# Optimization of the power output from ducted turbines

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**Abstract:** For many years there has been interest in the possibility of enhancing the performance of wind turbines by encasing them in a duct or 'shroud', but practical considerations have prevented commercial realization. While this possibility is not ruled out, more beneficial schemes for shrouding may well arise for water turbines in tidal streams. In this paper, the performance of a shrouded turbine has been analysed using one-dimensional theory by treating the ducts upstream and downstream of the turbine as contractions or expansions having specified diffusion efficiencies. It is shown that, for given diffuser efficiencies, there is an optimum turbine resistance for generating maximum power, because the swallowing capacity of the duct is increased as the resistance decreases. Simple experiments with static resistance elements in the duct have confirmed this theory for the swallowing capacity and shown that there is little to be gained by controlled diffusion at inlet. However, efficient controlled diffusion at outlet can lead to power enhancements of more than 30 per cent over an optimized turbine of the same area in the freestream. To achieve this, a more lightly loaded turbine must be utilized. Claims in the literature of much greater enhancements than this are sometimes made in relation to non-optimized conditions for the freestream case. The alternative requirement, that maximum power should be extracted from a given inlet area, is also analysed and shown to require a highly loaded turbine.

**Keywords:** wind turbines, tidal turbines, ducted turbines, shrouded turbines, swallowing capacity

## NOTATION

| | |
|---|---|
| $A$ | area |
| $c$ | blade chord |
| $C_L$ | lift coefficient of a blade section |
| $C_p$ | power coefficient, referred to the freestream velocity, defined by equation (5) |
| $C_p^*$ | power coefficient, referred to the incident velocity, defined by equation (7) |
| $C_{P_b}$ | base pressure coefficient, defined by equation (10) |
| $C_t$ | thrust coefficient, referred to the freestream velocity, defined by equation (3) |
| $C_t^*$ | thrust coefficient, referred to the incident velocity $= K$ |
| $K$ | resistance coefficient, defined by equation (1) |
| $n$ | rotational speed of the turbine |
| $N$ | number of turbine blades |
| $P$ | static pressure |
| $r$ | representative radius of a blade section |
| $U$ | one-dimensional flow velocity along the axis of the turbine |
| $W$ | mechanical power developed by the turbine |
| $X$ | ratio of representative blade speed to freestream velocity |
| $X^*$ | ratio of representative blade speed to incident axial velocity |
| $\varepsilon$ | ratio of drag to lift for a blade section |
| $\eta$ | efficiency of diffusion, defined by equation (8) and (9) |
| $\eta_t$ | turbine efficiency, defined by equation (2) |
| $\lambda$ | blade loading coefficient |
| $\rho$ | fluid density |
| $\phi$ | angle of the blade to the plane of rotation, the pitch angle |

### Subscripts

| | |
|---|---|
| 0 | relates to the flow at the inlet to the shroud |
| 0i | relates to the shroud at the inlet |
| 01 | relates to the inlet diffuser/contraction |
| 1 | upstream of the turbine or resistance |
| 2 | downstream of the turbine or resistance |
| 23 | relates to the outlet diffuser |
| 3 | at the shroud outlet |

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# 1 INTRODUCTION

The encasing of wind turbines in ducts, sometimes known as shrouds, has been considered for many years, but there are no commercial examples. Although some enhancement of performance under ideal conditions is well established, there are considerable disadvantages in the ducted design owing to the additional weight and drag to be carried at the top of the pylon. Moreover, the performance enhancements are only achieved if the duct is sufficiently aligned with the wind, and the wind is not too gusty. With the current development of turbines for tidal streams, the same considerations will apply, but the direction of the flow will be much more certain and the weight of the ducting will be of less concern.

From a detailed aerodynamic analysis of the particular configuration where the duct wall has an aerofoil cross-section, Politis and Koras [1] show that power enhancements of more than a factor of 2 are possible, largely confirming the earlier results from the Athens school [2]. Recently, researchers at the University of Rijeka in Croatia [3] have claimed that a shrouded turbine will produce 3 times more energy than a conventional machine. Vortec Energy Limited of New Zealand had intended to commercialize the world's first ever diffuser-augmented wind turbine, having constructed a prototype on the Waikato coastline in 1998 [4], but they have now ceased trading. A water turbine version of the Vortec diffuser has been tested by the University of Reading, and power augmentation by a factor of more than 2.2 has been obtained [5]. However, all these comparisons are for the same turbine, whereas in reality different turbines should be chosen.

It is thus of interest, and possibly of commercial advantage, to be able to calculate the optimum match of ducting to turbine, and the power enhancement when considering optimum conditions for both unshrouded and shrouded cases. No consistent theory for choosing the inlet and outlet to the turbine, or the optimum turbine characteristics, appears to have been published, although Hansen [6] and Wortman [7] include short chapters on shrouding in their books.

This paper explores the effect of turbine resistance and configuration of shrouding on the power generated. There are two alternative requirements:

(a) to generate the greatest power from a given rotor diameter,
(b) to generate the greatest power from a given area of stream.

The second may be particularly relevant for tidal streams. The analysis is done with simple one-dimensional theory, and some results are checked against experimental data for a shrouded resistance in a wind tunnel. Finally, some implications for turbine design in practical applications are discussed, although the objective is limited to laying out the fundamental principles by which design decisions may be made.

# 2 ANALYSIS

Many textbooks (e.g. reference [8]) treat the case of incompressible flow through a bare, unshrouded, turbine in terms of actuator disc theory. This assumes that the flow up to and away from the plane of the disc (turbine) is isentropic and that therefore Bernouilli's theory can be applied to those sections. Across the disc there is a step fall in pressure, and therefore the flow must diffuse into the area of the disc from upstream and away from the disc downstream so as to recover to the pressure of the freestream (see Fig. 1). Thus, the stream tube passing through the disc initially has a smaller area than the disc but eventually has a larger one. For optimum power generation (the Betz limit), the velocity through the turbine is two-thirds of that far upstream, and the velocity downstream in the wake is one-third of that upstream. Thus, the wake stream tube is twice the area of the disc.

Clearly, relative to this optimum unshrouded case, the flow downstream of the turbine can be diffused to a greater extent with a shroud by expanding the area of flow by more than a factor of 2. This reduces the pressure at the turbine and hence increases the mass flow and power. Moreover, the blockage in the flow imposed by the casing can induce a lower pressure in the wake than that of the freestream (a 'base pressure'), again augmenting the flow through the turbine.

The shrouded turbine can be analysed as an inlet section, which is either a contraction or an expansion (a 'diffuser') with an inlet area $A_{0i}$, a straight section containing the turbine of cross-section $A_1$ and a diffuser after the turbine from area $A_2 \, (= A_1)$ to area $A_3$ (Fig. 2). In practice, these three sections would be merged into one another to give a smooth internal profile of venturi form. The outer profile is
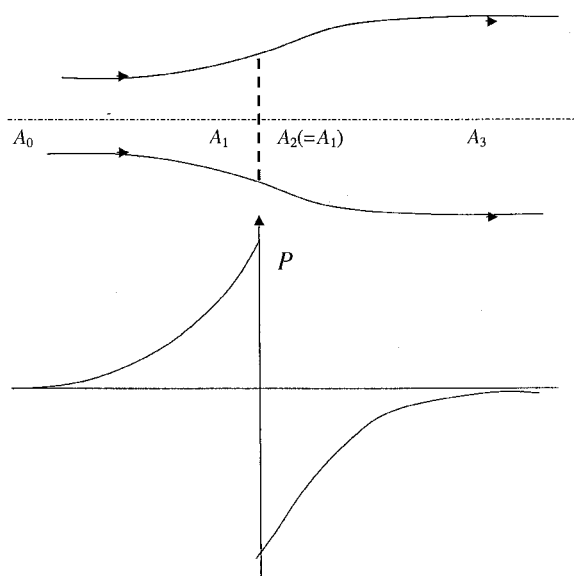


**Fig. 1** Flow pattern and pressure distribution for a turbine in a freestream

Experimental and
theoretical model



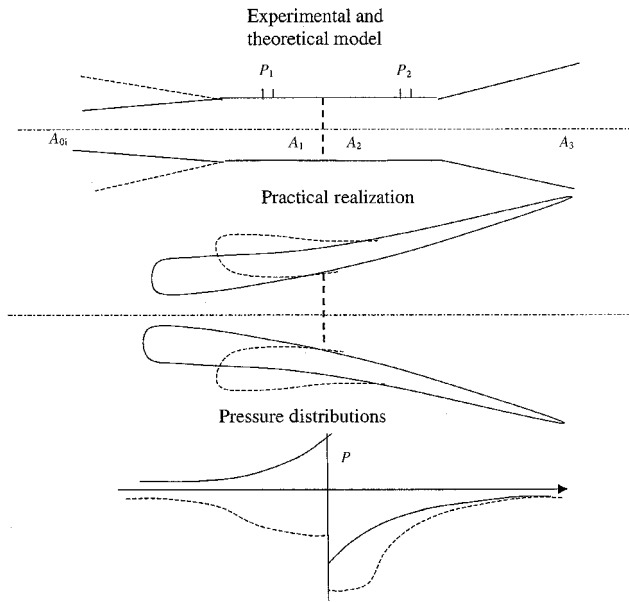Practical realization

Pressure distributions

**Fig. 2** Experimental and theoretical model compared with the practical realization of a ducted turbine: ------, low resistance configuration; ——, high-resistance configuration



**Fig. 3** Flow regimes for inlet contraction/expansion: (a) low-resistance duct, $A_{0i} > A_0$; (b) low-resistance duct, $A_{0i} < A_0$; (c) high-resistance duct, $A_{0i} > A_0$; (d) high-resistance duct, $A_{0i} < A_0$

also of significance because it determines the pressure in the ventilated base of this obstruction in the flow.

The qualitative effect of this shrouding can be easily deduced from simple principles, although it is not always fully appreciated. If the turbine presents only a small resistance to the flow, then the outlet diffuser leads to a greater flow through the turbine than if there were no diffuser, since some of the kinetic energy of the exhaust is recovered as a rise in pressure to that in the 'base' of the obstruction. By continuity, this implies that, upstream, the area of the stream tube to be drawn into the turbine, $A_0$, may be greater than the area of the turbine (see Figs 3a and b). A contracting section, or at least a rounded inlet, may be useful in smoothing this flow, but no obvious advantage is gained by making $A_{0i} > A_0$ because the area $A_0$ is not affected and the intake process may be no more efficient.

However, as the resistance of the turbine is increased, the swallowing capacity is reduced and the area of the upstream stream tube is diminished (Figs 3c and d). If the area of the inlet is not diminished correspondingly, the flow must still be slowed to expand in area as it enters the turbine, so there must be flow reversal in the inlet, which is likely to be detrimental to the diffusion process. The question then arises as to whether the diffusion upstream of the turbine can be improved by putting a physical diffusing section into that position, rather than relying on nature's own diffusion process with a natural shear layer. Since the shearing effect of the freestream is similar to that produced when wall jets are used to energize the boundary layer on the diffuser walls and hence prevent separation [9], a good natural performance is to be expected. Moreover, since inlet efficiencies
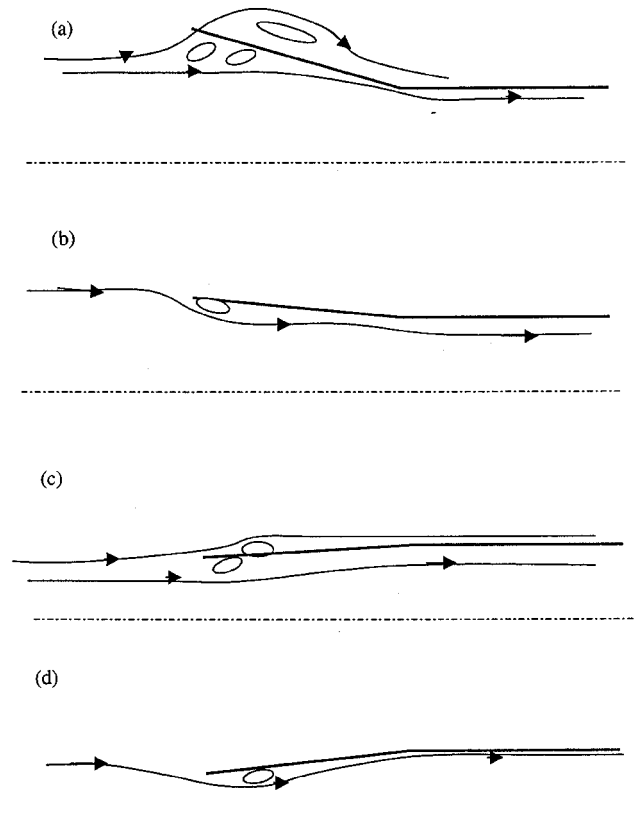
very close to unity have been measured for aero gas turbine inlets under high inlet velocity conditions when the flow into the fan is slowed down, it is unlikely that much improvement can be achieved.

To analyse this flow quantitatively, let the ducted turbine present a resistance of coefficient $K$ to the flow with velocity $U_1$ incident on it, so that (referring to the stations in Fig. 2)

$$K \equiv \frac{P_1 - P_2}{\frac{1}{2}\rho U_1^2} \tag{1}$$

Since the power generated by the turbine is proportional to the product of the pressure loss across it and the volumetric flow

$$W = \eta_t (P_1 - P_2) U_1 A_1 = \frac{1}{2}\eta_t K \rho U_1^3 A_1 \tag{2}$$

where $\eta_t$ is the turbine efficiency and $U_1$ will be diminished as $K$ increases, there may be an optimum resistance giving rise to the maximum power from a given turbine area.

The normal definition of the thrust coefficient for a turbine is one referenced to the freestream velocity $U_0$, so that

$$C_t \equiv \frac{P_1 - P_2}{\frac{1}{2}\rho U_0^2} \tag{3}$$

Thus

$$C_t = K \left( \frac{U_1}{U_0} \right)^2 \tag{4}$$

Similarly, the normal definition of the power coefficient is

$$C_p \equiv \frac{W}{\frac{1}{2} \rho U_0^3 A_1} \tag{5}$$

i.e. one referenced to the power density of the freestream, but to the area of the turbine. Substituting expression (2) for the power generated in terms of the turbogenerator efficiency $\eta_t$ and the volumetric flow yields

$$C_p = \frac{\eta_t (P_1 - P_2) A_1 U_1}{\frac{1}{2} \rho U_0^3 A_1} = \eta_t C_t \frac{U_1}{U_0} \tag{6}$$

However, if the power is referenced to the actual velocity incident on the turbine, a new power coefficient $C_p^*$ can be defined and simply related to the loss coefficient $K$, which might consistently be designated $C_t^*$

$$C_p^* \equiv \frac{W}{\frac{1}{2} \rho U_1^3 A_1} = \eta_t K (= \eta_t C_t^*) \tag{7}$$

For a give turbine design, the efficiency and the loss coefficient (and hence $C_p^*$) are functions of the ratio of the tip speed to $U_1$. The loss coefficient usually falls continuously with this ratio, while the efficiency peaks at the tip speed ratio that gives the design angle of incidence to the blades, and $C_p^*$ peaks at a slightly smaller ratio. This means that $C_p$ and $C_t$ are not unique functions of the blade speed ratio but depend on the ducting in which the blades are placed. Most data naturally refer to the unshrouded case.

To determine the dependence of the ratio $U_1/U_0$ on the loss coefficient of the turbine, it is necessary to compute the pressure distribution through the shroud. This is done through pressure coefficients or diffuser efficiencies defined by

$$C_{P_{01}} \equiv \frac{P_1 - P_0}{\frac{1}{2} \rho U_0^2} \equiv \eta_{01} \left\{ 1 - \left( \frac{U_1}{U_0} \right)^2 \right\} \tag{8}$$

and

$$C_{P_{23}} \equiv \frac{P_3 - P_2}{\frac{1}{2} \rho U_1^2} \equiv \eta_{23} \left\{ 1 - \left( \frac{U_3}{U_1} \right)^2 \right\} \tag{9}$$

for which plenty of data are available in the literature (e.g. reference [10]). Less plentiful are estimates of base pressure coefficients for the type of ventilated obstruction presented by the shroud, defined by

$$C_{P_b} \equiv \frac{P_3 - P_0}{\frac{1}{2} \rho U_0^2} \tag{10}$$

for which empirical data may be required. Summing the pressure differences and equating to zero to return to the freestream pressure

$$\eta_{01} \left\{ \left( \frac{U_1}{U_0} \right)^2 - 1 \right\} + (K - C_{P_{23}}) \left( \frac{U_1}{U_0} \right)^2 + C_{P_b} = 0 \tag{11}$$

with the solution

$$\frac{U_1}{U_0} = \sqrt{\frac{\eta_{01} - C_{P_b}}{\eta_{01} + K - C_{P_{23}}}} \tag{12}$$

This velocity ratio is also a measure of the mass flow through a turbine of given area in a given freestream, in other words, of the 'swallowing capacity'.

Thus, the general result for the shrouded case is

$$C_p = \eta_t K \left( \frac{\eta_{01} - C_{P_b}}{\eta_{01} + K - C_{P_{23}}} \right)^{3/2} \tag{13}$$

and maximization depends on the efficiency of diffusion into and out of the duct, as well as on the resistance coefficient.

This solution can be related to the actuator disc treatment of the unshrouded case through a momentum balance, assuming negligible shearing forces ($\eta_{01} = \eta_{23} = 1$) and $C_{P_b} = 0$

$$\frac{(P_1 - P_2) A_1}{\frac{1}{2} \rho U_1^2 A_1} = \frac{\rho U_0^2 A_0 - \rho U_3^2 A_3}{\frac{1}{2} \rho U_1^2 A_1} \tag{14}$$

Invoking continuity together with definition (1), this becomes

$$K = 2 \left( \frac{U_0}{U_1} - \frac{U_3}{U_1} \right) \tag{15}$$

so that, from definition (9)

$$C_{P_{23}} = - \left( \frac{U_0}{U_1} - \frac{K}{2} - 1 \right) \left( \frac{U_0}{U_1} - \frac{K}{2} + 1 \right) \tag{16}$$

and, from equation (11), after some algebra

$$C_p = \frac{\eta_t K}{(K/4 + 1)^3} \tag{17}$$

Maximization of this power coefficient (with $\eta_t = 1$) with respect to variations in $K$ shows that $K = 2$ and $C_p = 16/27 = 0.593$, the famous Betz result for the maximum power from a turbine in the freestream.

## 3 RESULTS OF COMPUTATIONS

Equations (12), (4) and (13) allow the ratio of velocities through the turbine and in the freestream, $U_1/U_0$, and the thrust and theoretical power coefficients, $C_t$ and $C_p/\eta_t$, to be computed as functions of the resistance coefficient, $K$, putting $C_{P_b} = 0$. These parameters are plotted in Figs 4 to 6, together with the actuator disc results, equations (12), (4) and (17).
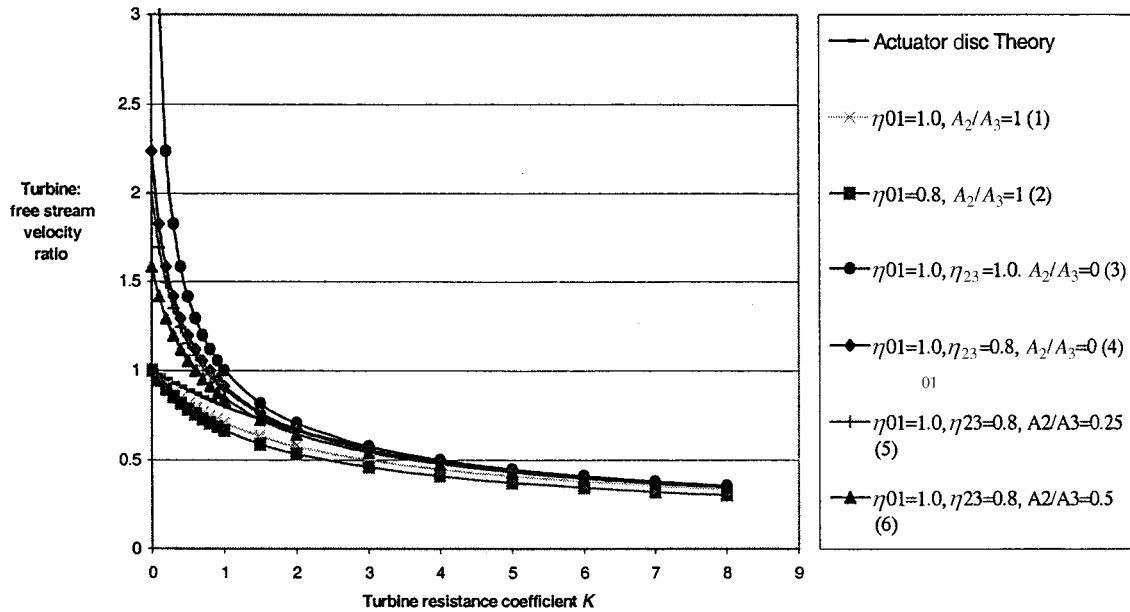
**Fig. 4** Ducted turbine velocity ratios

The duct cases have a different dependence on the velocity ratio from the freestream case, even when the inlet diffusion is perfect. This is because, in the freestream case, the outlet diffusion coefficient varies in a manner determined by equation (16), rather than being fixed by the outlet geometry. When $K = 4$ and the velocity ratio $U_1/U_0 = 0.5$, the actuator disc theory predicts the complete diffusion of the flow to recover the static pressure, so the result coincides with this limiting case for controlled diffusion to an infinite area (case 3). For $K > 4$, the actuator disc theory breaks down

for the freestream case because there is recirculation in the wake behind the turbine, and the calculations for the duct may also need to take into account a substantial contribution from the reduction in the base pressure at outlet. Naturally, failure to provide an outlet diffuser reduces the velocity through the duct (case 1), as does imperfect diffusion in the inlet (case 2). Without diffusion at the outlet (cases 1 and 2), as for the freestream case, the velocity in the duct cannot be greater than in the freestream, and it decreases continuously with increasing turbine resistance.
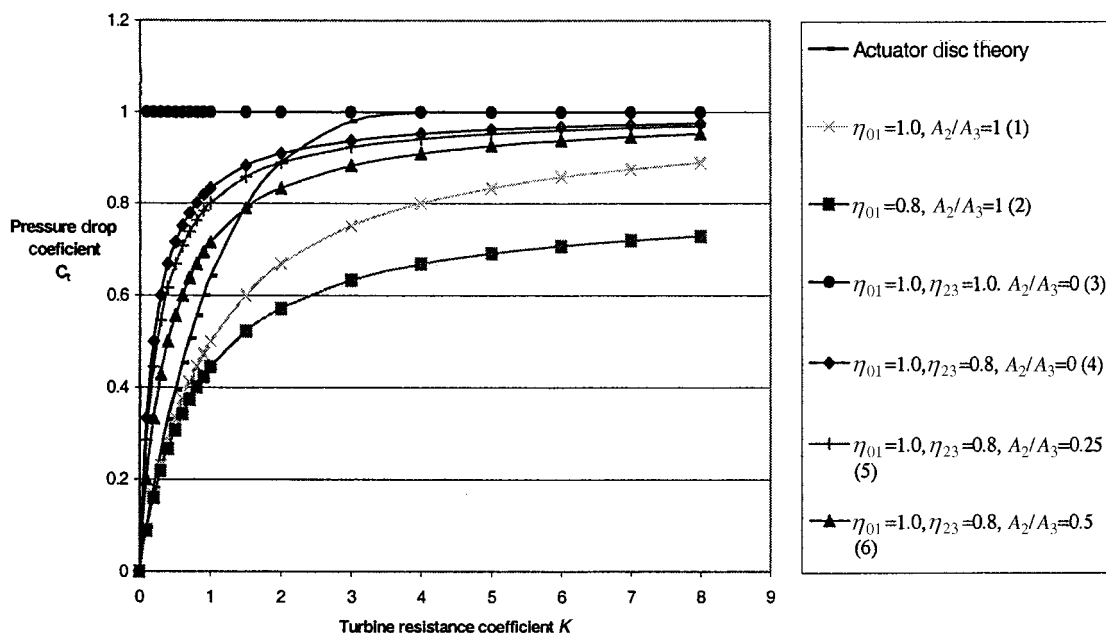


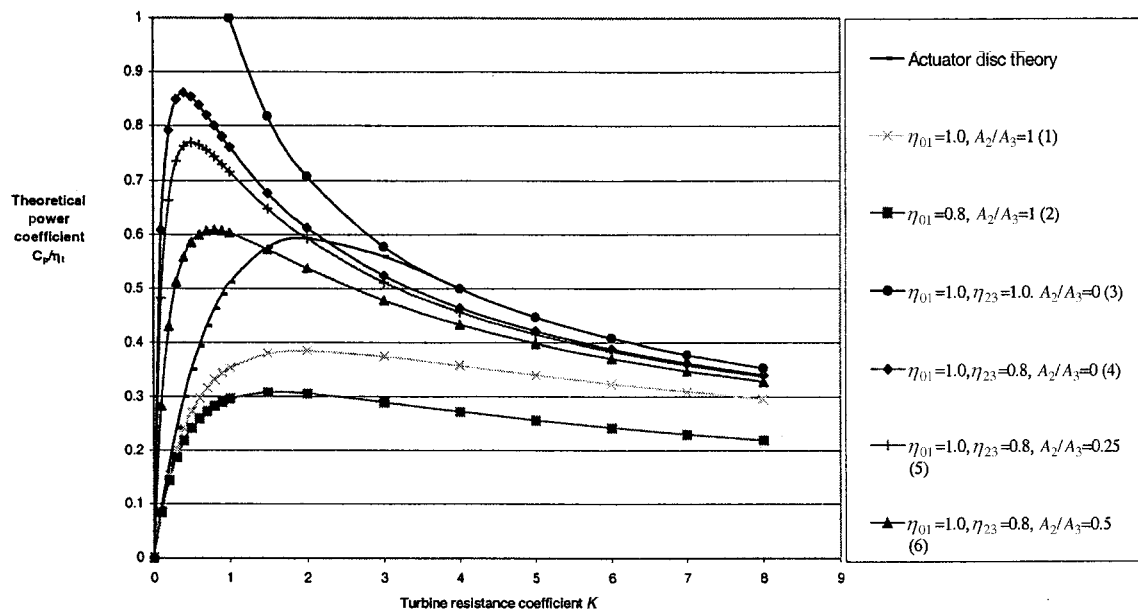**Fig. 5** Ducted turbine pressure drop coefficients

**Fig. 6** Ducted turbine theoretical power coefficients

With perfect diffusion at inlet and outlet (case 3), the duct always generates the freestream stagnation pressure across the turbine (Fig. 5), and, in the limit of no resistance, the velocity ratio (Fig. 4) could theoretically be infinite because the venturi section is collecting from, and expanding to, an infinite area.

Cases 4 to 6 show the sensitivity of the velocity through the duct to the degree of diffusion at outlet, with a realistic value (0.8) for the efficiency of the process. It can be seen that, for $K$ less than about 0.75, it will be possible to obtain a 'venturi effect', accelerating the flow from a larger freestream cross-sectional area into the turbine. In all cases the thrust coefficient tends to unity as the resistance of the turbine increases.

Since the power coefficient is the product of the parameters in Figs 4 and 5 [equation (6)], all these cases have a $C_p/\eta_t$ variation that peaks at some value of $K$ (Fig. 6). For the actuator disc case, this occurs at $K = 2$, as noted above. However, with an outlet diffuser it is possible to achieve $C_p/\eta_t$ considerably greater than 0.593, and, for the cases chosen, this occurs at a $K$ value of about 0.5. This shows that, for maximum power extraction from a given turbine blade area, a much more lightly loaded design should be chosen for the ducted case. Enhancements in power of more than 30 per cent are thus conceivable, assuming the turbine efficiency is no worse, even without the benefit of significant base pressure reduction. However, it is also clear that this result can be degraded significantly by an inefficient inlet.

## 4 EXPERIMENTAL DATA

Simple experiments were conducted in a wind tunnel with a model in which a gauze screen, wire lattice or one of two perforated plates represented the resistance of the turbine, in the geometry of Fig. 2. The resistance was installed half-way along a 500 mm duct, 100 mm in diameter, with diffusers or contractions at inlet, and sometimes a diffuser at outlet. Easily variable diffuser angles and area ratios, with the possibility of flow visualization inside them, were achieved by making the diffusers from acetate sheets rolled into a cone. The length of the diffusers/contractions was 350 mm.

Measurements of velocity and static pressure were made with a pitot static tube on the centre-line of the model at inlet and outlet, and the pressures at tapping points in the duct 150 mm upstream and 200 mm downstream of the resistance were also recorded. The coefficient for each resistance, $K$, was determined from the pressure difference with no diffuser, taking 90 per cent of the velocity on the centre-line at outlet as the mean velocity (allowing for boundary layer development). This coefficient was then taken to apply to the data with various diffusers in place.

The objectives of the experiments were:

(a) to obtain an estimate of the efficiency of diffusion from the freestream to the duct inlet when no diffuser was in place;

(b) to show that the mass flow through the duct was not significantly affected as the diffuser inlet area was reduced, until this area, $A_{0i}$, became more restrictive than the area of the swallowed stream tube with an open inlet (see Fig. 3d);

(c) to obtain an estimate of the base pressure coefficient under various conditions;

(d) to show that the velocity ratios predicted in Fig. 4 are approximately correct.

Two of the theoretical velocity ratio plots for no outlet diffuser from Fig. 4 are reproduced in Fig. 7, together with a
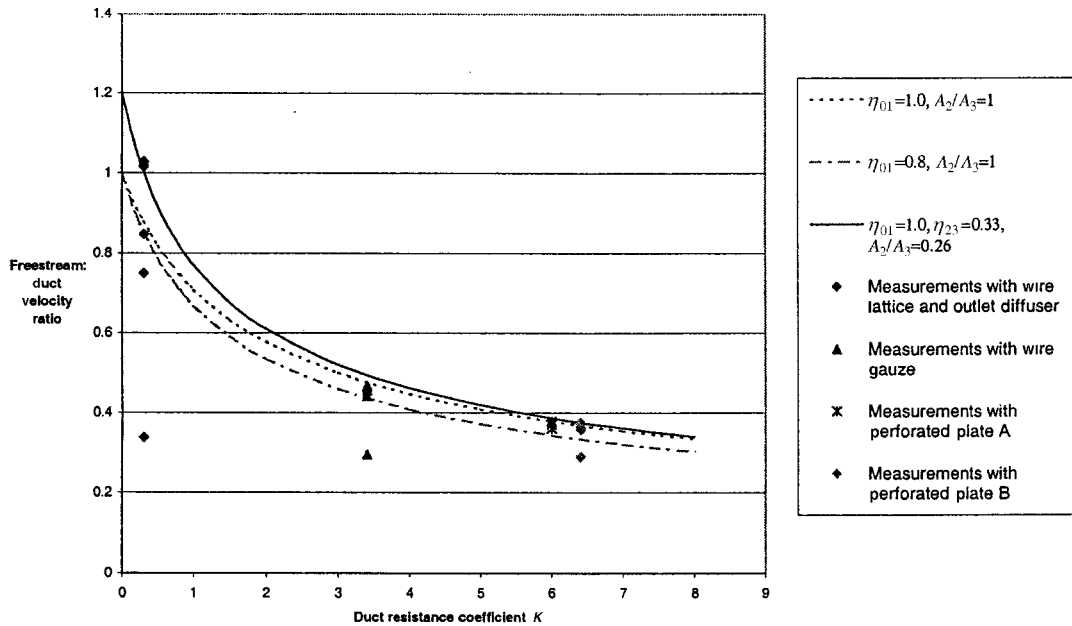
**Fig. 7**   Measured velocity ratios for a ducted resistance compared with theory

new plot for an outlet diffuser area ratio that was actually tested. Experimental data with a varying inlet area ratio are also plotted in Fig. 7, showing that the maximum measured values are in agreement with the theory for inlet diffuser efficiencies of 0.8–1.0. A plot of the velocity ratio against the inlet area ratio (Fig. 8) shows that the fall-off from the theoretical value for a given $K$ only becomes significant when the area ratio approaches the velocity ratio determined by the swallowing capacity.

The experimental data for the inlet diffusion efficiency (Fig. 9) include those for no diffuser, when $> 0.89$ was always achieved. There is some evidence of a degradation in efficiency in the one set of results for an inlet contraction. However, levels of $> 0.8$ were initially maintained as the physical area of the inlet, $A_{0i}$, was reduced, presumably because the inlet was not greatly affecting the diffusion of the flow entering the straight duct. Flow visualization with smoke injection demonstrated that the approaching streamlines were indeed perturbed as in Fig. 3c. As the inlet was restricted, the half-angle of the diffuser increased but, even with an area ratio of 0.25, it was still only 4°, so this factor was not responsible for the
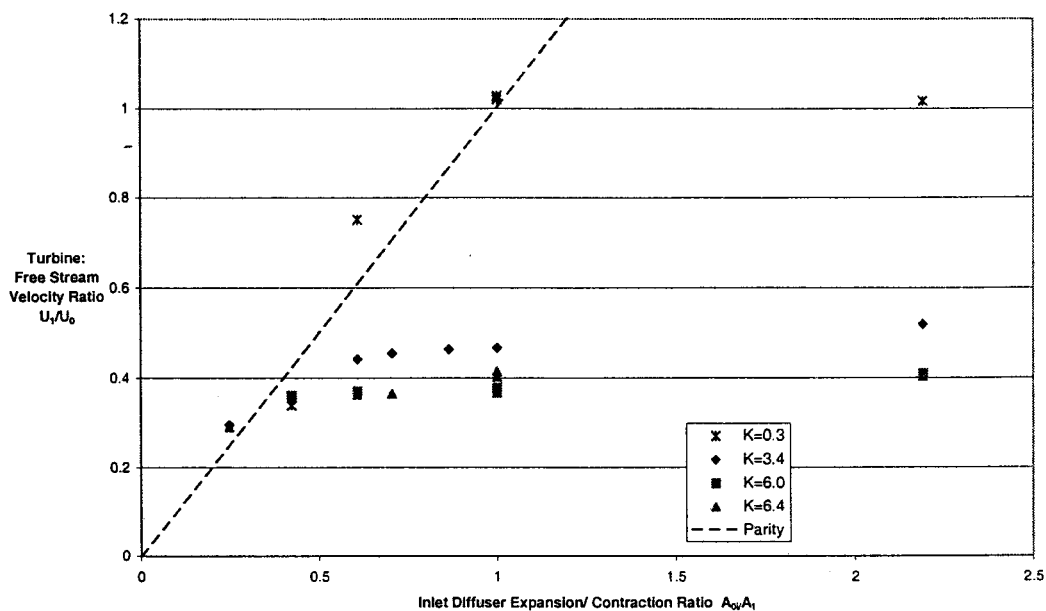


**Fig. 8**   Dependence of swallowed mass flow on inlet area

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**Fig. 9** Dependence of diffuser efficiency on inlet area

poor efficiency observed in that case [**10**]. Rather, the efficiency with this area ratio (allowing less flow than could be swallowed) was probably diminished by losses in the flow out of the vena contracta (Fig. 3d), there being no properly profiled inlet.

Base pressure coefficients determined from the measured suction at the end of the outlet diffuser are shown in Fig. 10 for a varying diffuser area ratio $A_3/A_2$, which determines the size of the wake, and a varying resistance coefficient $K$, as a function of the measured outlet velocity ratio $U_3/U_0$. As the outlet velocity increases, so does the divergence of the

streamlines at outlet and hence also the size of the wake and the base pressure coefficient.

## 5 DISCUSSION

In his book, Hansen [**6**] quotes his CFD analysis [**11**] of the flow through a diffusing shroud with an aerofoil cross-section, with the suction surface forming the internal wall, and the turbine represented by an actuator disc at the throat. This form of diffuser can be related to the more rudimentary



**Fig. 10** Variation in base pressure coefficient

geometry considered here (see Fig. 2) by noting that maximum diffusion coupled with a high base pressure coefficient, which are the conditions for the greatest mass flow, do correspond to the features of a high lift surface with the greatest possible circulation around it. However, Hansen compares his shrouded and unshrouded cases by examining the relative values of $C_p$ for the same $C_t$. A more useful comparison is on the basis of a common value of $K$, since this parameter expresses the thrust relative to the local conditions and not those upstream (which depend on the presence of the shroud), and hence are characteristic of a given turbine design. The comparison is still not perfect because the interaction of the blade tips with the casing changes the turbine aerodynamics to some extent.

The prime outcome of the present analysis is to show how the enhancement of performance to be gained by shrouding the turbine depends on the diffusion efficiency of the shroud and on the resistance presented by the turbine. With a realistic choice for the outlet diffuser and no special diffusion at inlet (but perhaps some profiling of the lip to prevent inlet separation, particularly in cross-winds), enhancements in obtainable power are predicted (Fig. 6) to range from high values with very little resistance (a factor of 5.2 at $K = 0.1$) to a factor of about 2.2 at the peak power condition for the shrouded case ($K = 0.5$) and to none at the peak power unshrouded (Bet$_Z$) condition of $K = 2$. However, these comparisons are a little academic since, at the design stage, different turbines should be selected for these applications so as to give optimum conditions for both unshrouded and shrouded cases.

Of particular interest for the shrouded case is a maximum theoretical power coefficient, $C_p/\eta_t$, of about 0.78 at $K = 0.5$, which occurs with diffusion to 4 times the turbine area with an efficiency of 0.8. This is achieved with a velocity

through the turbine that is about 15 per cent greater than the freestream, so it demands a rounded inlet allowing this degree of convergence of the streamlines without major inefficiency. The turbine is mounted in the throat of the venturi section so formed. In this case, the air is effectively sucked through the low-resistance turbine by the outlet diffusion. No account is taken in the calculations of base pressure reductions at the diffuser outlet, since the experiments indicate that $|C_{P_b}| < 0.1$ if there is a high degree of diffusion. This factor may be more significant with a carefully designed shroud profile (as in reference [11]), offsetting the degree of optimism in taking, $\eta_{01} = 1$ in equation (13).

In order to carry out a preliminary design of a 'propeller' turbine for these applications, the solutions to Glauert's equations for an optimal blade element given by Wortman [7] were used to evaluate the performance. (The solutions are also discussed in detail by Eggleston and Stoddard [12]). The approximation $\varepsilon \tan \phi \ll 1$, where $\phi$ is the pitch angle of the blade and $\varepsilon$ is the drag–lift ratio, was used in the present calculations, greatly simplifying the solution. There was little loss of accuracy for blade angles less than 50°, as seen from Figs 11 to 13, where the full solutions are compared with the approximate ones. These figures differ from the usual presentation in that the results for angles, blade loading and efficiency are presented in relation to the local blade speed ratio $X^* \equiv (2\pi nr)/U_1$, rather than the more usual $X \equiv (2\pi nr)/U_0$.

For a loss coefficient giving optimum unshrouded power ($K$ close to 2) with a drag–lift ratio $\varepsilon = 0.01$ and a blade speed ratio $X = 3$ or $X^* = 4.5$, the blade angle $\phi$ must be about 12° (Fig. 11) and the loading coefficient $\lambda \equiv (NcC_L)/(8\pi r) = 0.022$ (see the Notation and Fig. 12). The associated turbine efficiency (Fig. 13) is $\eta_t = 0.93$, and $C_p = 0.593 \times 0.93 = 0.55$.
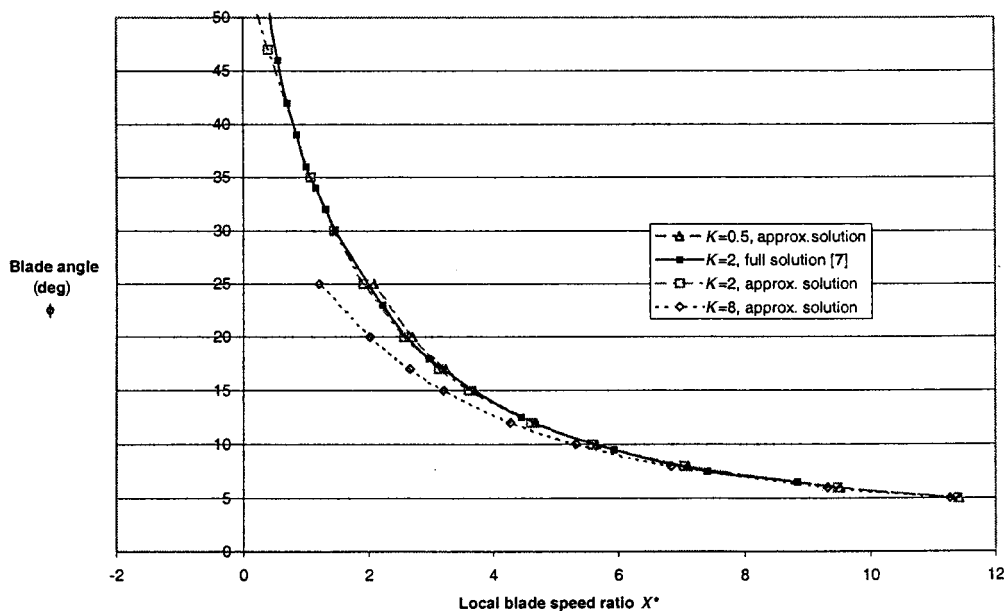


**Fig. 11** Blade angles for a 'propeller' turbine with drag–lift = 0.01
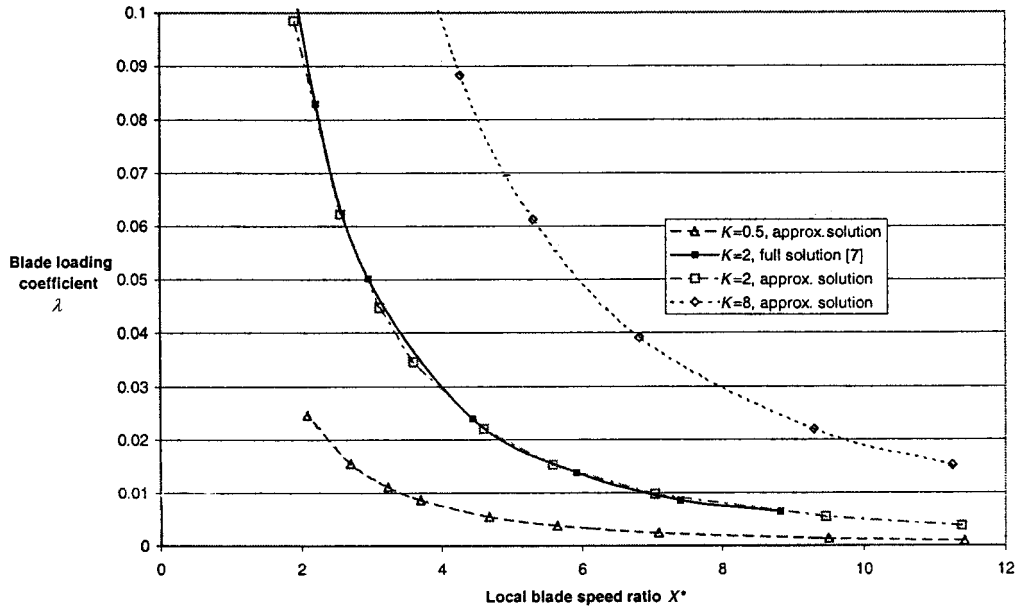
**Fig. 12**  Blade loading coefficient for a 'propeller' turbine with drag–lift = 0.01

Preserving the rotational speed $(X = 3)$ in the optimum shrouded condition of $K = 0.5$ so that $X^* = 2.6$ is seen to require a 21° blade with $\lambda = 0.016$. Alternatively, if the loading coefficient is to be preserved $(\lambda = 0.022)$, the blade angle should be 24° and $X^* = 2.2$. Although $C_p$ would be only about 0.34 under such conditions for the unshrouded case, in the shrouded case it follows from Figs 6 and 13 that $C_p = 0.78 \times 0.95 = 0.73$ with a practically achievable diffusion efficiency. Thus, theoretically there is a 33 per cent enhancement in achievable power at the optimum shrouded conditions, over the optimum unshrouded ones, with the same diameter of turbine.

If, on the other hand, it is desired to extract the maximum power from a given cross-section of stream, then since from equation (2)

$$W = \frac{1}{2}\eta_t K \rho U_0^3 A_0 \left(\frac{U_1}{U_0}\right)^2 = \eta_t C_t \frac{1}{2}\rho U_0^3 A_0 \tag{18}$$

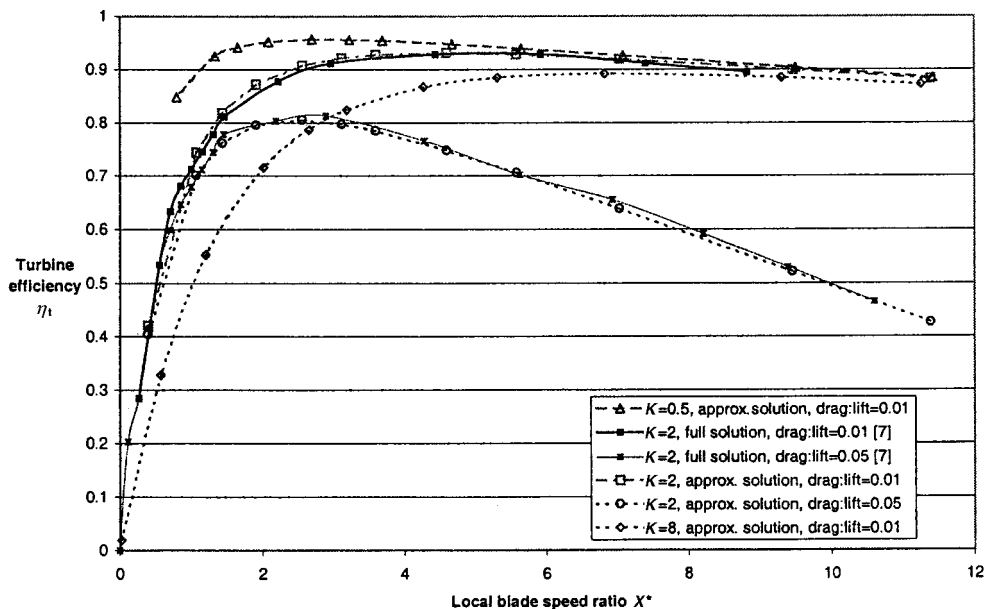it follows that the thrust coefficient must be maximized, provided the turbine efficiency is not degraded. Figure 5



**Fig. 13**  Calculated efficiency curves for a 'propeller' turbine

shows that $C_t$ increases monotonically with resistance $K$, even though $U_1/U_0$ decreases. The corollary is that the turbine area must increase, and this limits the practical $C_t$ and $K$. If there were an application for a turbine having the characteristics in Figs 11 to 13, a reasonable compromise might be found at about $K = 8$, when $U_0/U_1 = A_1/A_0 \approx 3$. Inspection of the turbine characteristics suggests that, with $X^* = 6$, $\lambda$ would be 0.05, the blade angle just 9° and $\eta_t = 0.89$ with $\varepsilon = 0.01$.

In the case of a water turbine not like a 'propeller', the efficiency characteristic might be quite different, but the same principles would apply. In some cases the objective might be to extract maximum power from a limited area of tidal stream, taking advantage of naturally occurring variations in bed depth, for example. In such cases, the greatest possible pressure drop across the turbine that was consistent with high efficiency (i.e maximum $C_t\eta_t$) would give the maximum power, provided the turbine area was correctly matched to the inlet flow area. The large size of this turbine, and the onset of cavitation, are two possible reasons for limiting this strategy, however.

In the case of wind turbines, the deployment of shrouds is always going to be limited by the considerations mentioned in section 1. The incremental costs will clearly be specific to the installation and depend on how light the shroud can be made. However, with average practical output benefits from shrouding unlikely to reach even 30 per cent, the case for shrouded blades relative to unshrouded blades of larger diameter may be difficult to make.

## 6 SUMMARY AND CONCLUSIONS

The performance of a shrouded turbine has been analyzed using one-dimensional theory by treating the ducts upstream and downstream of the turbine as contractions or expansions (diffusers) having specified diffusion efficiencies. It was found, for an achievable (but not necessarily economic) configuration, that:

1. An enhancement in the power coefficient of more than 30 per cent over the optimum for an unshrouded turbine can be gained, provided the turbine resistance is reduced appropriately by choosing a more lightly loaded design.
2. Such an enhancement is associated with a 15 per cent increase in velocity through the turbine over the freestream value.

3. The significantly greater enhancements in power in the literature have been associated with non-optimum conditions for the unshrouded turbine.
4. If the rotational speed has to be preserved because of the requirements of the generator, the correct turbine characteristic for the shrouded application is achieved with more highly pitched but lightly loaded blades.
5. On the other hand, if the turbine inlet area has to be minimized, then the turbine thrust coefficient must be increased by utilizing more highly loaded blades, up to the optimum point where there is an offsetting loss of turbine efficiency.
6. This inlet area is much smaller than that of the turbine, so efficient diffusion is essential, but this is achieved naturally if there is no interference from the walls of the inlet duct.

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