

Optimization of Thermally Actuated Bimorph Cantilevers for Maximum Deflection

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ABSTRACT

The tip displacement of thermally actuated bimorph cantilevers is examined analytically to determine the optimum combination of material properties and structure dimensions to achieve maximum deflection. Two cases are considered: constant combined layer thickness (varying the layer thickness ratio) and constant single layer thickness (varying the second layer thickness). Analytical results are verified by comparison with finite element simulations. Several design optimization examples are presented to illustrate the usefulness of the analytical approach over conventional numerical and simulation methods.

Keywords: bimorph cantilever, optimization, maximum deflection, thermal actuation, MEMS

1. INTRODUCTION

Bimorph structures have long been studied and used in Micro-Electro-Mechanical Systems (MEMS) for sensors and actuators because of their sensitivity, fast response time and ease of integration into semiconductor technology [1]. They can also provide important information about the material properties of device components such as residual stress, elastic modulus, thermal expansion and piezoresistivity. In some applications, desirable large deflections are designed using numerical, simulation or experimental approaches. Despite their wide use, these methods are still far from optimum. Recently however, several studies have addressed analytical optimization of bimorph cantilevers [2, 3], but a solution has only been given for structures with constant combined layer thickness. To our knowledge, an analytical optimization solution has not yet been published for the less limited problem of a bimorph cantilever with one constant and one variable layer thickness.

In this paper, an analytical model is derived which allows theoretical investigation and quantitative optimization of bimorph cantilever tip displacement based on material properties and structure dimensions. The optimization equations relating bimorph thickness,

width and elasticity parameters, and expressions for the resulting maximum tip deflection are then obtained for the two cases. We find that for bimorph cantilevers with a constant total thickness, the maximum deflection is independent of the relative width, thickness and Young's modulus of the two layers, and the optimization equation is a quadratic expression. For bimorph cantilevers with one constant and one variable layer thickness, the maximum deflection is found to be inversely proportional to the thickness of the variable layer, and the optimization equation is a cubic expression.

2. BIMORPH CANTILEVERS

A bimorph cantilever of length L is sketched in Figure 1. In general, the two layers can have different width, thickness, Young's modulus and thermal expansion coefficient (TEC). Temperature changes induce cantilever deflection due to a bending moment generated by the TEC difference for the two materials. The radius of curvature r of a bent cantilever can be derived by minimizing the total energy [2] or balancing forces [4], which gives:

$$k = \frac{1}{r} = \frac{6b_1b_2E_1E_2t_1t_2(t_1+t_2)\Delta\alpha\Delta T}{(b_1E_1t_1^2)^2 + (b_2E_2t_2^2)^2 + 2b_1b_2E_1E_2t_1t_2(2t_1^2 + 3t_1t_2 + 2t_2^2)} \quad (1)$$

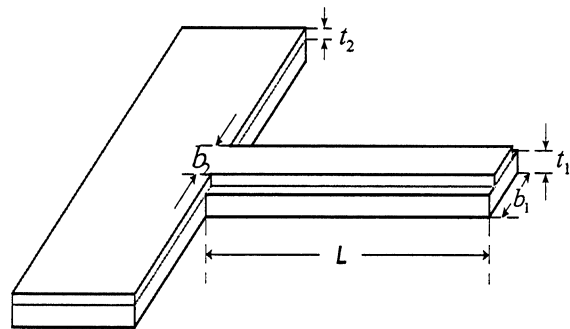


Figure 1: A typical bimorph cantilever.

where b_1 , b_2 , t_1 , t_2 , E_1 , and E_2 are the width, thickness and Young's modulus of each material, respectively, $\Delta\alpha$ is the TEC difference for the two materials and ΔT is the temperature change from the value at which the cantilever is not deflected.

The tip deflection, in terms of the radius of curvature and cantilever length, can be derived using Figure 2:

$$d = 2r \sin^2\left(\frac{\theta}{2}\right) \quad (2)$$

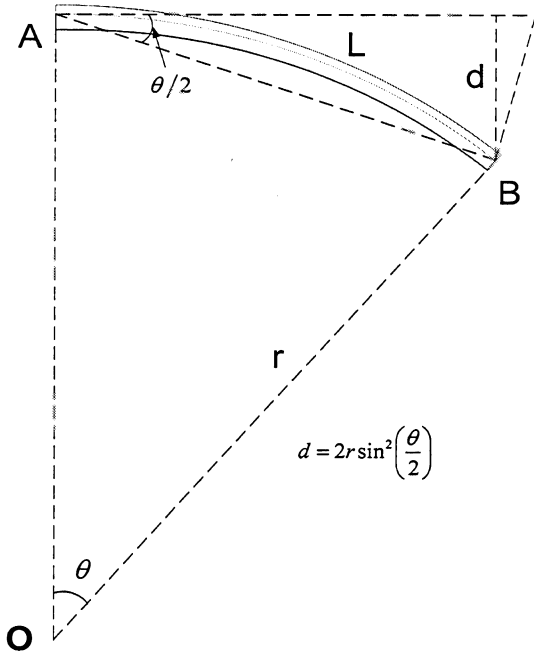


Figure 2: Tip deflection d of a bent cantilever.

For small angles we use the approximation $\sin^2(\theta/2) \approx (\theta/2)^2 = (L/2r)^2$, which introduces less than 1% error for values of $\theta/2$ up to 10° . Thus in this approximation, combining Equations (1) and (2) yields an expression for the deflection equation:

$$d \cong \frac{L^2}{2r} = \frac{3b_1b_2E_1E_2t_1t_2(t_1+t_2)\Delta\alpha\Delta TL^2}{(b_1E_1t_1^3)^2 + (b_2E_2t_2^3)^2 + 2b_1b_2E_1E_2t_1t_2(2t_1^2 + 3t_1t_2 + 2t_2^2)} \quad (3)$$

3. OPTIMIZATION

3.1 Bimorph Cantilevers with Constant Total Thickness

To simplify the deflection equation, we define $t_2/t_1 = x$, $b_2/b_1 = y$ and $E_2/E_1 = c$, all positive ratios. For cantilevers with constant total thickness, Equation (3) now can be rewritten as:

$$d_1 \cong \frac{cxy(1+x)^2}{c^2x^4y^2 + 4cx^3y + 6cx^2y + 4cxy + 1} \cdot \frac{3\Delta\alpha\Delta TL^2}{t_1 + t_2} \quad (4)$$

$$= f_1(x, y) \cdot \frac{3\Delta\alpha\Delta TL^2}{t_{total}}$$

This expression reveals that the deflection is dependent on only relative values of Young's modulus, width and thickness, not absolute values. For a bimorph cantilever with a specified material combination and length, the second part of Equation (4) is constant, so the deflection is maximized when $f_1(x, y)$ has its maximum. The function $f_1(x, y)$ is plotted in Figure 3, showing the existence of a constant maximum.

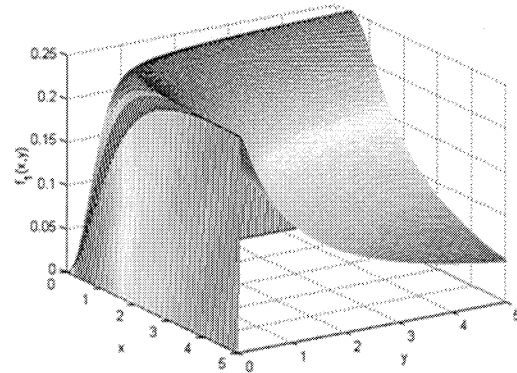


Figure 3: The function $f_1(x, y)$.

Taking the partial derivative of $f_1(x, y)$ with respect to x , the optimization equation is found to be:

$$cx^2y = 1 \quad (5)$$

Thus $f_{1(max)} = 1/4$ and the maximum deflection equation reduces to:

$$d_{1(max)} = \frac{3\Delta\alpha\Delta TL^2}{4t_{total}} \quad (6)$$

This interesting result shows that for any given combination of materials where the total bimorph thickness is held constant, the maximum attainable deflection is independent of the relative values of Young's modulus, width and thickness, and is determined solely by the TEC difference, ΔT , the cantilever length and the combined thickness. Gehring, *et al.*, reached the same conclusion in [2].

3.2 Bimorph Cantilevers with One Constant and One Variable Layer Thickness

For the less limited case where t_1 is constant and t_2 is variable, the deflection equation can be rewritten as:

$$d_2 \cong \frac{cxy(1+x)}{c^2x^4y^2 + 4cx^3y + 6cx^2y + 4cxy + 1} \cdot \frac{3\Delta\alpha\Delta TL^2}{t_1} \quad (7)$$

$$= f_2(x, y) \cdot \frac{3\Delta\alpha\Delta TL^2}{t_1}$$

The function $f_2(x, y)$ is plotted in Figure 4. In this case, in contrast to the constant total thickness example, the maximum value is not a constant.

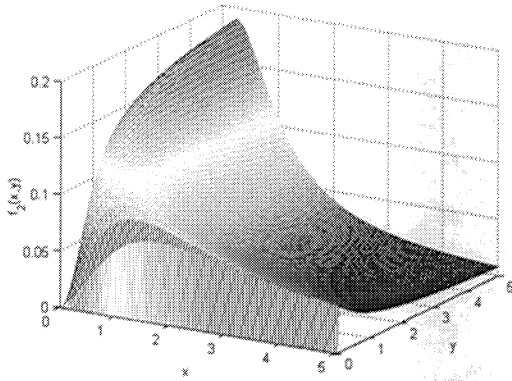


Figure 4: The function $f_2(x, y)$.

Taking the partial derivative of $f_2(x, y)$ with respect to x , the optimization equation is found to be:

$$2x^3 + 3x^2 - \frac{1}{cy} = 0 \quad (8)$$

Therefore,

$$f_{2(\max)}(x) = \frac{2x+3}{12(x+1)^2} \quad (9)$$

In Equation (9), $f_{2(\max)}(x)$ is a function of the relative thickness: the smaller the x the larger the value of $f_{2(\max)}(x)$. When $x \rightarrow 0$, $f_{2(\max)} \rightarrow 1/4$. By combining Equations (7) and (9), the maximum deflection equation reduces to:

$$d_{2(\max)} = \frac{2x+3}{12(x+1)^2} \cdot \frac{3\Delta\alpha\Delta TL^2}{t_1} \quad (10)$$

In practical terms, this result indicates that to approach the ultimate maximum value of $f_{2(\max)}$, one would choose a relatively thin, wide and stiff variable layer compared to the fixed thickness layer.

Equation (8) can be solved analytically. Since c and y are positive, it is clear that there is one and only one positive root [5]:

$$x = \sqrt[3]{\frac{1}{4cy} - \frac{1}{8} + \frac{\sqrt{1-cy}}{4cy}} + \sqrt[3]{\frac{1}{4cy} - \frac{1}{8} - \frac{\sqrt{1-cy}}{4cy}} - \frac{1}{2} \quad (11)$$

This form of the solution is valid for values of $cy \leq 1$. For $cy > 1$, a real solution is also obtained.

Defining $\frac{1}{4cy} - \frac{1}{8} = m$ and $\frac{\sqrt{cy-1}}{4cy} = n$ gives:

$$x = \sqrt[3]{m+ni} + \sqrt[3]{m-ni} - \frac{1}{2}$$

$$= \sqrt[3]{\sqrt{m^2+n^2} \cdot e^{j\phi}} + \sqrt[3]{\sqrt{m^2+n^2} \cdot e^{-j\phi}} - \frac{1}{2} \quad (12)$$

$$= \cos \frac{\phi}{3} - \frac{1}{2}$$

where $\phi = \tan^{-1} \frac{n}{m} = \tan^{-1} \left(\frac{2\sqrt{cy-1}}{2-cy} \right)$ and $\phi \in [0, \pi)$.

Figure 5 plots x versus cy , obtained from Equations (11) and (12), along with the resulting values of $f_{2(\max)}$. Higher values of cy lead to lower values of x and higher $f_{2(\max)}$. This figure suggests a useful procedure for optimizing bimorph tip deflection when one layer thickness is held constant. The designer would begin by choosing materials from those available to maximize the TCE difference. This choice of materials sets the Young's modulus ratio c , and in most cases the product cy as well, since typically $y_{\max}=1$ due to fabrication constraints. Figure 5 then provides the thickness ratio x to give maximum tip deflection, and it also determines $f_{2(\max)}$.

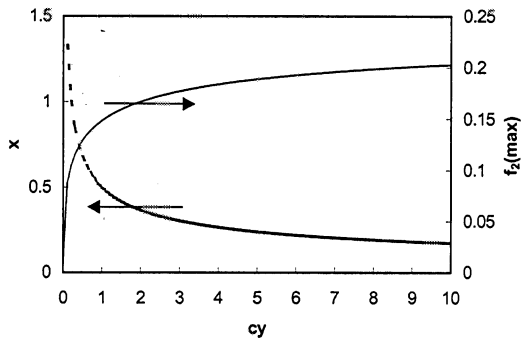


Figure 5: x and $f_{2(\max)}$ versus cy .

4. VERIFICATION

In this section, simulation results obtained from CoventorWare are used to verify the analytical results. The initial dimensions and material properties of a non-optimized bimorph cantilever are given in Table 1.

Table 1: Materials properties and initial dimensions of a bimorph cantilever used in simulation.

	Layer 1 Polysilicon	Layer 2 Aluminum
Initial Dimensions $L \times W \times T$ (μm)	1000 \times 100 \times 10	1000 \times 50 \times 5
Young's Modulus (GPa)	165	70
TEC ($1e-6$)	2.8	23.2

Optimization is made for either constant total thickness ($15\mu\text{m}$) or constant polysilicon layer thickness ($10\mu\text{m}$). The tip deflection before optimization is $52.6\mu\text{m}$ for a 100K temperature change. After optimization, the calculated deflections are $102\mu\text{m}$ and $76.1\mu\text{m}$, respectively. These outcomes are summarized in Table 2, and are found to compare with good agreement to the simulation results. For the constant total thickness case, many width and thickness ratios will give the same maximum deflection. For the constant single layer thickness case, the maximum deflection increases with increasing second layer width and an associated decreasing second layer thickness. As illustrated in Figure 6, the analytical approach provides a method for determining the second layer thickness for maximum deflection that is much more straightforward than using repeated simulations.

Table 2: Cantilever dimensions and deflection before and after optimization.

	Width \times Thickness (μm)		Deflection (μm)	
	Poly-Si	Al	Calcu.	Simu.
Before optimization	100 \times 10	50 \times 5	52.6	54.4
When $t_1+t_2 = \text{const}$	100 \times 4.73	50 \times 10.27	102	102.8
	100 \times 5.92	100 \times 9.08	102	104.1
When $t_1 = \text{const}$	100 \times 10	50 \times 9.76	64.7	65.9
	100 \times 10	100 \times 7.27	76.1	78.3

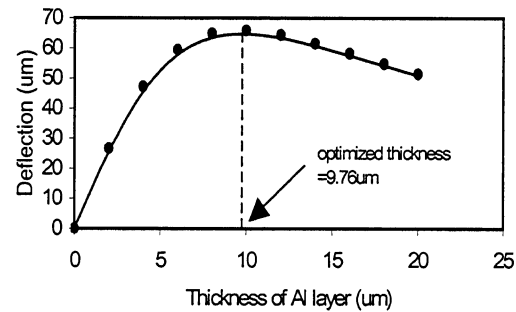


Figure 6: Comparison of simulation (dots) and analytical (line) methods for finding the optimum thickness for the bimorph of row 5 in Table 2.

5. CONCLUSIONS

Theoretical analysis of thermally actuated bimorph cantilevers for maximum deflection is presented. An analytical model is derived which allows quantitative optimization of bimorph cantilever performance based on material properties and structure dimensions. This model is verified by simulation, showing that substantial improvements in the design approach for bimorph cantilevers are still possible.

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