     T-LOW     P-LOW     STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
              (DEG R)    (PSIA)     (DEG R)   (PSIA)    EFFICIENCY     QUALITY                       (LBM/HR PER KW)

      1       1350.0       88.8     1029.7      4.71     .17639         .8871           .4209        64.214
      2        929.7      512.9      575.0      1.47     .29462         .8021           .5791         8.087

                                 OVERALL EFFICIENCY =  .41904
```

Table 10. (Continued)

```
STAGE 1 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION      TEMPERATURE
                              1              .0231            1411.54
                              2              .0230            1323.07
                              3              .0230            1234.61
                              4              .0230            1146.15

STAGE 2 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION      TEMPERATURE
                              1              .0967             881.15
                              2              .0825             804.61
                              3              .0722             728.07
                              4              .0646             651.54

STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE         TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY    QUALITY                     (LBM/HR PER KW)

  1        1500.0      225.7     1057.7       6.51      .21857        .8668           .4765        58.599
  2         957.7      665.9      575.0       1.47      .30733        .7899           .5235         7.248

                                   OVERALL EFFICIENCY =  .45872

STAGE 1 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION      TEMPERATURE
                              1              .0333            1582.97
                              2              .0328            1455.94
                              3              .0325            1328.91
                              4              .0323            1201.88

STAGE 2 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION      TEMPERATURE
                              1              .1033             894.88
                              2              .0869             814.91
                              3              .0752             734.94
                              4              .0667             654.97

STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE         TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY    QUALITY                     (LBM/HR PER KW)

  1        1710.0      624.5     1074.9       7.91      .27394        .8440           .5453        53.365
  2         974.9      775.9      575.0       1.47      .31461        .7820           .4547         6.294

                                   OVERALL EFFICIENCY =  .50236
```

Table 10. (Continued)

```
STAGE 1 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION     TEMPERATURE
                              1             .0419           1729.80
                              2             .0409           1569.59
                              3             .0401           1409.39
                              4             .0395           1249.18

STAGE 2 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION     TEMPERATURE
                              1             .1091            906.18
                              2             .0906            823.39
                              3             .0777            740.59
                              4             .0683            657.80

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT      P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                          (LBM/HR PER KW)

  1       1890.0     1236.7     1089.0       9.25      .31029          .8346            .5842          50.347
  2        989.0      876.3      575.0       1.47      .32024          .7754            .4158           5.774

                                   OVERALL EFFICIENCY =  .53116

STAGE 1 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION     TEMPERATURE
                              1             .0591           2024.02
                              2             .0559           1798.05
                              3             .0535           1572.07
                              4             .0517           1346.09

STAGE 2 HAS 4 EXTRACTIONS AS FOLLOWS
                         EXTRACTION     MASS FRACTION     TEMPERATURE
                              1             .1232            931.09
                              2             .0990            842.07
                              3             .0829            753.05
                              4             .0717            664.02

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT      P/P-TOTAL      M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                          (LBM/HR PER KW)

  1       2250.0     3433.6     1120.1      12.86      .36233          .8342            .6314          46.341
  2       1020.1     1132.8      575.0       1.47      .33171          .7596            .3686           5.207

                                   OVERALL EFFICIENCY =  .57385
```

Table 11. Hg (Stage 1)/H_2O (Stage 2) with Five Extractions with Regeneration on Each Stage

```
STAGE 1 HAS 5 EXTRACTIONS AS FOLLOWS
                      EXTRACTION      MASS FRACTION       TEMPERATURE
                          1                .0130              1299.85
                          2                .0131              1249.70
                          3                .0131              1199.55
                          4                .0131              1149.39
                          5                .0132              1099.24

STAGE 2 HAS 5 EXTRACTIONS AS FOLLOWS
                      EXTRACTION      MASS FRACTION       TEMPERATURE
                          1                .0802               886.74
                          2                .0702               824.39
                          3                .0625               762.05
                          4                .0566               699.70
                          5                .0518               637.35

STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                       (LBM/HR PER KW)

  1       1350.0      88.8      1049.1      5.94      .16593          .8944           .3945         64.200
  2        949.1     615.5       575.0      1.47      .30537          .7937           .6055          8.486

                              OVERALL EFFICIENCY =  .42063
```

Table 11. (Continued)

```
  STAGE 1 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION      MASS FRACTION      TEMPERATURE
                        1              .0190              1427.39
                        2              .0189              1354.78
                        3              .0189              1282.16
                        4              .0189              1209.55
                        5              .0189              1136.94
  STAGE 2 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION      MASS FRACTION      TEMPERATURE
                        1              .0853               899.44
                        2              .0738               834.55
                        3              .0652               769.66
                        4              .0585               704.78
                        5              .0533               639.89

  STAGE     T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
            (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY    QUALITY                       (LBM/HR PER KW)

    1       1500.0    225.7     1064.3     7.02     .21566         .8690          .4683         58.609
    2        964.3    706.9      575.0     1.47     .31218         .7869          .5317          7.442

                               OVERALL EFFICIENCY =  .46052

  STAGE 1 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION      MASS FRACTION      TEMPERATURE
                        1              .0275              1605.39
                        2              .0272              1500.78
                        3              .0269              1396.17
                        4              .0267              1291.56
                        5              .0266              1186.95
  STAGE 2 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION      MASS FRACTION      TEMPERATURE
                        1              .0918               914.45
                        2              .0782               846.56
                        3              .0683               778.67
                        4              .0607               710.78
                        5              .0549               642.89

  STAGE     T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE         TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
            (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY    QUALITY                       (LBM/HR PER KW)

    1       1710.0    624.5     1082.3     8.59     .27133         .8462          .5380         53.466
    2        982.3    827.9      575.0     1.47     .31975         .7785          .4620          6.485

                               OVERALL EFFICIENCY =  .50432
```

Table 11. (Continued)

```
 STAGE 1 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1               .0348           1757.65
                        2               .0341           1625.30
                        3               .0335           1492.95
                        4               .0329           1360.60
                        5               .0326           1228.25
 STAGE 2 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1               .0970            925.75
                        2               .0816            855.60
                        3               .0706            785.45
                        4               .0624            715.30
                        5               .0561            645.15

 STAGE    T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY     QUALITY                        (LBM/HR PER KW)

   1      1890.0    1236.7    1095.9     9.97     .30845         .8364            .5784         50.508
   2       995.9     929.1     575.0     1.47     .32513         .7720            .4216          5.950
                               OVERALL EFFICIENCY =  .53330

 STAGE 1 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1               .0495           2062.53
                        2               .0472           1875.06
                        3               .0453           1687.59
                        4               .0438           1500.12
                        5               .0427           1312.65
 STAGE 2 HAS 5 EXTRACTIONS AS FOLLOWS
                    EXTRACTION     MASS FRACTION     TEMPERATURE
                        1               .1096            950.15
                        2               .0895            875.12
                        3               .0757            800.09
                        4               .0658            725.06
                        5               .0584            650.03

 STAGE    T-HIGH    P-HIGH    T-LOW     P-LOW     STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)   (PSIA)    (DEG R)   (PSIA)    EFFICIENCY     QUALITY                        (LBM/HR PER KW)

   1      2250.0    3433.6    1125.2    13.55     .36200         .8354            .6281         46.609
   2      1025.2    1179.3     575.0     1.47     .33597         .7569            .3719          5.362
                               OVERALL EFFICIENCY =  .57635
```

Table 12. Comparison of Optimum Binary Cycles with Organic Upper Stage with and Without Regeneration

Organic Stage	ΔT	Diphenyl						Water			
		T_h	p_h	T_ℓ	p_ℓ	$T_C'-T_A$	Regenerator Effectiveness	T_h	p_h	T_ℓ	p_ℓ
Regenerative ($T_o-T_E'=50°$)	100°	1260	222	762	.72	90.7	.860	662	12.0	575	1.47
Nonregenerative	50°	1260	222	1066	49.9	--	--	1016	1099	575	1.47

η	Quality, X		P/P_{Tot}		$\dot{m}/P_{Tot}$	
	$(C_6H_5)_2$	H_2O	$(C_6H_5)_2$	H_2O	$(C_6H_5)_2$	H_2O
.355	1.000	.930	.796	.204	39.3	6.48
.357	1.000	.762	.275	.725	45.7	7.84

Table 13. K(Stage 1)/$(C_6H_5)_2$ (Stage 2)/H_2O (Stage 3) with Regeneration in Stage 2 (T_o-T_E'=50°)

```
T(C-PRIME) - T(A) =   91.8          REGENERATOR EFFECTIVENESS =   .864


STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                        (LBM/HR PER KW)

  1       1620.0        3.6     1374.0        .50      .11138          .9095         .2584          8.605
  2       1274.0      242.0      759.1        .69      .28987         1.0000         .5977         28.471
  3        659.1       11.3      575.0       1.47      .09823          .9317         .1438          4.704
                                 OVERALL EFFICIENCY =  .43095



T(C-PRIME) - T(A) =   91.8          REGENERATOR EFFECTIVENESS =   .864


STAGE     T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT     P/P-TOTAL     M-DOT/P-TOTAL
          (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                        (LBM/HR PER KW)

  1       1800.0       10.9     1374.0        .50      .17072          .8620         .3641          7.786
  2       1274.0      242.0      759.1        .69      .28987         1.0000         .5126         24.416
  3        659.1       11.3      575.0       1.47      .09823          .9317         .1233          4.034
                                 OVERALL EFFICIENCY =  .46895
```

Table 13. (Continued)

```
T(C-PRIME) - T(A) =   91.8          REGENERATOR EFFECTIVENESS =   .864


STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                     (LBM/HR PER KW)

  1        1980.0       26.6     1374.0        .50      .21764          .8254          .4362         7.209
  2        1274.0      242.0      759.1        .69      .28987         1.0000          .4545        21.648
  3         659.1       11.3      575.0       1.47      .09823          .9317          .1094         3.576

                                OVERALL EFFICIENCY =   .49899
```

```
T(C-PRIME) - T(A) =   91.8          REGENERATOR EFFECTIVENESS =   .864


STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                     (LBM/HR PER KW)

  1        2160.0       55.6     1374.0        .50      .25519          .7974          .4879         6.777
  2        1274.0      242.0      759.1        .69      .28987         1.0000          .4128        19.661
  3         659.1       11.3      575.0       1.47      .09823          .9317          .0993         3.248

                                OVERALL EFFICIENCY =   .52304
```

```
T(C-PRIME) - T(A) =   91.8          REGENERATOR EFFECTIVENESS =   .864


STAGE      T-HIGH     P-HIGH     T-LOW      P-LOW      STAGE          TURBINE EXIT    P/P-TOTAL     M-DOT/P-TOTAL
           (DEG R)    (PSIA)     (DEG R)    (PSIA)     EFFICIENCY     QUALITY                     (LBM/HR PER KW)

  1        2340.0      103.5     1374.0        .50      .28575          .7764          .5266         6.435
  2        1274.0      242.0      759.1        .69      .28987         1.0000          .3916        18.175
  3         659.1       11.3      575.0       1.47      .09823          .9317          .0918         3.303

                                OVERALL EFFICIENCY =   .54261
```

Table 13. (Continued)

T(C-PRIME) - T(A) = 91.8 REGENERATOR EFFECTIVENESS = .864

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2520.0	174.0	1374.0	.50	.31082	.7605	.5564	6.156
2	1274.0	242.0	759.1	.69	.28987	1.0000	.3576	17.033
3	659.1	11.3	575.0	1.47	.09823	.9317	.0860	2.814

OVERALL EFFICIENCY = .55867

T(C-PRIME) - T(A) = 91.8 REGENERATOR EFFECTIVENESS = .864

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2700.0	271.6	1374.0	.50	.33166	.7493	.5798	5.918
2	1274.0	242.0	759.1	.69	.28987	1.0000	.3387	16.132
3	659.1	11.3	575.0	1.47	.09823	.9317	.0815	2.665

OVERALL EFFICIENCY = .57201

Table 14. K(Stage 1)/$(C_6H_5)_2$ (Stage 2)/H_2O (Stage 3) without Regeneration in Stage 2 (ΔT=50° between Stages 2 and 3)

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1620.0	3.6	1374.0	.50	.11138	.9095	.2576	8.575
2	1274.0	242.0	1066.7	50.05	.10393	1.0000	.2136	30.774
3	1016.7	1102.3	575.0	1.47	.28722	.7614	.5289	5.720

OVERALL EFFICIENCY = .43244

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1800.0	10.9	1374.0	.50	.17072	.8620	.3630	7.763
2	1274.0	242.0	1066.7	50.05	.10393	1.0000	.1832	26.404
3	1016.7	1102.3	575.0	1.47	.28722	.7614	.4538	4.908

OVERALL EFFICIENCY = .47035

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1980.0	26.6	1374.0	.50	.21764	.8254	.4350	7.190
2	1274.0	242.0	1066.7	50.05	.10393	1.0000	.1625	23.419
3	1016.7	1102.3	575.0	1.47	.28722	.7614	.4025	4.353

OVERALL EFFICIENCY = .50031

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2160.0	55.6	1374.0	.50	.25519	.7974	.4867	6.761
2	1274.0	242.0	1066.7	50.05	.10393	1.0000	.1476	21.275
3	1015.7	1102.3	575.0	1.47	.28722	.7614	.3656	3.955

OVERALL EFFICIENCY = .52429

Table 14. (Continued)

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STAGE    T-HIGH     P-HIGH    T-LOW      P-LOW     STAGE           TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)    (PSIA)    (DEG R)    (PSIA)    EFFICIENCY      QUALITY                      (LBM/HR PER KW)

  1      2340.0      103.5    1374.0        .50     .28575          .7764          .5255         6.421
  2      1274.0      242.0    1066.7      50.05     .10393         1.0000          .1365        19.670
  3      1016.7     1102.3     575.0       1.47     .28722          .7614          .3380         3.656

                              OVERALL EFFICIENCY =  .54381


STAGE    T-HIGH     P-HIGH    T-LOW      P-LOW     STAGE           TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)    (PSIA)    (DEG R)    (PSIA)    EFFICIENCY      QUALITY                      (LBM/HR PER KW)

  1      2520.0      174.0    1374.0        .50     .31082          .7605          .5552         6.143
  2      1274.0      242.0    1066.7      50.05     .10393         1.0000          .1279        18.436
  3      1016.7     1102.3     575.0       1.47     .28722          .7614          .3168         3.427

                              OVERALL EFFICIENCY =  .55982


STAGE    T-HIGH     P-HIGH    T-LOW      P-LOW     STAGE           TURBINE EXIT    P/P-TOTAL    M-DOT/P-TOTAL
         (DEG R)    (PSIA)    (DEG R)    (PSIA)    EFFICIENCY      QUALITY                      (LBM/HR PER KW)

  1      2700.0      271.6    1374.0        .50     .33166          .7493          .5787         5.906
  2      1274.0      242.0    1066.7      50.05     .10393         1.0000          .1212        17.464
  3      1016.7     1102.3     575.0       1.47     .28722          .7614          .3001         3.246

                              OVERALL EFFICIENCY =  .57313
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Table 15. Cs (Stage 1)/$(C_6H_5)_2$ (Stage 2)/H_2O (Stage 3) without Regeneration in Stage 2 (ΔT=50° between Stages 2 and 3)

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1440.0	2.6	1252.3	.51	.09554	.9219	.2393	36.910
2	1152.3	103.7	1057.9	46.08	.07062	1.0000	.1600	46.465
3	1007.9	1026.3	575.0	1.47	.28527	.7663	.6007	6.521

OVERALL EFFICIENCY = .39920

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1620.0	8.4	1251.2	.50	.16375	.8672	.3686	32.596
2	1151.2	102.8	1057.6	45.93	.07022	1.0000	.1322	38.691
3	1007.6	1023.5	575.0	1.47	.28519	.7661	.4992	5.420

OVERALL EFFICIENCY = .44422

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1800.0	21.6	1259.1	.54	.21246	.8306	.4439	29.780
2	1159.1	109.3	1058.3	46.24	.07359	1.0000	.1211	33.068
3	1008.3	1029.4	575.0	1.47	.28535	.7658	.4350	4.721

OVERALL EFFICIENCY = .47861

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	1980.0	46.1	1274.9	.63	.24736	.8066	.4892	27.803
2	1174.9	123.2	1063.8	48.72	.07933	1.0000	.1181	29.036
3	1013.8	1076.8	575.0	1.47	.28659	.7629	.3927	4.253

OVERALL EFFICIENCY = .50565

Table 15. (Continued)

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2160.0	86.1	1276.7	.64	.28004	.7860	.5309	26.169
2	1176.7	124.8	1062.6	48.18	.08039	1.0000	.1097	26.362
3	1012.6	1066.6	575.0	1.47	.28633	.7635	.3594	3.693

OVERALL EFFICIENCY = .52749

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2340.0	145.6	1279.8	.66	.30570	.7726	.5610	24.866
2	1179.8	127.8	1063.1	48.40	.08145	1.0000	.1038	24.415
3	1013.1	1070.8	575.0	1.47	.28644	.7633	.3352	3.631

OVERALL EFFICIENCY = .54493

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2520.0	225.4	1281.7	.67	.32700	.7633	.5848	23.780
2	1181.7	129.5	1063.2	48.44	.08210	1.0000	.0988	22.938
3	1013.2	1071.6	575.0	1.47	.28646	.7632	.3164	3.427

OVERALL EFFICIENCY = .55921

STAGE	T-HIGH (DEG R)	P-HIGH (PSIA)	T-LOW (DEG R)	P-LOW (PSIA)	STAGE EFFICIENCY	TURBINE EXIT QUALITY	P/P-TOTAL	M-DOT/P-TOTAL (LBM/HR PER KW)
1	2700.0	327.4	1329.9	1.05	.33346	.7685	.5841	23.576
2	1223.9	181.6	1066.6	50.00	.09680	1.0000	.1130	19.557
3	1016.6	1101.3	575.0	1.47	.28720	.7615	.3029	3.276

OVERALL EFFICIENCY = .57088