Optimization of tether geometry to achieve low anchor loss in Lamé-mode resonators

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***Abstract*—In this paper, we study the fundamental cause of anchor dissipation in Lamé- or wineglass-mode resonators and show that by carefully optimizing the resonator tether geometry low anchor losses can be achieved, making it possible to reach the intrinsic *f*x*Q* limit of the resonating material. Through analytical and finite element investigation, we demonstrate that the anchor loss is most significant when the flexural-mode resonance frequency of the tether is in sync with the Lamé-mode frequency of the resonating plate. This has a significant bearing on the design of Lamé-mode resonators with release holes or temperature-compensation trenches, where the modification of the resonator structure requires modification to the tether geometry to maintain a high *Q*. We verify the predicted results through experimental measurements and present an optimized design that exhibits a mechanical *Q* of 408,270 for the fundamental Lamé mode at 41.57 MHz. The *f*x*Q* for this device is 1.7x10^{13}, one of the highest values reported in silicon.**

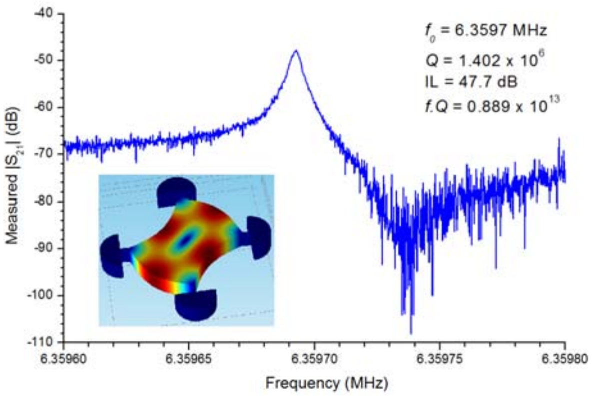

Fig. 1: Measured response of a fundamental Lamé mode at 6.3597 MHz showing a *Q* of 1.4 million.

I. INTRODUCTION

Low-loss resonators operating in the low- to mid-frequency range (kHz to few MHz) are critical in the implementation of low-power precision timing references in real time clocks and inertial measurement units (IMUs). At these frequencies, most silicon-based resonator topologies suffer from increased thermoelastic damping (TED) [1]. Due to their isochoric nature, pure Lamé-mode resonators are immune to TED and are ideally suited for such low-frequency timing applications. Straight tether supported Lamé- or wineglass-mode resonators operating in the low MHz frequency range have been shown to exhibit extremely high $f \times Q$ values on the order of 10^{13}, close to the fundamental phonon-phonon loss limit in silicon (see Fig. 1) [2], [3]. However, experimental results suggest increased anchor dissipation exists for certain tether geometries, resulting in *Q* degradation by over an order of magnitude [3]. In light of the utility of these resonators in timing applications, the design of tethers for low anchor loss is critical and merits further investigation.

Anchor loss, also known as clamping loss, has been widely investigated in a variety of resonator geometries due to its significance in the design of high-*Q* resonators. Unlike intrinsic losses which are primarily set by the material of the resonator and its operating frequency [1], [4], anchor loss is design dependent and therefore can be minimized through appropriate design choices. Analytical models estimating the clamping loss in simple beam geometries have been presented in literature [5]. Recently, more exotic ring resonators have also been analyzed for their substrate damping effects [6]. Unfortunately, most of these analytical estimates require precise knowledge of the stress profile at the anchoring area, which is difficult in the case of complex resonator geometries. In addition, these methods involve the use of fitting parameters, which need to be extracted from measured results. Recognizing this limitation, a significant effort has been put towards the estimation of anchor *Q* using finite element methods. Bindel *et al.* applied the method of perfectly matched layers (PML) to the problem of anchor dissipation in stem-supported disk resonators [7], motivated by their utility in electromagnetic analysis [8]. Since then, there has been additional work validating the PML-based approach for anchor *Q* estimation [9]. In both these works, the finite element analysis (FEA) has been implemented using custom built codes, making it difficult to generically adopt. In this work, COMSOL®, a commercial software package, is utilized to model the anchor loss in resonators. The COMSOL

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simulation parameters are chosen such that the simulated anchor Qs match reported analytical estimates of anchor Q for different beam resonance modes. This model is then used to predict the anchor loss in Lamé-mode resonators. In the subsequent analysis, it is shown that the anchor Q degradation is primarily due to the co-existence of the tether resonance modes near the frequency of the primary Lamé mode. This has important consequences in the design of Lamé-mode resonatorswith passive compensation or etch holes for wet release compatibility,since such modifications can lead to a reduction in the resonator frequency.

For the purpose of experimental verification, a 100 µm × 100 µm Lamé-mode resonator is analyzed for its tether-dependent anchor loss and a good agreement between the predicted and measured results is shown. An optimized Lamé-mode resonator is presented with a measured mechanical Q of 408,270 at a center frequency of 41.57 MHz, attaining an $f \times Q$ of 1.7×10^{23}, which is close to the intrinsic phonon-phonon limit in silicon [4].

II. ESTIMATION OF ANCHOR *Q* USING FINITE ELEMENT MODELING

Fig. 2 shows the finite element model of a beam using a PML to model the substrate as a semi-infinite layer [7]. The material parameters for the PML are same as those for the substrate layer, which usually are the same as those for the beam. It should be noted that the parameters of the PML are critical in achieving accurate anchor Q estimates. Similar to the observations in [9], the choice of alpha, *i.e.* the PML scaling factor, determines the magnitude of the simulated anchor Q and should be optimized for the specific resonator frequency and geometry. This idea is elaborated in Fig. 3, which shows a large variation in the simulated Q as a function of alpha.

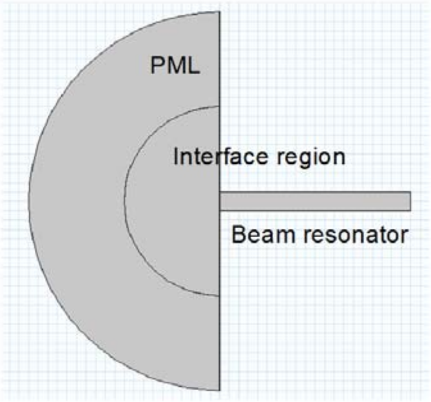


Fig. 2: Model of a beam resonator for anchor loss estimation.

From Fig. 3, we can clearly see that the minimum value of the estimated anchor Q converges to the analytical estimate from [5]. A similar study is performed for all the beam geometries and modes analyzed in [5]. The results are summarized in Table 1, demonstrating an excellent agreement between the analytical and the FEM estimates of minimum anchor Q.

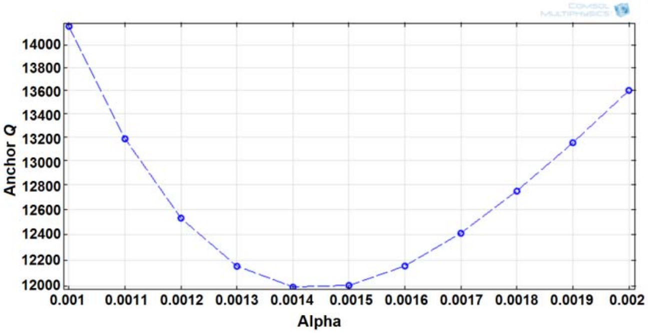


Fig. 3: Simulated anchor Q for a silicon beam resonator as a function of alpha. The silicon beam is 500 µm long and 7.2 µm wide. The analytical estimate of anchor Q for the third-order flexural mode (clamped-clamped boundary) is calculated to be 11,177 [5].

Table 1: Comparison of the simulated anchor Q with analytically calculated estimates for flexural modes of a silicon beam resonator [5].

Length (µm)	**Width (µm)**	**Frequency (kHz)**	**Anchor *Q***	
			Analytical	***Simulated***
Fixed Free Cantilever (first-order flexure)				
700	6.45	15.94	2.6626×10^6	2.65×10^6
500	4	19	4.0684×10^6	4.1×10^6
500	5	24.59	2.083×10^6	2.1×10^6
500	6.2	30.14	1.0925×10^6	1.1×10^6
Fixed- Fixed Beam (first-order flexure)				
900	7.9	74.8	248,470	247,612
700	5.15	80.3	421,980	419,165
500	3.8	117	382,800	380,418
300	3.4	288.4	115,440	115700
Fixed- Fixed Beam (third-order flexure)				
700	8.7	740	17,365	18,940
500	6.1	1030	18,357	20,061
500	7.2	1210	11,177	11,976

III. RESONATOR DESIGN

Fig. 4 shows a scanning electron microscope (SEM) image and critical tether parameters of a Lamé-mode resonator. The frequency of the fundamental Lamé mode is set by the characteristic plate length L as marked in Fig. 4 and is given as,

$$f_0 = \frac{1}{2\mathrm{L}}\sqrt{\frac{G}{\rho}} \tag{1}$$

where G is the shear modulus (Pa) and ρ is the density (kg/m^3). Fig. 5 plots the simulated anchor Q loaded with the intrinsic phonon-phonon loss limit in silicon [4] as a function of tether geometry for a 100 µm × 100 µm square plate resonating in the fundamental Lamé mode.

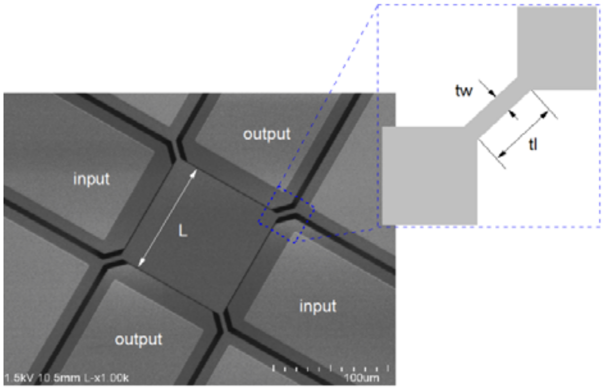


Fig. 4: (Left) A SEM image of a Lamé-mode resonator and (right) the critical tether parameters. The characteristic length L sets the frequency of the fundamental Lamé mode.

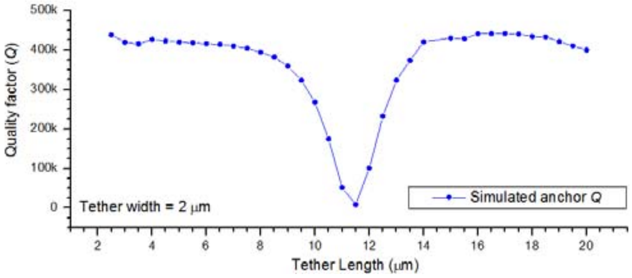


Fig. 5: Simulated anchor Q as a function of tether length for a 100 μm × 100 μm Lamé-mode resonator having a tether width of 2 μm. The simulated Q is loaded by the phonon-phonon loss limit in silicon [4].

IV. ORIGIN OF ANCHOR LOSS IN LAMÉ RESONATORS

In order to support the Lamé-mode resonance, the tethers are required to undergo forced flexural vibrations. As a consequence, the anchor loss in such resonators is strongly dictated by the flexural resonance frequency of the tether which can be expressed as,

$$f_n = \frac{1}{2\pi}\beta_n^2 \frac{t_w}{t_l^2}\sqrt{\frac{EI}{\rho A}}, \tag{2}$$

where f_n is the resonance frequency of the nth mode, β_n is the mode constant, t_w is the tether width (μm), t_l is the tether length (μm), E is the Young's modulus (Pa), I is the area moment of inertia for the tether cross-section (m^4) and A is the cross-section area of the tether (m^2). For a cantilever loaded with a mass at its free-end, the resonance frequency is known to be lower and can be estimated using the formulation in [10]. The effect of mass loading is accounted for by the lowering of the mode constants which affects the resonance frequency through (2). Table 2 lists the mode constants and the critical tether length at which the flexural resonance frequency for the tether equals the frequency of the fundamental Lamé-mode of a 100 μm × 100 μm square plate when the tether width is fixed at 2 μm. While the results agree well for longer tether lengths, at shorter tether lengths, the tether aspect ratio is much smaller than 10:1 and the Euler-Bernoulli approximation introduces significant inaccuracies. The FEA simulation results for the Lamé mode shape and the first two flexural modes of the tether along with the associated tether lengths are shown in Fig. 6.

Table 2: Analytically estimated critical tether length for a 100 μm × 100 μm Lamé-mode resonator.

	β	Critical tether length (μm)	
		Analytical	Simulated
Mode 1	0.425	1.44	3.5
Mode 2	3.925	13.34	11.5
Mode 3	7.065	24.01	23
Mode 4	10.215	34.72	34

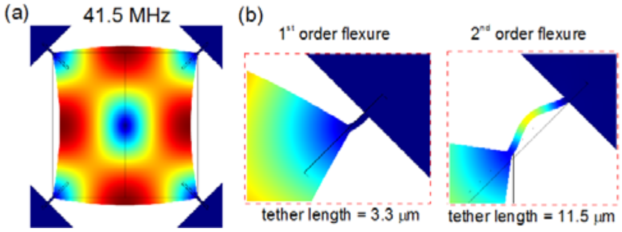


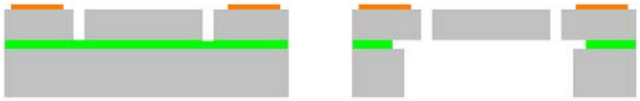
Fig. 6: (a) Fundamental Lamé mode of a 100 μm × 100 μm silicon square plate. (b) First- and second-order flexural modes of the tether. The tether width is 2 μm and the critical tether length is 3.3 μm and 11.5 μm for the first- and second-order mode, respectively. With these critical anchor dimensions, maximum anchor loss of the resonator is found.

From the presented analysis, it is clear that when the tether resonance frequency is near the frequency of the Lamé mode, the resonator Q is degraded due to the additional energy lost from the tether's resonance. In other words, to enable high-Q Lamé-mode resonators, the tether geometry should be set such that the flexural mode of the tethers is far from the fundamental Lamé-mode frequency.

V. EXPERIMENTAL VERIFICATION

A. *Device Fabrication*

In order to experimentally investigate the dependence of the Lamé-mode anchor Q on its tether length, devices were fabricated using the process flow shown in Fig. 7. The starting substrate is a silicon-on-insulator (SOI) wafer with a 25 μm thick low resistivity (0.02 Ω.cm) device layer to enable electrostatic actuation. The choice of electrostatic actuation is made to ensure that the resonator Q is not degraded by interface losses [11].



(a) DRIE to create resonator profile. Lift-off top metal (Ti/Au) for electrical contact.

(b) Backside DRIE to release device. BHF etch to remove BOX layer.

Fig. 7: Fabrication process flow used to experimentally investigate anchor loss in Lamé-mode resonators.

B. *Measured Results*

The fabricated devices are measured using an Agilent E5061B network analyzer in a vacuum probe station. Multiple dies of resonators were tested and the extracted mechanical Q as a function of the tether length is plotted in

Fig. 8. The data for the simulated anchor *Q* is overlaid for comparison. In order to obtain the unloaded resonator *Q*, the series electrical resistance and the resonator motional impedance is estimated using standard extraction techniques [12]. From the analysis, it is estimated that series electrical resistance of the contacts to the resonator is 3.9 kΩ.

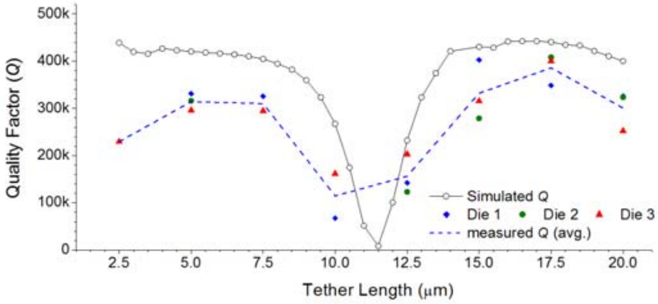


Fig. 8: Extracted unloaded *Qs* of multiple Lamé-mode resonators as a function of tether length for a tether width of 2 µm. The data for simulated *Q* is overlaid for comparison.

Fig. 9 plots the measured response of a Lamé-mode resonator with optimized tether geometry: a tether length of 17.5 µm and tether width of 2 µm. The measured *Q* for this device is 302,120 at 41.57 MHz. Using the extraction method in [12], the unloaded *Q* of the resonator is extracted to be 408,270 giving an *f*×*Q* of 1.7 × 10^{13}.

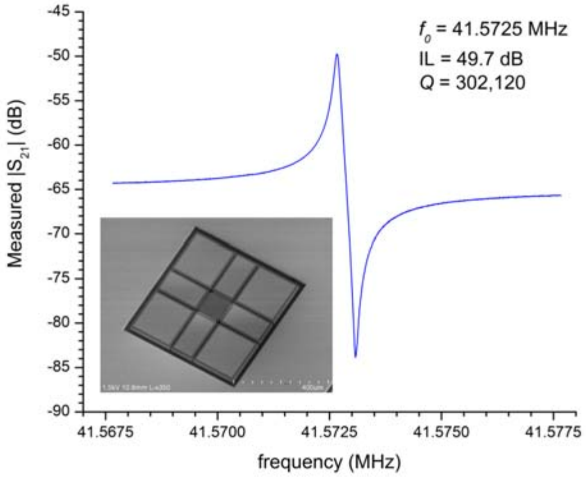


Fig. 9: Measured response of an optimized Lamé-mode resonator with an applied bias of 250 V. (Inset) SEM image of the device. The device has a tether length of 17.5 µm and tether width of 2 µm.

The results in Fig. 8 suggest that for optimum tether geometries the resonator *Q* is primarily limited by the intrinsic phonon-phonon interactions in silicon. Secondly, we can clearly see the accuracy of prediction using finite element methods. With the validation of the anchor loss model, a similar analysis can be performed for the tether optimization of passively compensated Lamé-mode resonators, and will aid in the development of low-power low-frequency precision timing references.

VI. CONCLUSION

In this paper we investigated the cause of anchor loss in Lamé-mode resonators and verified the role of the tether resonance modes in degrading the Lamé-mode resonator *Q*. Finite element analysis was used to estimate the resonator *Q* and was shown to agree well with both analytical estimates and experimental measurements. An optimized tether design was presented with an unloaded *Q* of 408,270 for the fundamental Lamé mode at 41.5 MHz attaining an *f*×*Q* of 1.7 × 10^{13}, which is close to the fundamental phonon-phonon loss limit in silicon.

For low-power low noise timing applications, it is important to design passively compensated resonators to reduce the frequency fluctuation with temperature. From the presented analysis, we can deduce that since the frequency of temperature-compensated Lamé-mode resonators is different from their uncompensated counter-parts, the optimum tether geometry will also be different. The presented tether optimization procedure will be invaluable for the design of low-loss temperature-compensated Lamé-mode resonators.

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